Subdivision Surfaces

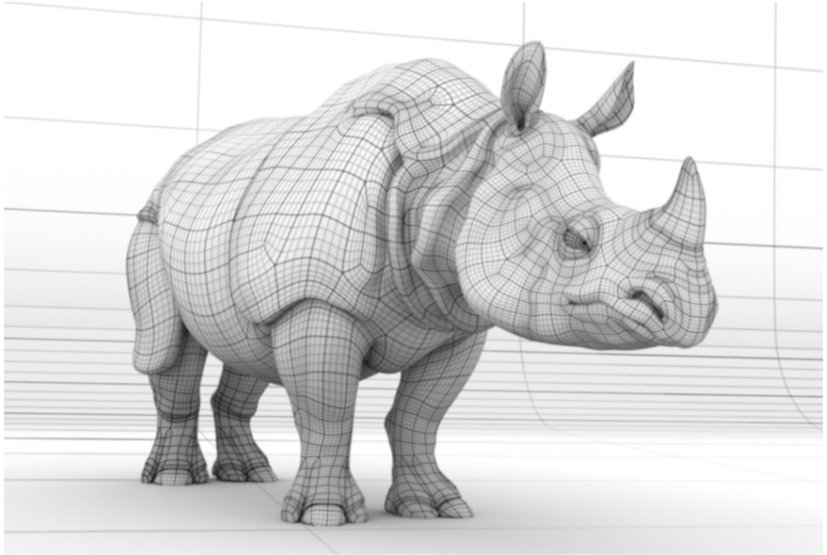
COS 426, Spring 2017
Princeton University

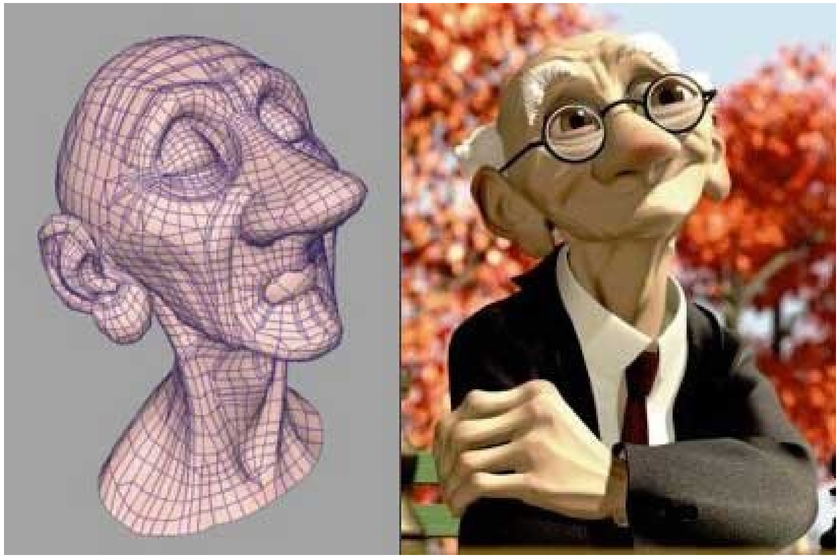
3D Object Representations

- Raw data
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - ➢ Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

Subdivision Surfaces

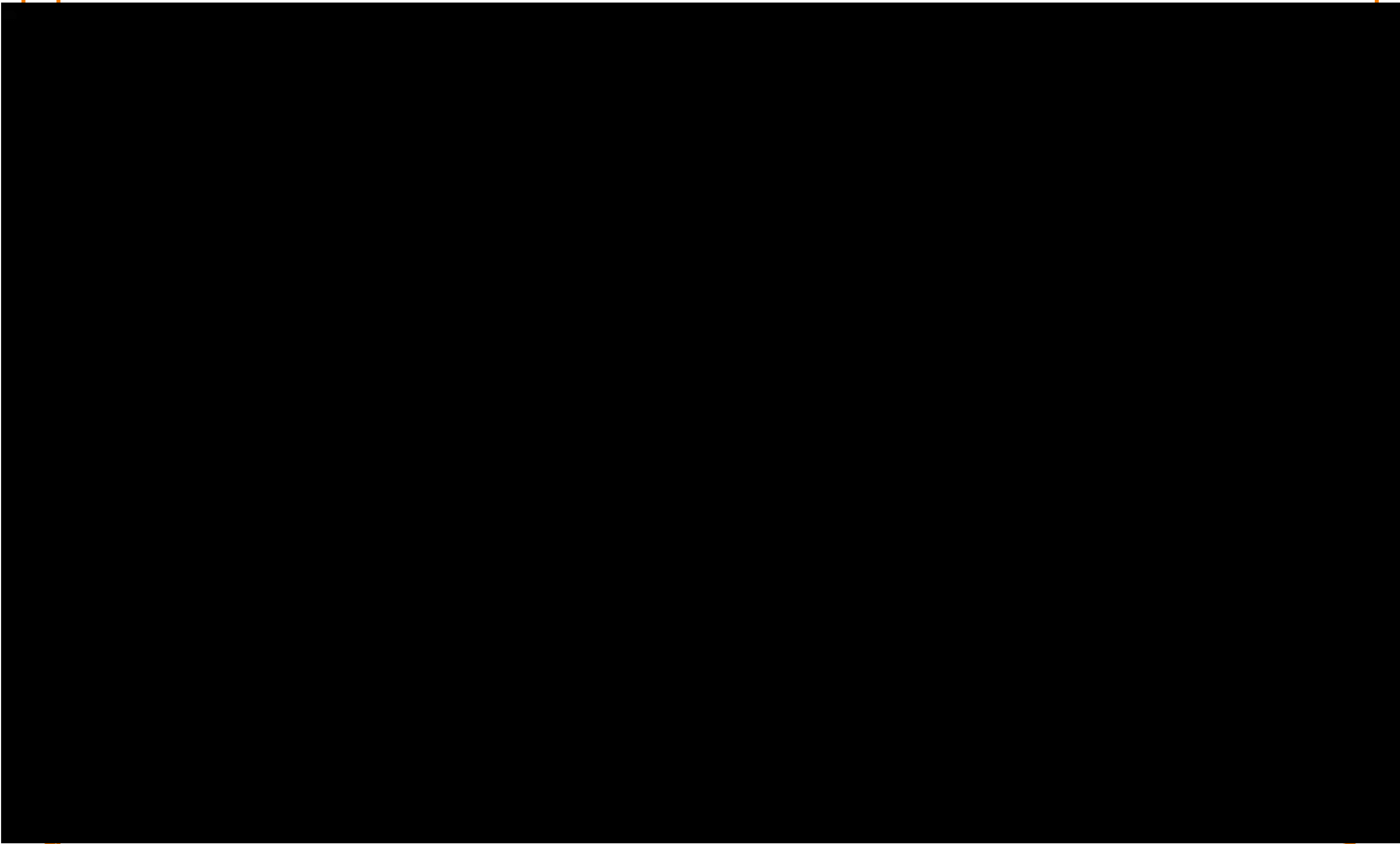
- Used in movie and game industries
- Supported by most 3D modeling software





Geri's Game © Pixar Animation Studios

Subdivision Surfaces



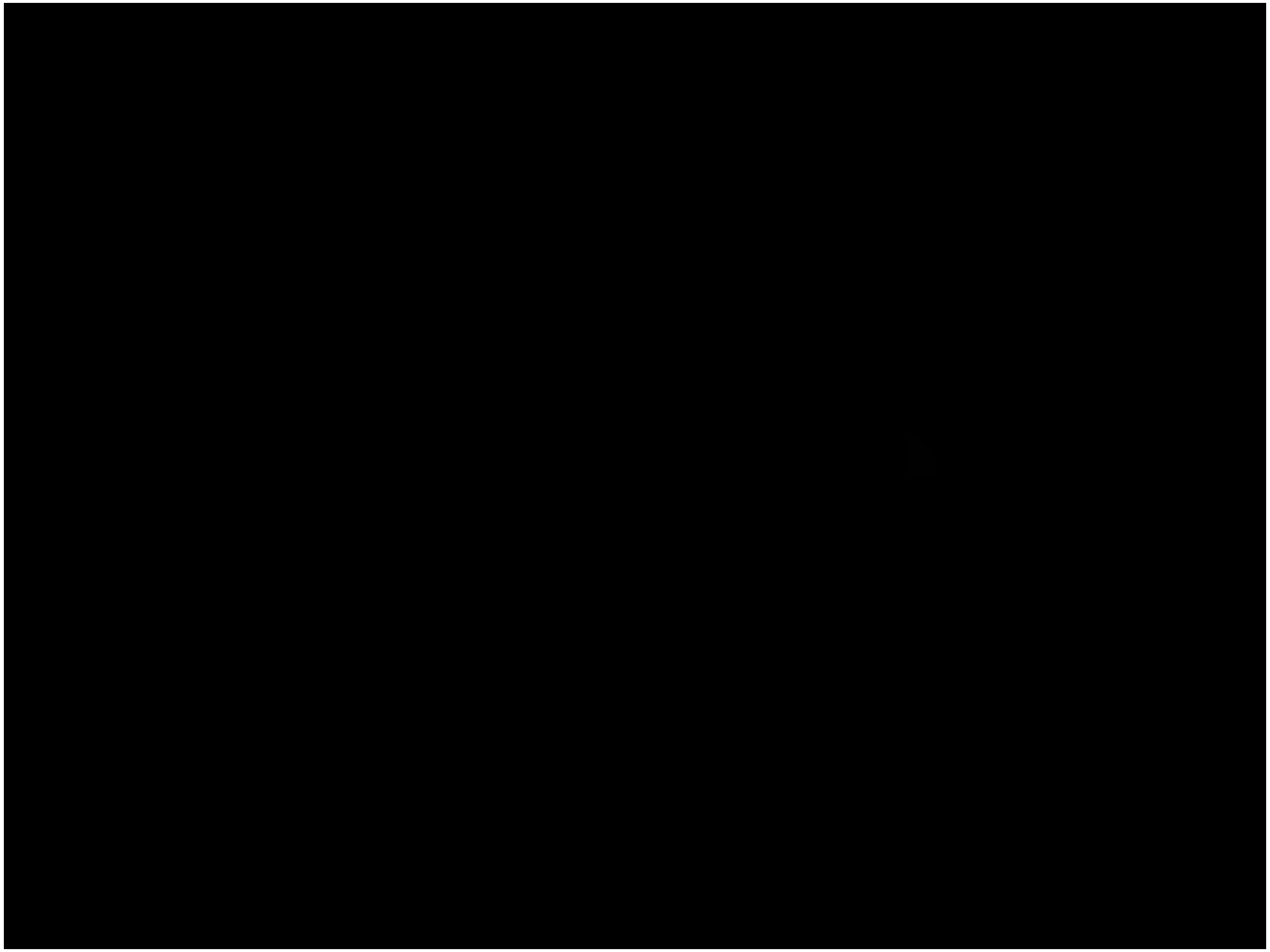
Geri's Game © Pixar Animation Studios

Geri’s Game

- “served as a demonstration of a new animation tool called subdivision surfaces” (Wikipedia)
- Subdivision used for head, hands & some clothing
- Academy Award winner

Geri's Game

- Guest performance in Toy Story 2



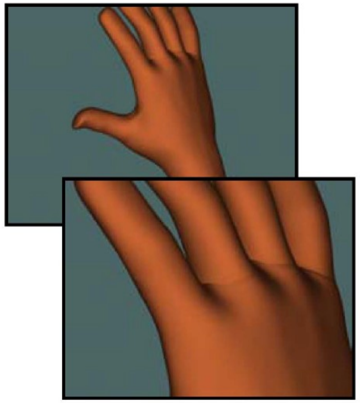
Toy Story 2 © Pixar Animation Studios

Subdivision Surfaces

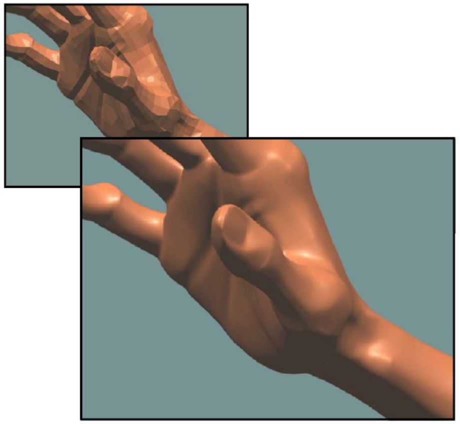
- An alternative to NURBS, overcoming:
  - Many patches
  - Difficult to mark sharp features

  - Irregularities after deformation

Woody’s hand (NURBS)



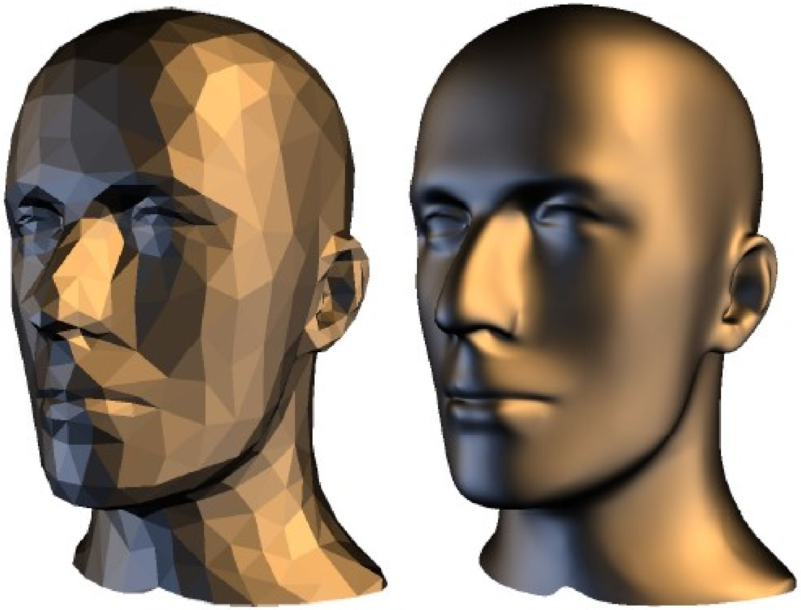
Geri’s hand (subdivision)



Stanford Graphics course notes

Subdivision Surfaces

- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections



Reif & Schroeder 2000

Subdivision Surfaces

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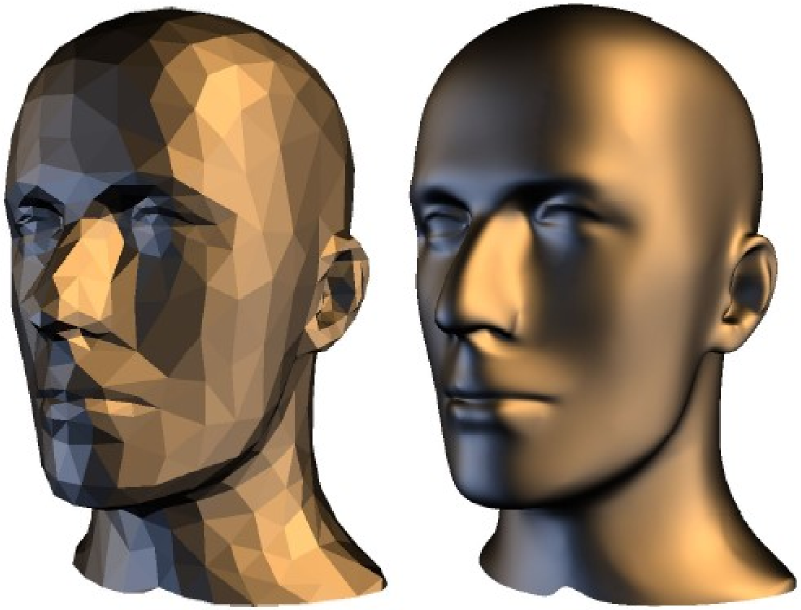
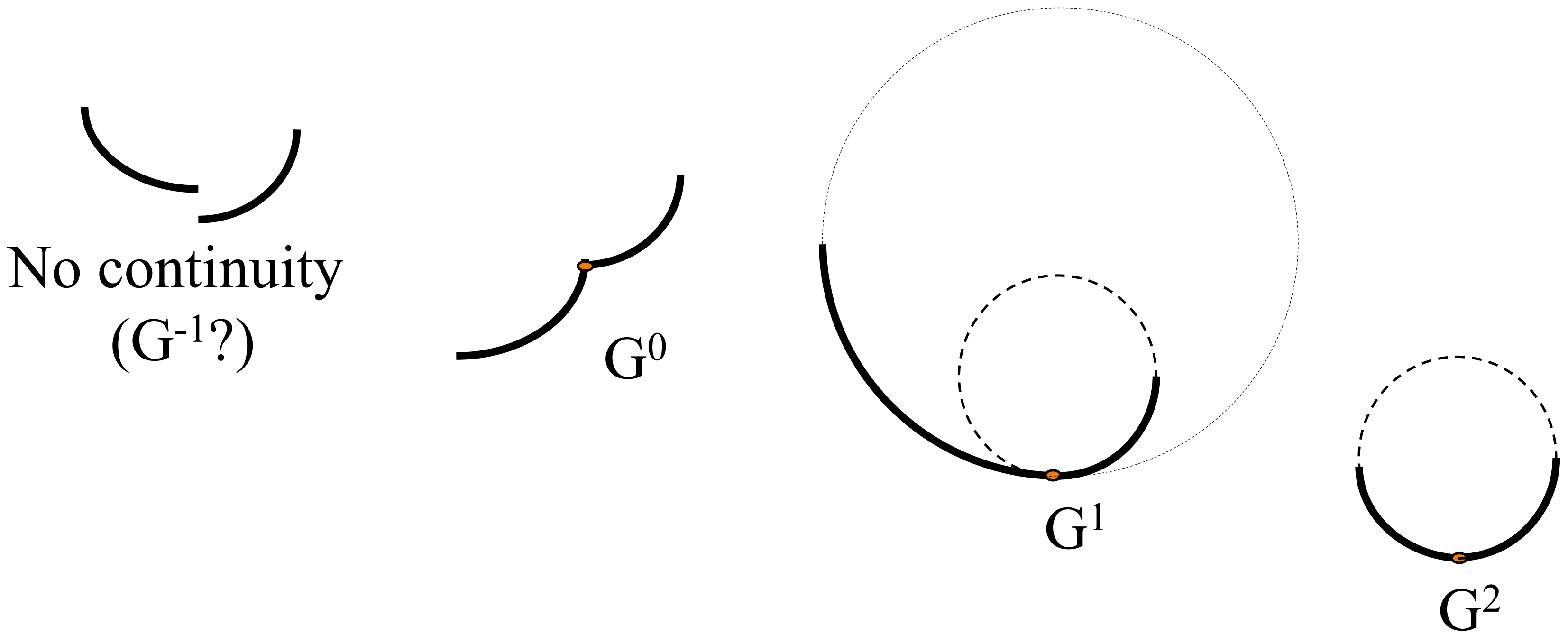


Reif & Schroeder 2000

Continuity

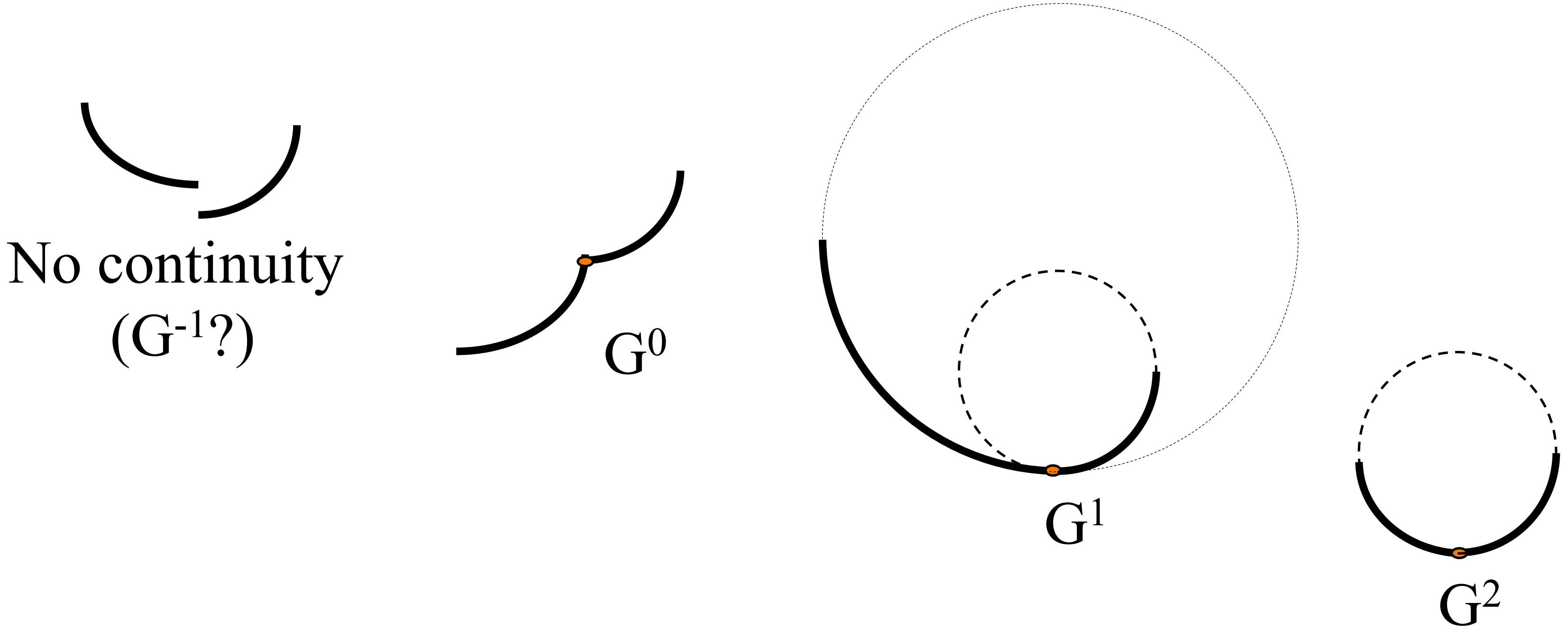
A curve / surface with G^k continuity has a continuous k-th derivative, geometrically.



Continuity

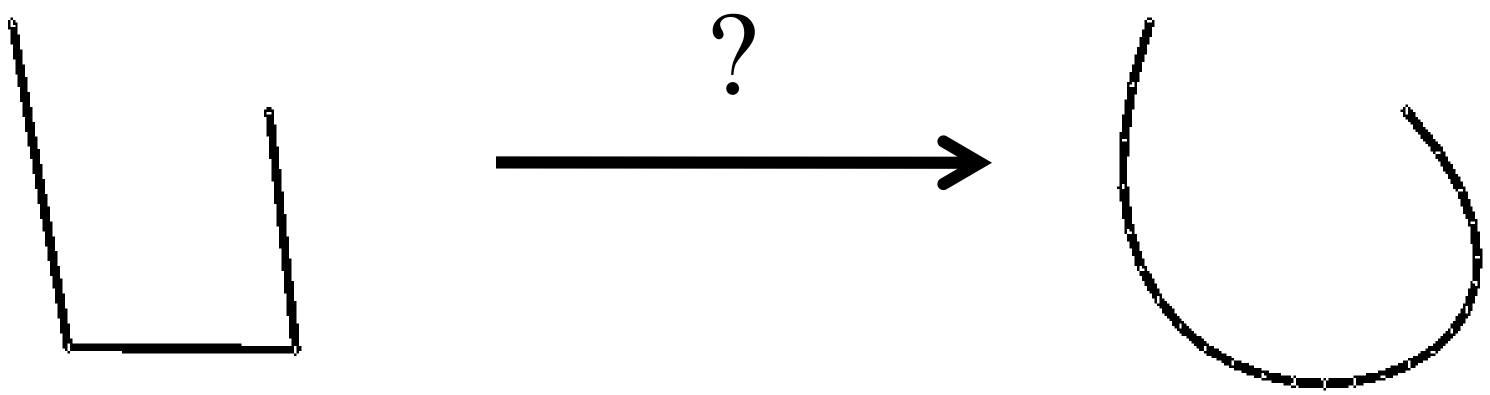
Similar, but not exactly C^k continuity

- Geometric, not of algebraic(e.g.: $f_x(u) = r_x \cos(2\pi u)$)
- Different only when algebraic gradient $\vec{\nabla} f = 0$



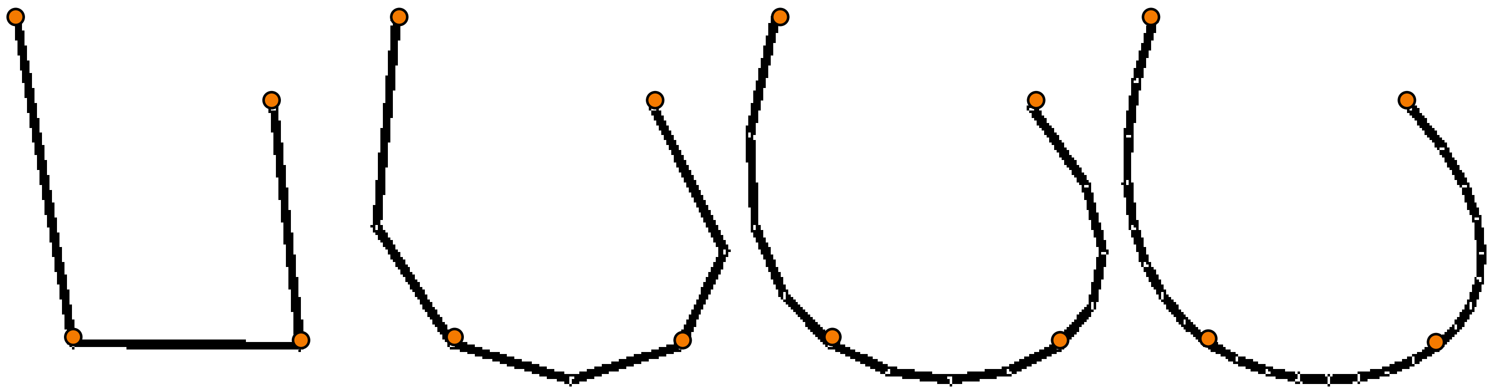
Subdivision

- How do you make a curve with guaranteed continuity?



Subdivision

- How do you make a curve with guaranteed continuity? …



Subdivision

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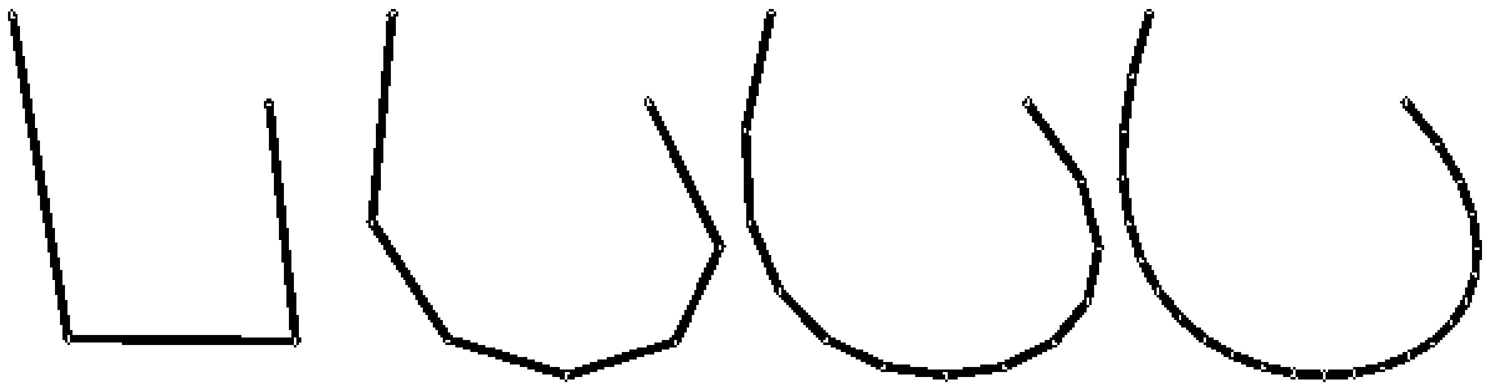
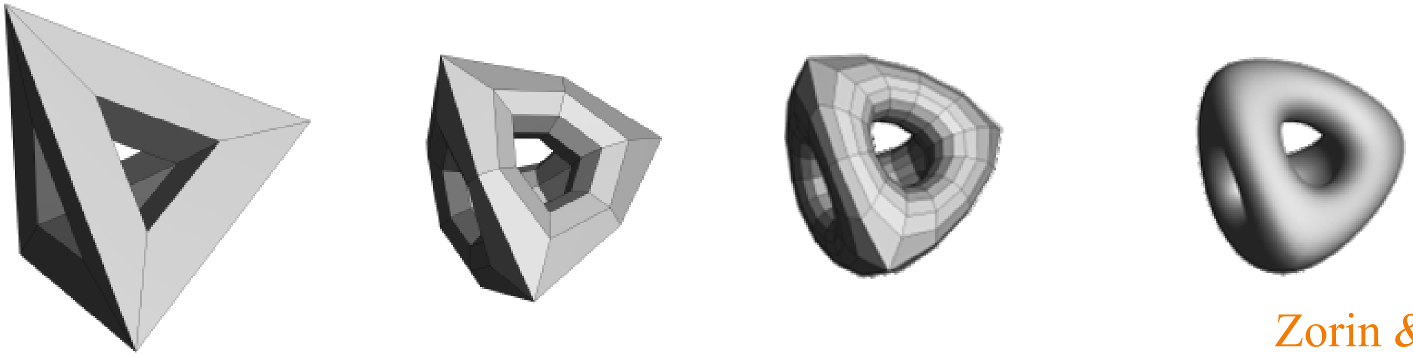
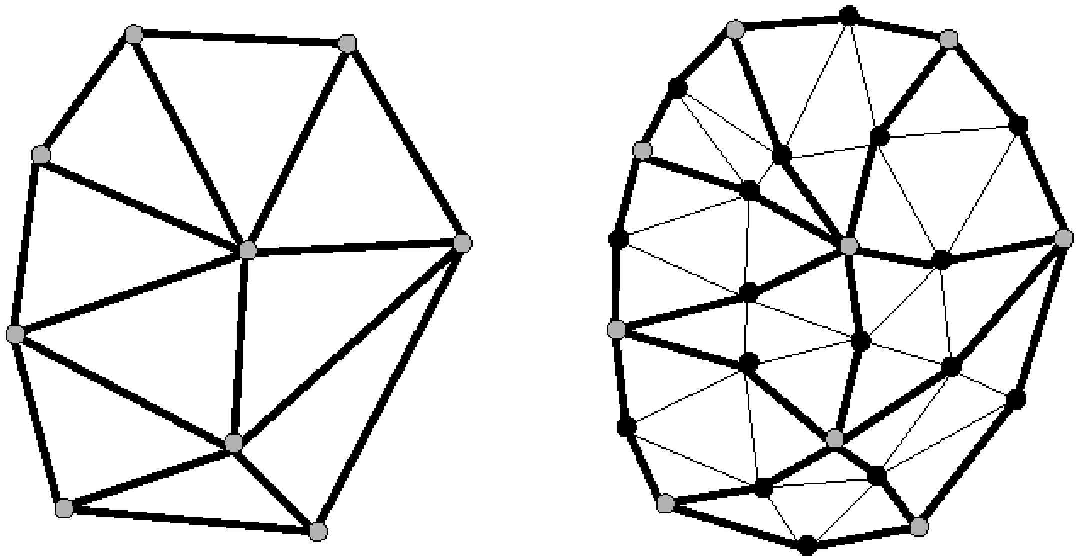




Zorin & Schroeder
SIGGRAPH 99
Course Notes

Subdivision Surfaces

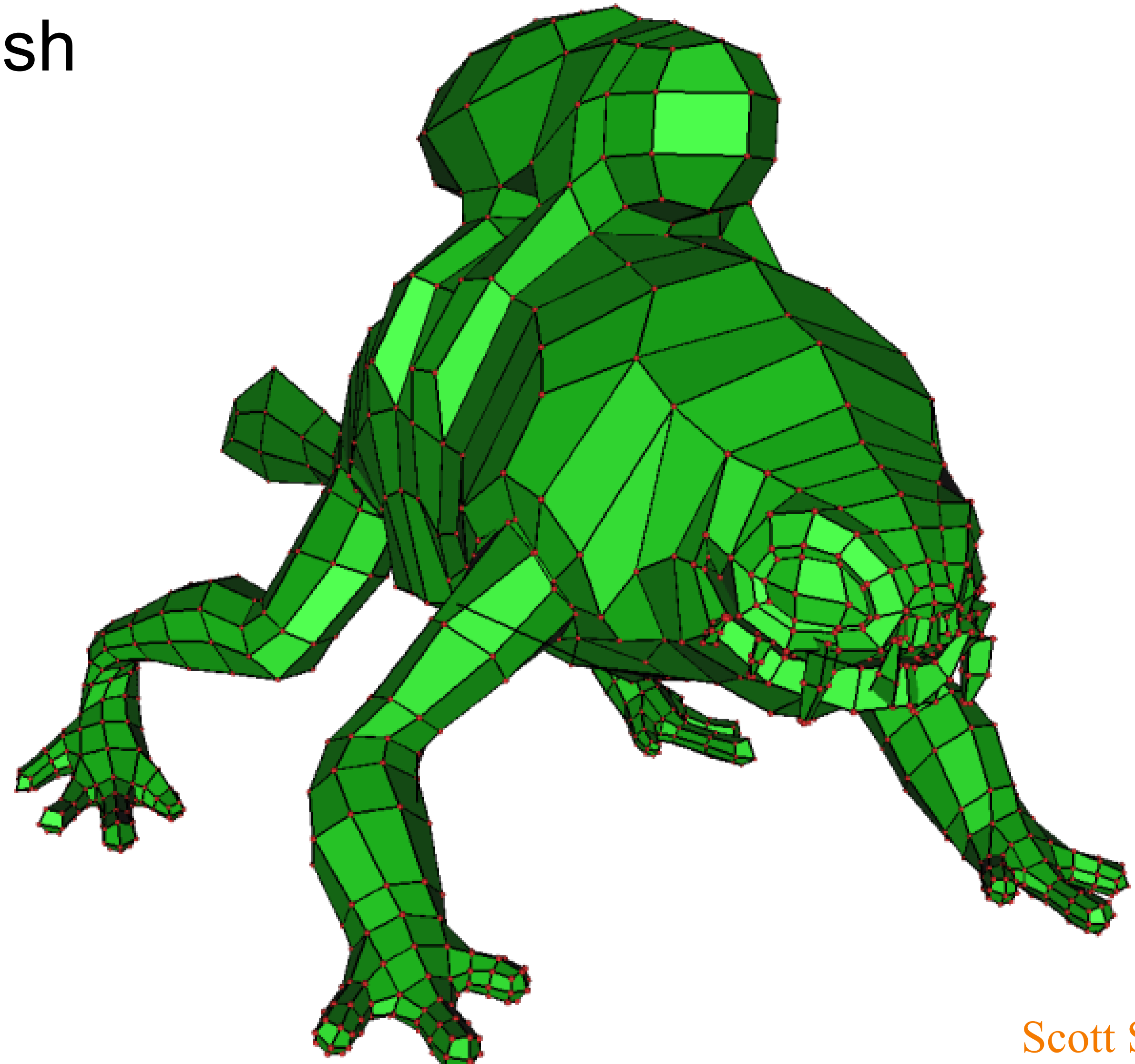
- Repeated application of
  - Topology refinement (splitting faces)
  - Geometry refinement (weighted averaging)



Zorin & Schroeder
SIGGRAPH 99
Course Notes

Subdivision Surfaces – Examples

- Base mesh



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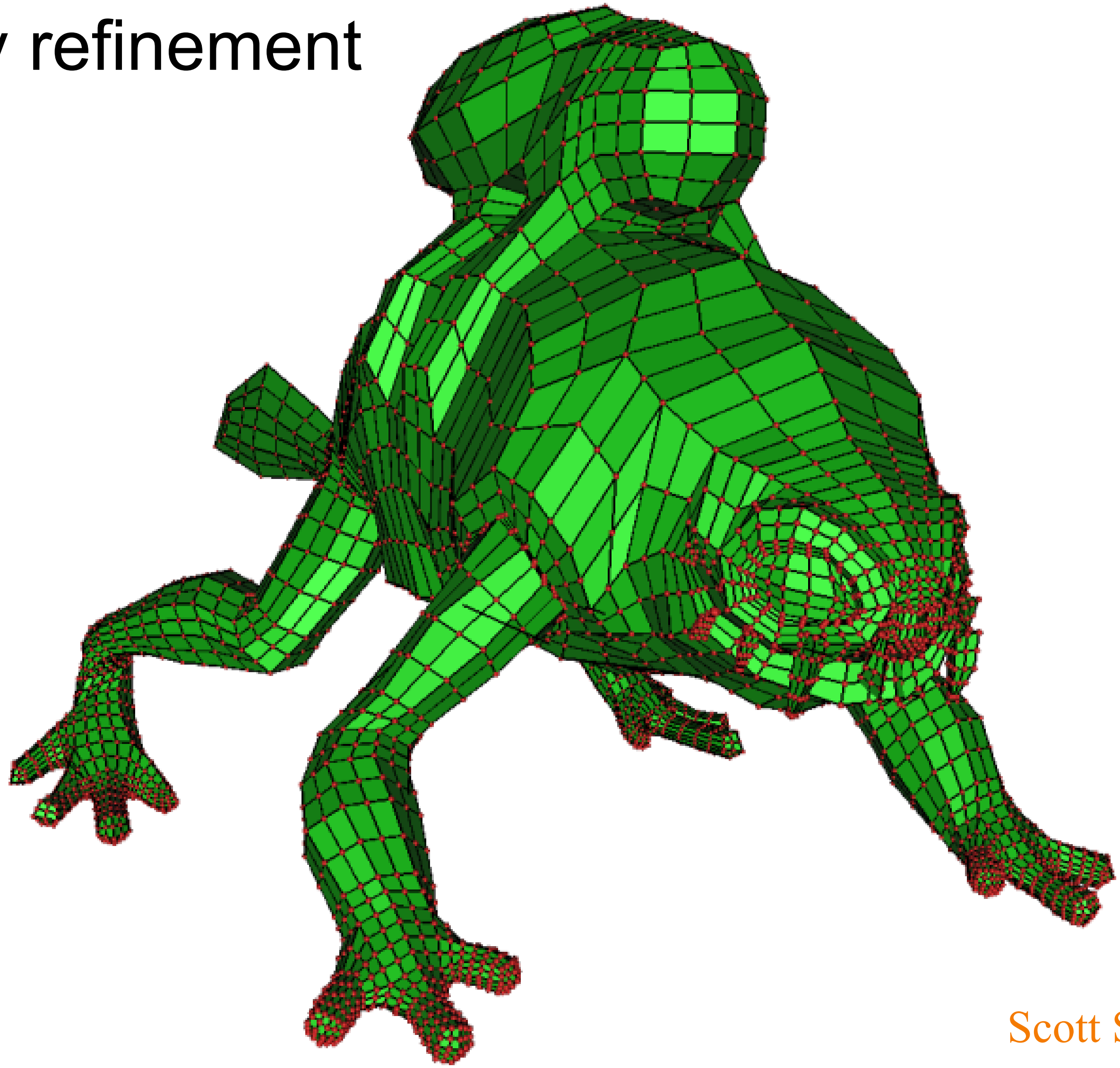
Subdivision Surfaces – Examples

- Topology refinement



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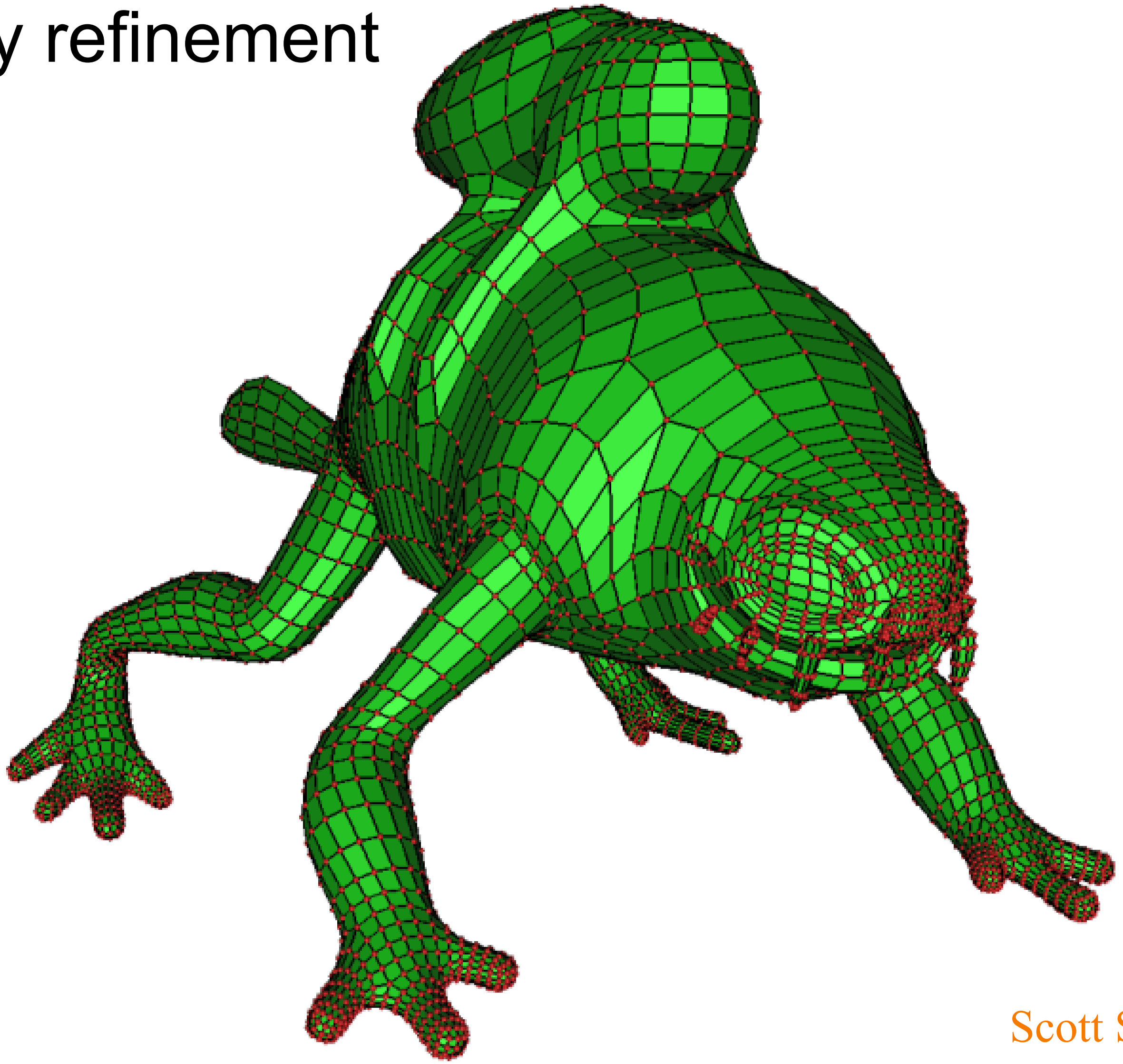
Subdivision Surfaces – Examples

- Geometry refinement



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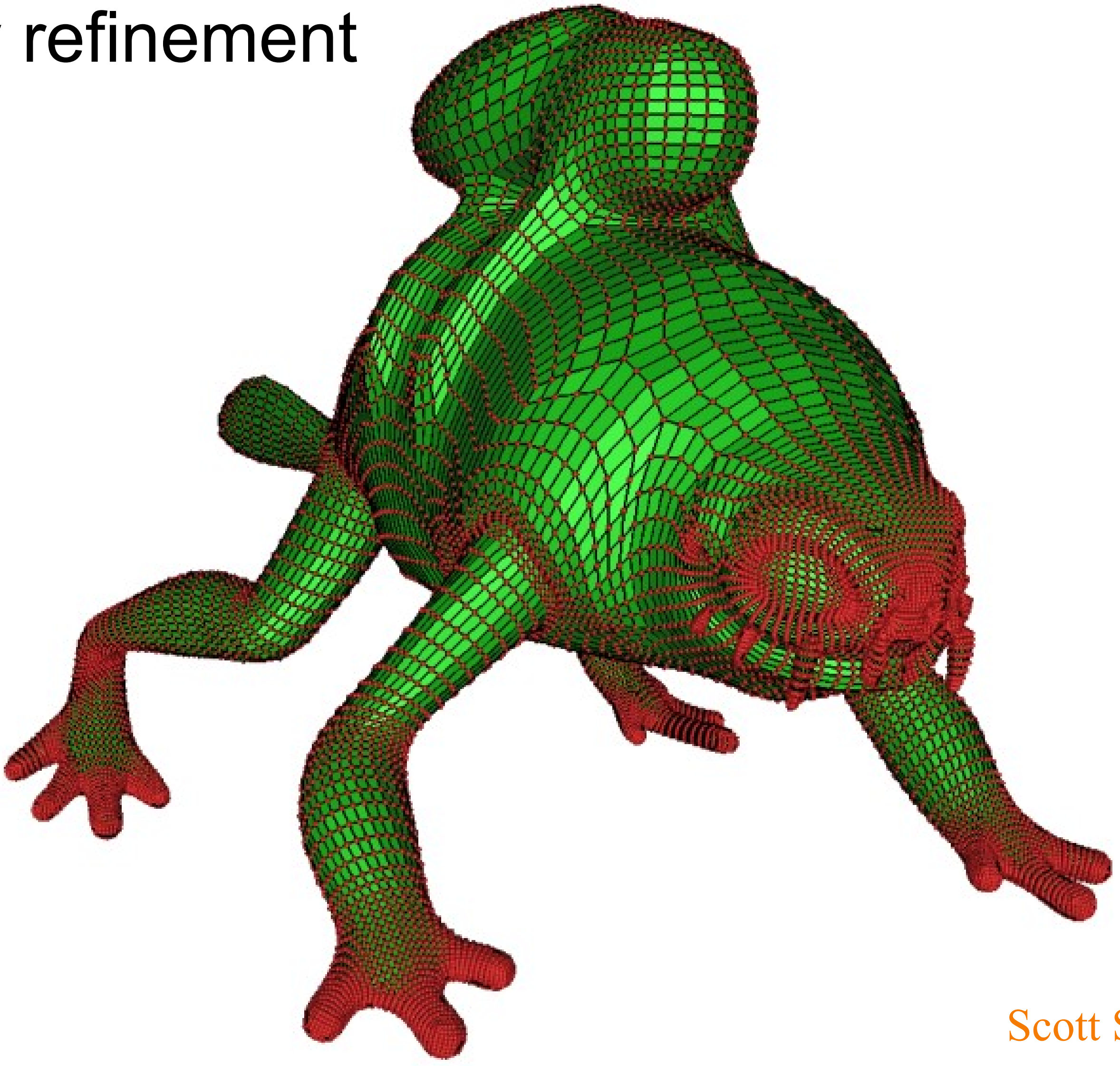
Subdivision Surfaces – Examples

- Topology refinement



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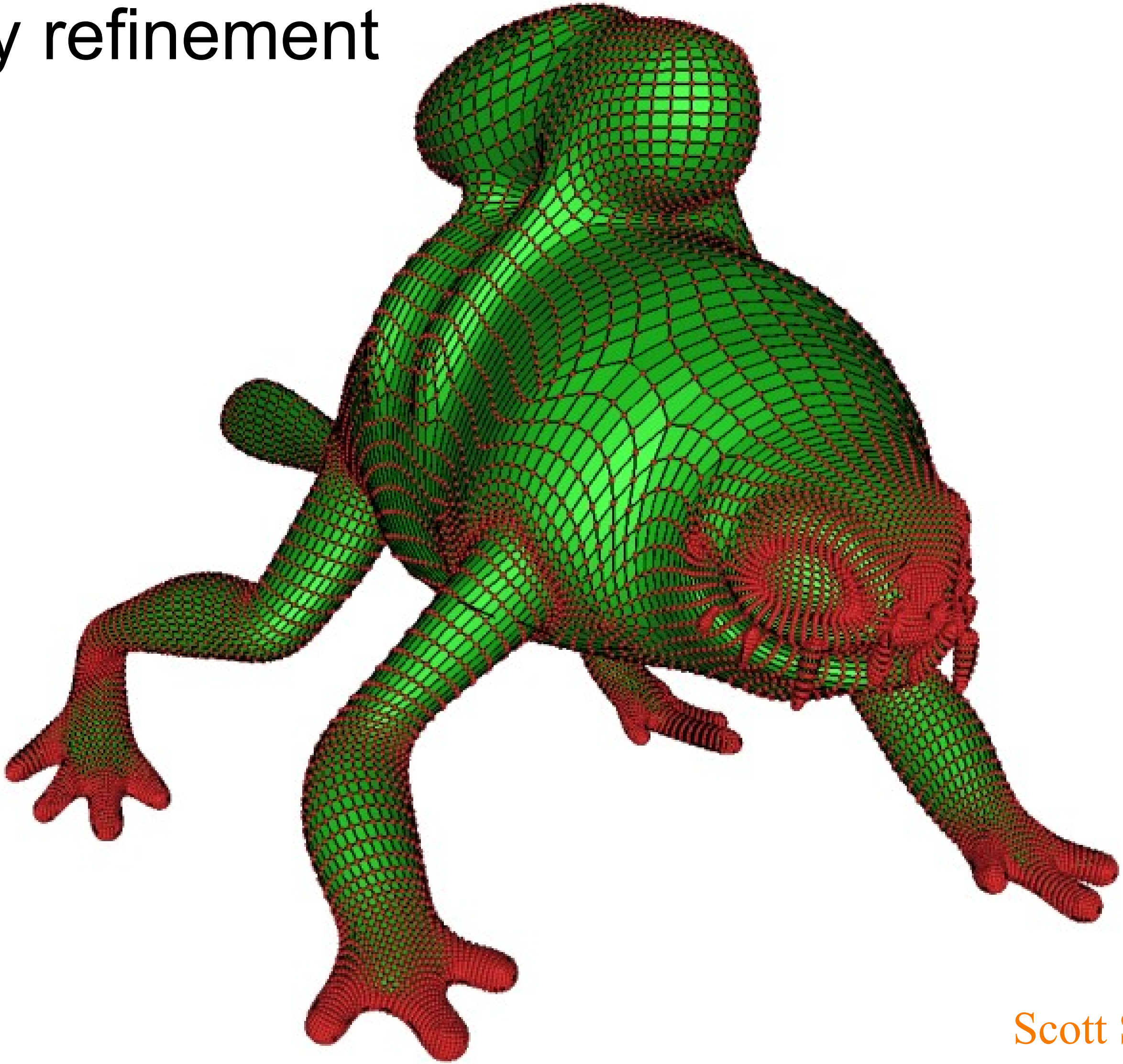
Subdivision Surfaces – Examples

- Geometry refinement



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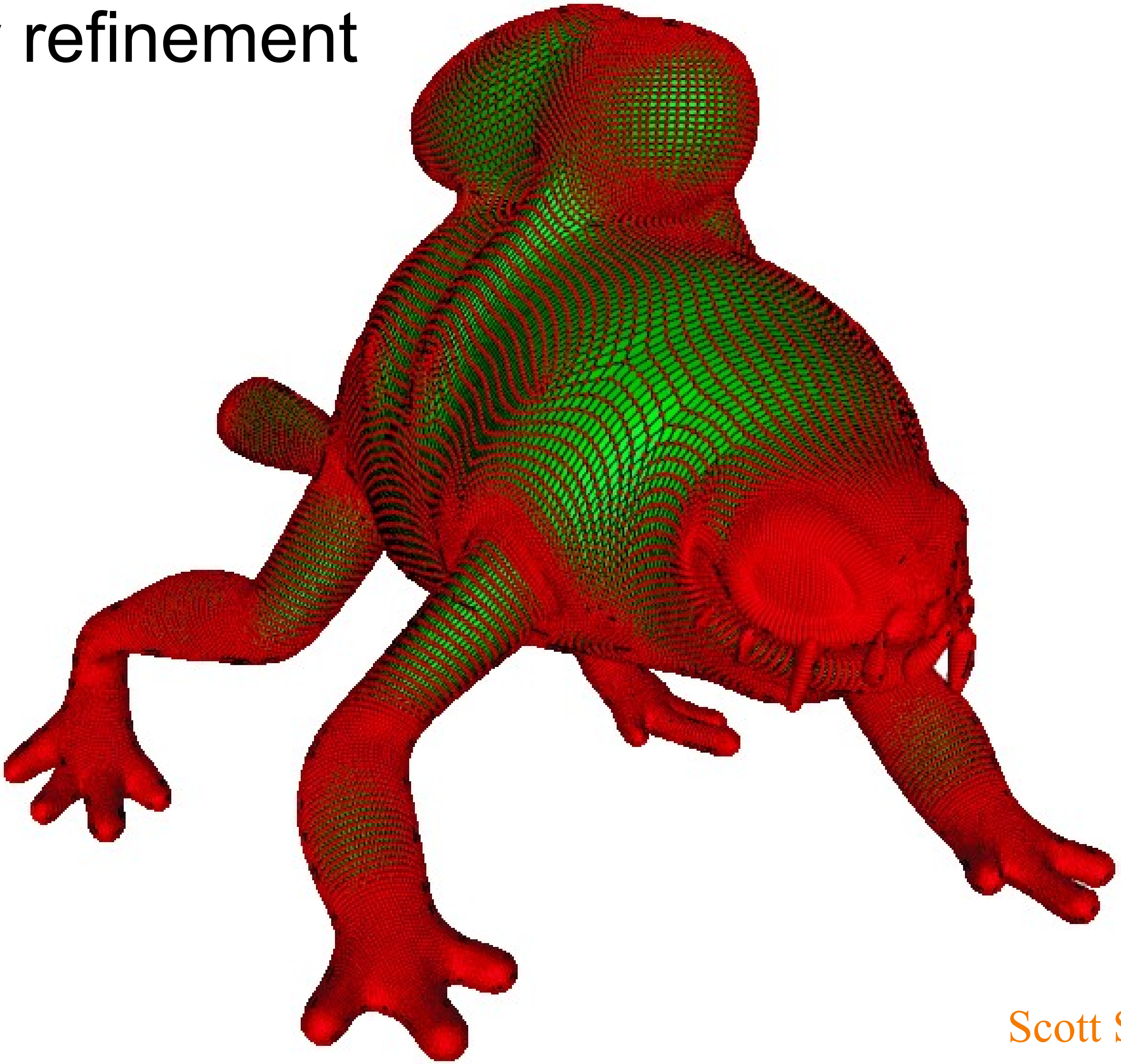
Subdivision Surfaces – Examples

- Topology refinement



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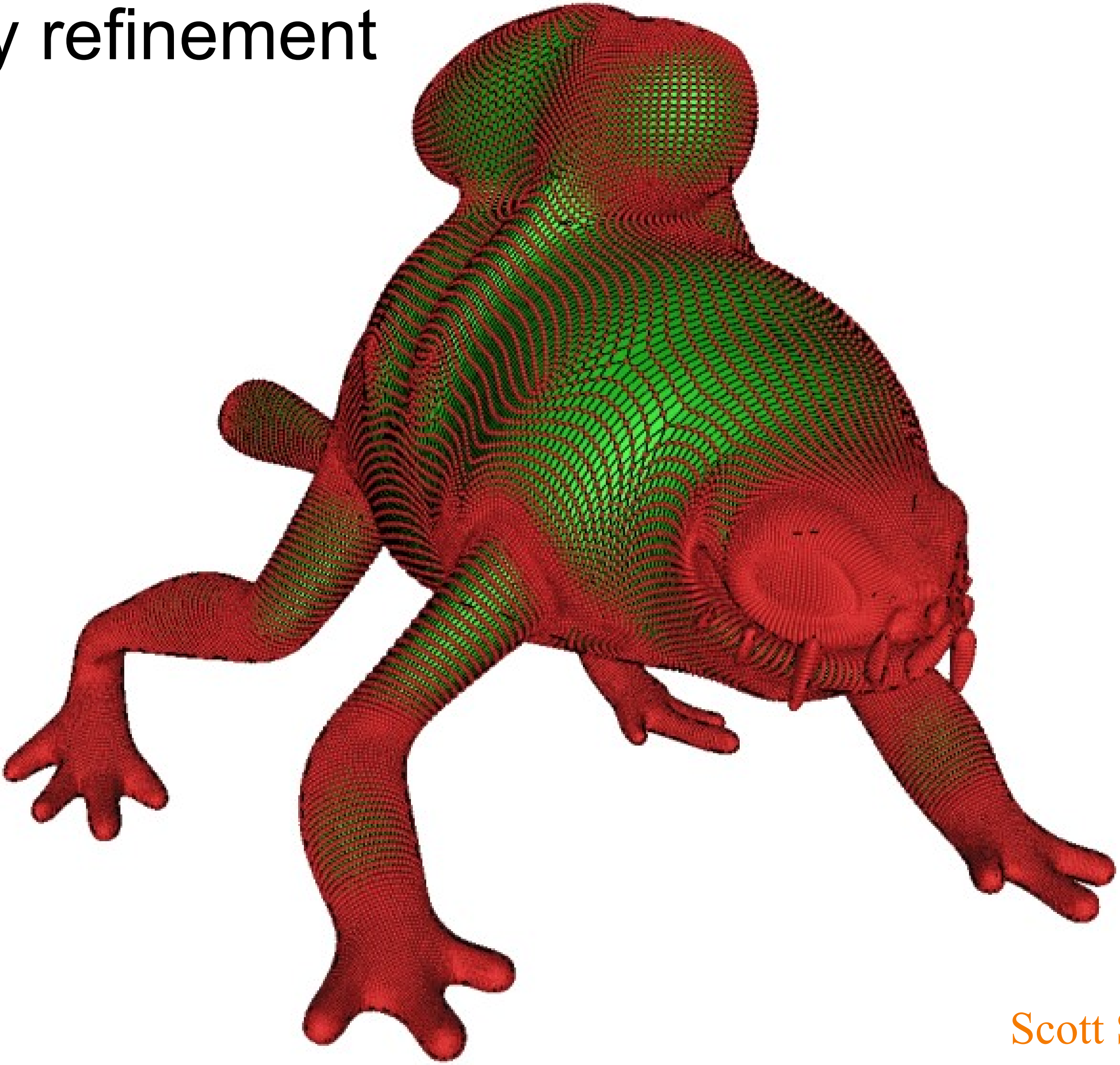
Subdivision Surfaces – Examples

- Geometry refinement



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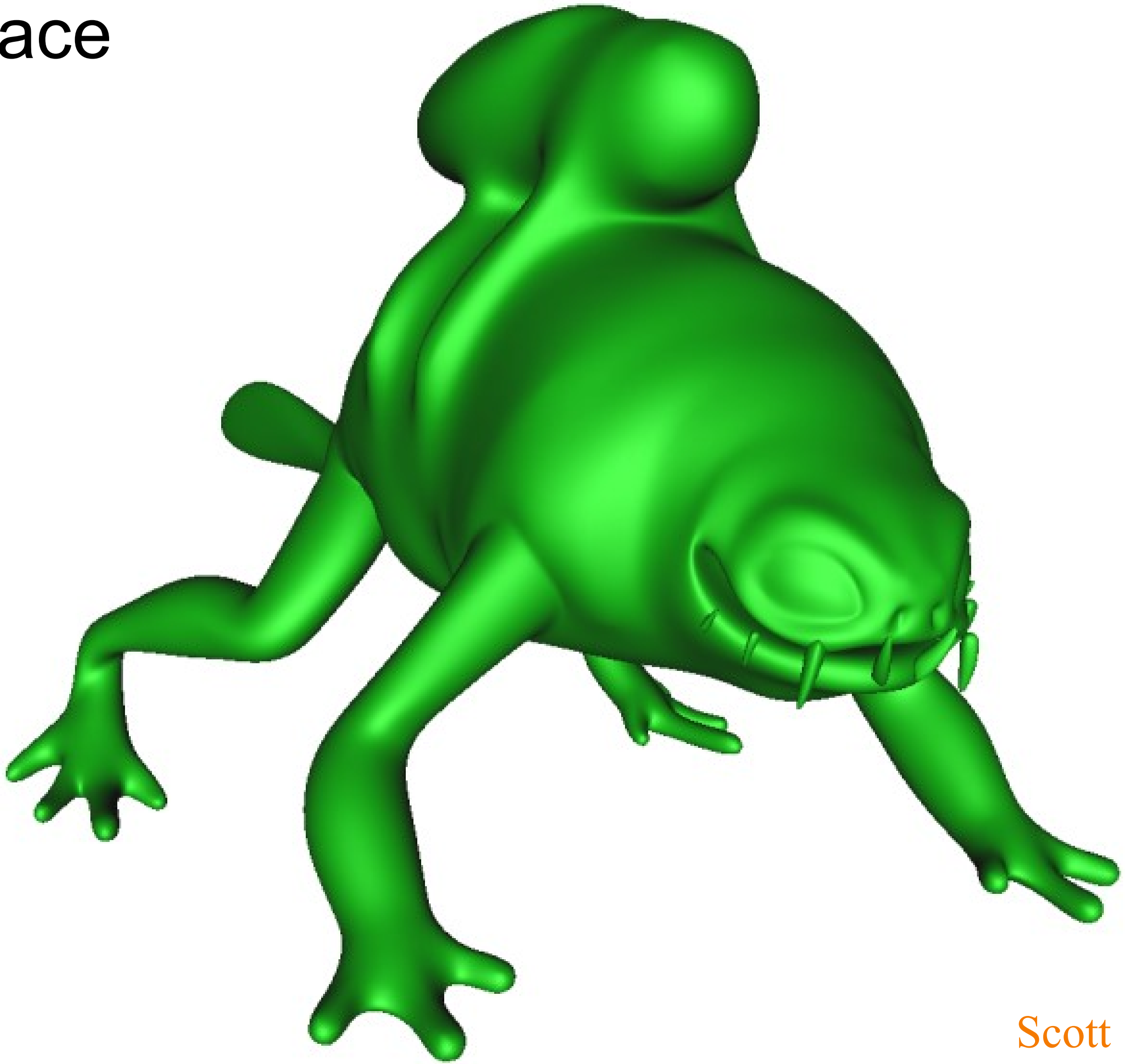
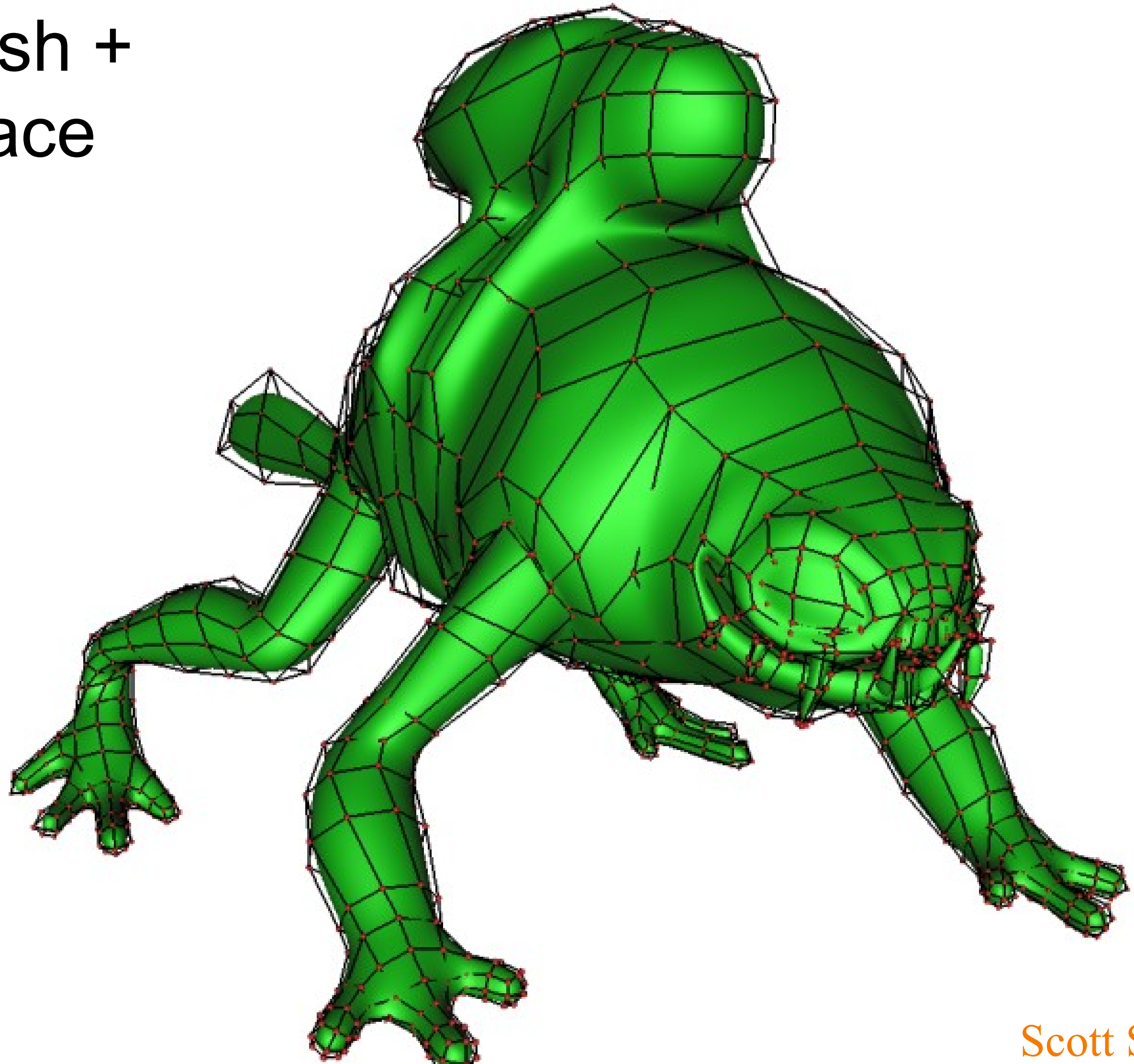
Subdivision Surfaces – Examples

- Limit surface



Subdivision Surfaces – Examples

- Base mesh + limit surface



Meshlab demo

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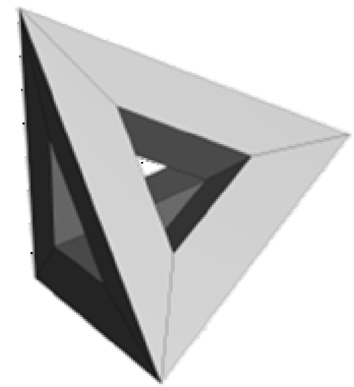
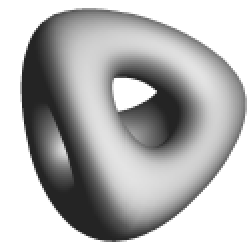
Design of Subdivision Rules



- What types of input?
  - Quad meshes, triangle meshes, etc.
- How to refine topology?
  - Simple implementations
- How to refine geometry?
  - Smoothness guarantees in limit surface
    - » Continuity (C^0, C^1, C^2, …?)
  - Provable relationships between limit surface and original control mesh
    - » Interpolation of vertices?

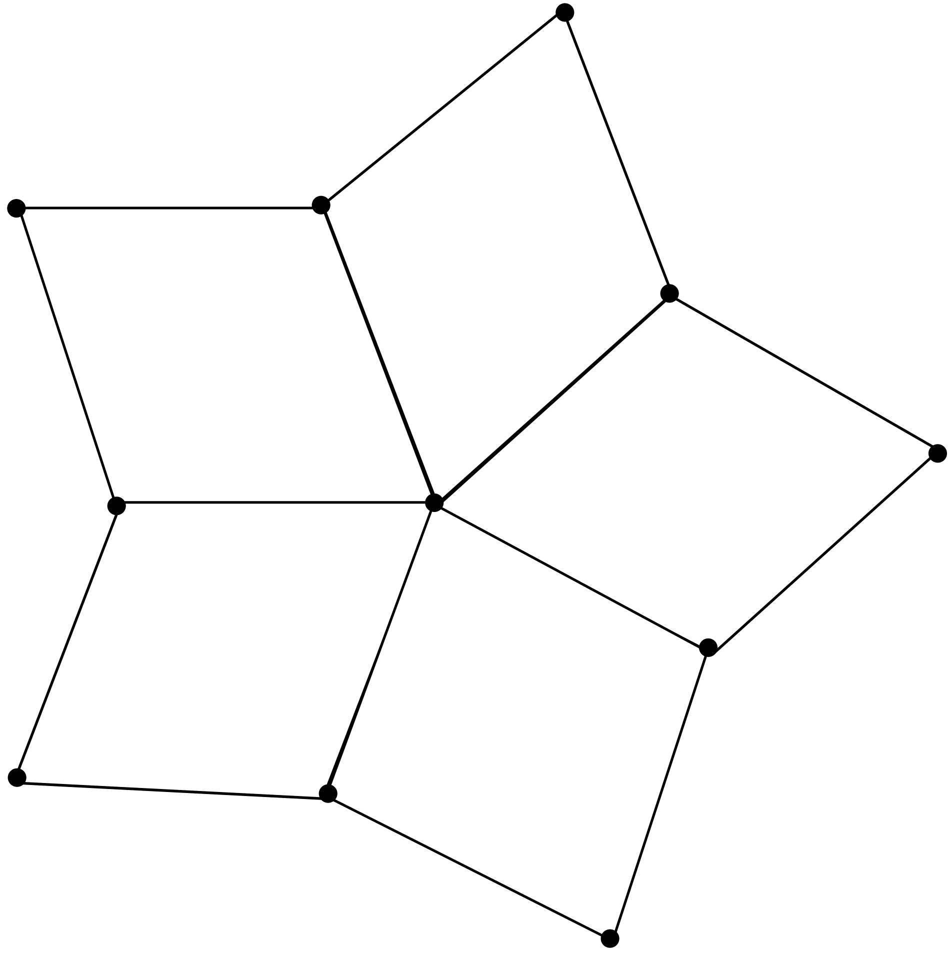


Linear Subdivision

- Type of input
  - Quad mesh -- four-sided polygons (*quads*)
  - Any number of quads may touch each vertex
- Topology refinement rule
  - Split every quad into four at midpoints
- Geometry refinement rule
  - Average vertex positions

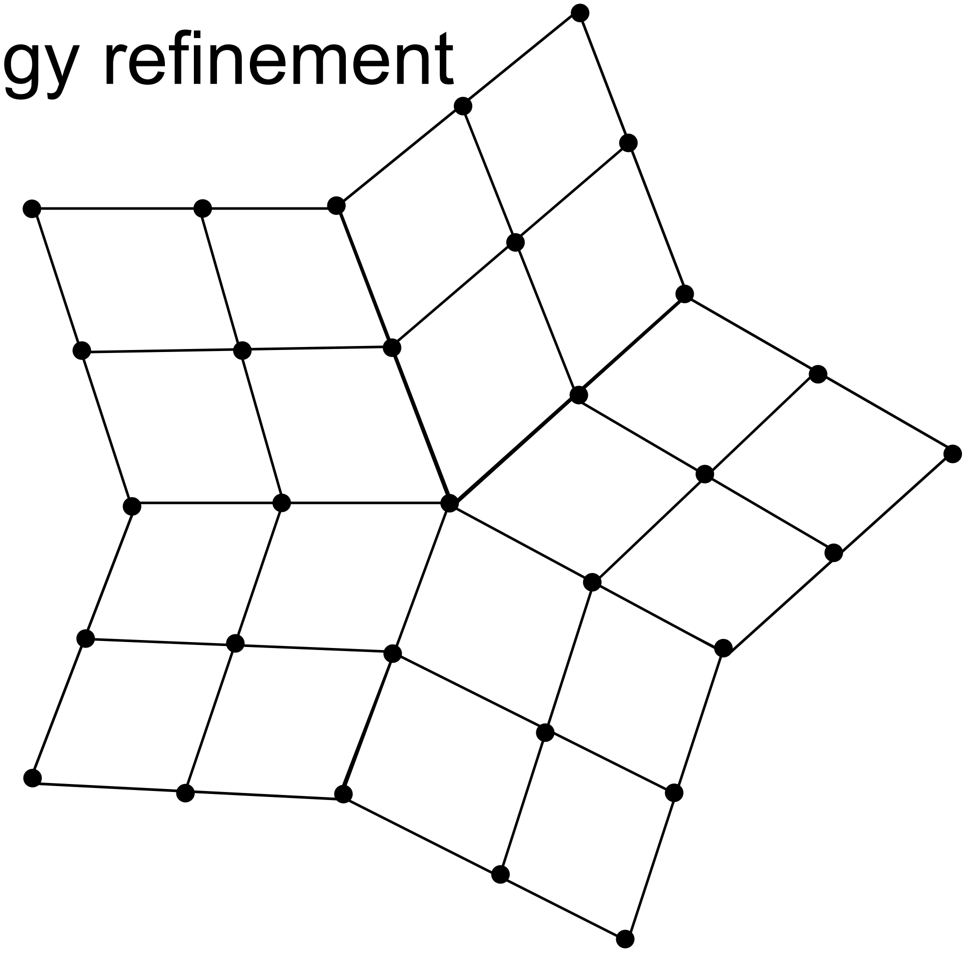
This is a simple example to demonstrate how subdivision schemes work

Linear Subdivision



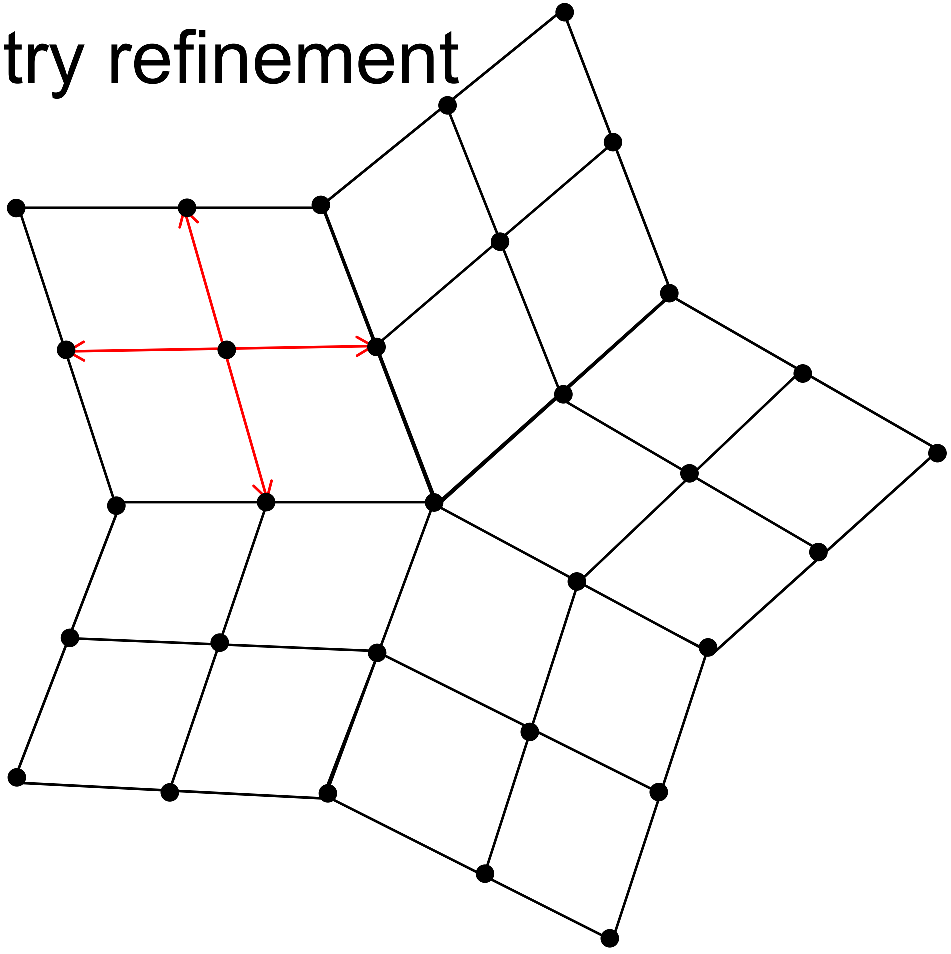
Linear Subdivision

- Topology refinement



Linear Subdivision

- Geometry refinement



Linear Subdivision

LinearSubivision (F_0, V_0, k)

for $\mathrm{i} = 1 \ldots \mathrm{k}$ levels

(F_i, V_i) = RefineTopology(F_{i-1}, V_{i-1})

RefineGeometry(F_i, V_i)

return (F_k, V_k)

Linear Subdivision

RefineTopology (F, V)

$newV$ = V

$newF$ = {}

for each face F_i

Insert new vertex c at centroid of F_i into $newV$

for j = 1 to 4

Insert in $newV$ new vertex e_j at centroid of each edge ($F_{i,j}$, $F_{i,j+1}$)

for j = 1 to 4

Insert new face ($F_{i,j}$, e_j, c, e_{j-1}) into $newF$

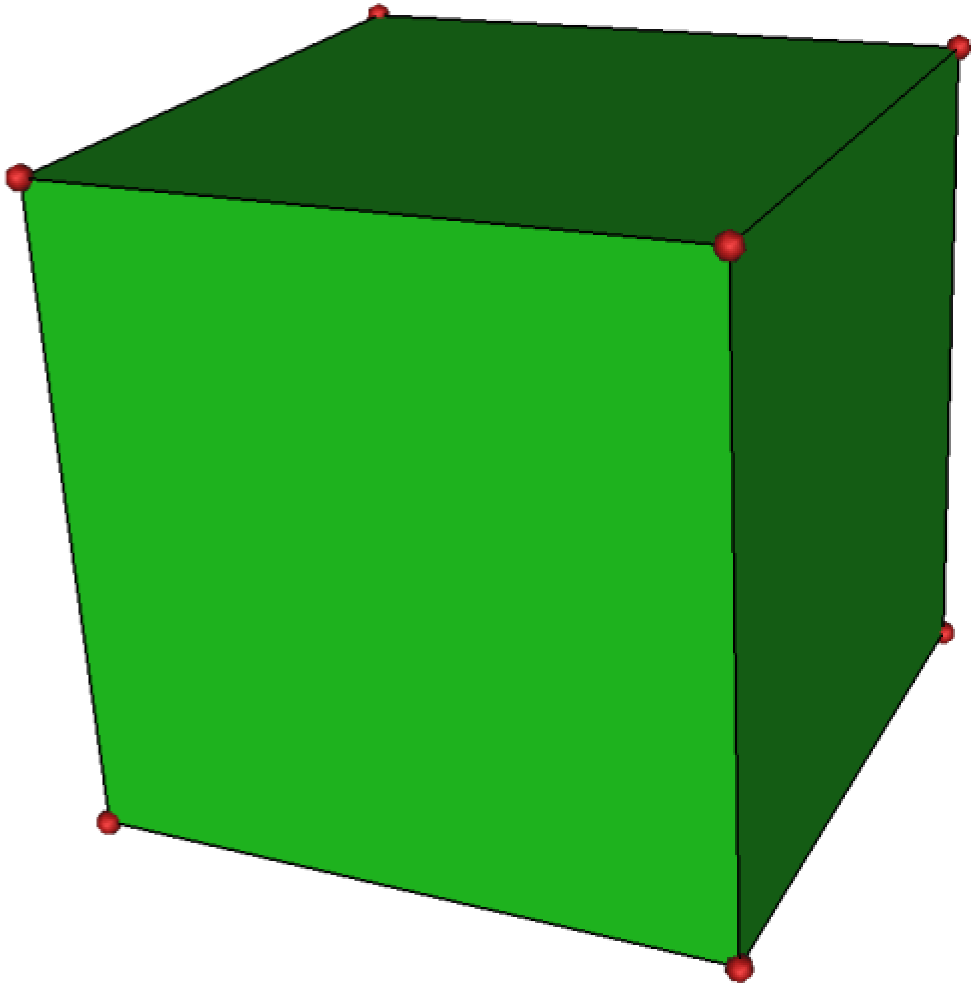
return ($newF$, $newV$)

Linear Subdivision

```
RefineGeometry( F, V )
    newV = V
    newF = F
    for each vertex V_i in newV
        weight = 0;
        newV[i] = (0,0,0)
        for each face F_j connected to V_i
            newV[i] += centroid of F_j
            weight += 1.0;
        newV[i] /= weight
    return (newF, newV)
```

Linear Subdivision

- Example



Input mesh

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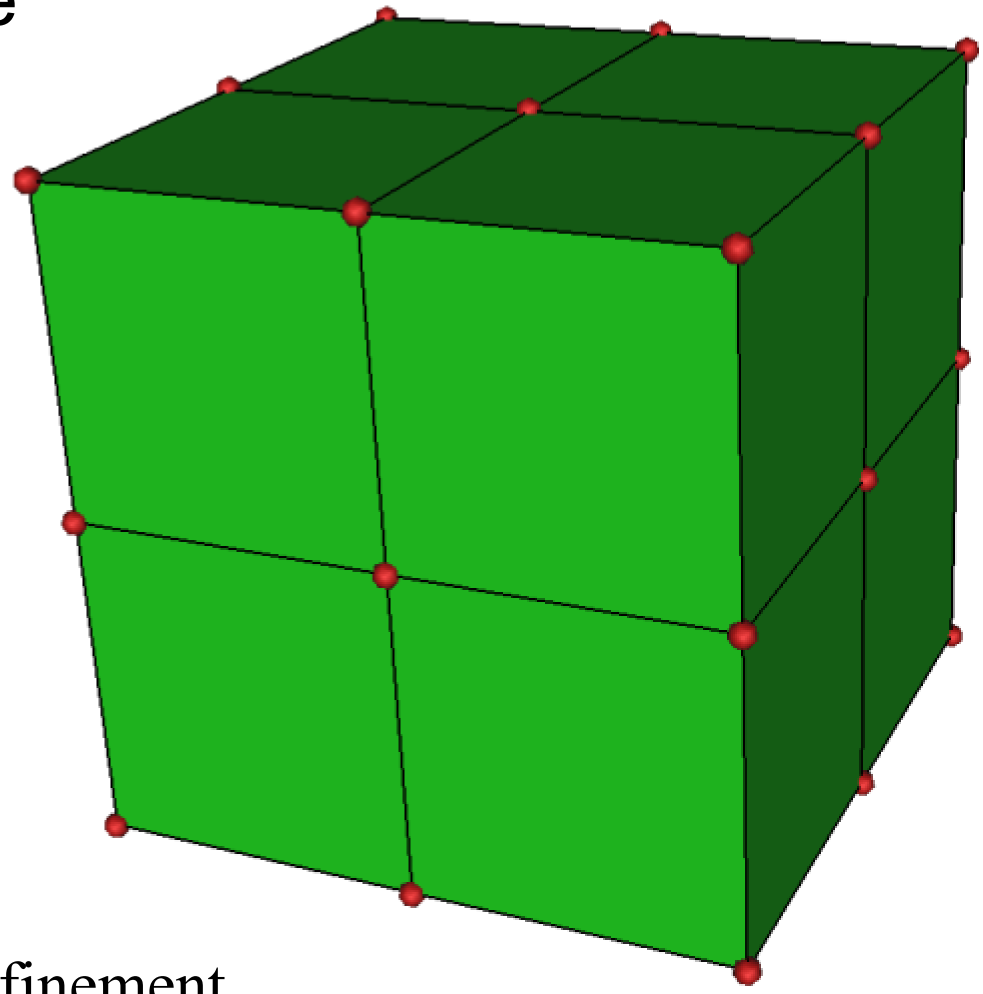
Linear Subdivision

- Example



Topology refinement

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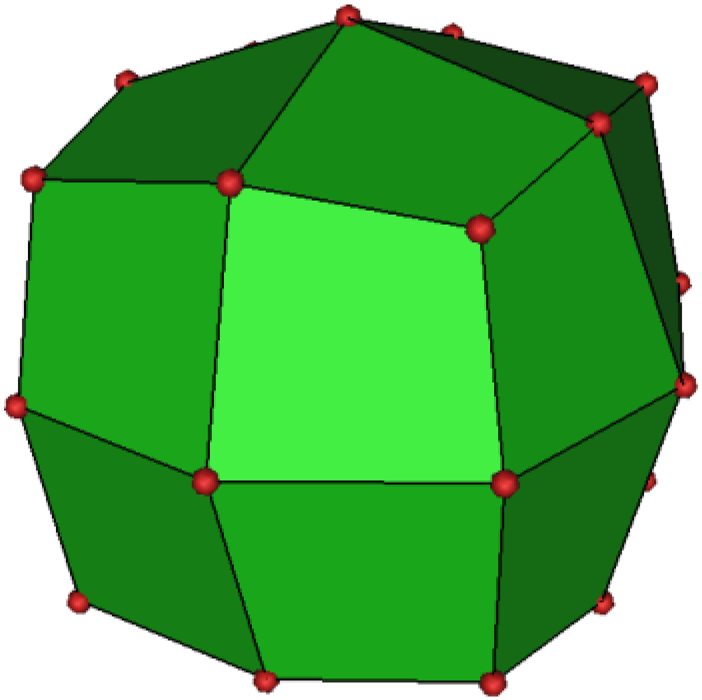
Linear Subdivision

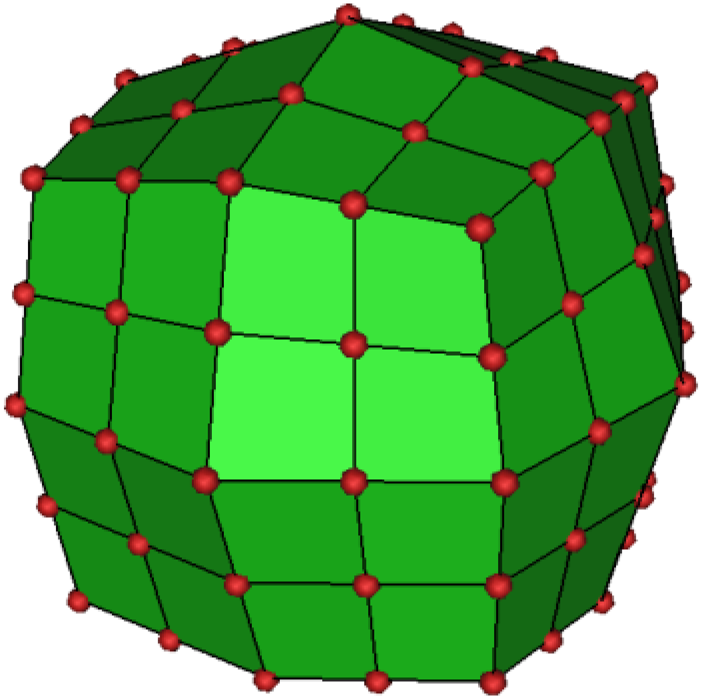
- Example



Geometry refinement

Linear Subdivision

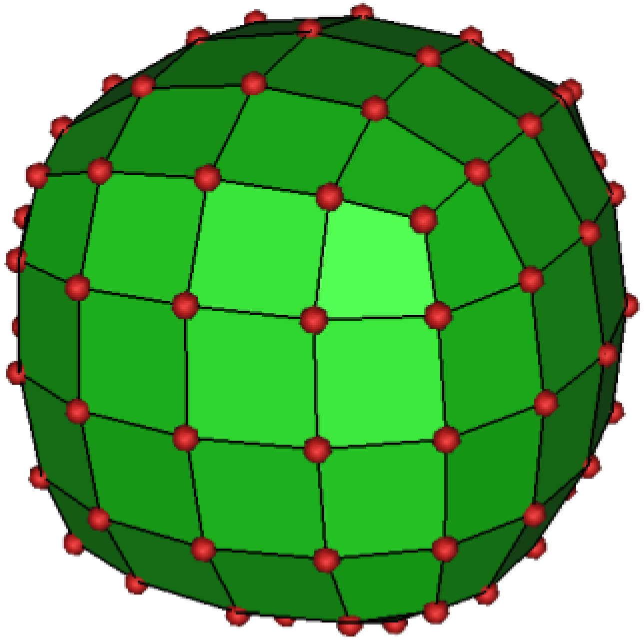
- Example



Topology refinement

Linear Subdivision

- Example



Geometry refinement

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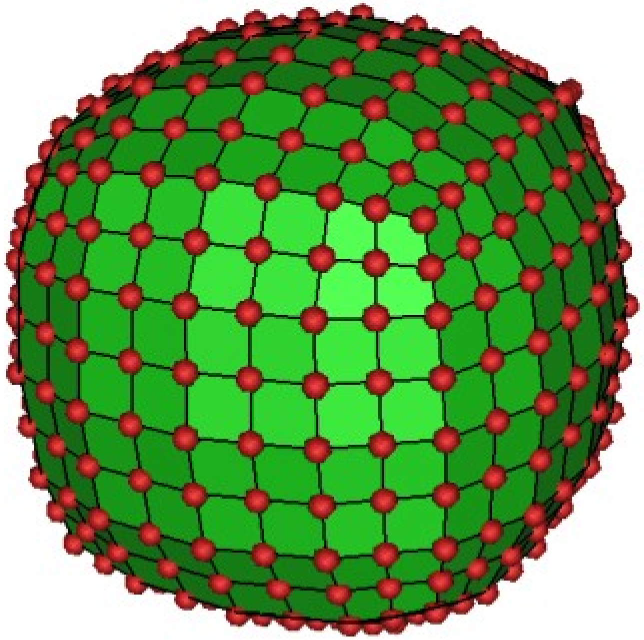
Linear Subdivision

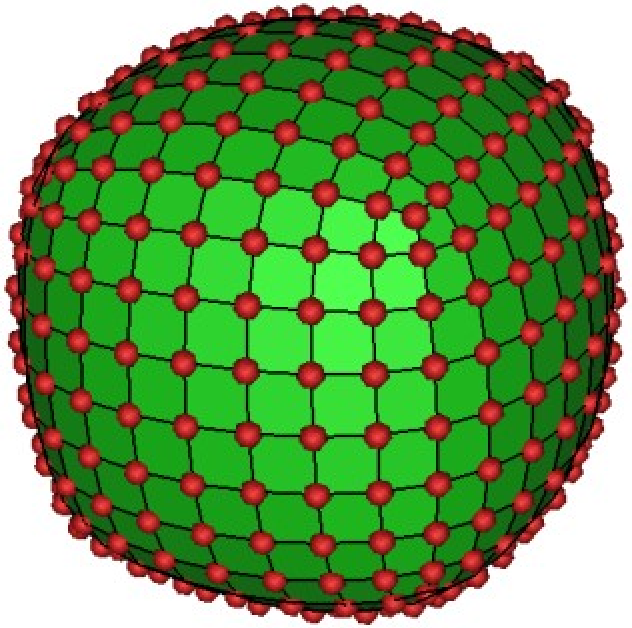
- Example



Topology refinement

Linear Subdivision

- Example



Geometry refinement

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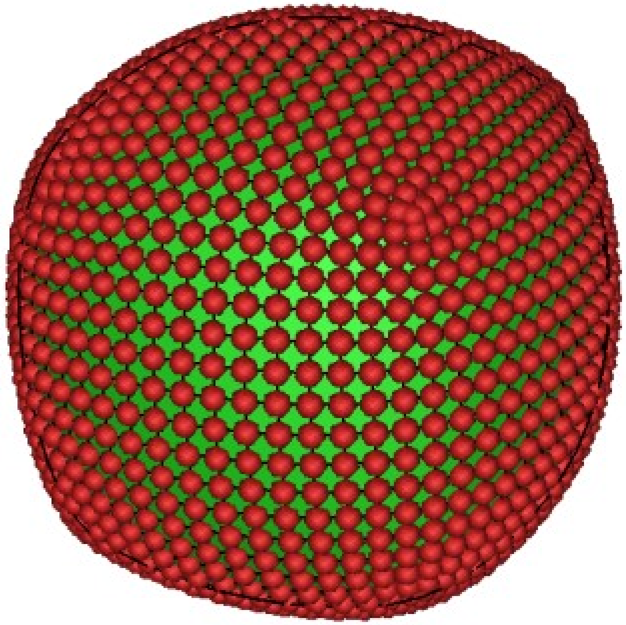
Linear Subdivision

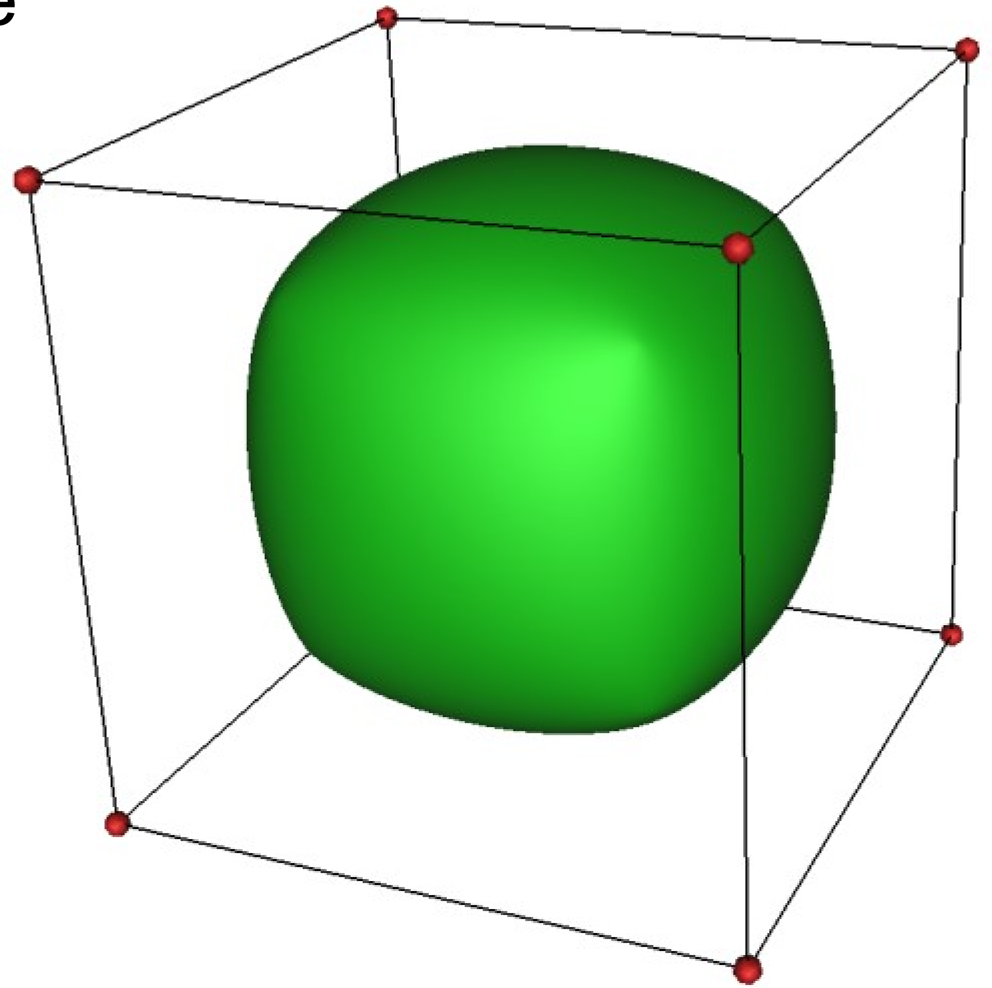
- Example



Topology refinement

Linear Subdivision

- Example



Final result

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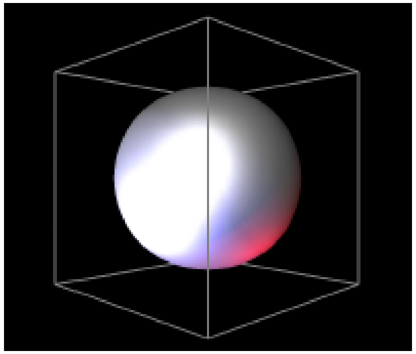
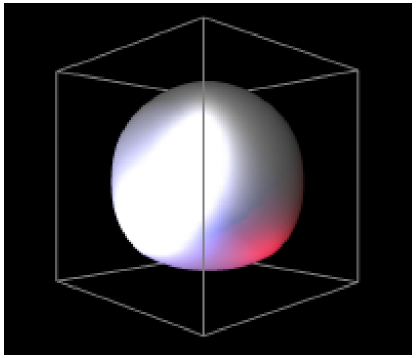
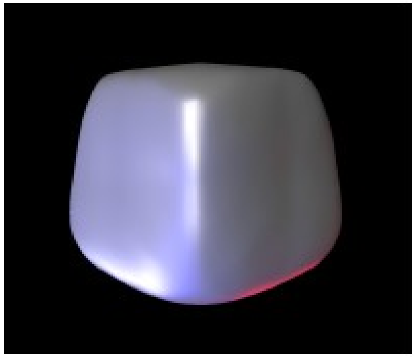
Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others
- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

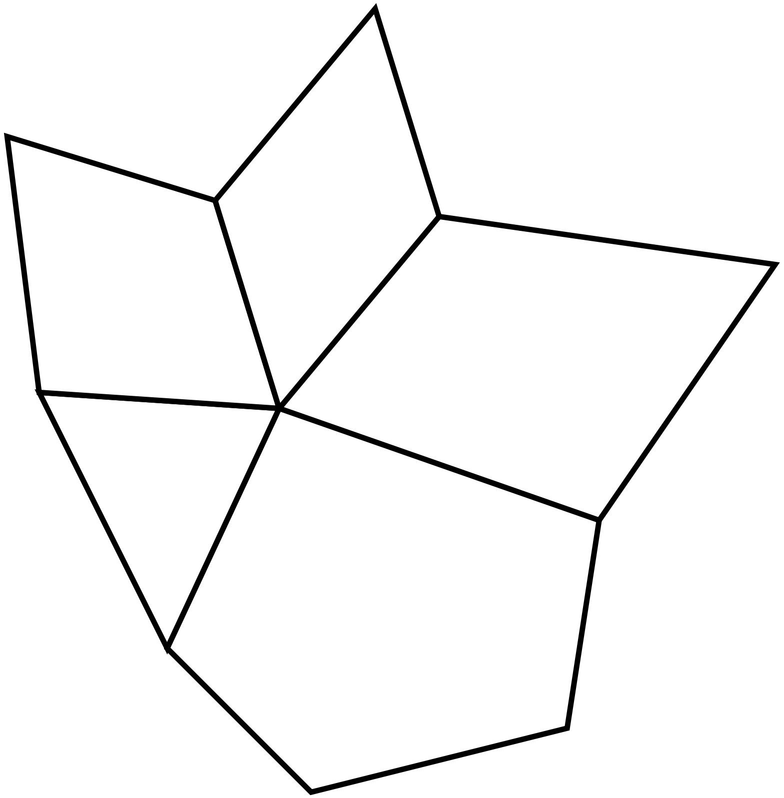
… which makes differences in …
  - Provable properties







Catmull-Clark Subdivision



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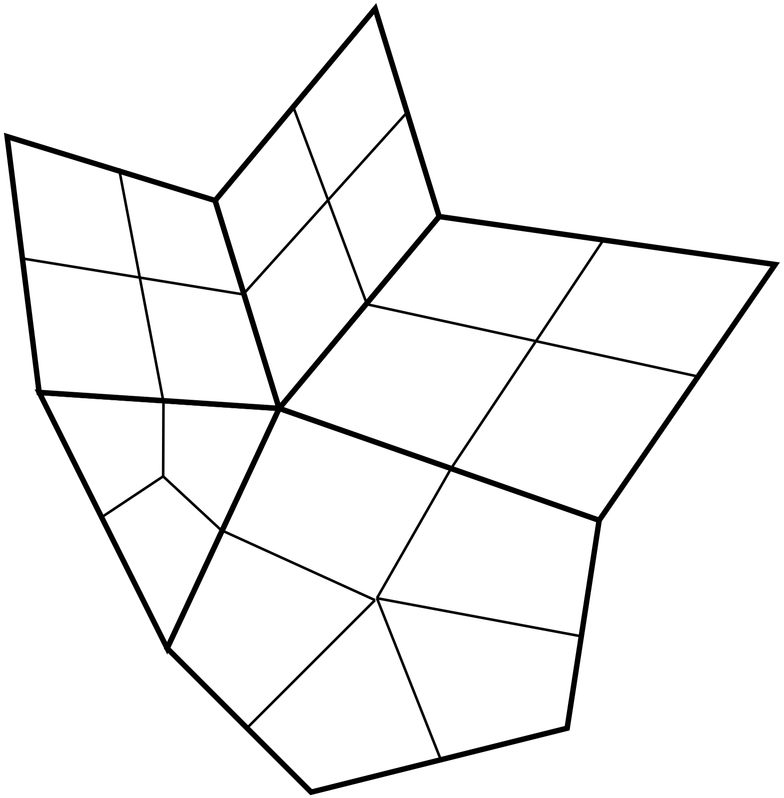

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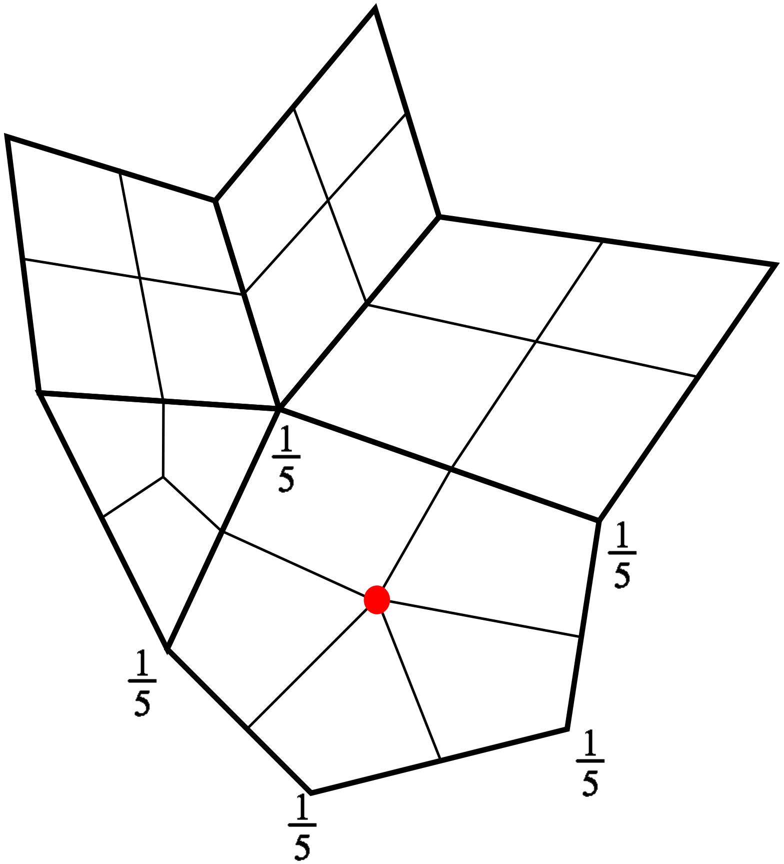

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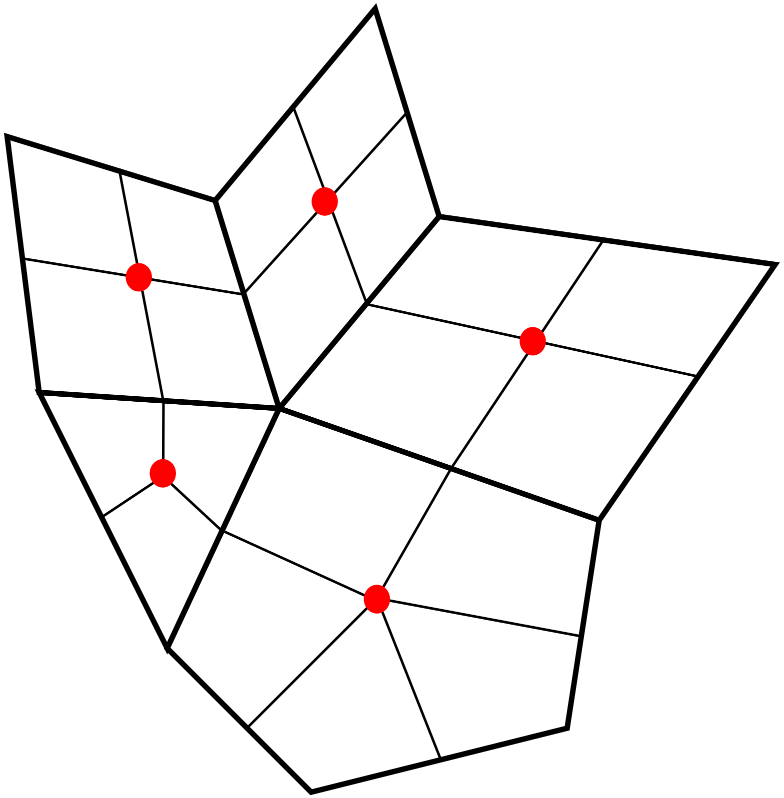

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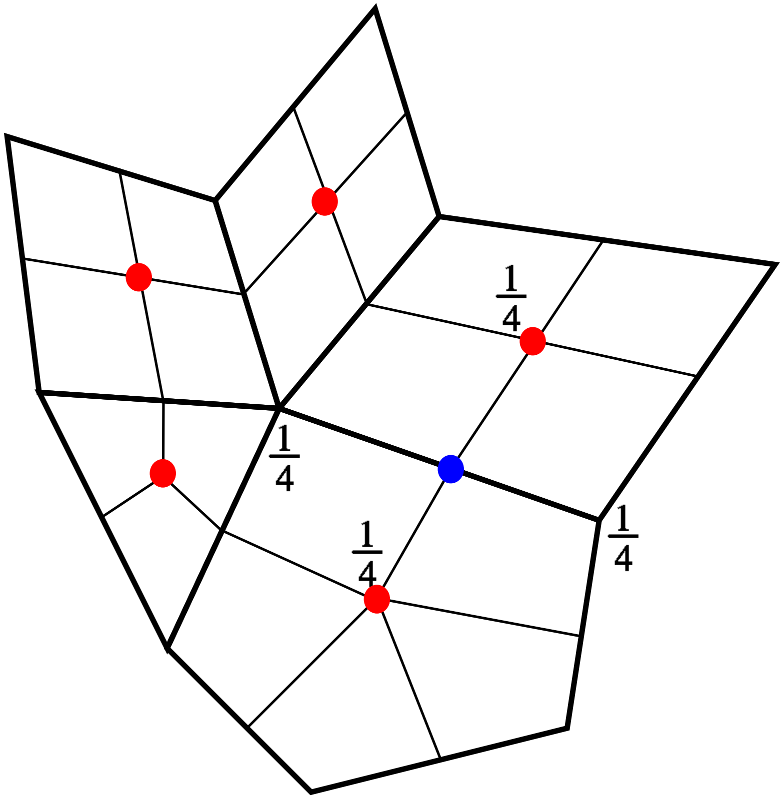
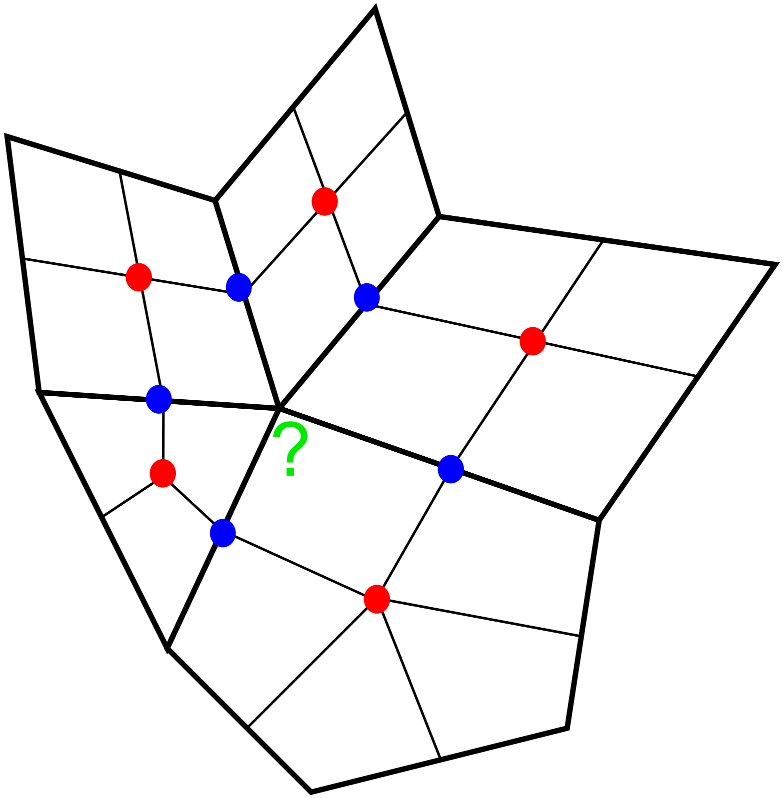
Catmull-Clark Subdivision

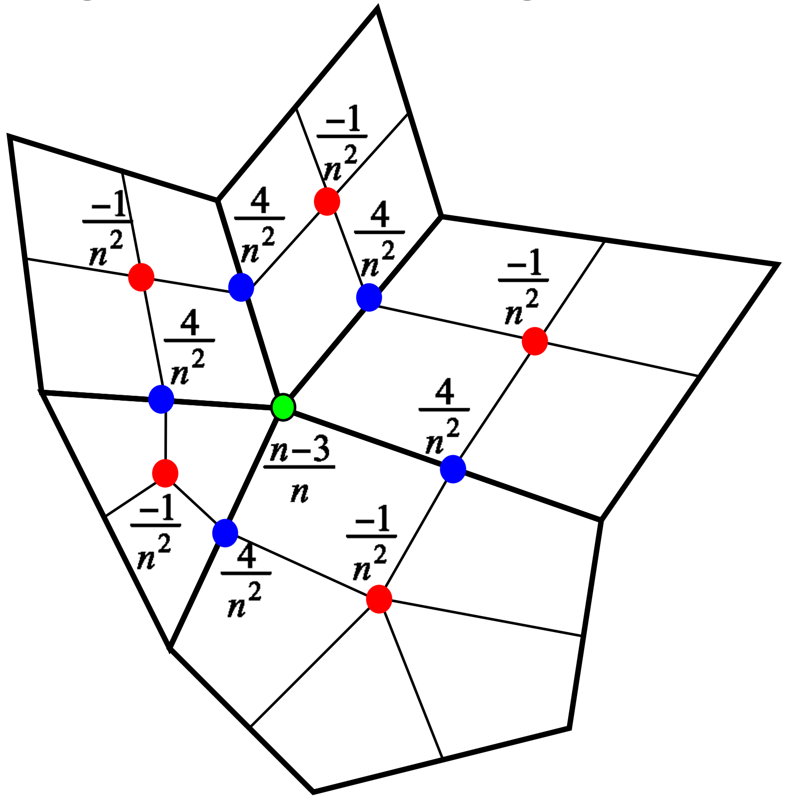


Catmull-Clark Subdivision



Catmull-Clark Subdivision

New ● = (4 * avg of ● –1 * avg of ● + (n-3) * ●) / n



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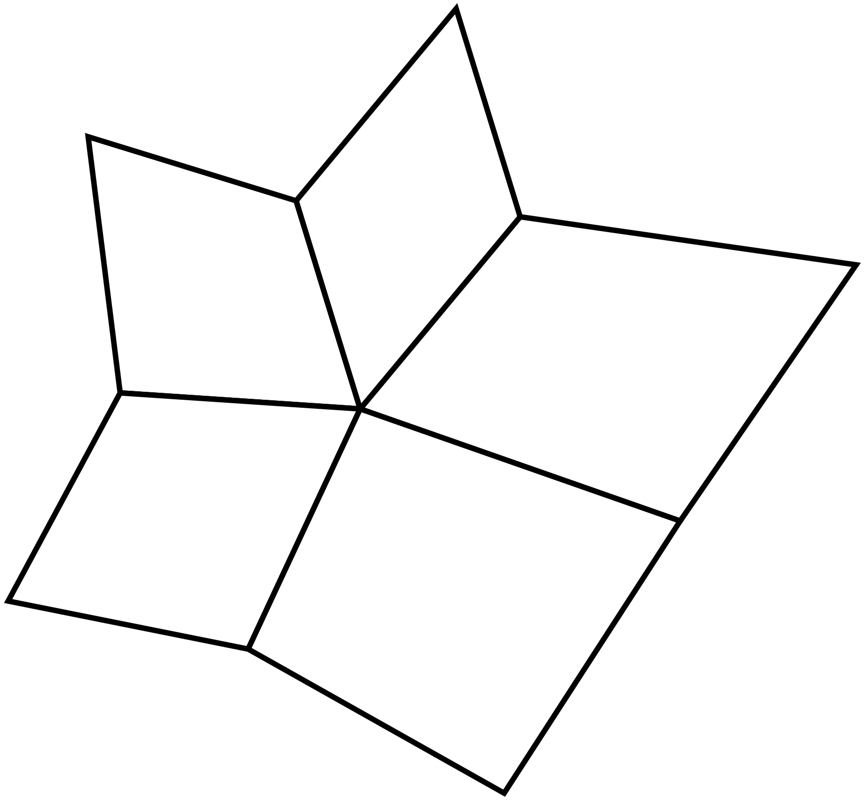
Catmull-Clark Subdivision



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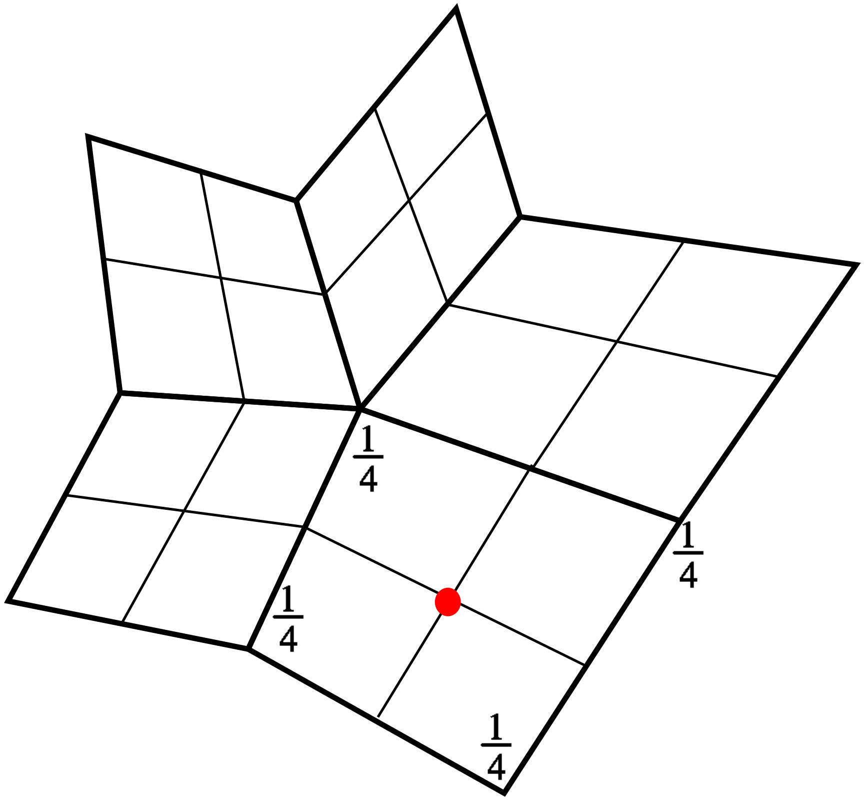
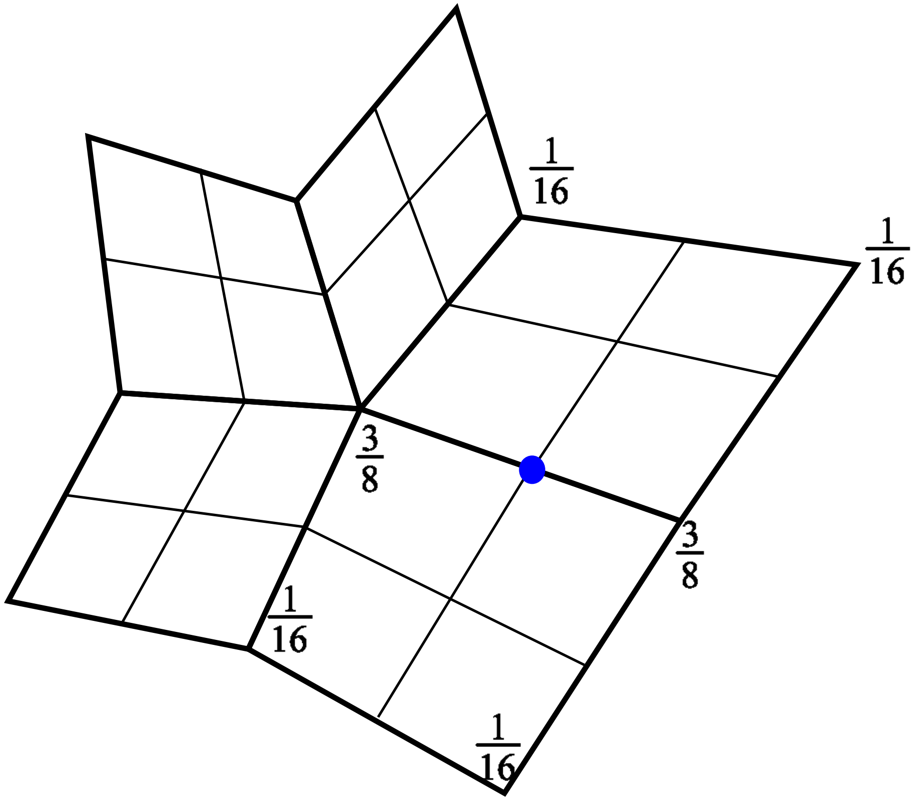
Catmull-Clark Subdivision



Catmull-Clark Subdivision



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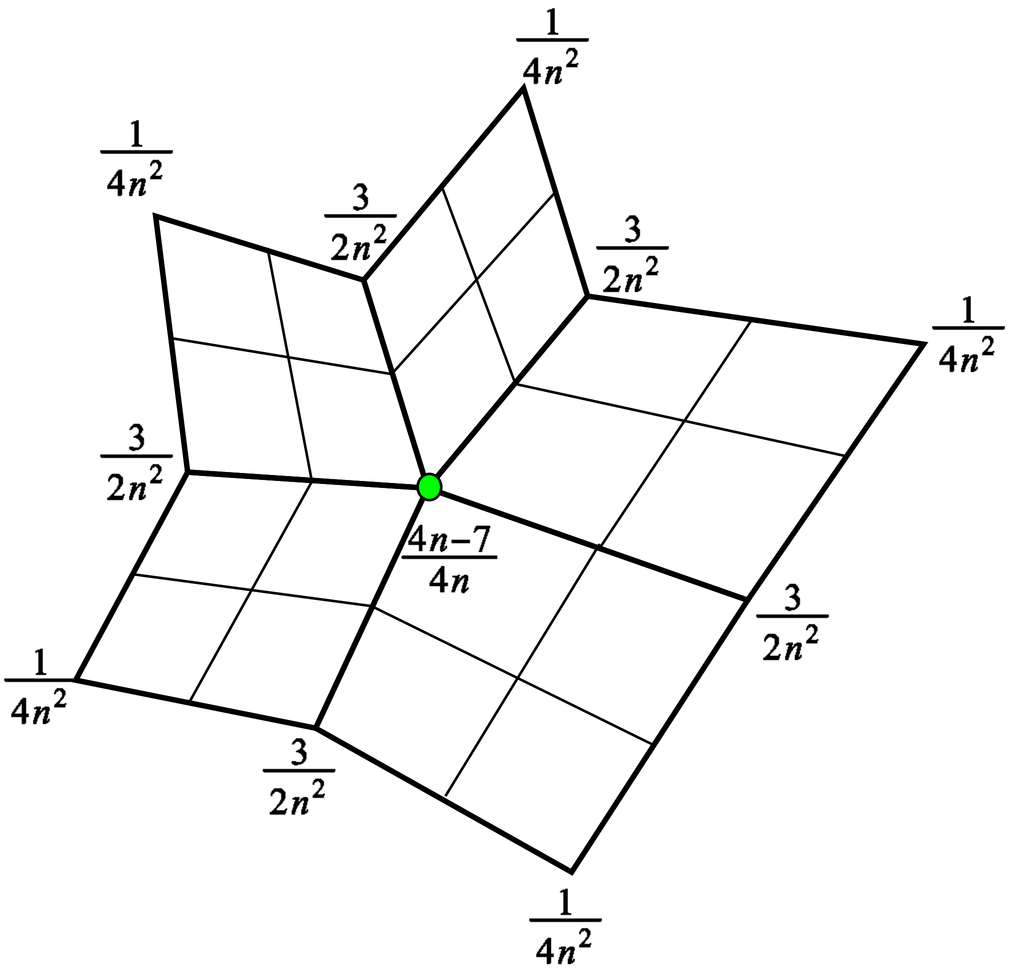

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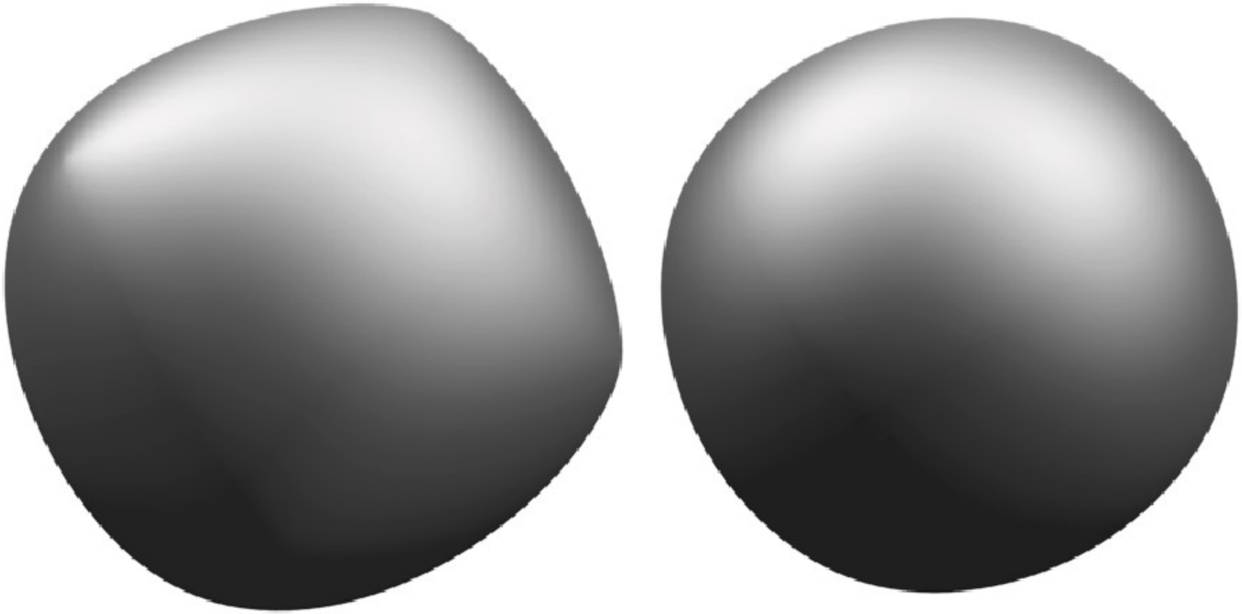

Linear Subdivision

Catmull-Clark Subdivision

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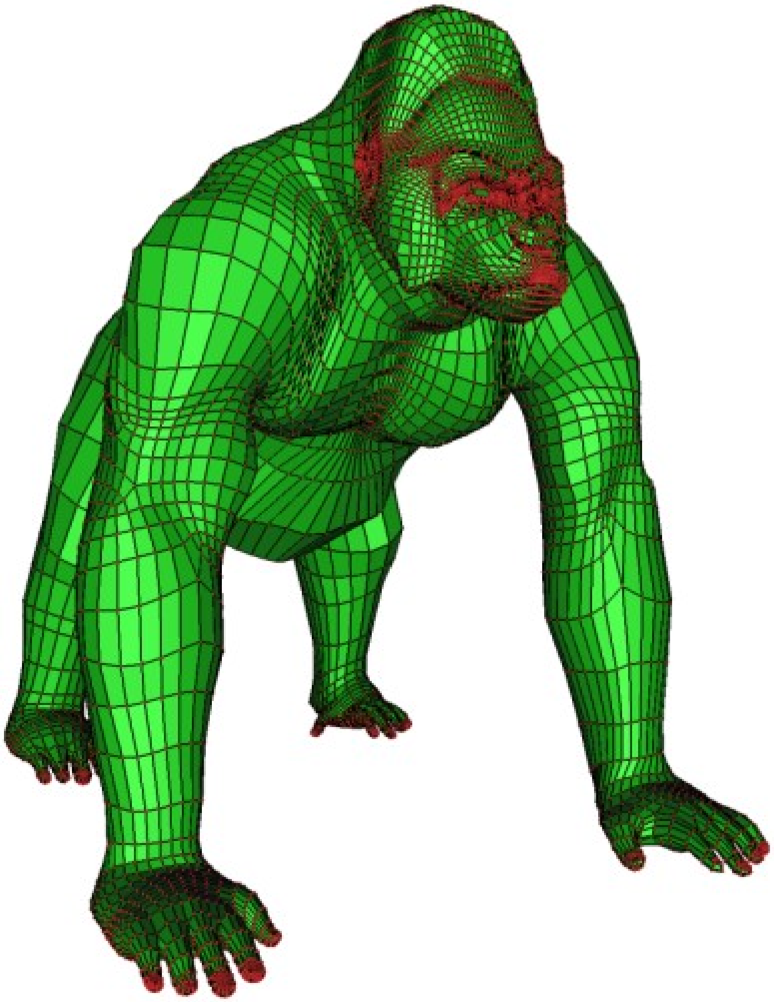
Catmull-Clark Subdivision



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Catmull-Clark Subdivision

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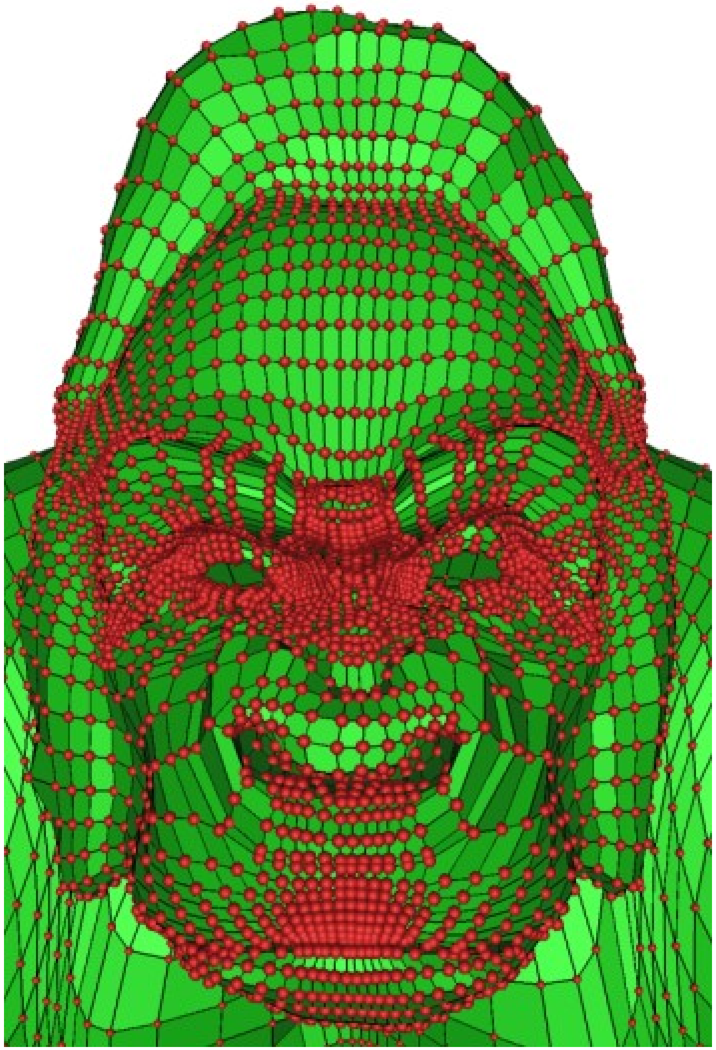
Catmull-Clark Subdivision



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Catmull-Clark Subdivision

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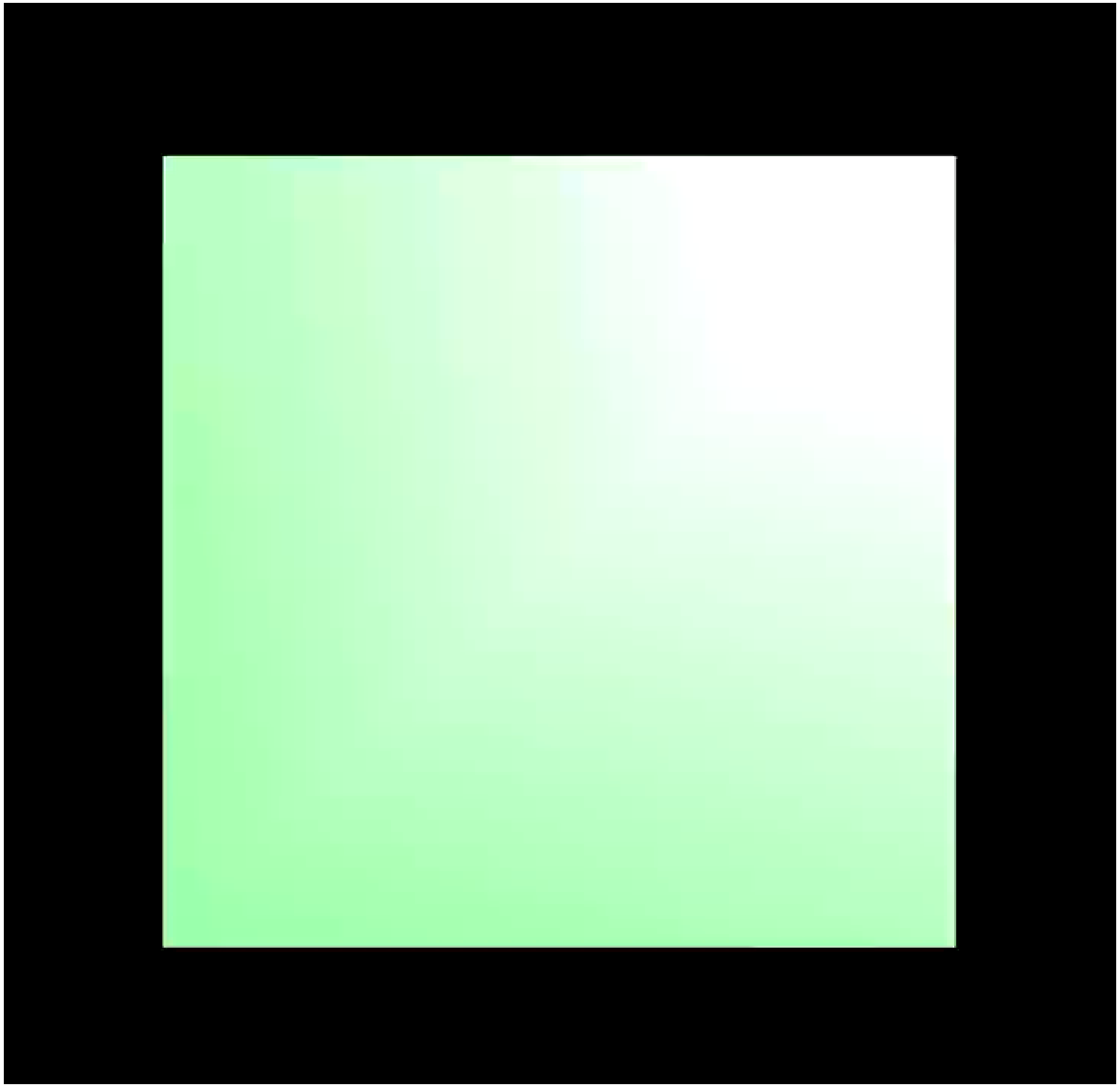
Catmull-Clark Subdivision



https://www.youtube.com/watch?v=4hbHa8deT90

Catmull-Clark Subdivision

- One round of subdivision produces all quads
- Smoothness of limit surface
  - C^2 almost everywhere
  - C^1 at vertices with valence $\neq$ 4
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull
- Most commonly used subdivision scheme in the movies…

Pixar

Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - ➢ Loop
  - Many others
- Differ in ...
  - Input topology
  - How refine topology
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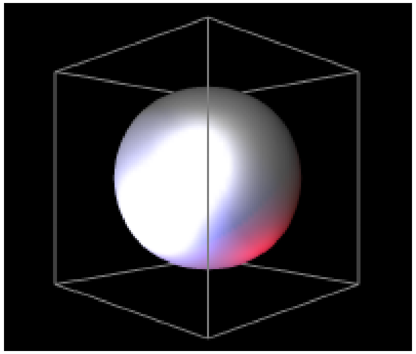
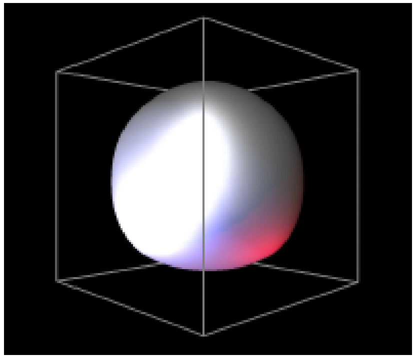
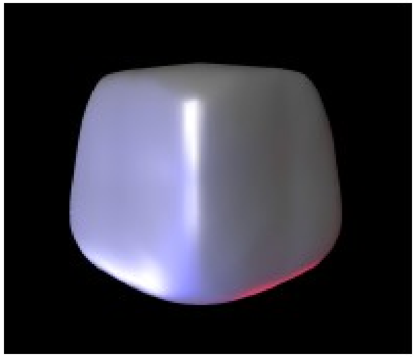
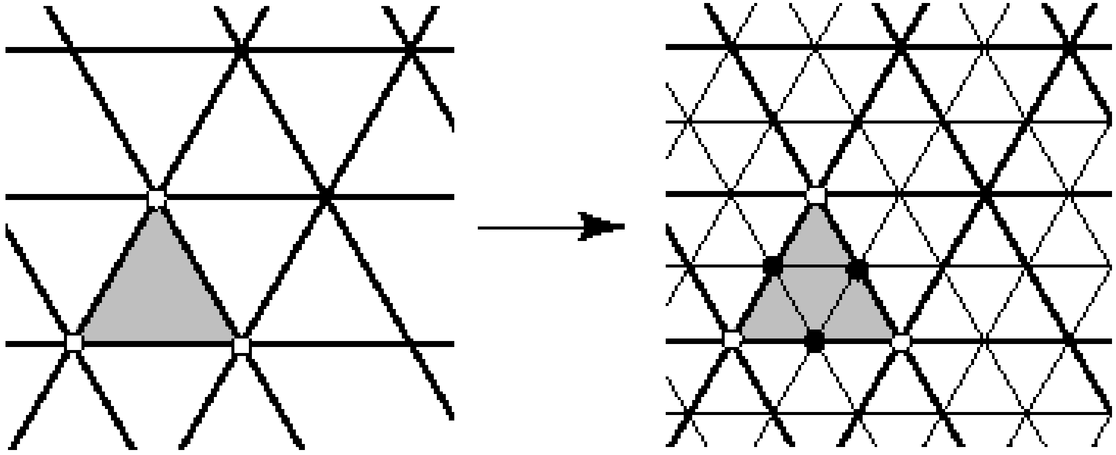
  - Provable properties

Loop Subdivision

- Operates on pure triangle meshes
- Subdivision rules
  - Linear subdivision
  - Averaging rules for “even / odd” (white / black) vertices



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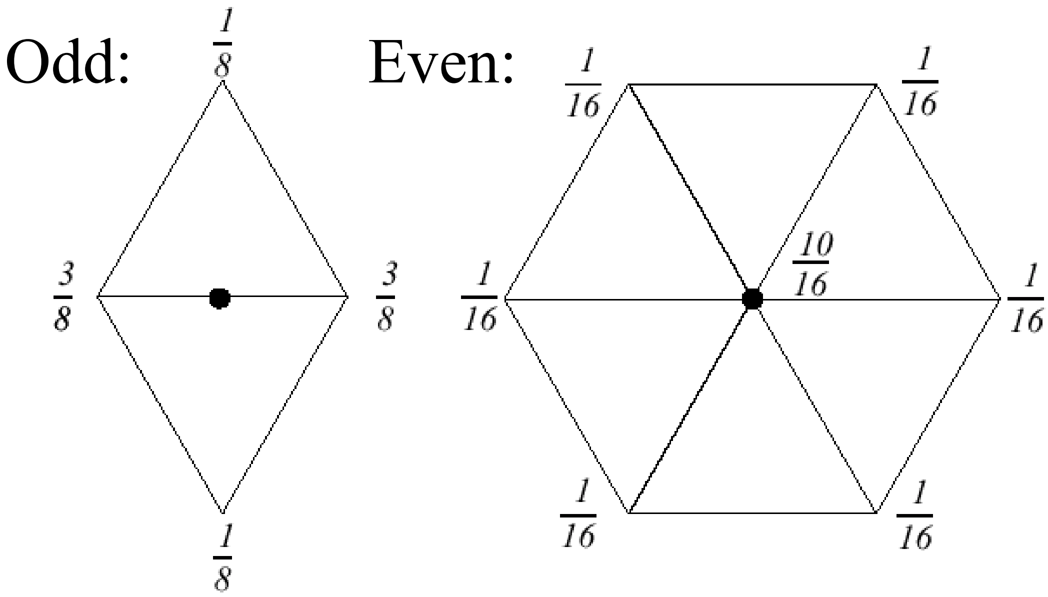
Loop Subdivision

- Averaging rules
  - Weights for “odd” and “even” vertices



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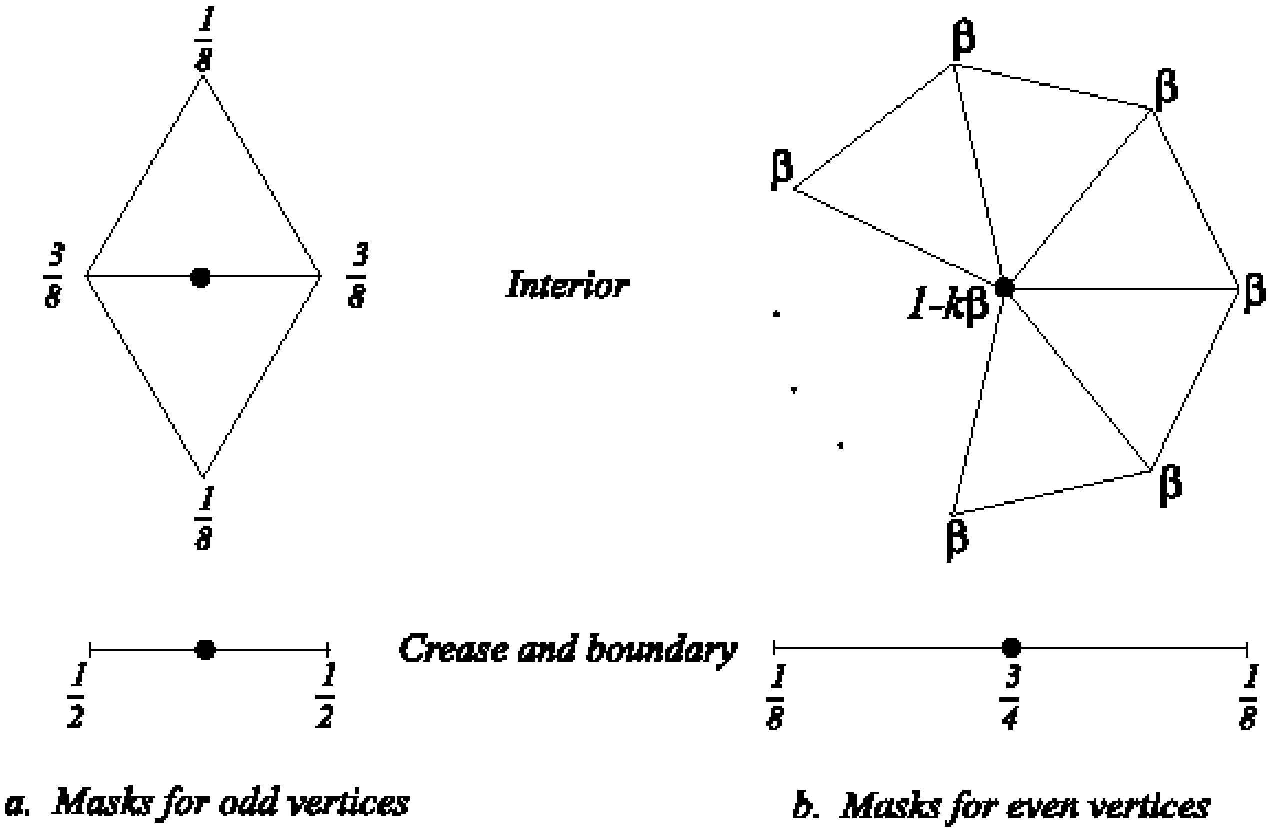
Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:



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Loop Subdivision

- How to choose β?
  - Analyze properties of limit surface
  - Interested in continuity of surface and smoothness
  - Involves calculating eigenvalues of matrices

» Original Loop

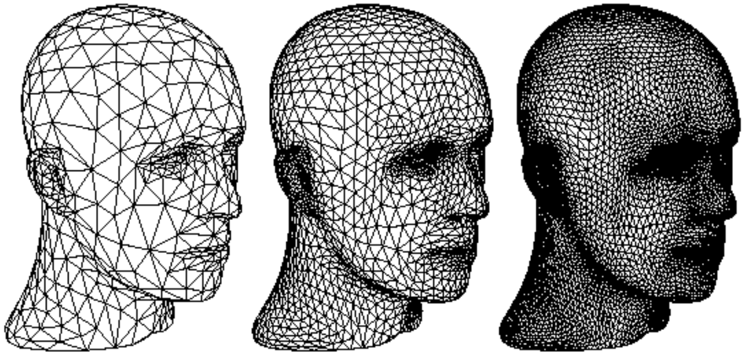
$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

» Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

Loop Subdivision

- Operates only on triangle meshes
- Smoothness of limit surface
  - C^2 almost everywhere
  - C^1 at vertices with valence $\neq 6$
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull



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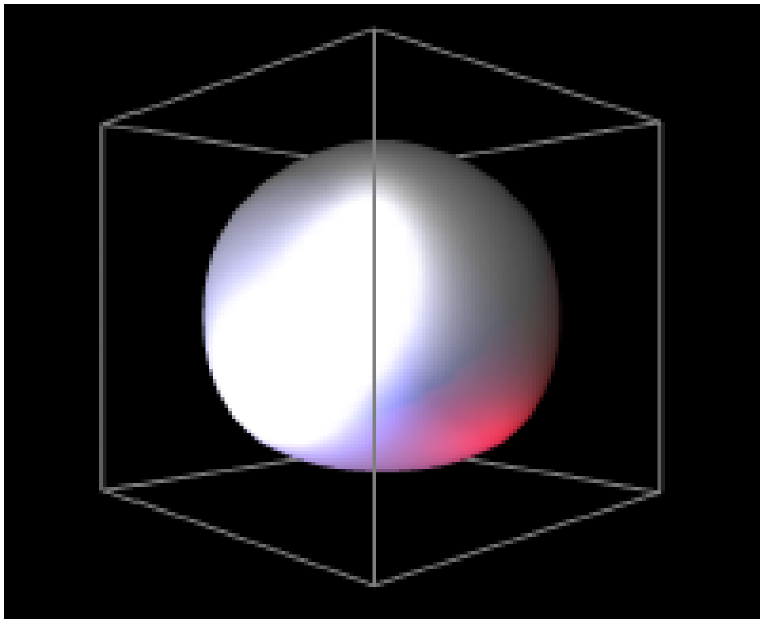
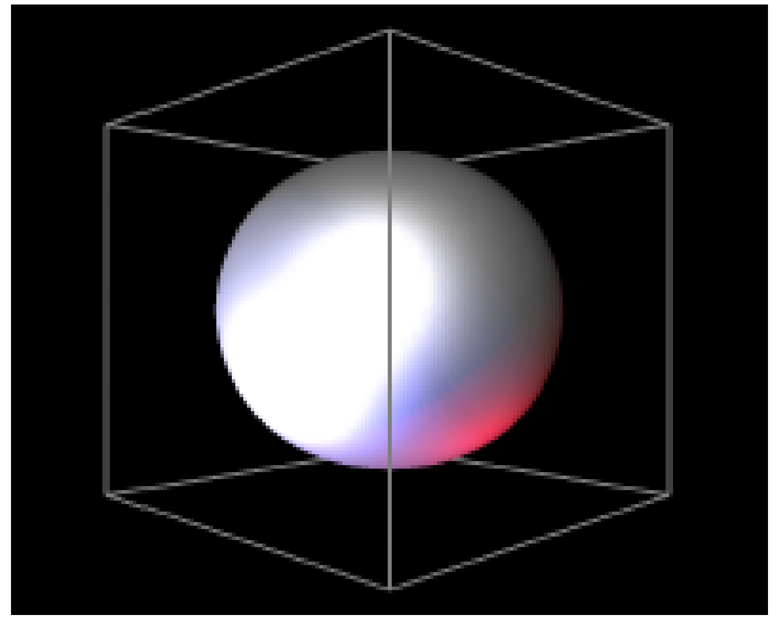
Subdivision Schemes

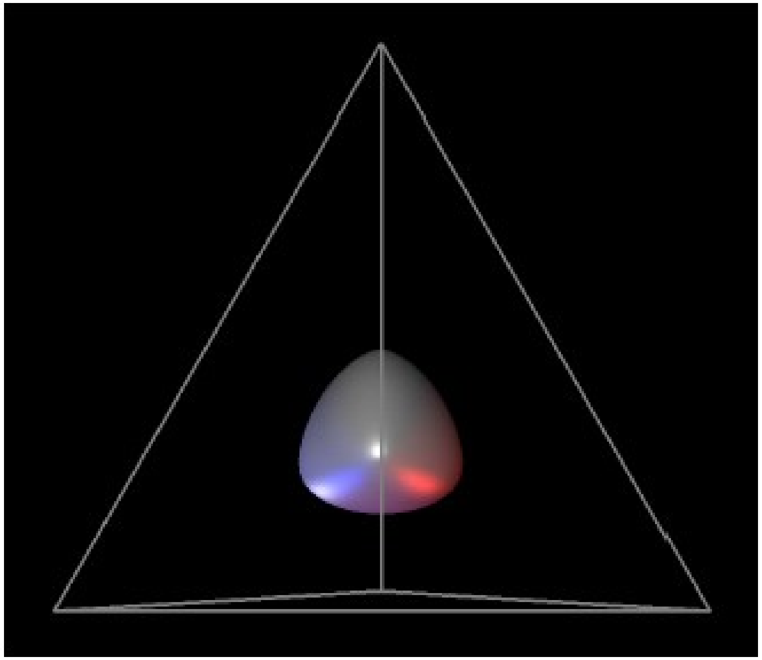


Loop

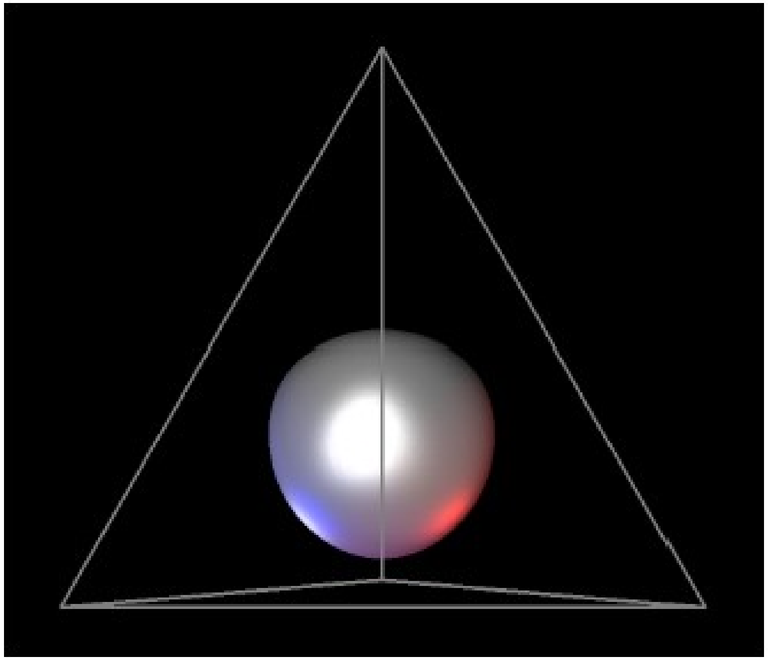


Catmull-Clark

Subdivision Schemes

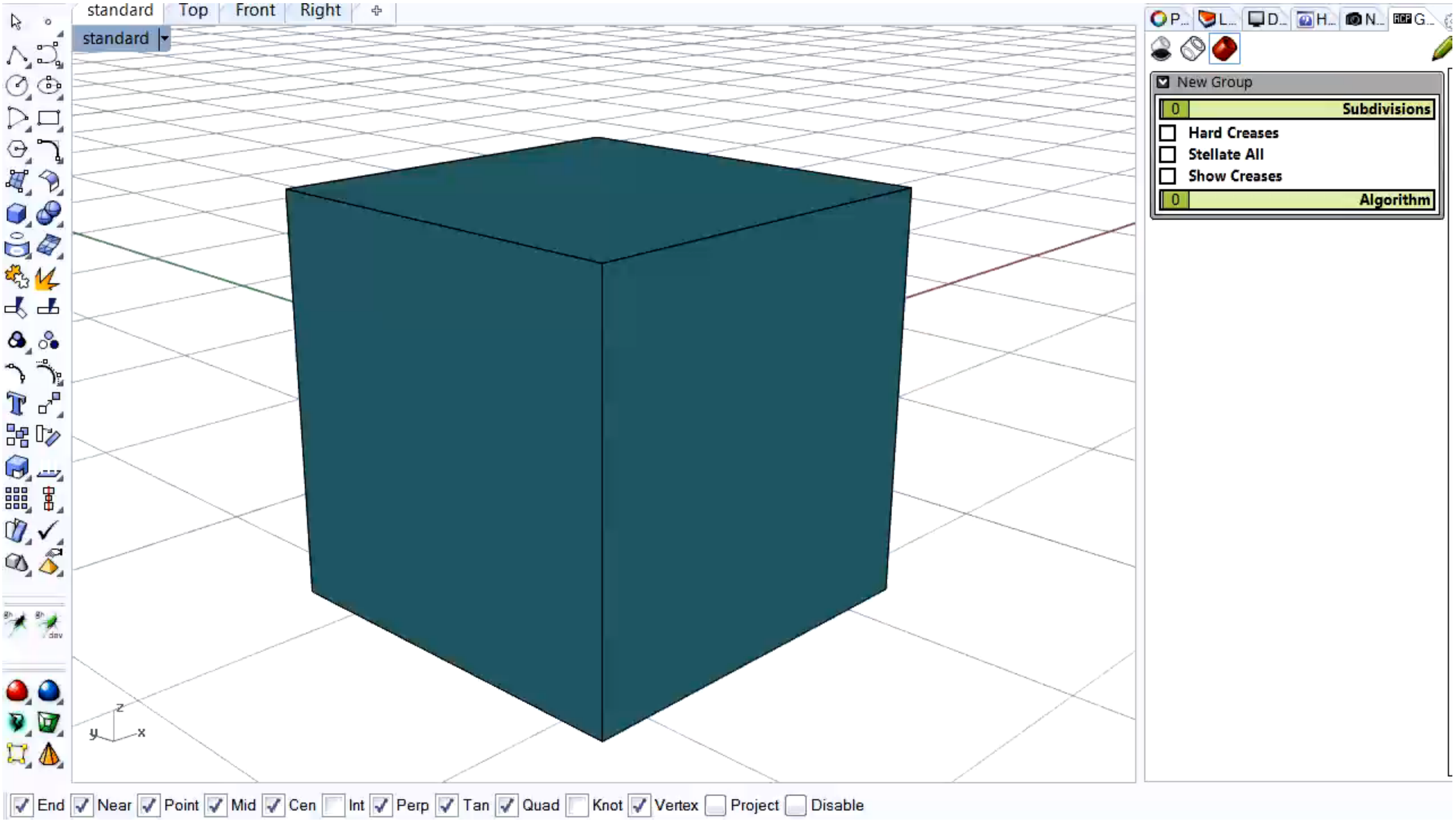


Loop



Catmull-Clark

Subdivision Schemes



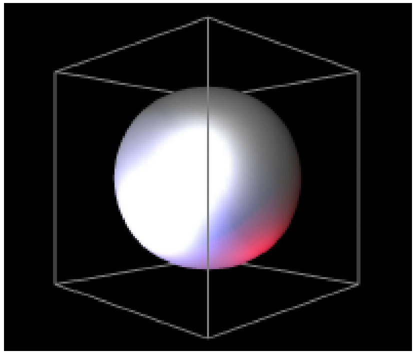
https://vimeo.com/118340176

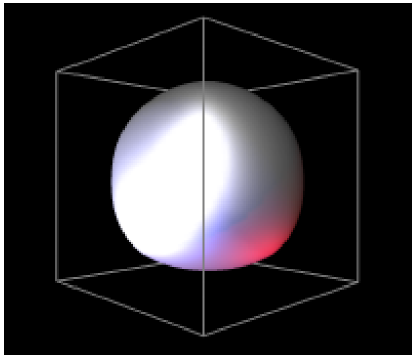
Subdivision Schemes

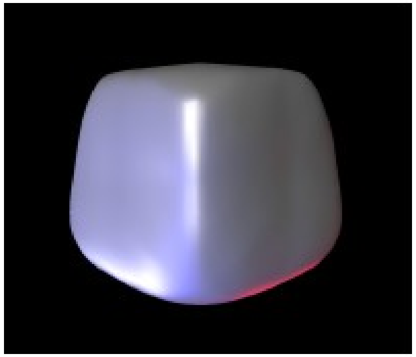
- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others
- Differ in …
  - Input topology
  - How refine topology
  - How refine geometry

… which makes differences in …

  - Provable properties

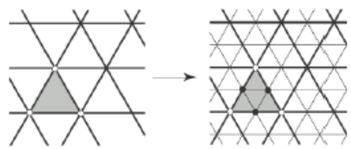
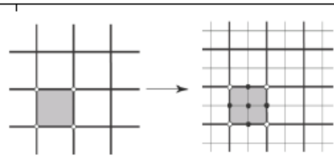


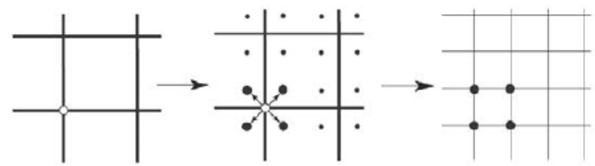




Subdivision Schemes

- Other subdivision schemes

Primal (face split)		
	 *Triangular meshes*	 *Quad Meshes*
Approximating	Loop(C^2)	Catmull-Clark(C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)



Dual (vertex split)
Doo-Sabin, Midedge(C^1)
Biquartic (C^2)

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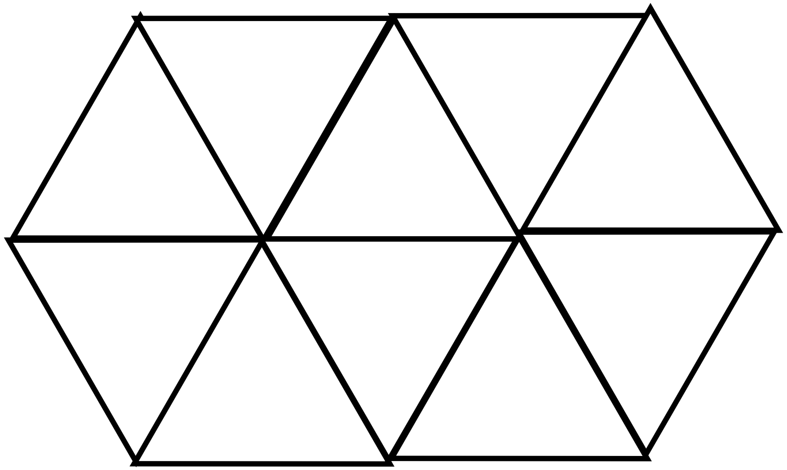
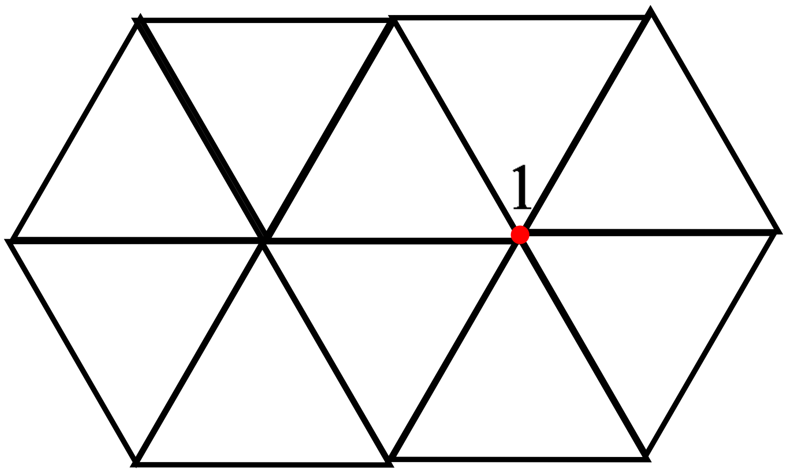
Other Subdivision Schemes

- Butterfly subdivision



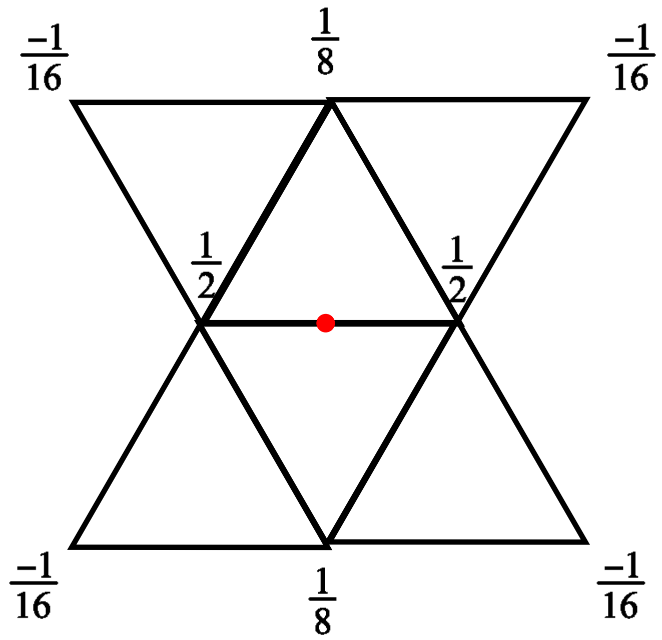
Other Subdivision Schemes

- Butterfly subdivision



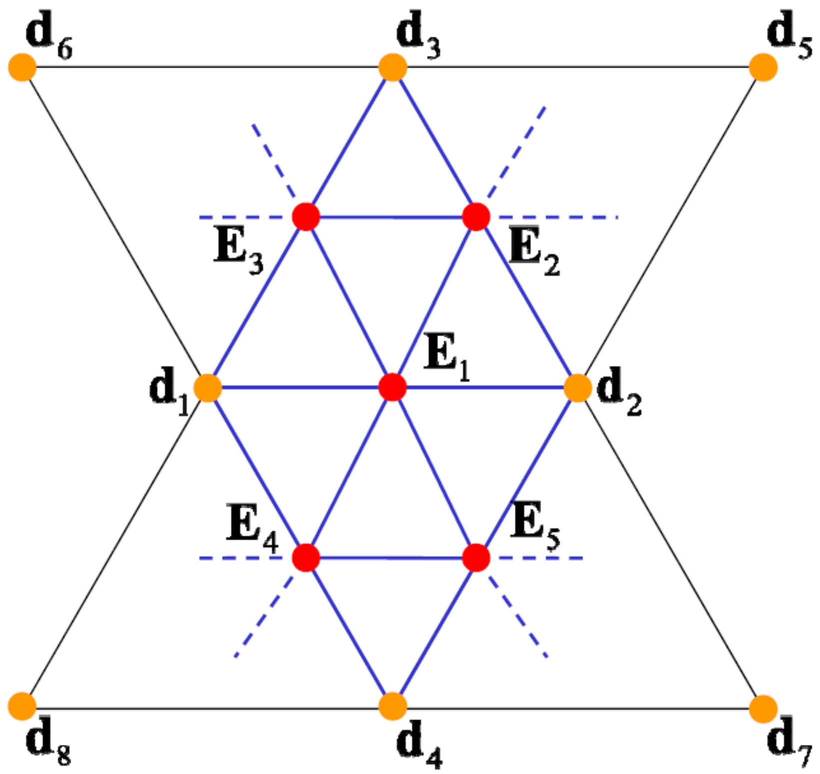
Other Subdivision Schemes

- Butterfly subdivision



Other Subdivision Schemes

- Butterfly subdivision



$$\mathbf{E}_1 = \frac{1}{2}\left(\mathbf{d}_1 + \mathbf{d}_2\right) + \omega\left(\mathbf{d}_3 + \mathbf{d}_4\right) - \frac{\omega}{2}\left(\mathbf{d}_5 + \mathbf{d}_6 + \mathbf{d}_7 + \mathbf{d}_8\right)$$

$$\mathbf{d}'_i = \mathbf{d}_i$$

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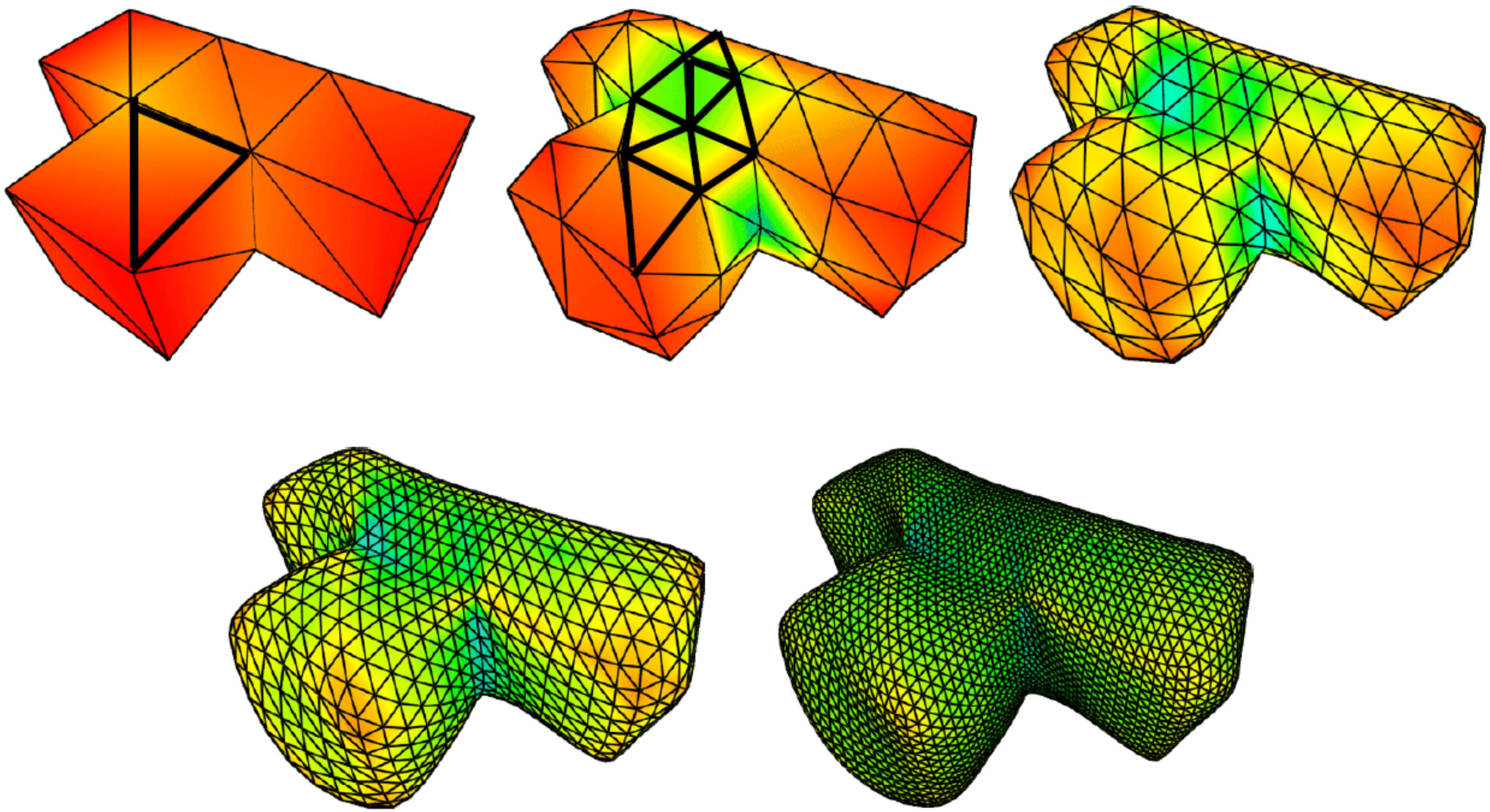
Other Subdivision Schemes

- Butterfly subdivision



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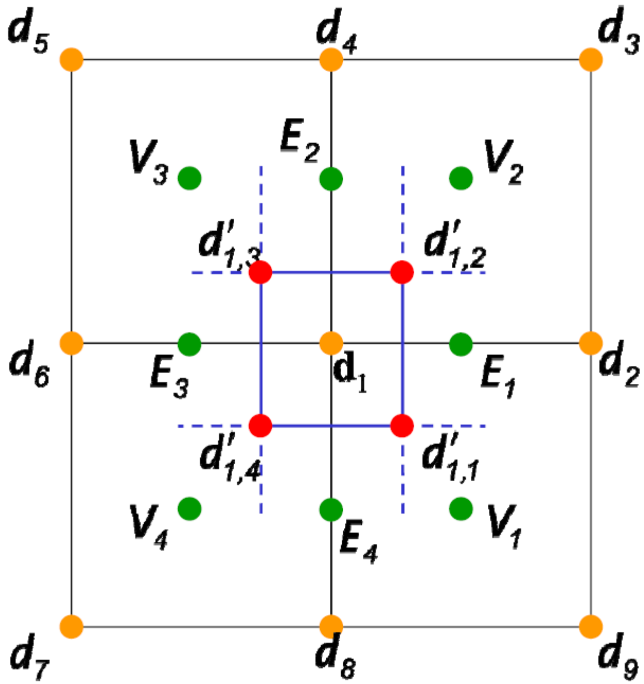
Other Subdivision Schemes

- Vertex-split subdivision
  (Doo-Sabin, Midedge, Biquartic)



$$V_2 = \frac{1}{n} \times \sum_{j=1}^{n} d_j$$

$$E_i = \frac{1}{2}(d_1 + d_{2i})$$

$$d'_{1,j} = \frac{1}{4}\left(d_1 + E_j + E_{j-1} + V_j\right)$$

One step of Doo-Sabin subdivision

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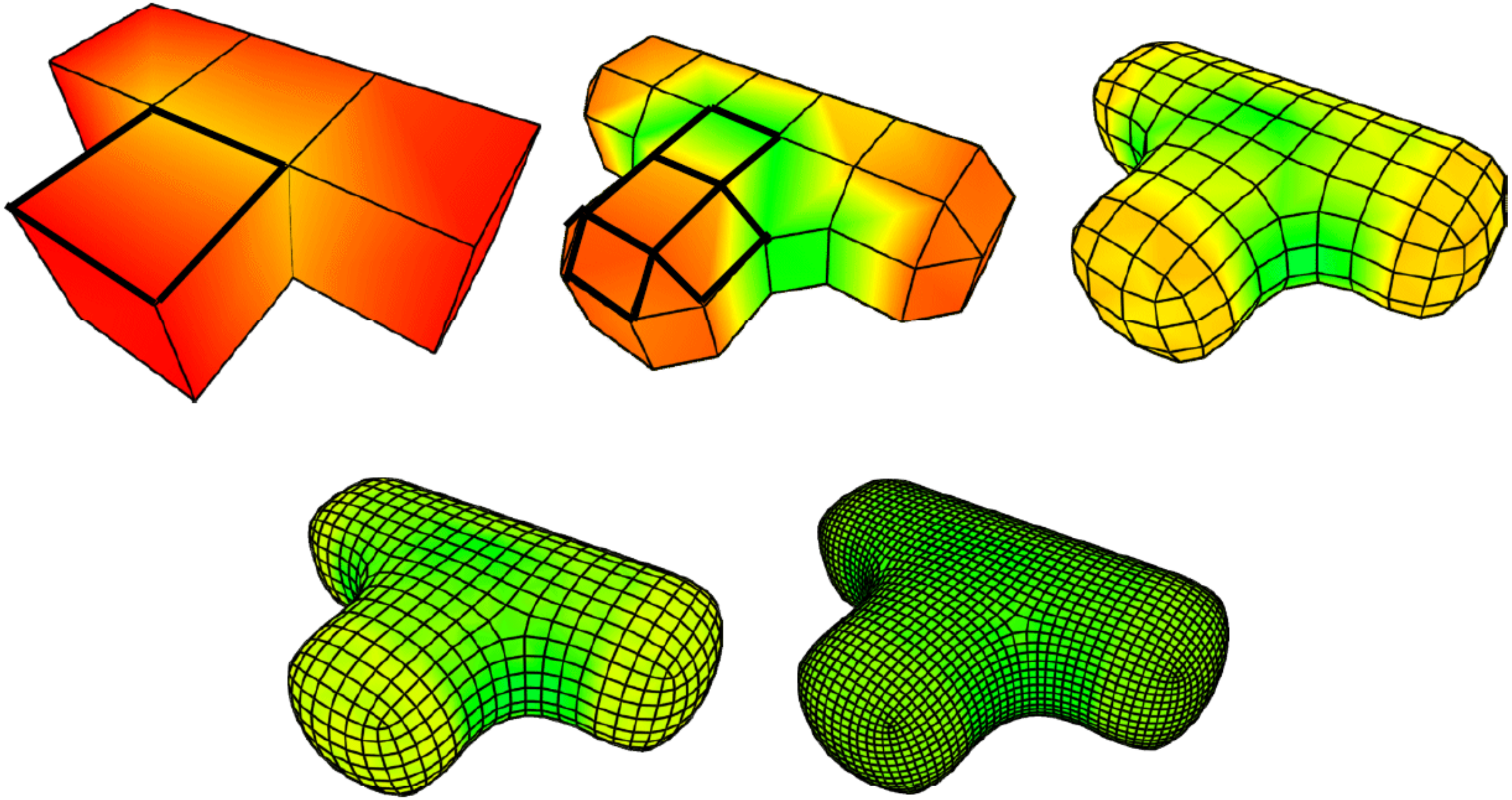
Other Subdivision Schemes

- Doo-Sabin Subidvision



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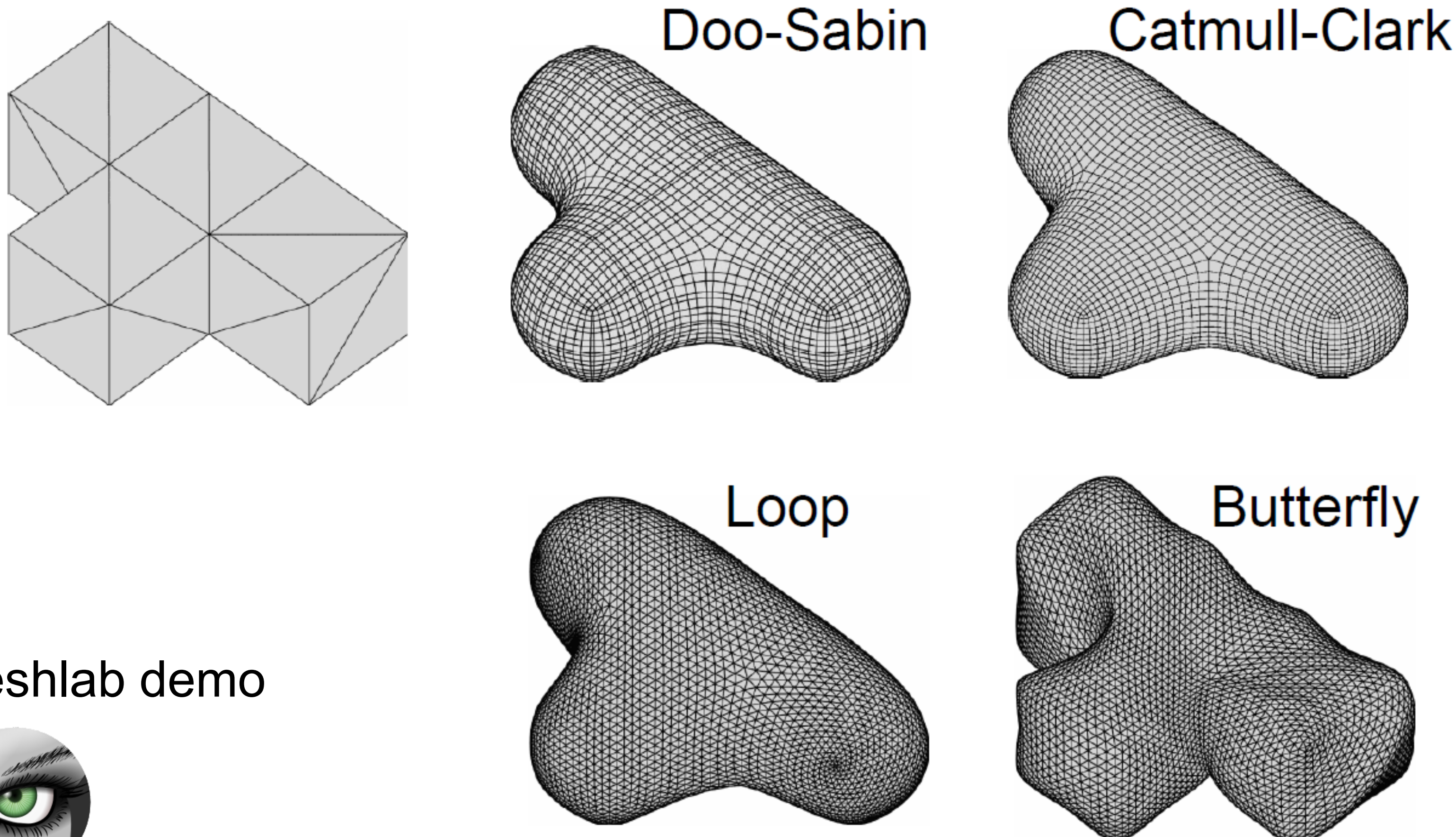
Other Subdivision Schemes

Comparison



Meshlab demo

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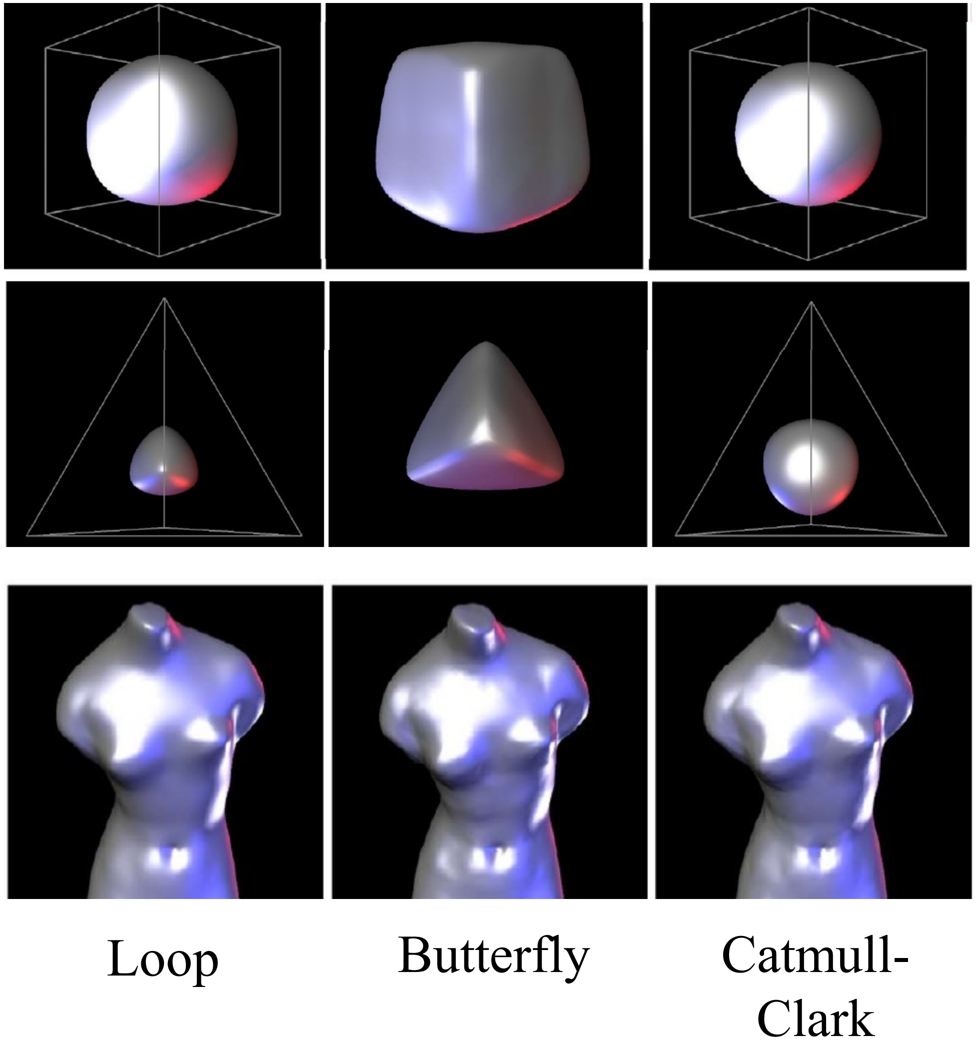
Other Subdivision Schemes

- Comparisons:
  - Interpolation vs. smoothness
  - Triangulation dependent asymmetry
  - Shrinking for approximating schemes
  - Similar for smooth meshes with uniform triangles



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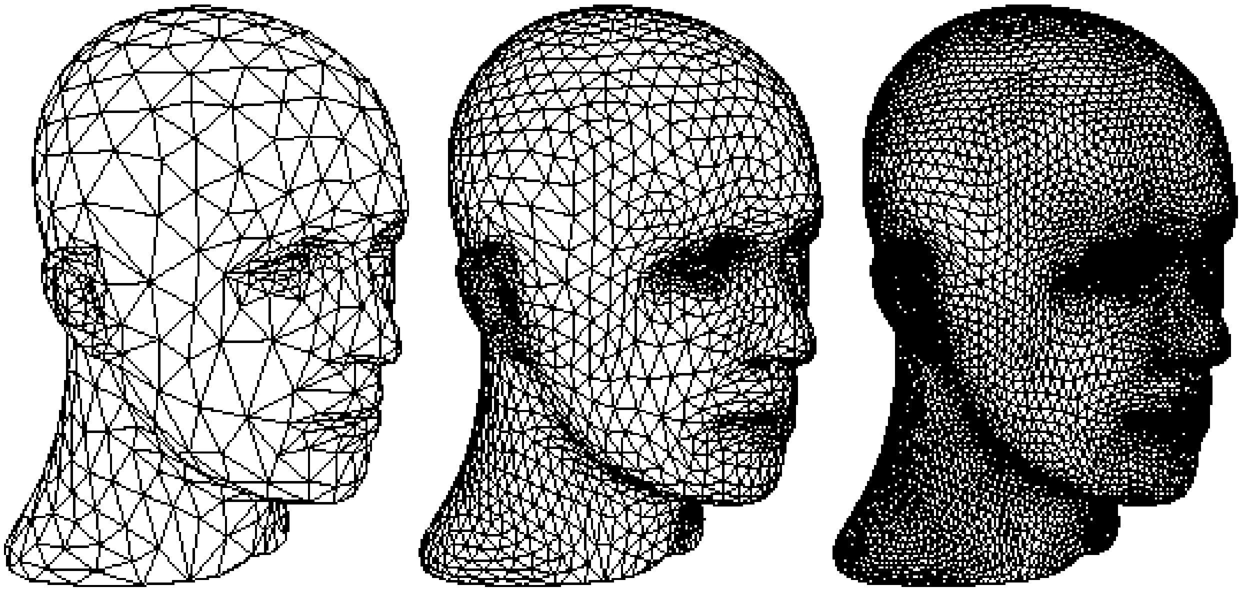
Drawing Subdivision Surfaces

- Goal:
  - Draw best approximation of smooth limit surface
  - With limited triangle budget



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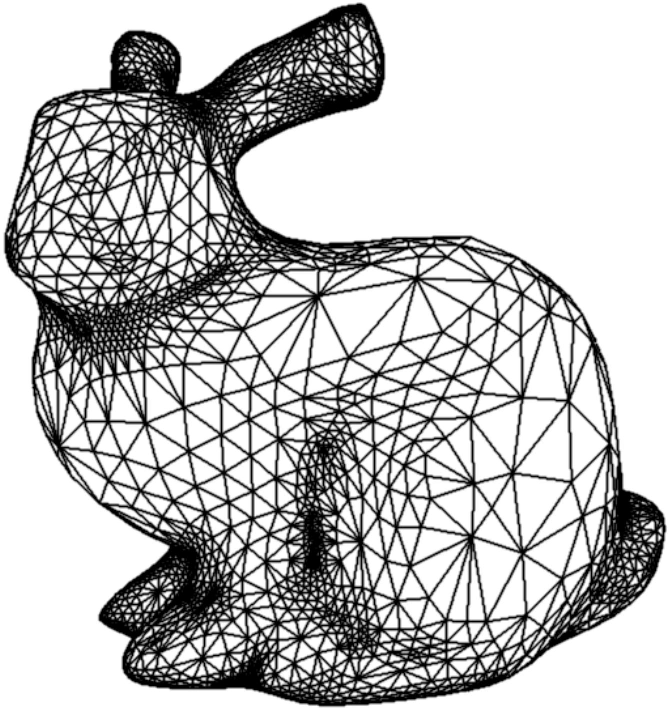
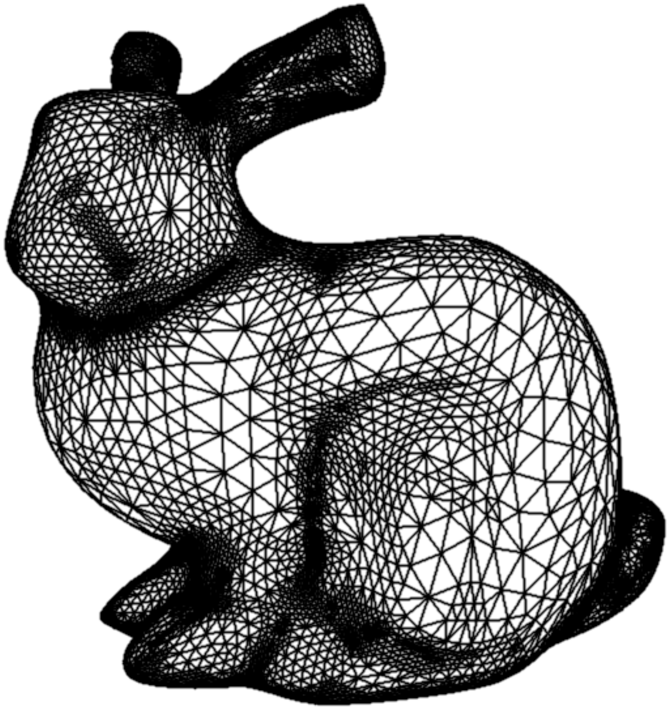
Drawing Subdivision Surfaces

- Goal:
  - Draw best approximation of smooth limit surface
  - With limited triangle budget
- Solution:
  - Stop subdivision at different levels across the surface
  - Stop-criterion depending on quality measure
- Quality of approximation can be defined by
  - Projected (screen) area of final triangles
  - Local surface curvature

Adaptive Subdivision



10072 Triangles

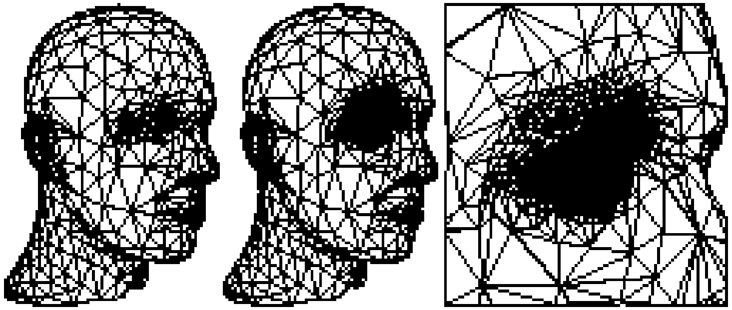


228654 Triangles

[Kobbelt 2000]

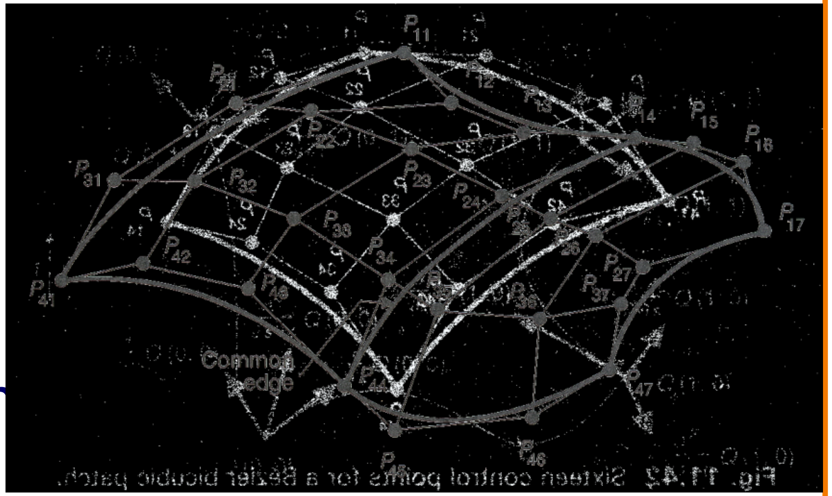
Subdivision Surface Summary

- Advantages:
  - Simple method for describing complex surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Intuitive specification
  - Local support
  - Guaranteed continuity
  - Multi-resolution



- Difficulties:
  - Parameterization
  - Intersections

Comparison



Parametric surfaces

- Provide parameterization
- More restriction on topology of control mesh
- Some require careful placement of control mesh vertices to guarantee continuity (e.g., Bezier)

Subdivision surfaces

- No parameterization
- Subdivision rules can be defined for arbitrary topologies
- Provable continuity for all placements of control mesh vertices

Comparison

Feature	Polygonal Mesh	Parametric Surface	Subdivision Surface
Accurate	No	Yes	Yes
Concise	No	Yes	Yes
Intuitive specification	No	Yes	Yes
Local support	Yes	Yes	Yes
Affine invariant	Yes	Yes	Yes
Arbitrary topology	Yes	No	Yes
Guaranteed continuity	No	Yes	Yes
Natural parameterization	No	Yes	No
Efficient display	Yes	Yes	Yes
Efficient intersections	No	No	No