Efficient Sorting Algorithms

- mergesort
- sorting complexity
- quicksort
- animations

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Two classic sorting algorithms

Critical components in the world's computational infrastructure.
- Full scientific understanding of their properties has enabled us to develop them into practical system sorts.
- Quicksort honored as one of top 10 algorithms of 20th century in science and engineering.

Mergesort.
- Java sort for objects.
- Perl, Python stable sort.

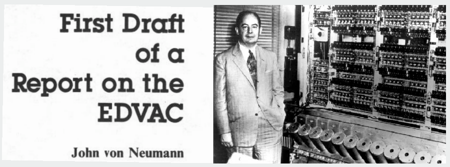
Quicksort.
- Java sort for primitive types.
- C qsort, Unix, g++, Visual C++, Python.

- mergesort
- sorting complexity
- quicksort
- animations

Mergesort

Basic plan.
- Divide array into two halves.
- Recursively sort each half.
- Merge two halves.

input	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
sort left half	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
sort right half	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge results	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X



Mergesort trace

	a[]															
lo hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 0, 0, 1)	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 2, 2, 3)	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 0, 1, 3)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 4, 4, 5)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 6, 6, 7)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 4, 5, 7)	E	G	M	R	E	O	R	S	T	E	X	A	M	P	L	E
merge(a, 0, 3, 7)	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
merge(a, 8, 8, 9)	E	E	G	M	O	R	R	S	E	T	X	A	M	P	L	E
merge(a, 10, 10, 11)	E	E	G	M	O	R	R	S	E	T	A	X	M	P	L	E
merge(a, 8, 9, 11)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, 12, 12, 13)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, 14, 14, 15)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	E	L
merge(a, 12, 13, 15	E	E	G	M	O	R	R	S	A	E	T	X	E	L	M	P
merge(a, 8, 11, 15)	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge(a, 0, 7, 15)	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

result after recursive all

Merging

Goal. Combine two sorted subarrays into a sorted whole.

Q. How to merge efficiently?

A. Use an auxiliary array.

		a[]												aux[]									
	k	0	1	2	3	4	5	6	7	8	9	i	j	0	1	2	3	4	5	6	7	8	9
input		E	E	G	M	R	A	C	E	R	T			-	-	-	-	-	-	-	-	-	-
copy left		E	E	G	M	R	A	C	E	R	T			E	E	G	M	R	-	-	-	-	-
reverse copy right		E	E	G	M	R	A	C	E	R	T			E	E	G	M	R	T	R	E	C	A
												0	9										
	0	A										0	8	E	E	G	M	R	T	R	E	C	A
	1	A	C									0	7	E	E	G	M	R	T	R	E	C	
	2	A	C	E								1	7	E	E	G	M	R	T	R	E		
	3	A	C	E	E							2	7		E	G	M	R	T	R	E		
	4	A	C	E	E	E						2	6			G	M	R	T	R	E		
merged result	5	A	C	E	E	E	G					3	6			G	M	R	T	R			
	6	A	C	E	E	E	G	M				4	6				M	R	T	R			
	7	A	C	E	E	E	G	M	R			5	6					R	T	R			
	8	A	C	E	E	E	G	M	R	R		5	5						T	R			
	9	A	C	E	E	E	G	M	R	R	T	6	5						T				
final result		A	C	E	E	E	G	M	R	R	T												

Merging: Java implementation

```
public static void merge(Comparable[] a, int lo, int m, int hi)
{
   for (int i = lo; i <= m; i++)
      aux[i] = a[i];                                          copy

   for (int j = m+1; j <= hi; j++)
      aux[j] = a[hi-j+m+1];                                   reverse copy

   int i = lo, j = hi;
   for (int k = lo; k <= hi; k++)
      if (less(aux[j], aux[i])) a[k] = aux[j--];              merge
      else                      a[k] = aux[i++];
}
```

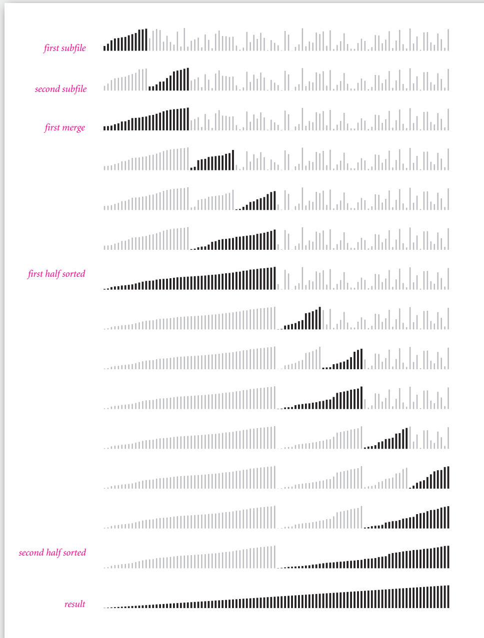
	lo			i	m		j			hi
aux[]	A	G	L	O	R	T	S	M	I	H
							k			
a[]	A	G	H	I	L	M				

Mergesort: Java implementation

```
public class Merge
{
   private static Comparable[] aux;

   private static void merge(Comparable[] a, int lo, int m, int hi)
   {  /* as before */  }

   private static void sort(Comparable[] a, int lo, int hi)
   {
      if (hi <= lo) return;
      int m = lo + (hi - lo) / 2;
      sort(a, lo, m);
      sort(a, m+1, hi);
      merge(a, lo, m, hi);
   }

   public static void sort(Comparable[] a)
   {
      aux = new Comparable[a.length];
      sort(a, 0, a.length - 1);
   }
}
```

lo					m				hi
10	11	12	13	14	15	16	17	18	19

Mergesort visualization



Mergesort: empirical analysis

Running time estimates:

- Home pc executes 10^8 comparisons/second.
- Supercomputer executes 10^{12} comparisons/second.

	insertion sort (N^2)			mergesort (N log N)		
computer	thousand	million	billion	thousand	million	billion
home	instant	2.8 hours	317 years	instant	1 second	18 min
super	instant	1 second	1 week	instant	instant	instant

Lesson. Good algorithms are better than supercomputers.

Mergesort: mathematical analysis

Proposition. Mergesort uses $\sim N \lg N$ compares to sort any array of size N.

Def. $T(N)$ = number of compares to mergesort an array of size N.

$$= \underbrace{T(N/2)}_{\text{left half}} + \underbrace{T(N/2)}_{\text{right half}} + \underbrace{N}_{\text{merge}}$$

Mergesort recurrence. $T(N) = 2\,T(N/2) + N$ for $N > 1$, with $T(1) = 0$.

- Not quite right for odd N.
- Same recurrence holds for many divide-and-conquer algorithms.

Solution. $T(N) \sim N \lg N$.

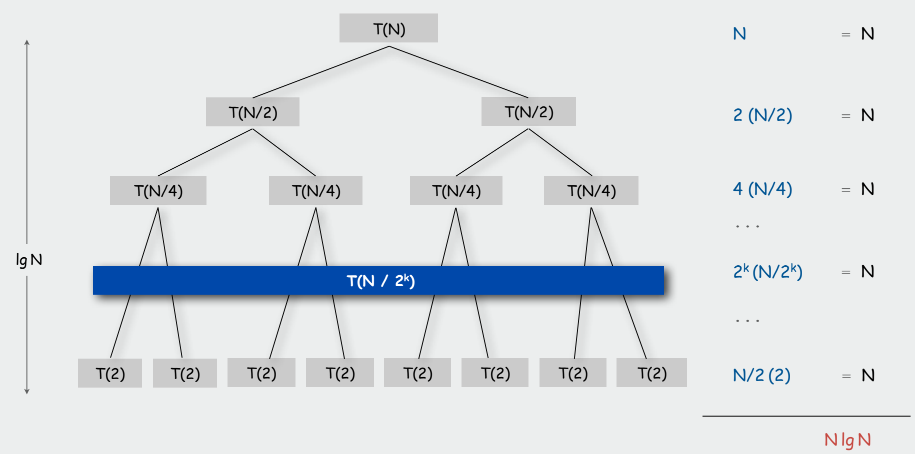
- For simplicity, we'll prove when N is a power of 2.
- True for all N. [see COS 340]

Mergesort recurrence: proof 1

Mergesort recurrence. $T(N) = 2\,T(N/2) + N$ for $N > 1$, with $T(1) = 0$.

Proposition. If N is a power of 2, then $T(N) = N \lg N$.

Pf.



Mergesort recurrence: proof 2

Mergesort recurrence. $T(N) = 2\,T(N/2) + N$ for $N > 1$, with $T(1) = 0$.

Proposition. If N is a power of 2, then $T(N) = N \lg N$.

Pf.

T(N) = 2 T(N/2) + N		given
T(N) / N = 2 T(N/2) / N + 1		divide both sides by N
= T(N/2) / (N/2) + 1		algebra
= T(N/4) / (N/4) + 1 + 1		apply to first term
= T(N/8) / (N/8) + 1 + 1 + 1		apply to first term again
. . .		
= T(N/N) / (N/N) + 1 + 1 + ... + 1		stop applying, T(1) = 0
= lg N		

Mergesort recurrence: proof 3

Mergesort recurrence. $T(N) = 2\,T(N/2) + N$ for $N > 1$, with $T(1) = 0$.

Proposition. If N is a power of 2, then $T(N) = N \lg N$.

Pf. [by induction on N]

- Base case: $N = 1$.
- Inductive hypothesis: $T(N) = N \lg N$.
- Goal: show that $T(2N) = 2N \lg (2N)$.

T(2N) = 2 T(N) + 2N	given
= 2 N lg N + 2 N	inductive hypothesis
= 2 N (lg (2N) - 1) + 2N	algebra
= 2 N lg (2N)	QED

Mergesort analysis: memory

Proposition G. Mergesort uses extra space proportional to N.

Pf. The array `aux[]` needs to be of size N for the last merge.

two sorted subarrays	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merged array	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Def. A sorting algorithm is in-place if it uses O(log N) extra memory.

Ex. Insertion sort, selection sort, shellsort.

Challenge for the bored. In-place merge. [Kronrud, 1969]

Mergesort: practical improvements

Use insertion sort for small subarrays.

- Mergesort has too much overhead for tiny subarrays.
- Cutoff to insertion sort for ≈ 7 elements.

Stop if already sorted.

- Is biggest element in first half ≤ smallest element in second half?
- Helps for nearly ordered lists.

biggest element in left half ≤ smallest element in right half

two sorted subarrays	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X
merged array	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Eliminate the copy to the auxiliary array. Save time (but not space) by switching the role of the input and auxiliary array in each recursive call.

Ex. See Program 8.4 or `Arrays.sort()`.

Bottom-up mergesort

Basic plan.

- Pass through array, merging subarrays of size 1.
- Repeat for subarrays of size 2, 4, 8, 16, ...

lo m hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 0, 0, 1)	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 2, 2, 3)	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 4, 4, 5)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 6, 6, 7)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 8, 8, 9)	E	E	G	M	O	R	R	S	E	T	X	A	M	P	L	E
merge(a, 10, 10, 11)	E	E	G	M	O	R	R	S	E	T	A	X	M	P	L	E
merge(a, 12, 12, 13)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, 14, 14, 15)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	E	L
merge(a, 0, 1, 3)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, 4, 5, 7)	E	G	M	R	E	O	R	S	T	E	X	A	M	P	L	E
merge(a, 8, 9, 11)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, 12, 13, 15)	E	E	G	M	O	R	R	S	A	E	T	X	E	L	M	P
merge(a, 0, 3, 7)	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
merge(a, 8, 11, 15)	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge(a, 0, 7, 15)	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

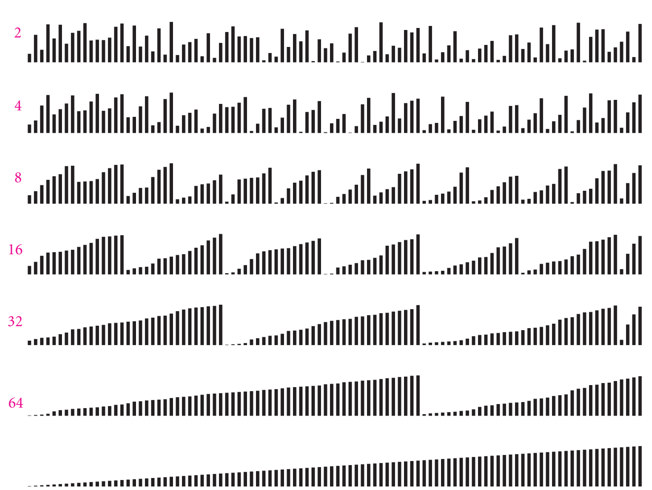
Bottom line. No recursion needed!

Bottom-up mergesort: Java implementation

```
public class MergeBU
{
   private static Comparable[] aux;

   private static void merge(Comparable[] a, int lo, int m, int hi)
   {  /* as before */  }

   public static void sort(Comparable[] a)
   {
      int N = a.length;
      aux = new Comparable[N];
      for (int m = 1; m < N; m = m+m)
         for (int i = 0; i < N-m; i += m+m)
            merge(a, i, i+m, Math.min(i+m+m, N));
   }
}
```

Bottom line. Concise industrial-strength code, if you have the space.

Bottom-up mergesort: visualization



Complexity of sorting

Computational complexity. Framework to study efficiency of algorithms for solving a particular problem X.

Machine model. Focus on fundamental operations.
Upper bound. Cost guarantee provided by some algorithm for X.
Lower bound. Proven limit on cost guarantee of any algorithm for X.
Optimal algorithm. Algorithm with best cost guarantee for X.

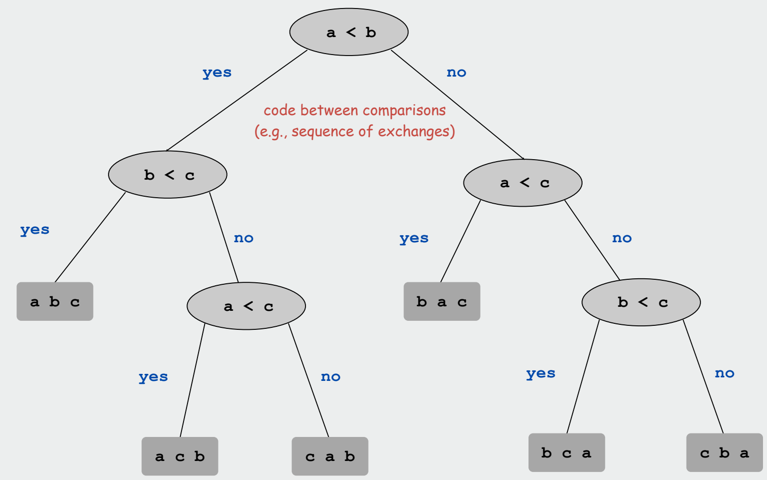
lower bound ~ upper bound

Example: sorting.

access information only through compares

- Machine model = # compares.
- Upper bound = N lg N from mergesort.
- Lower bound = N lg N ?
- Optimal algorithm = mergesort ?

Decision tree

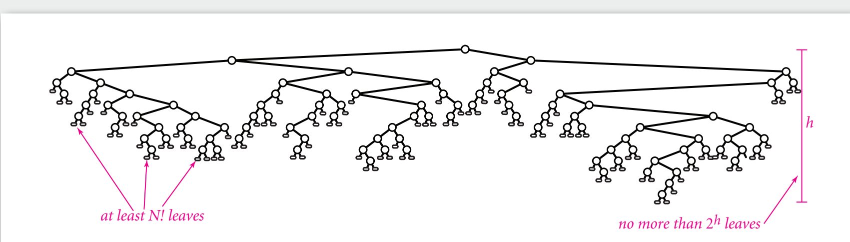


Compare-based lower bound for sorting

Proposition. Any compare-based sorting algorithm must use more than $N \lg N - 1.44\, N$ comparisons in the worst-case.

Pf.

- Assume input consists of N distinct values a_1 through a_N.
- Worst case dictated by height h of decision tree.
- Binary tree of height h has at most 2^h leaves.
- $N\,!$ different orderings $\Rightarrow$ at least $N\,!$ leaves.



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- Assume input consists of N distinct values a_1 through a_N.
- Worst case dictated by height h of decision tree.
- Binary tree of height h has at most 2^h leaves.
- $N\,!$ different orderings $\Rightarrow$ at least $N\,!$ leaves.

$$
\begin{aligned}
2^h &\ge N\,! \\
h &\ge \lg N\,! \\
&\ge \lg (N/e)^N \\
&= N \lg N - N \lg e \\
&\ge N \lg N - 1.44\, N
\end{aligned}
$$

Stirling's formula

Complexity of sorting

Machine model. Focus on fundamental operations.
Upper bound. Cost guarantee provided by some algorithm for X.
Lower bound. Proven limit on cost guarantee of any algorithm for X.
Optimal algorithm. Algorithm with best cost guarantee for X.

Example: sorting.
- Machine model = # compares.
- Upper bound = N lg N from mergesort.
- Lower bound = N lg N - 1.44 N.
- Optimal algorithm = mergesort.

First goal of algorithm design: optimal algorithms.

Complexity results in context

Other operations? Mergesort optimality is only about number of compares.

Space?
- Mergesort is not optimal with respect to space usage.
- Insertion sort, selection sort, and shellsort are space-optimal.
- Is there an algorithm that is both time- and space-optimal?

Lessons. Use theory as a guide.
Ex. Don't try to design sorting algorithm that uses $\frac{1}{2} N \lg N$ compares.

Complexity results in context (continued)

Lower bound may not hold if the algorithm has information about
- The key values.
- Their initial arrangement.

Partially ordered arrays. Depending on the initial order of the input, we may not need N lg N compares.

insertion sort requires O(N) compares on an already sorted array

Duplicate keys. Depending on the input distribution of duplicates, we may not need N lg N compares.

stay tuned for 3-way quicksort

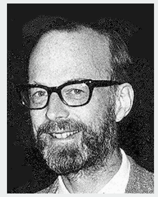
Digital properties of keys. We can use digit/character comparisons instead of key comparisons for numbers and strings.

stay tuned for radix sorts

Quicksort

Basic plan.

- Shuffle the array.
- Partition so that, for some `i`
  - element `a[i]` is in place
  - no larger element to the left of `i`
  - no smaller element to the right of `i`
- Sort each piece recursively.



Sir Charles Antony Richard Hoare
1980 Turing Award

input	Q	U	I	C	K	S	O	R	T	E	X	A	M	P	L	E
shuffle	E	R	A	T	E	S	L	P	U	I	M	Q	C	X	O	K
partition	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S
sort left	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
sort right	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
result	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X

partitioning element · *not greater* · *not less*

Quicksort partitioning

Basic plan.

- Scan from left for an item that belongs on the right.
- Scan from right for item item that belongs on the left.
- Exchange.
- Continue until pointers cross.

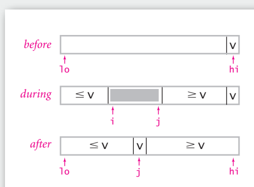
	i	j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial values	-1	15	E	R	A	T	E	S	L	P	U	I	M	Q	C	X	O	K
scan left, scan right	1	12	E	R	A	T	E	S	L	P	U	I	M	Q	C	X	O	K
exchange	1	12	E	C	A	T	E	S	L	P	U	I	M	Q	R	X	O	K
scan left, scan right	3	9	E	C	A	T	E	S	L	P	U	I	M	Q	R	X	O	K
exchange	3	9	E	C	A	I	E	S	L	P	U	T	M	Q	R	X	O	K
scan left, scan right	5	5	E	C	A	I	E	S	L	P	U	T	M	Q	R	X	O	K
final exchange	5	5	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S
result			E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S

a[i] · v

Partitioning trace (array contents before and after each exchange)

Quicksort: Java code for partitioning

```
private static int partition(Comparable[] a, int lo, int hi)
{
   int i = lo - 1;
   int j = hi;
   while(true)
   {

      while (less(a[++i], a[hi]))            find item on left to swap
         if (i == hi) break;

      while (less(a[hi], a[--j]))            find item on right to swap
         if (j == lo) break;


      if (i >= j) break;                     check if pointers cross

      exch(a, i, j);                         swap
   }

   exch(a, i, hi);                           swap with partitioning item
   return i;                 return index of item now known to be in place
}
```



Quicksort: Java implementation

```
public class Quick
{
   public static void sort(Comparable[] a)
   {
      StdRandom.shuffle(a);
      sort(a, 0, a.length - 1);
   }

   private static void sort(Comparable[] a, int lo, int hi)
   {
      if (hi <= lo) return;
      int i = partition(a, lo, hi);
      sort(a, lo, i-1);
      sort(a, i+1, hi);
   }
}
```

Quicksort trace

	lo	i	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial values				Q	U	I	C	K	S	O	R	T	E	X	A	M	P	L	E
random shuffle				E	R	A	T	E	S	L	P	U	I	M	Q	C	X	O	K
	0	5	15	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S
	0	2	4	A	C	E	I	E	K	L	P	U	T	M	Q	R	X	O	S
	0	1	1	A	C	E	I	E	K	L	P	U	T	M	Q	R	X	O	S
	0		0	A	C	E	I	E	K	L	P	U	T	M	Q	R	X	O	S
	3	3	4	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	4		4	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	6	12	15	A	C	E	E	I	K	L	P	O	R	M	Q	S	X	U	T
no partition for subarrays of size 1	6	10	11	A	C	E	E	I	K	L	P	O	M	Q	R	S	X	U	T
	6	7	9	A	C	E	E	I	K	L	M	O	P	Q	R	S	X	U	T
	6		6	A	C	E	E	I	K	L	M	O	P	Q	R	S	X	U	T
	8	9	9	A	C	E	E	I	K	L	M	O	P	Q	R	S	X	U	T
	8		8	A	C	E	E	I	K	L	M	O	P	Q	R	S	X	U	T
	11		11	A	C	E	E	I	K	L	M	O	P	Q	R	S	X	U	T
	13	13	15	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	14	15	15	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	14		14	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
result				A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X

Quicksort: implementation details

Partitioning in-place. Using a spare array makes partitioning easier, but is not worth the cost.

Terminating the loop. Testing whether the pointers cross is a bit trickier than it might seem.

Staying in bounds. The `(i == hi)` test is redundant, but the `(j == lo)` test is not.

Preserving randomness. Shuffling is key for performance guarantee.

Equal keys. When duplicates are present, it is (counter-intuitively) best to stop on elements equal to the partitioning element.

Quicksort: empirical analysis

Running time estimates:

- Home pc executes 10^8 comparisons/second.
- Supercomputer executes 10^{12} comparisons/second.

	insertion sort (N^2)			mergesort (N log N)			quicksort (N log N)		
computer	thousand	million	billion	thousand	million	billion	thousand	million	billion
home	instant	2.8 hours	317 years	instant	1 second	18 min	instant	0.3 sec	6 min
super	instant	1 second	1 week	instant	instant	instant	instant	instant	instant

Lesson 1. Good algorithms are better than supercomputers.
Lesson 2. Great algorithms are better than good ones.

Quicksort: average-case analysis

Proposition I. The average number of compares C_N to quicksort an array of N elements is ~ 2N ln N (and the number of exchanges is ~ ⅓ N ln N).

Pf. C_N satisfies the recurrence $C_0 = C_1 = 0$ and for $N \geq 2$:

$$C_N = \underbrace{(N+1)}_{\text{partitioning}} + \underbrace{(C_0 + C_1 + \ldots + C_{N-1})}_{\text{left}} / N + \underbrace{(C_{N-1} + C_{N-2} + \ldots + C_0)}_{\text{right}} \underbrace{/ N}_{\text{partitioning probability}}$$

- Multiply both sides by N and collect terms:

$$NC_N = N(N+1) + 2(C_0 + C_1 + \ldots + C_{N-1})$$

- Subtract this from the same equation for N-1:

$$NC_N - (N-1)C_N = 2N + 2C_{N-1}$$

- Rearrange terms and divide by N(N+1):

$$C_N / (N+1) = (C_{N-1} / N) + 2 / (N+1)$$

Quicksort: average-case analysis

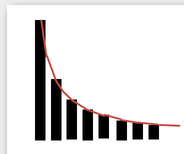
- From before:

$$C_N \,/\, (N+1) = C_{N-1} \,/\, N + 2 \,/\, (N+1)$$

- Repeatedly apply above equation:

$$\begin{aligned} C_N / (N+1) &= C_{N-1} / N + 2 / (N+1) \\ &= C_{N-2} / (N-1) + 2/N + 2/(N+1) \\ &= C_{N-3} / (N-2) + 2/(N-1) + 2/N + 2/(N+1) \\ &= 2\,(1 + 1/2 + 1/3 + \ldots + 1/N + 1/(N+1)) \end{aligned}$$

- Approximate by an integral:



$$\begin{aligned} C_N &\approx 2(N+1)\,(1 + 1/2 + 1/3 + \ldots + 1/N) \\ &= 2(N+1)\, H_N \approx 2(N+1) \int_1^N dx/x \end{aligned}$$

- Finally, the desired result:

$$C_N \approx 2(N+1) \ln N \approx 1.39\, N \lg N$$

Quicksort: summary of performance characteristics

Worst case. Number of compares is quadratic.
- $N + (N-1) + (N-2) + \ldots + 1 \sim N^2 / 2$.
- More likely that your computer is struck by lightning.

Average case. Number of compares is $\sim 1.39\, N \lg N$.
- 39% more compares than mergesort.
- But faster than mergesort in practice because of less data movement.

Random shuffle.
- Probabilistic guarantee against worst case.
- Basis for math model that can be validated with experiments.

Caveat emptor. Many textbook implementations go quadratic if input:
- Is sorted or reverse sorted
- Has many duplicates (even if randomized!) [stay tuned]

Quicksort: practical improvements

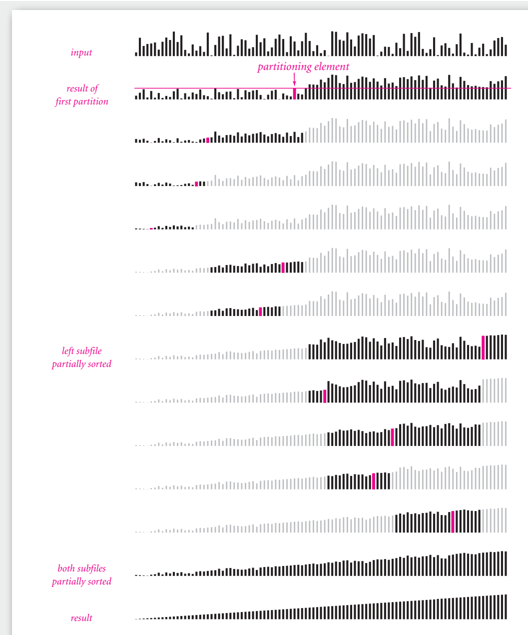
Median of sample.
- Best choice of pivot element = median.
- Estimate true median by taking median of sample.

Insertion sort small files.
- Even quicksort has too much overhead for tiny files.
- Can delay insertion sort until end.

Optimize parameters.
~ 12/7 N lg N comparisons
- Median-of-3 random elements.
- Cutoff to insertion sort for ≈ 10 elements.

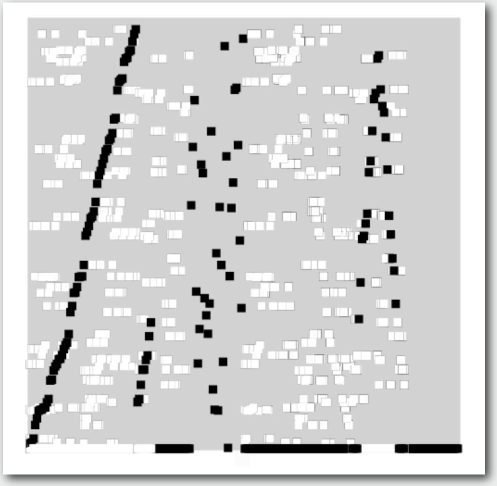
Non-recursive version.
guarantees O(log N) stack size
- Use explicit stack.
- Always sort smaller half first.

Quicksort with cutoff to insertion sort: visualization

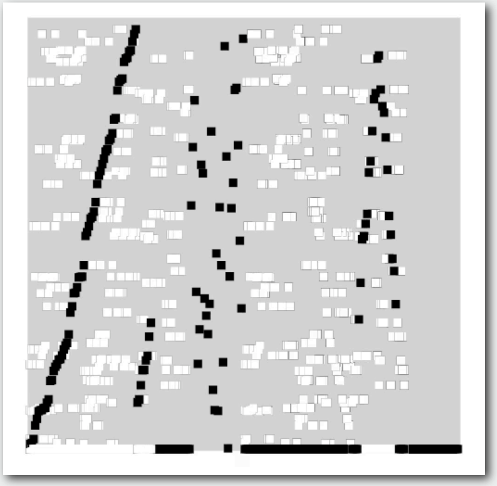


- mergesort
- sorting complexity
- quicksort
- **animations**

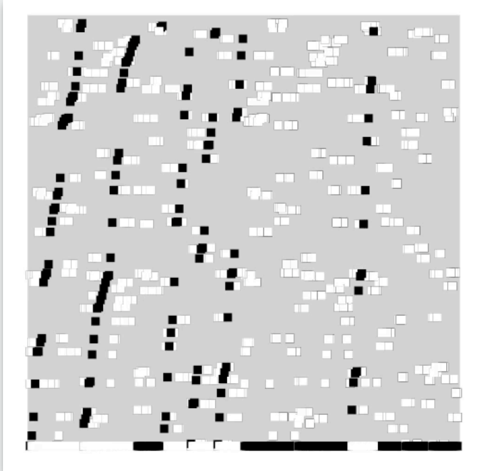
Mergesort animation



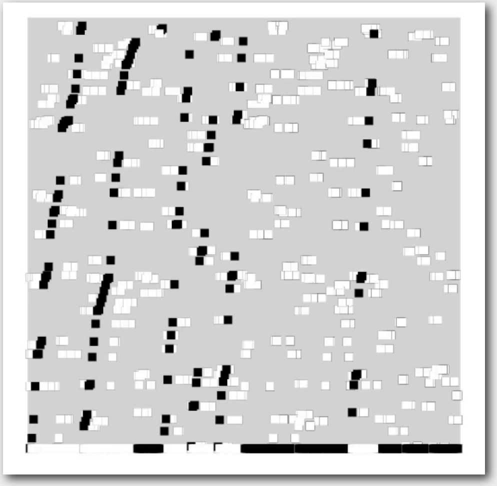
Mergesort animation



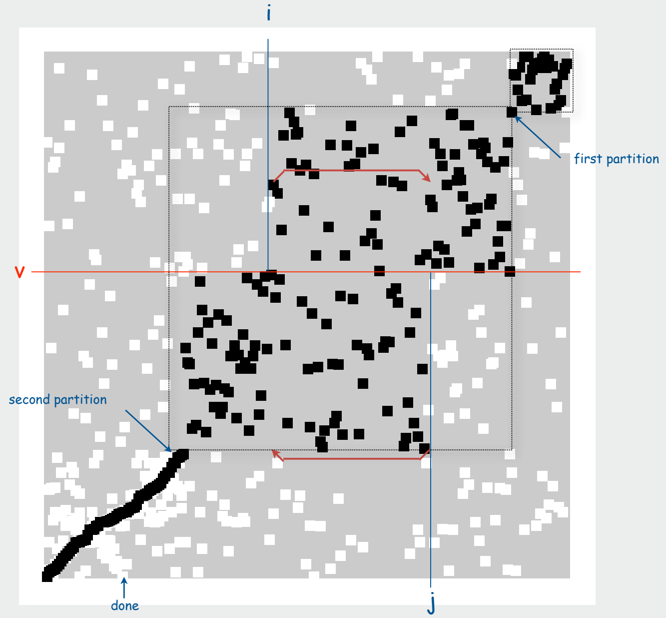
Botom-up mergesort animation



Botom-up mergesort animation



Quicksort animation



Quicksort animation