More on Vectors and Matrices

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Perspectives on Vectors and Matrices

- Physicist vs. Mathematician vs. Computer Scientist

- Things we do in Computer Science and Machine Learning that make Physicists and Mathematicians go hmm…

Vectors with Different Units Per Dimension

- Housing

$$\begin{bmatrix} 2600\ \text{ft}^2 \\ \$300{,}000 \end{bmatrix}$$

Vectors with Different Units Per Dimension

- Web page ranking

$$\begin{bmatrix} 3 \text{ keyword mentions} \\ 42 \text{ incoming links} \end{bmatrix}$$

Vectors with Different Units Per Dimension

- Health monitoring

$$\begin{bmatrix} 72 \text{ beats/min} \\ 123 \text{ mm Hg} \end{bmatrix}$$

Linear Operations "Accommodate" Units

- Components with different "units" add

$$\begin{bmatrix} 3 \text{ keyword mentions} \\ 42 \text{ incoming links} \end{bmatrix} + \begin{bmatrix} 5 \text{ keyword mentions} \\ 17 \text{ incoming links} \end{bmatrix} = \begin{bmatrix} 8 \text{ keyword mentions} \\ 59 \text{ incoming links} \end{bmatrix}$$

Web page #1 Web page #2 Web page collection

Linear Operations "Accommodate" Units

- One way of thinking about dot product is "weighting" dimensions or "probing" individual components, while accommodating units

$$\begin{bmatrix} 72 \text{ beats/min} \\ 123 \text{ mm Hg} \end{bmatrix} \cdot \begin{bmatrix} 1 \text{ unit risk / (beat/min)} \\ 0.5 \text{ units risk / (mm Hg)} \end{bmatrix} = 133.5 \text{ units risk}$$

$$\begin{bmatrix} 72 \text{ beats/min} \\ 123 \text{ mm Hg} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 72 \text{ beats/min}$$

Matrices vs. Arrays of Numbers

- We will often go back and forth between arrays of numbers, matrices, and “unrolled” vectors

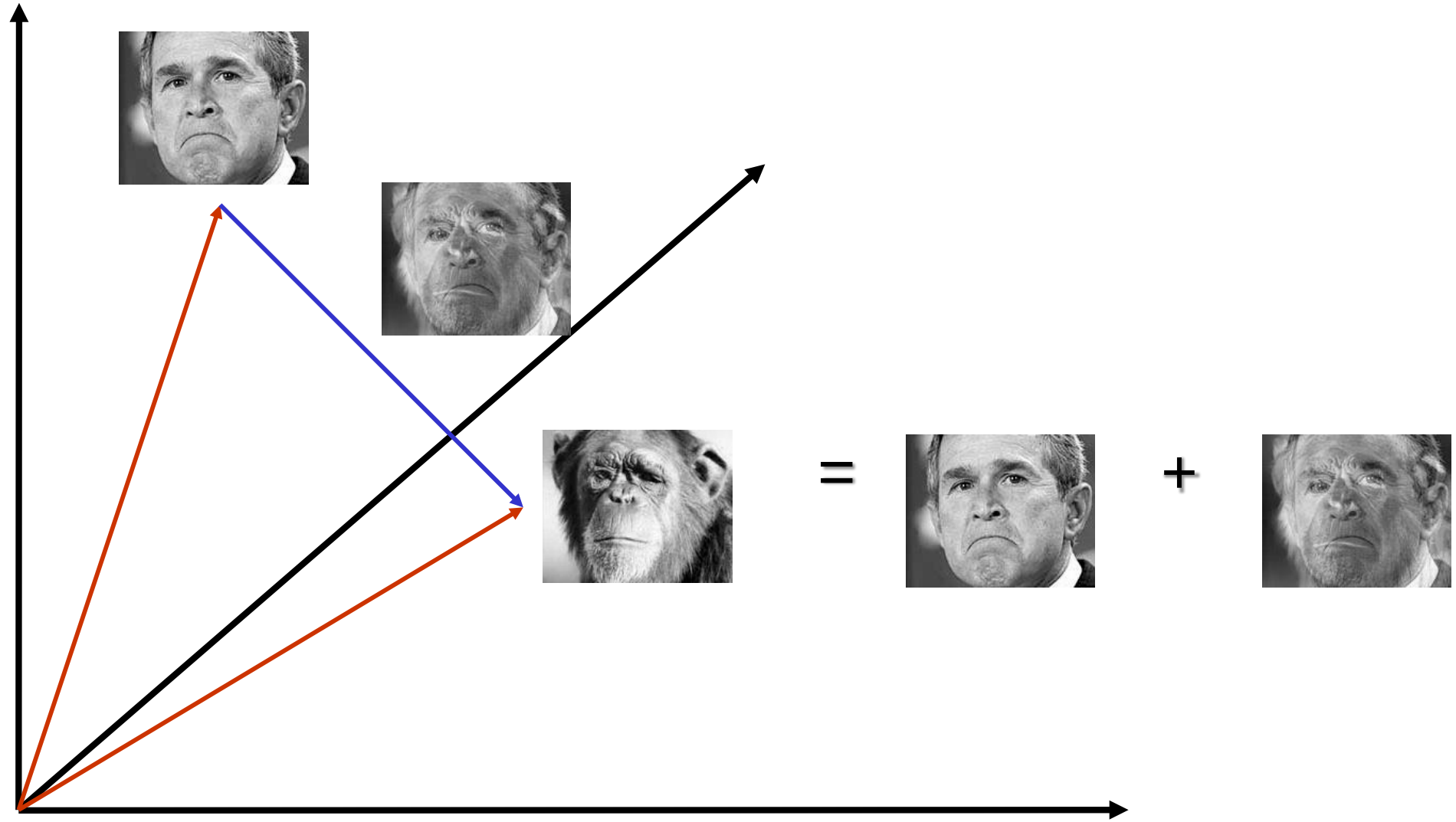
Linear Algebra on Images

- Digital images are arrays of pixels
  - Value at each pixel is intensity of light
  - For color, store intensity in Red, Green, Blue channels
    (3 channels enough because of human visual system)
- Can "unroll" an image, treat it as a vector in a (high-dimensional) vector space
  - Light is linear! *(But images are often nonlinearly mapped)
  - Can perform usual manipulations, such as …

Average Princetonian Face

- From 2005 BSE thesis project by Clay Bavor and Jesse Levinson

Vector Spaces of Images



[Seitz]

Detecting Princetonians

[Bavor & Levinson]