Matching and Recognition in 3D

Based on slides by Tom Funkhouser and Misha Kazhdan

From 2D to 3D: Some Things Easier

No occlusion (but sometimes missing data instead)

Segmenting objects often simpler

From 2D to 3D: Many Things Harder

Rigid transform has 6 degrees of freedom vs. 3

- Brute-force algorithms much less practical

Rotations do not commute

- Difficult to parameterize, search over

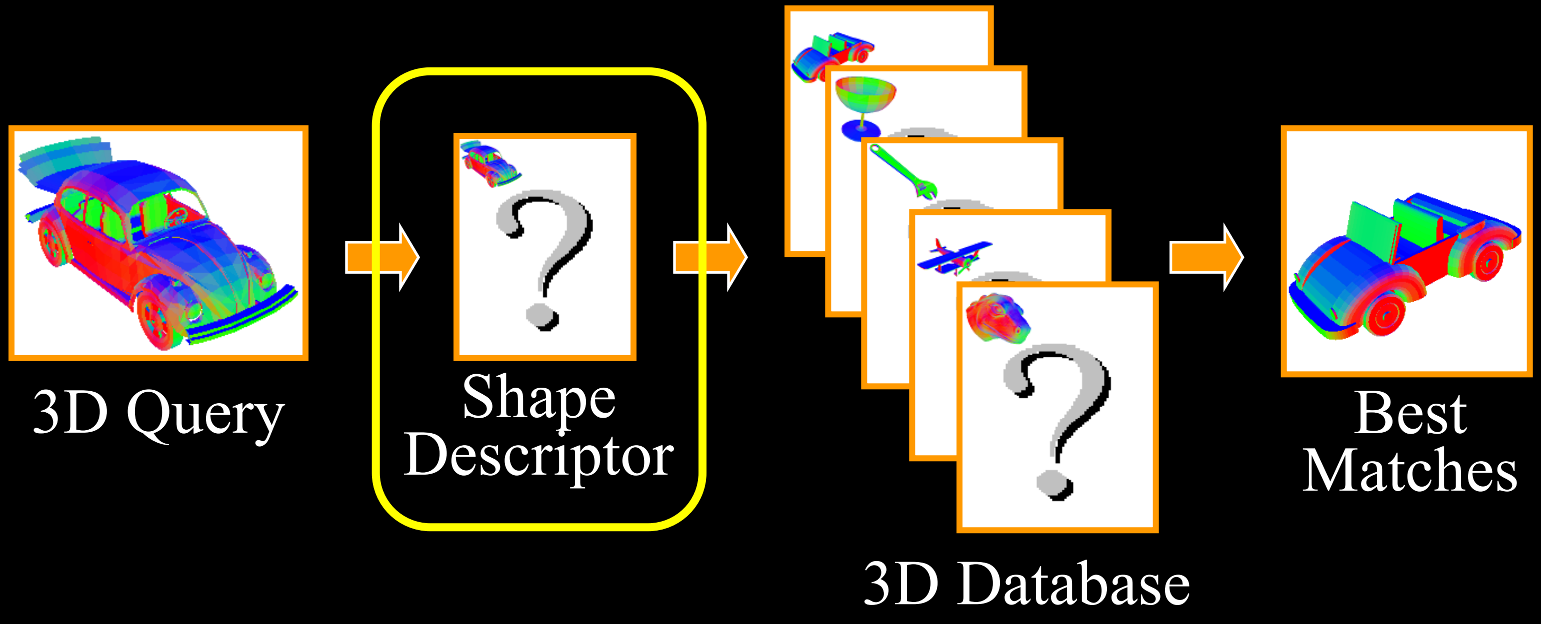
No natural parameterization for surfaces in 3D

- Hard to do FFT, convolution, PCA
- Exception: range images (which are view dependent)

Shape Matching Challenge

Need shape descriptor & matching method that is:

- Concise to store
- Quick to compute
- Efficient to match
- Discriminating



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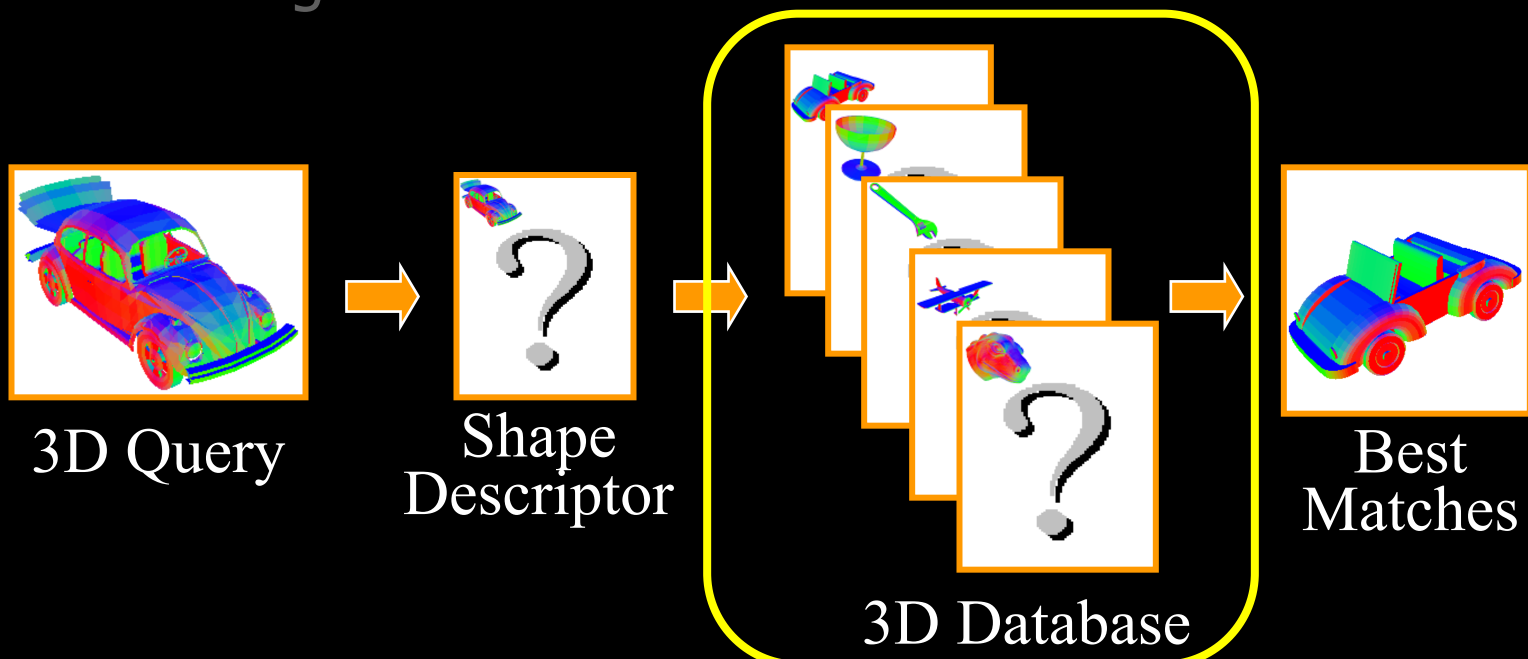

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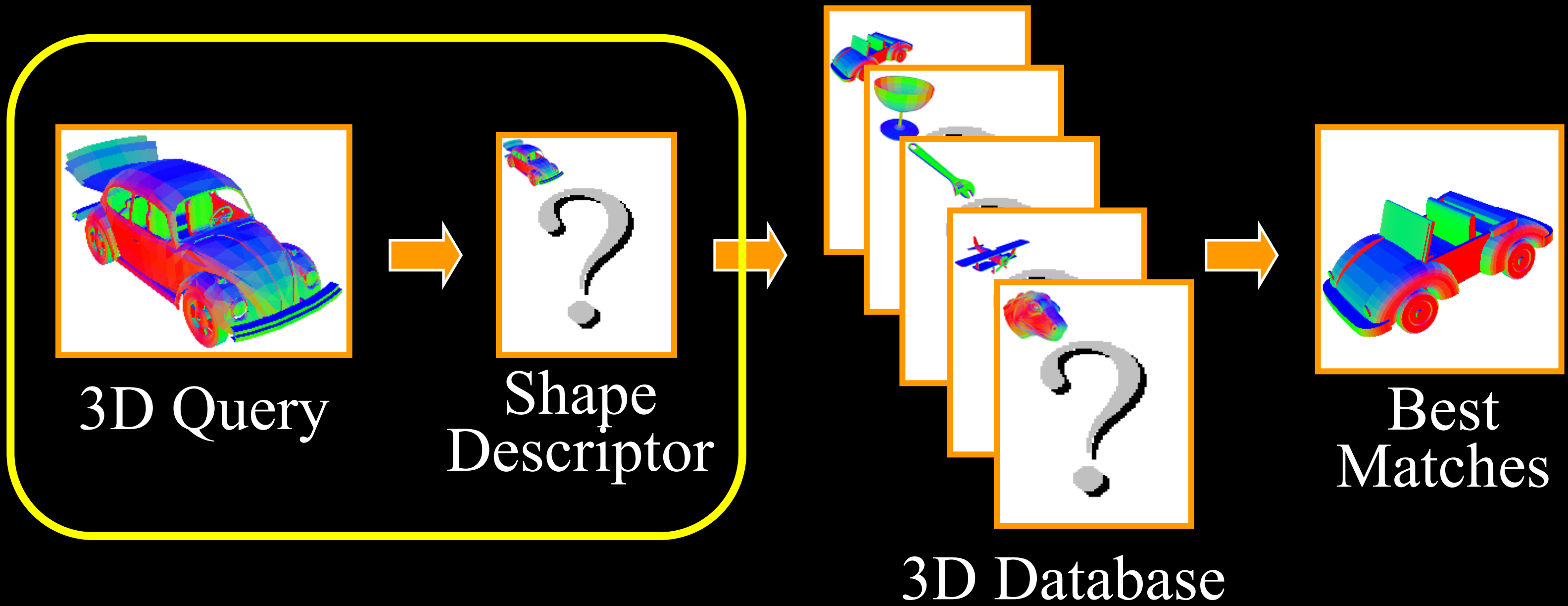

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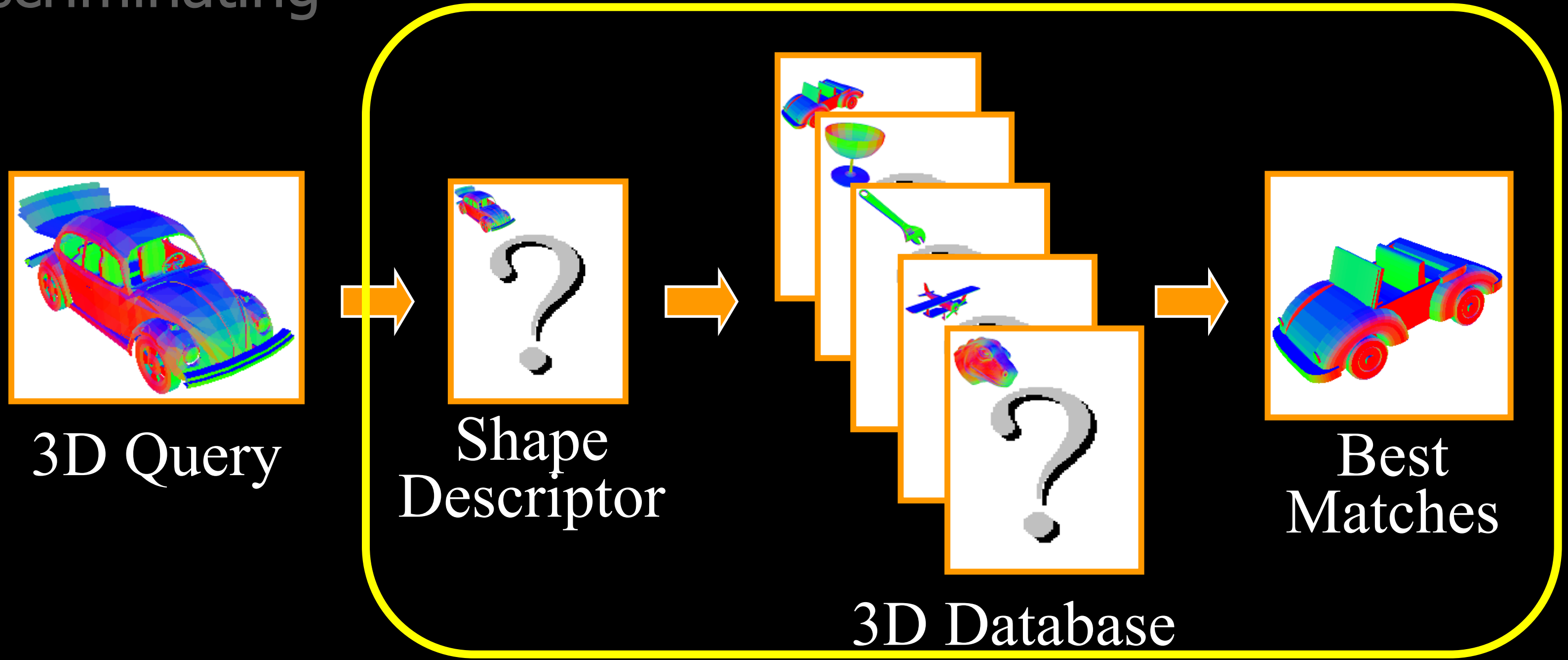

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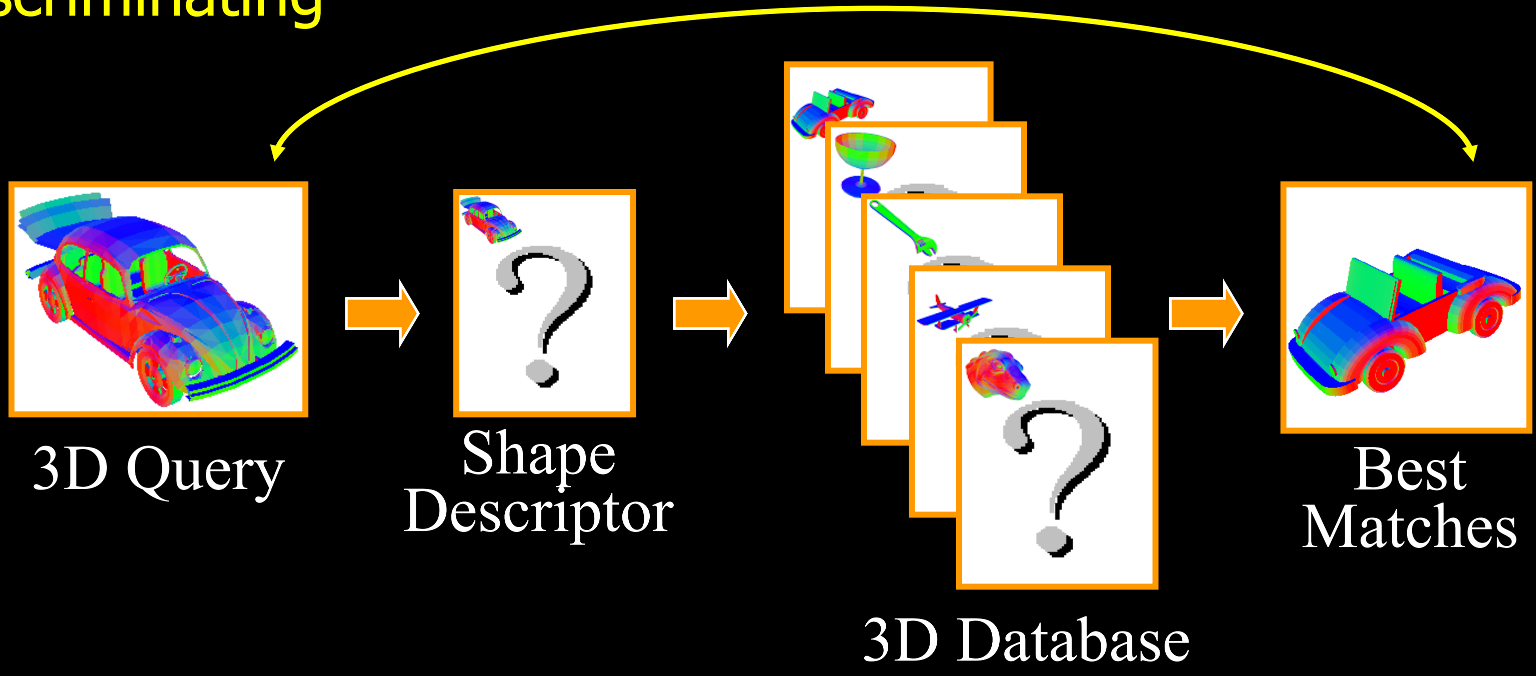

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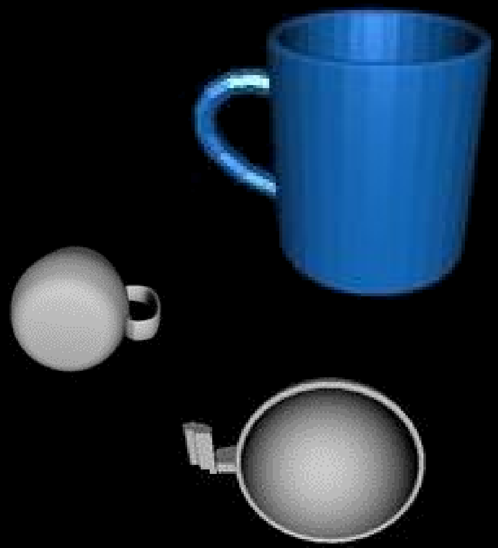

Different Transformations
(translation, scale, rotation, mirror)

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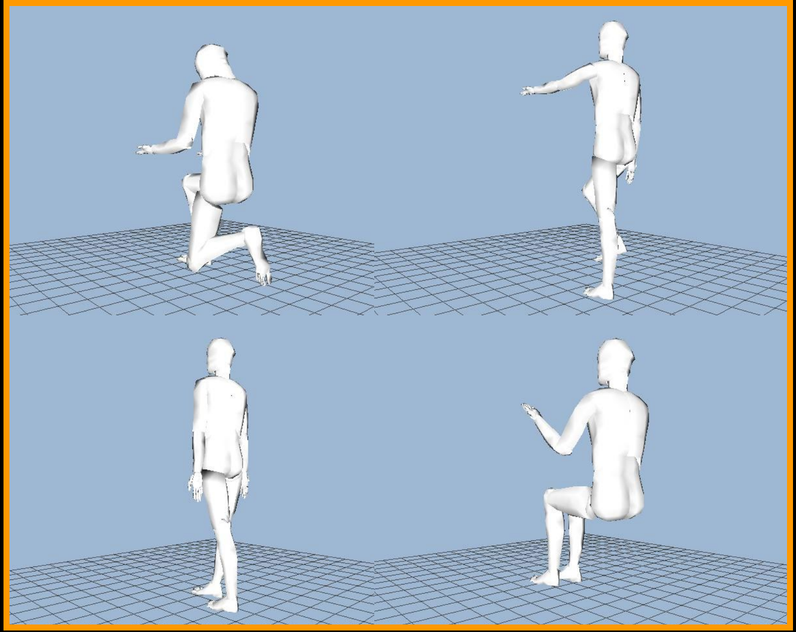

Different Articulated Poses

Shape Matching Challenge

Image courtesy of Ramamoorthi et al.

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Scanned Surface

Shape Matching Challenge

Images courtesy of Viewpoint & Stanford

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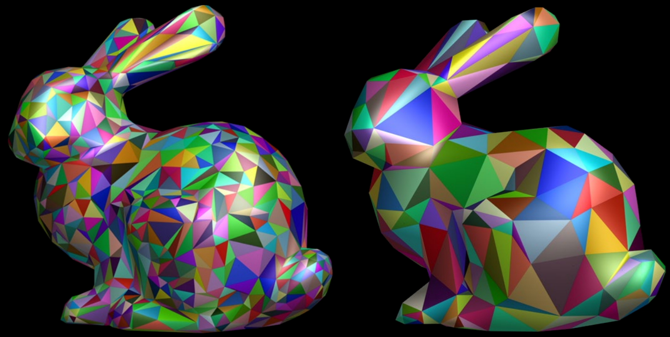
Different Genus



Different Tessellations

Shape Matching Challenge

Images courtesy of
Utah & De Espona

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No Bottom!

&*Q?@#A%!

Taxonomy of 3D Matching Methods

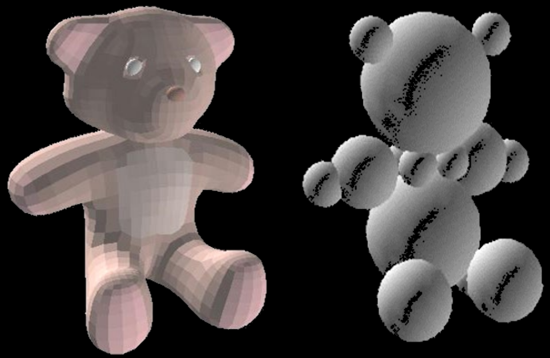
Images courtesy of Amenta & Osada

Structural representations

- Skeletons
- Part-based methods
- Feature-based methods

Statistical representations

- Attribute feature vectors
- Volumetric methods
- Surface-based methods
- View-based methods



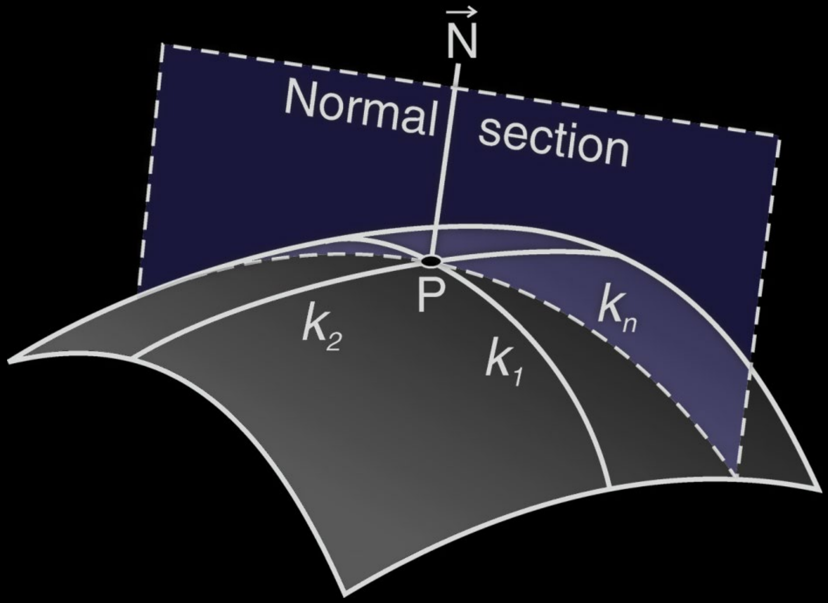
Features on Surfaces

Can construct edge and corner detectors

Analogue of 1st derivative: surface normal

Analogue of 2nd derivative: curvature

- Curvature at each point in each direction
- Minimum and maximum: "principal curvatures"
- Can threshold or do nonmaximum suppression



Using Curvatures for Recognition/Matching

Curvature histograms: compute κ_1 and κ_2 throughout surface, create 2D histograms

Invariant to translation, rotation

Alternative: use κ_2 / κ_1 – also invariant to scale

- Shape index: $S = \frac{1}{2} - \frac{1}{\pi} \tan^{-1} \frac{\kappa_1 + \kappa_2}{\kappa_1 - \kappa_2} \in [0..1]$

Curvatures sensitive to noise (2nd derivative…), so sometimes just use sign of curvatures

Using Curvatures for Segmentation

Sharp creases in surface (i.e., where $|\kappa_1|$ is large) tend to be good places to segment

Option #1: look for maxima of curvature in the first principal direction

- Much like Canny edge detection
- Nonmaximum suppression, hysteresis thresholding

Option #2: optimize for both high curvature and smoothness using graph cuts, snakes, etc.

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Image courtesy of Mao Chen

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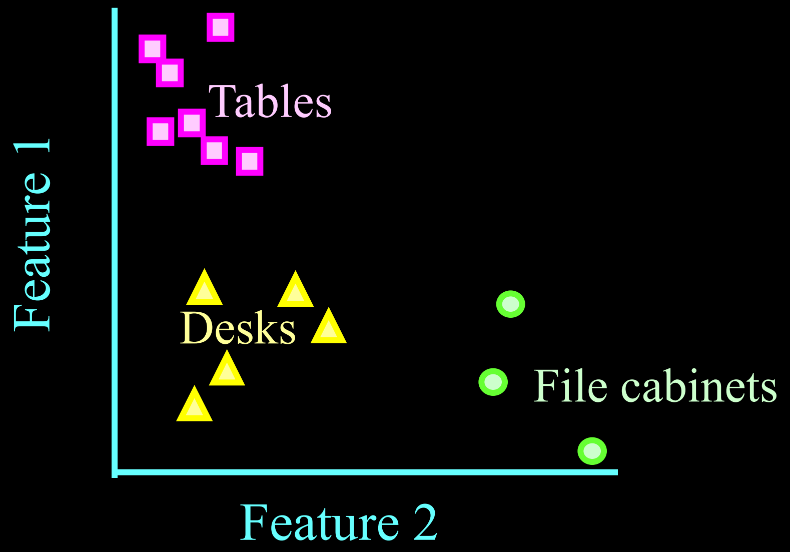


Example

Shape distributions

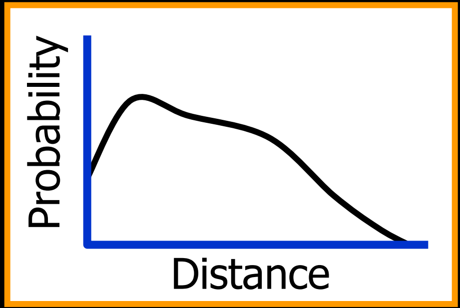
- Shape representation: probability distributions
- Distance measure: difference between distributions
- Evaluation method: classification performance

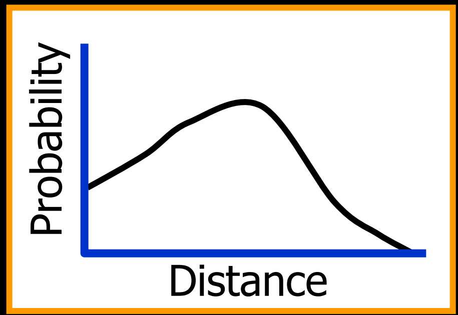
Shape Distributions

Key idea: map 3D surfaces to common parameterization by randomly sampling shape function

3D Models

Randomly sample shape function



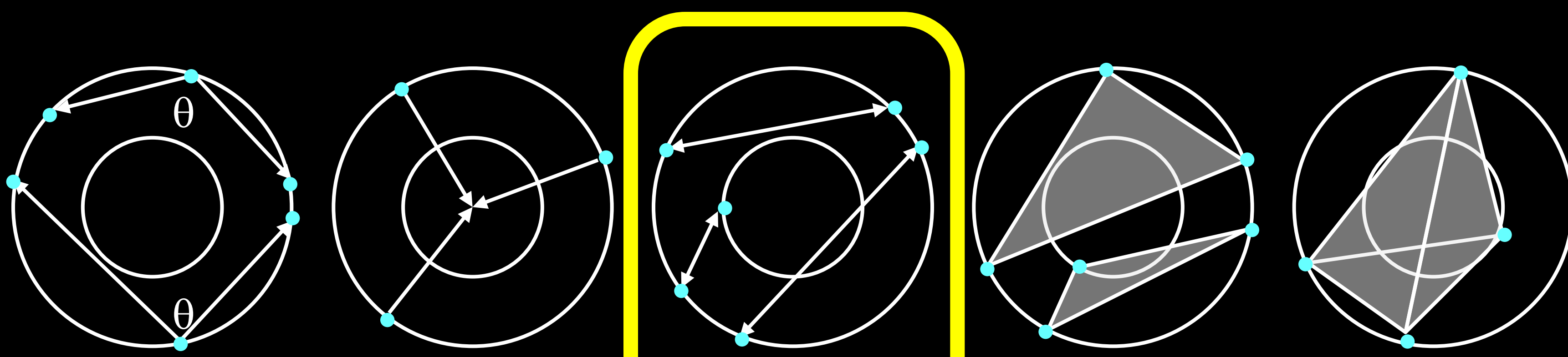


D2 Shape Distributions

Similarity Measure

Which Shape Function?

Implementation: simple shape functions based on angles, distances, areas, and volumes



A3 (angle) | D1 (distance) | D2 (distance) | D3 (area) | D4 (volume)

[Ankerst 99]

D2 Shape Distribution

Properties

- Concise to store?
- Quick to compute?
- Invariant to transforms?
- Efficient to match?
- Insensitive to noise?
- Insensitive to topology?
- Robust to degeneracies?
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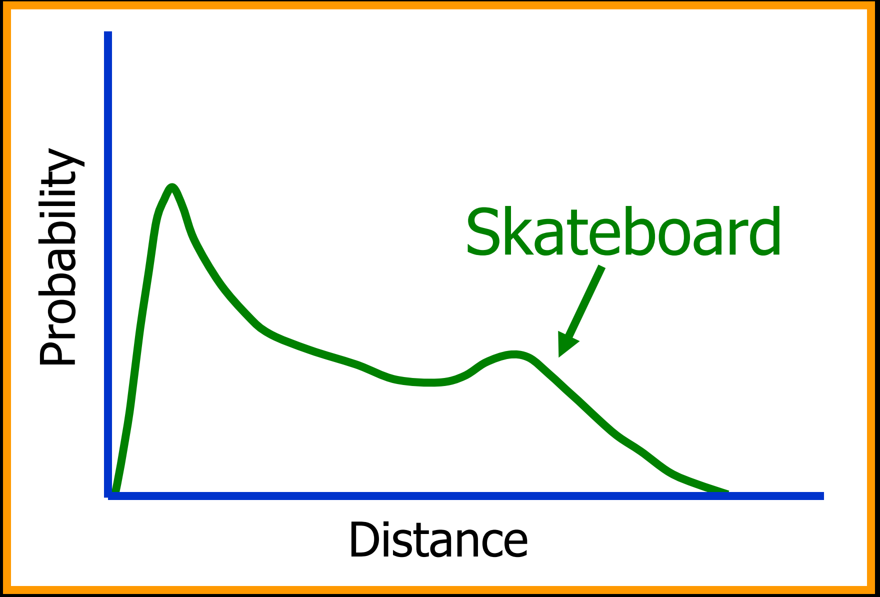

512 bytes (64 values)

0.5 seconds (10^6 samples)

D2 Shape Distribution

Properties

- ✓ Concise to store
- ✓ Quick to compute
- ➢ Invariant to transforms?
  - ✓ Translation
  - ✓ Rotation
  - ✓ Mirror
  - ✓ Scale (w/ normalization)
- Efficient to match?
- Insensitive to noise?
- Insensitive to topology?
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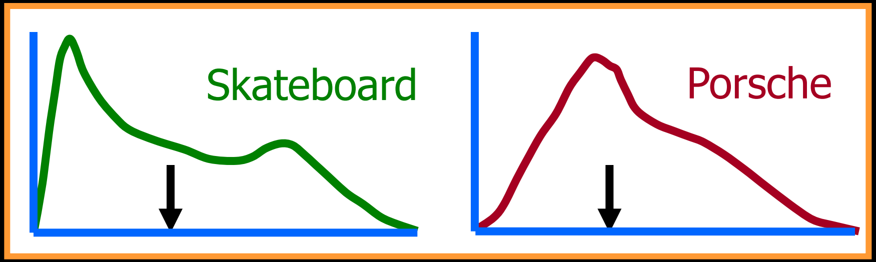

Normalized Means

D2 Shape Distribution

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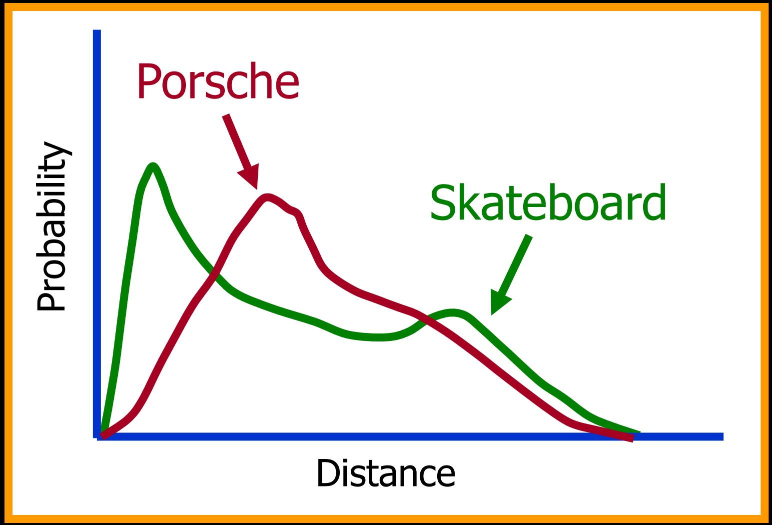

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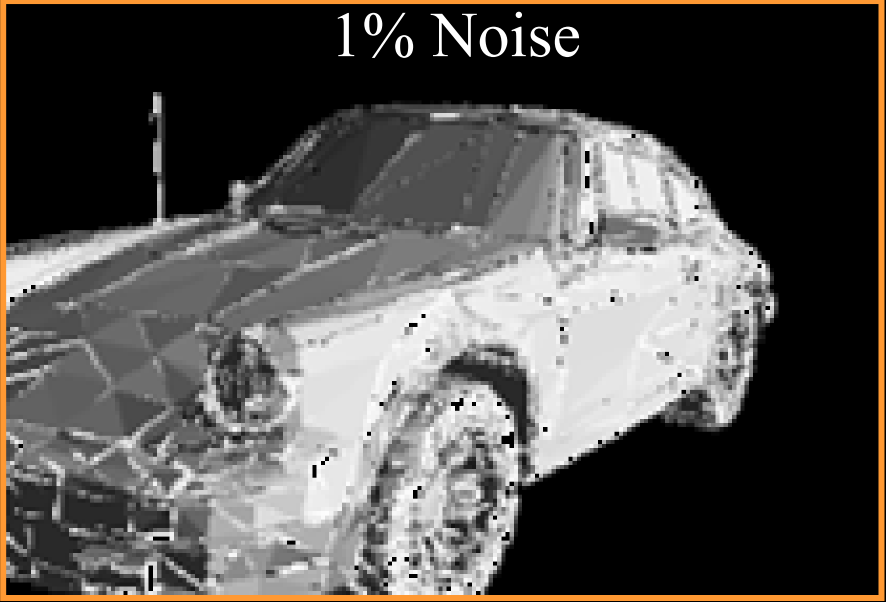

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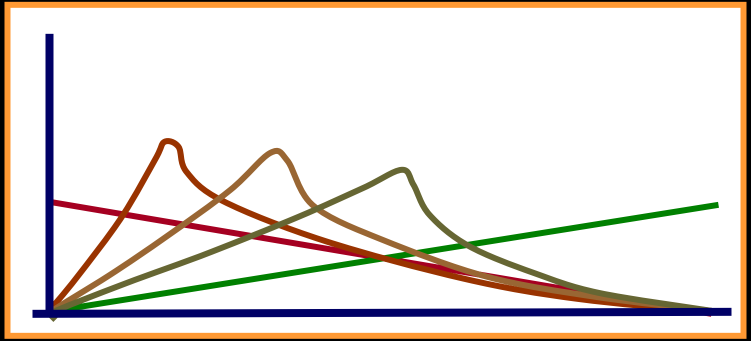

Ellipsoids with Different Eccentricities

D2 Shape Distribution

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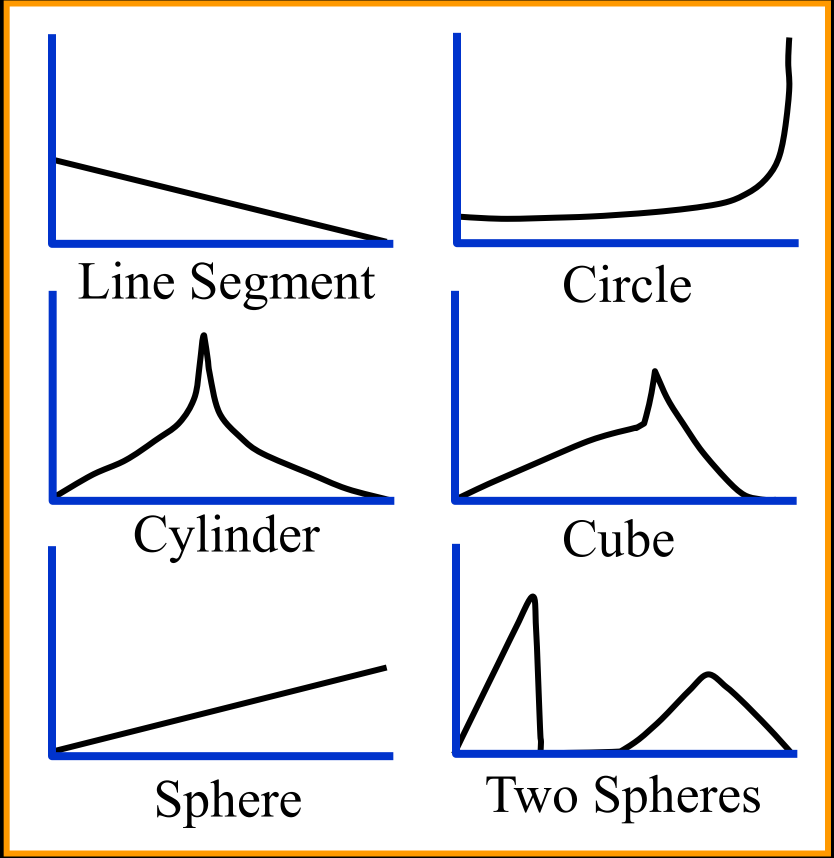


D2 Shape Distribution Results

Question

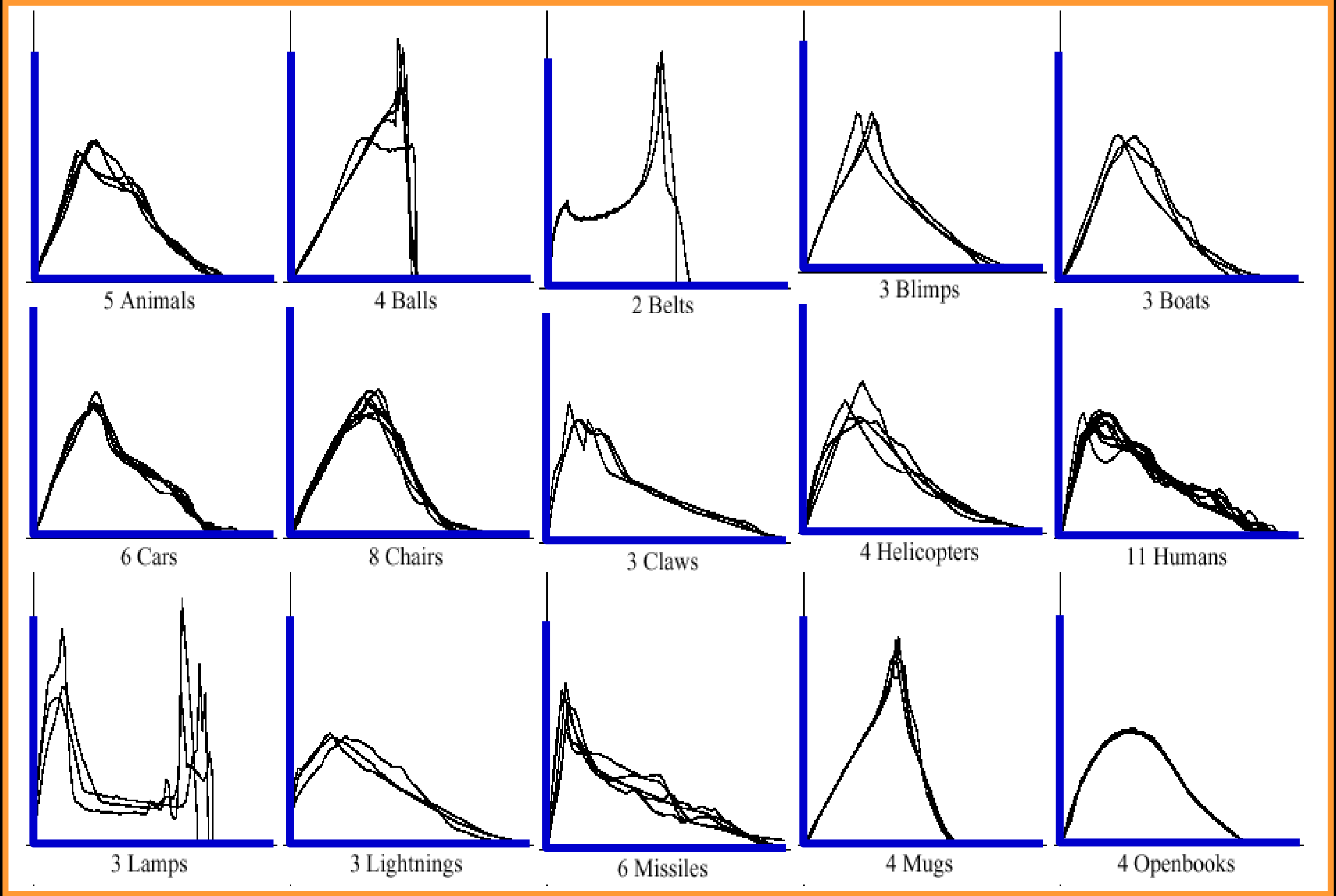
- How discriminating are D2 shape distributions?

Test database

- 133 polygonal models
- 25 classes

D2 Shape Distribution Results

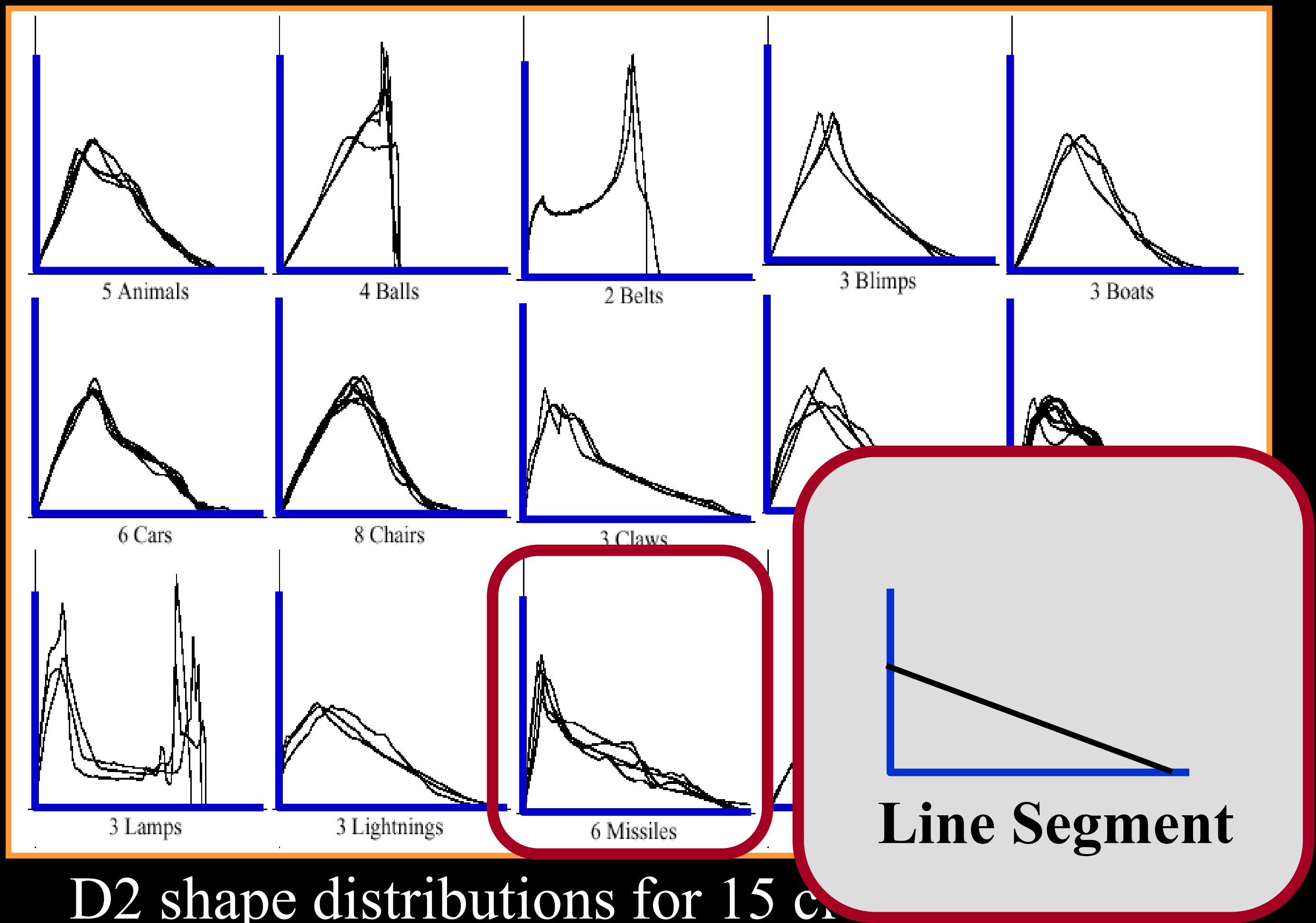
D2 distributions are different across classes



D2 shape distributions for 15 classes of objects

D2 Shape Distribution Results

D2 distributions reveal gross shape of object



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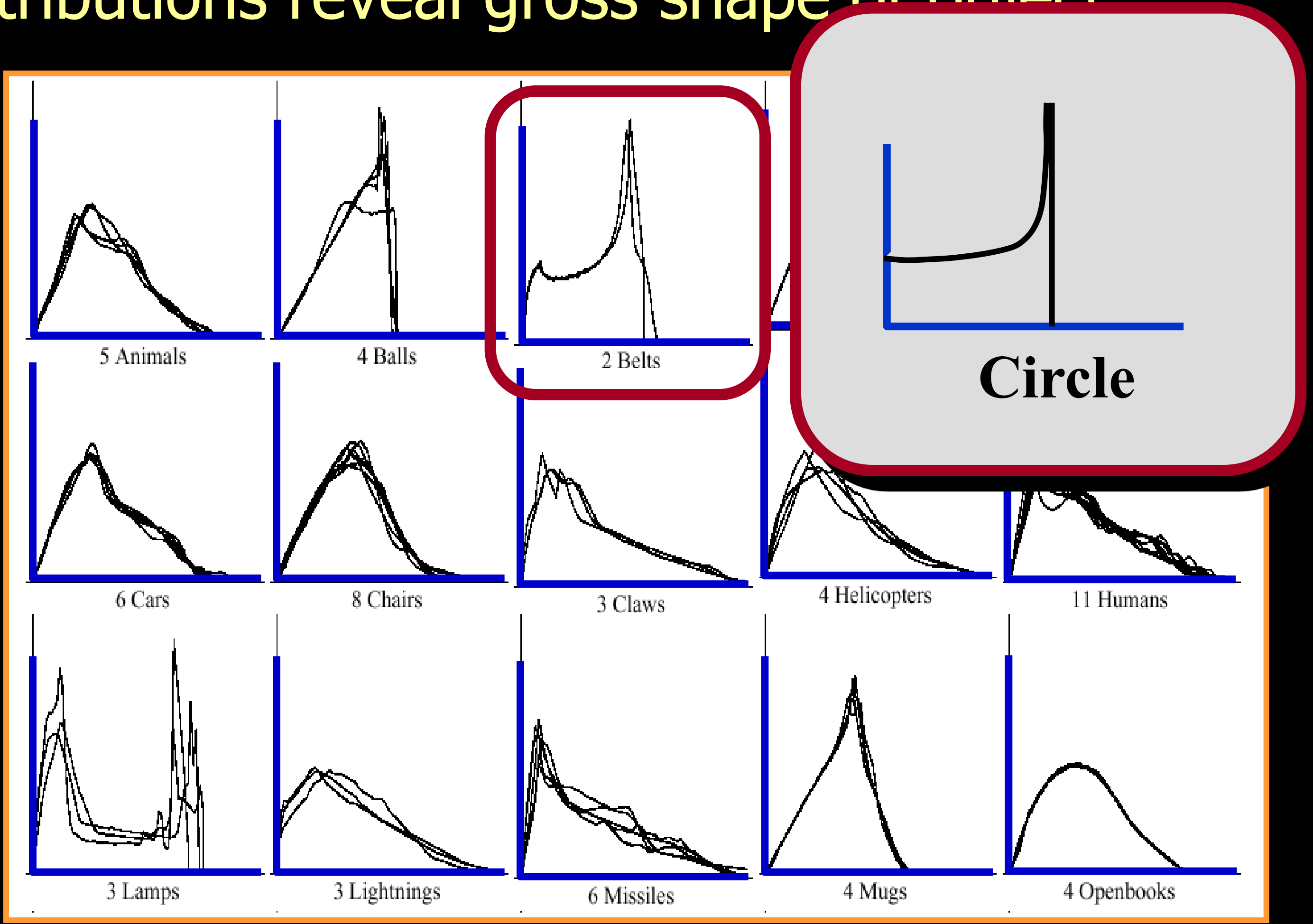

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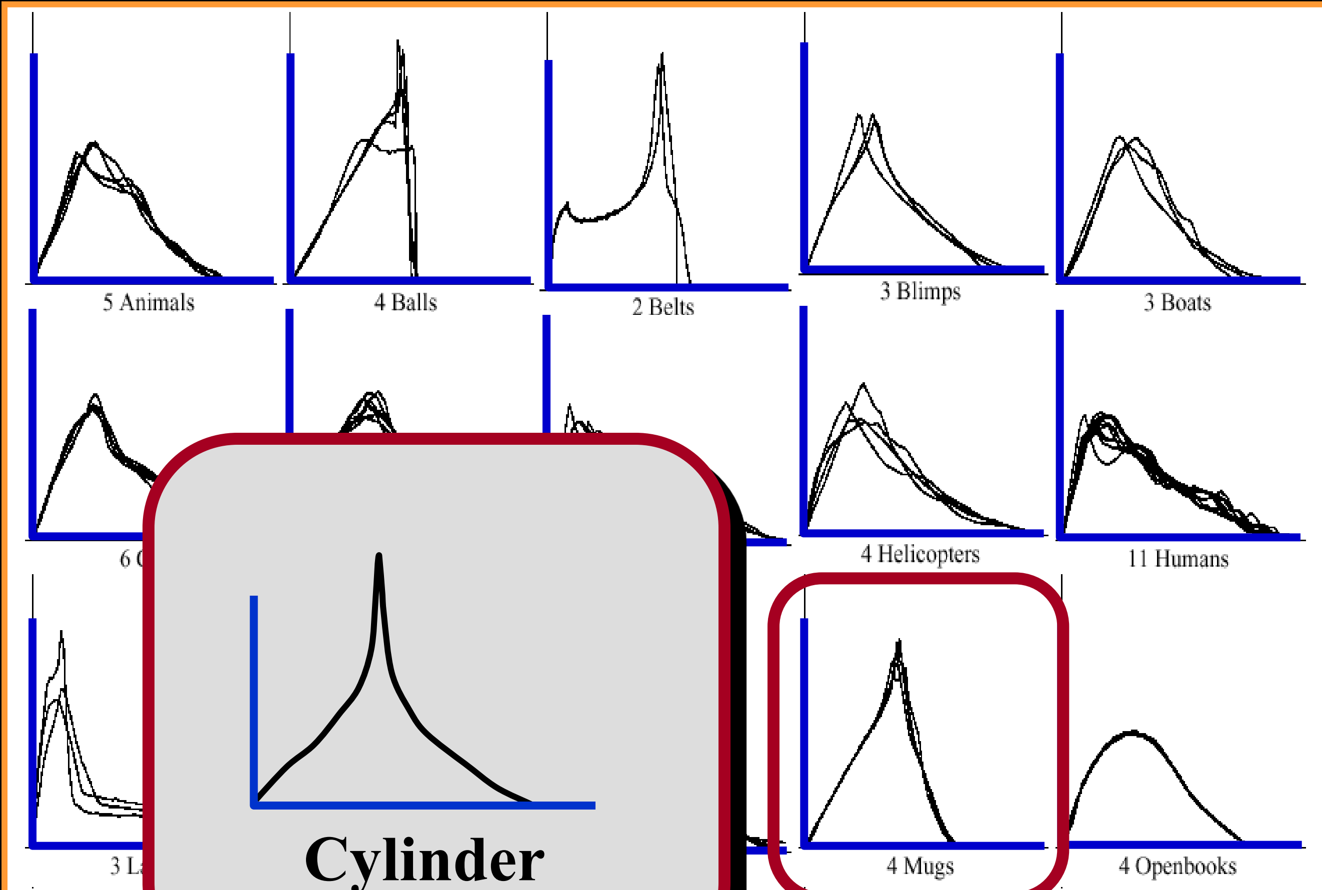

D2 … 15 classes of objects

D2 Shape Distribution Results

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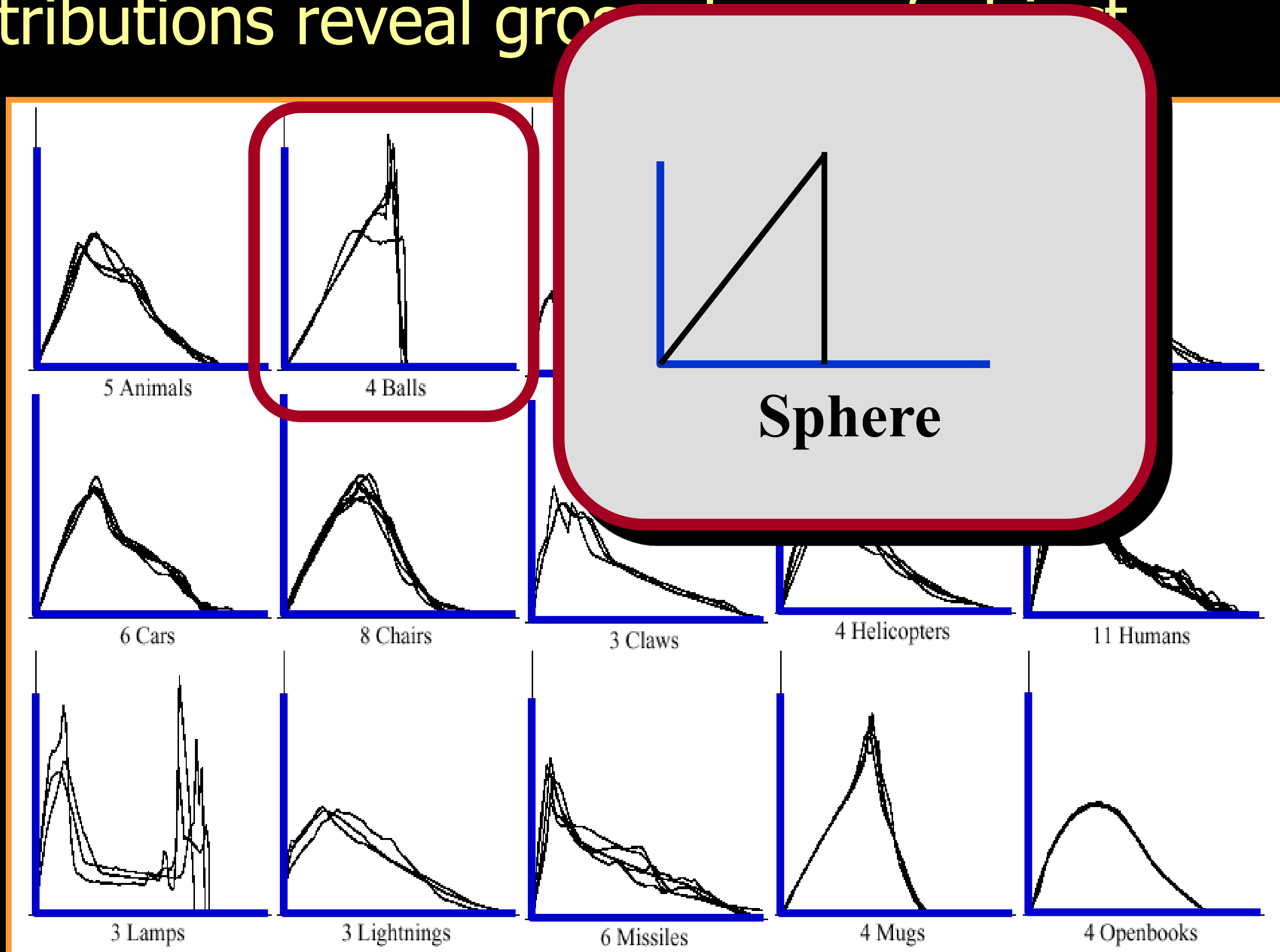

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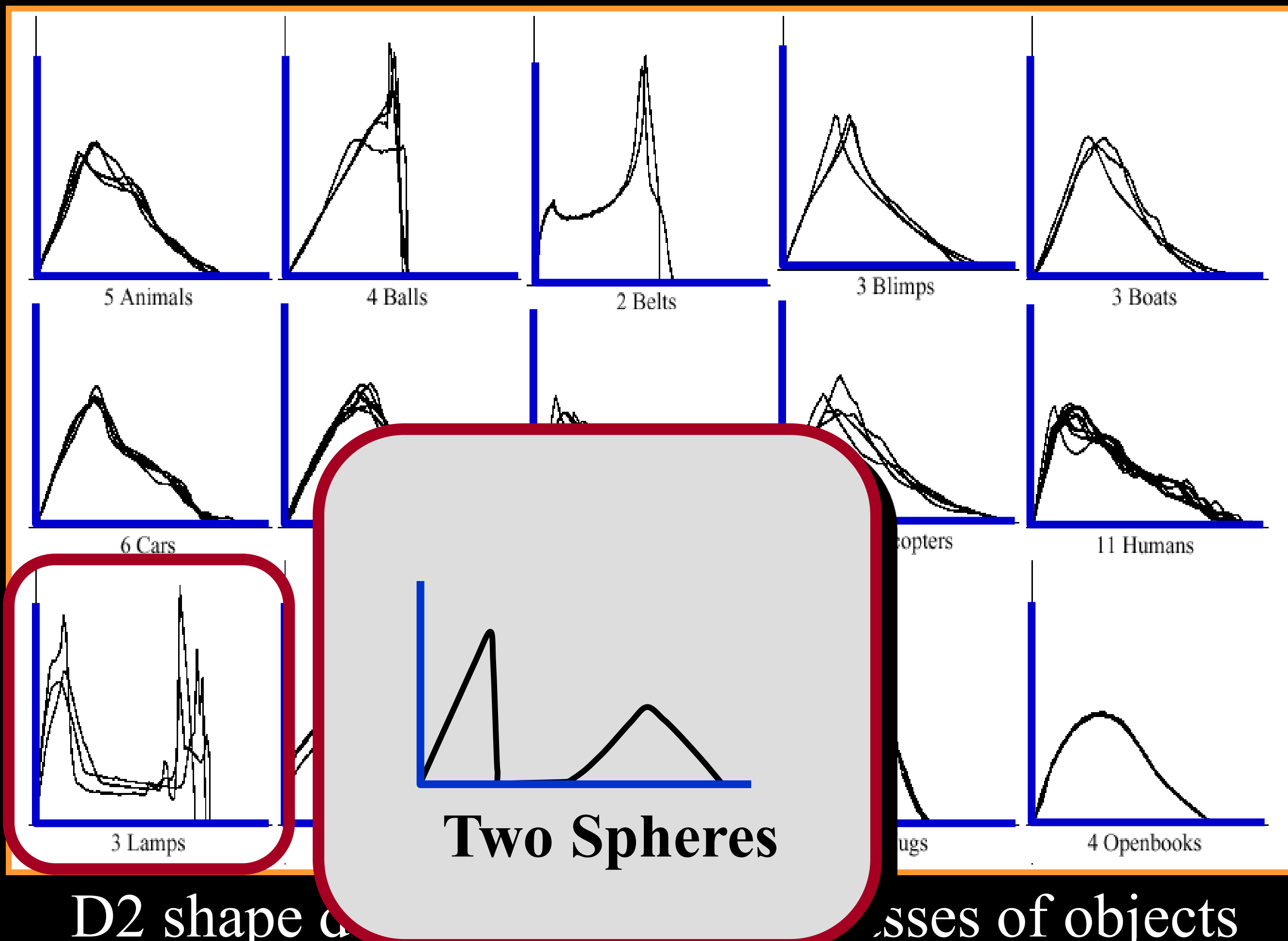
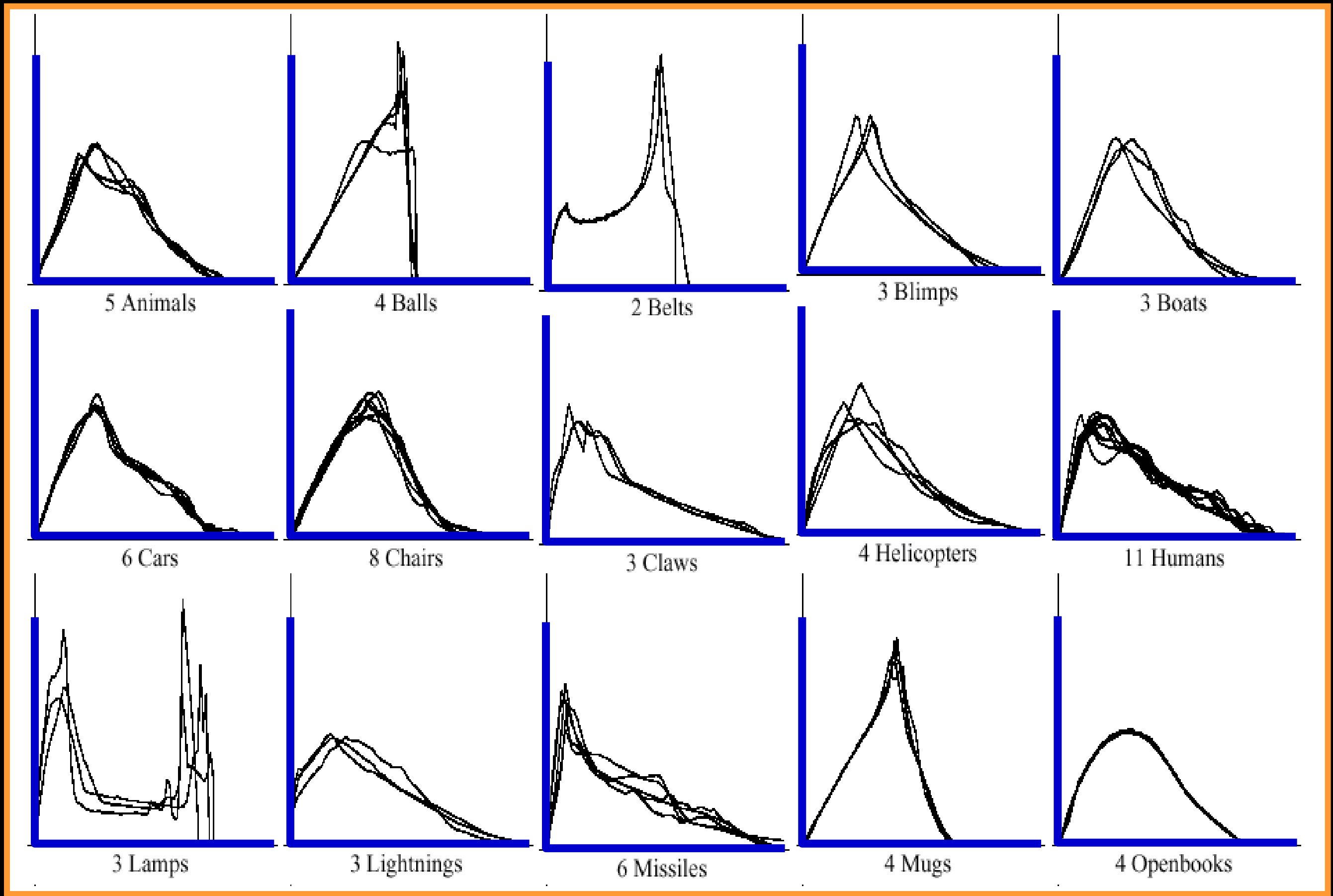


D2 shape d… …sses of objects

D2 Shape Distribution Results

But … are D2 distributions discriminating?



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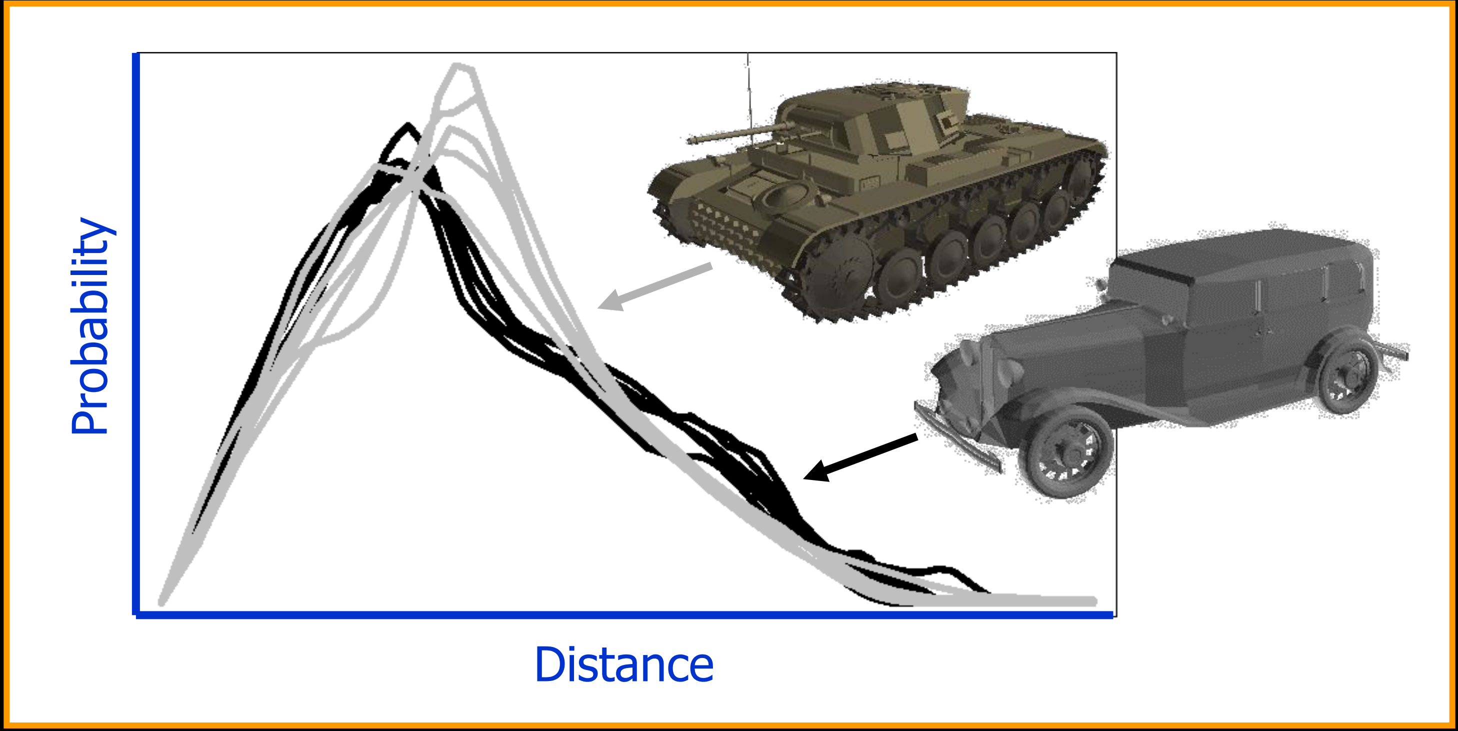


D2 distributions for 5 tanks (gray) and 6 cars (black)

Evaluation Methods

For each model (the query):

- Compute match score for all models
- Rank matches from best to worst
- Measure how often models in same class as query appear near top of ranked list

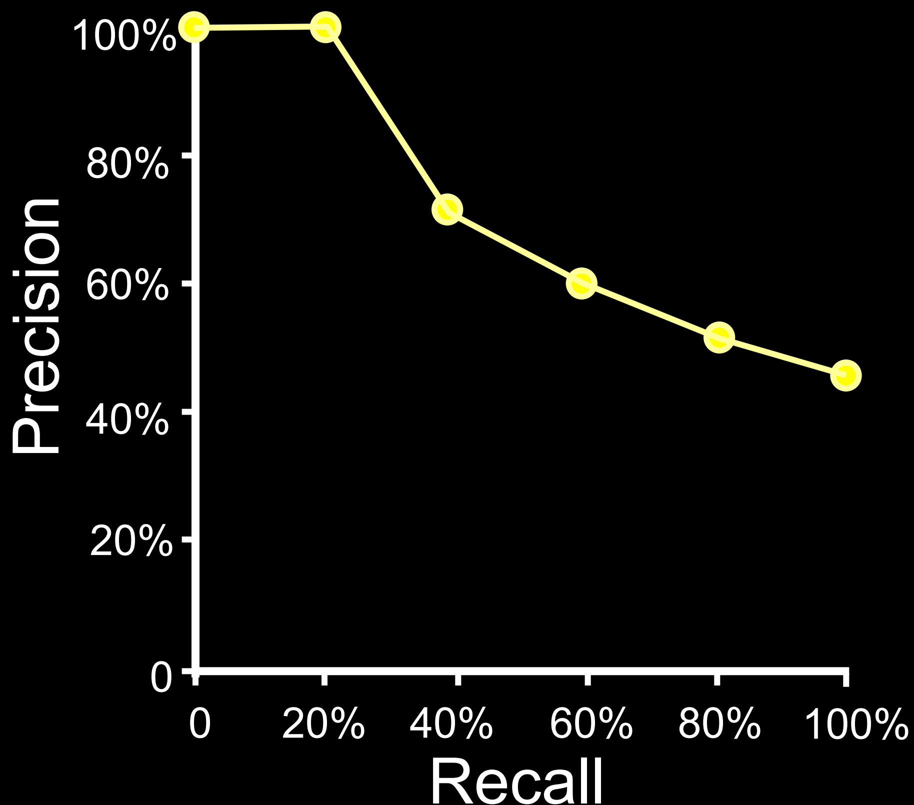
Query

Ranked Matches

Evaluation Methods

Precision-recall curves

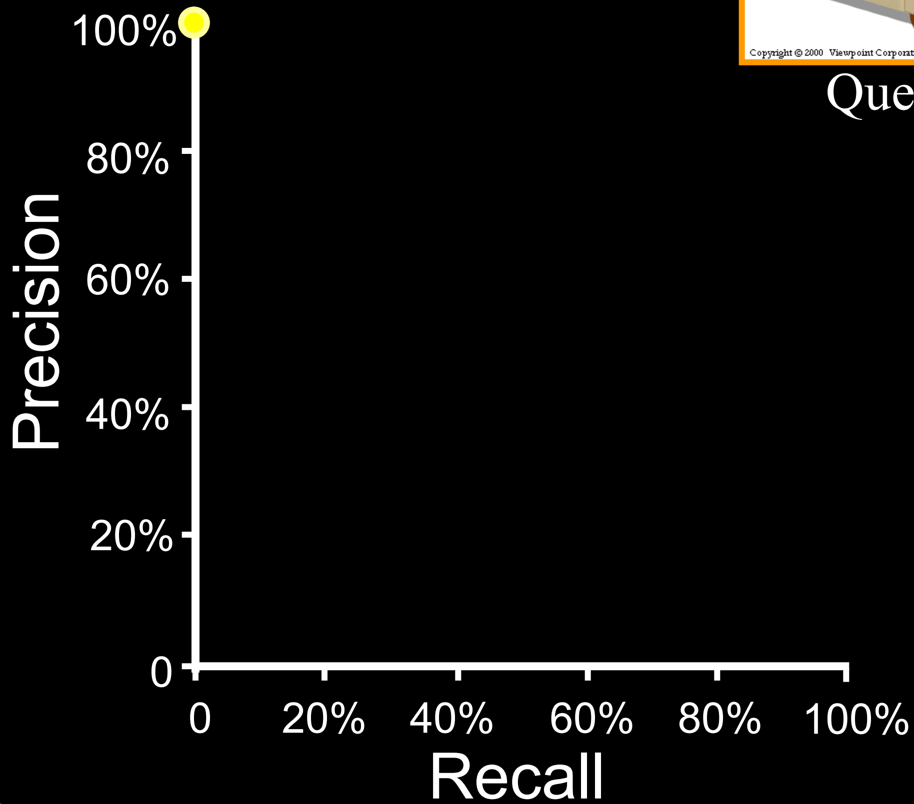
- Precision = retrieved_in_class / total_retrieved
- Recall = retrieved_in_class / total_in_class



Evaluation Methods

Precision-recall curve example

- Precision = 0 / 0
- Recall = 0 / 5



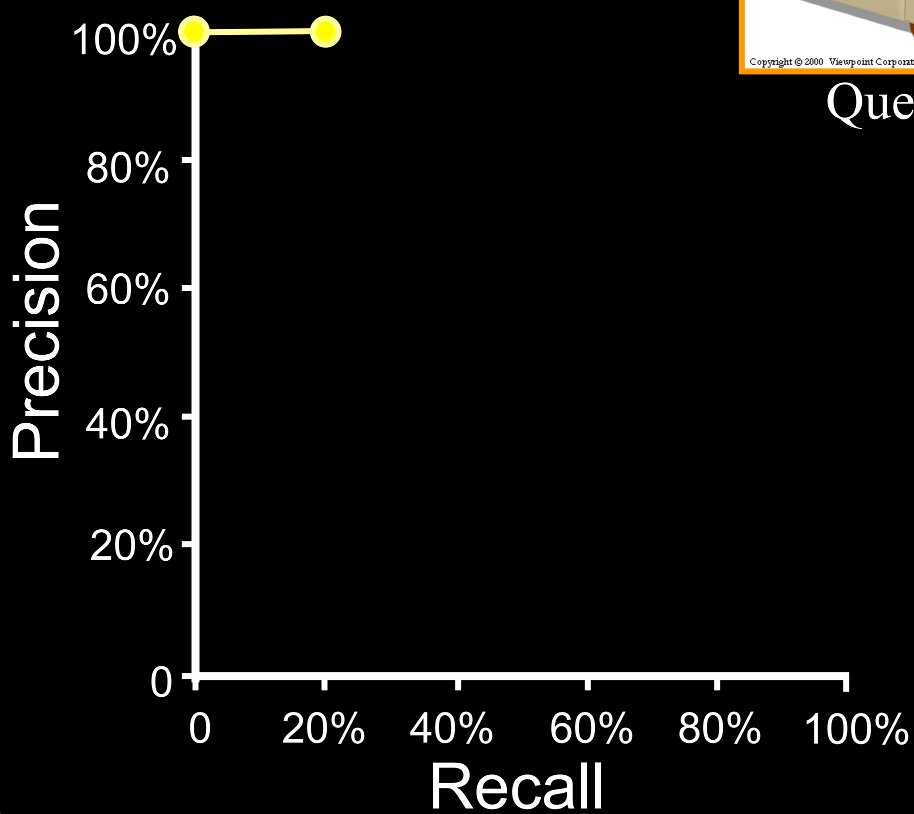
Query

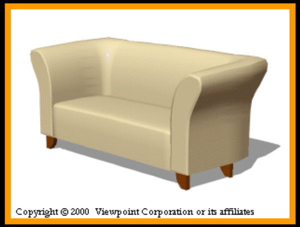
Ranked Matches

Evaluation Methods

Precision-recall curve example

- Precision = 1 / 1
- Recall = 1 / 5





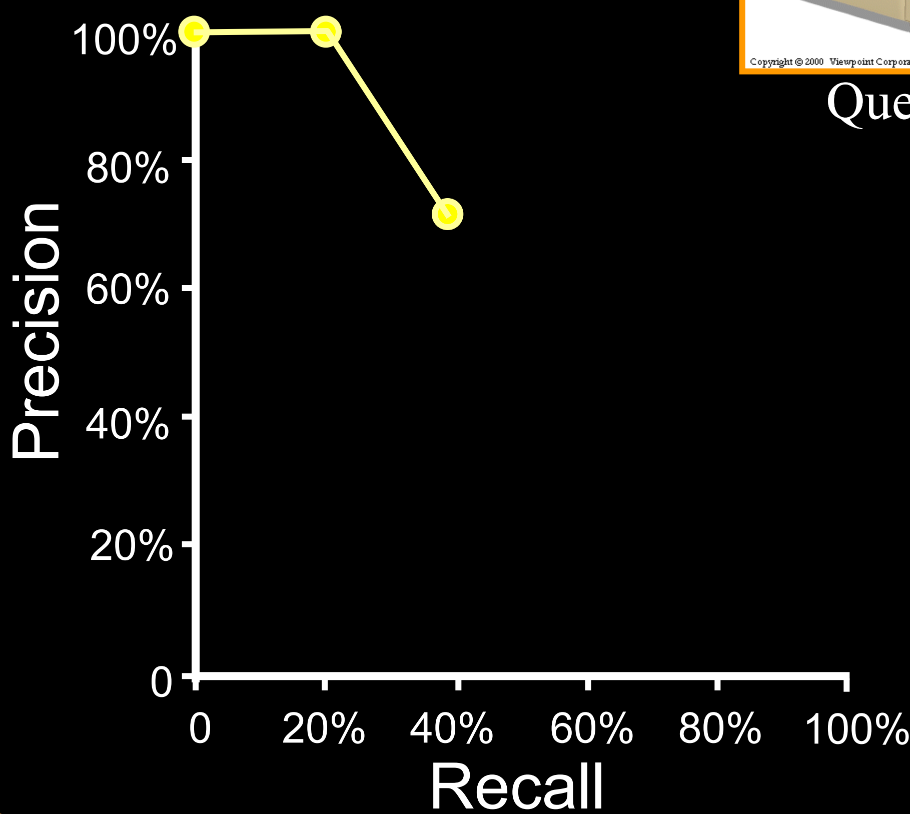
Query

Ranked Matches

Evaluation Methods

Precision-recall curve example

- Precision = 2 / 3
- Recall = 2 / 5



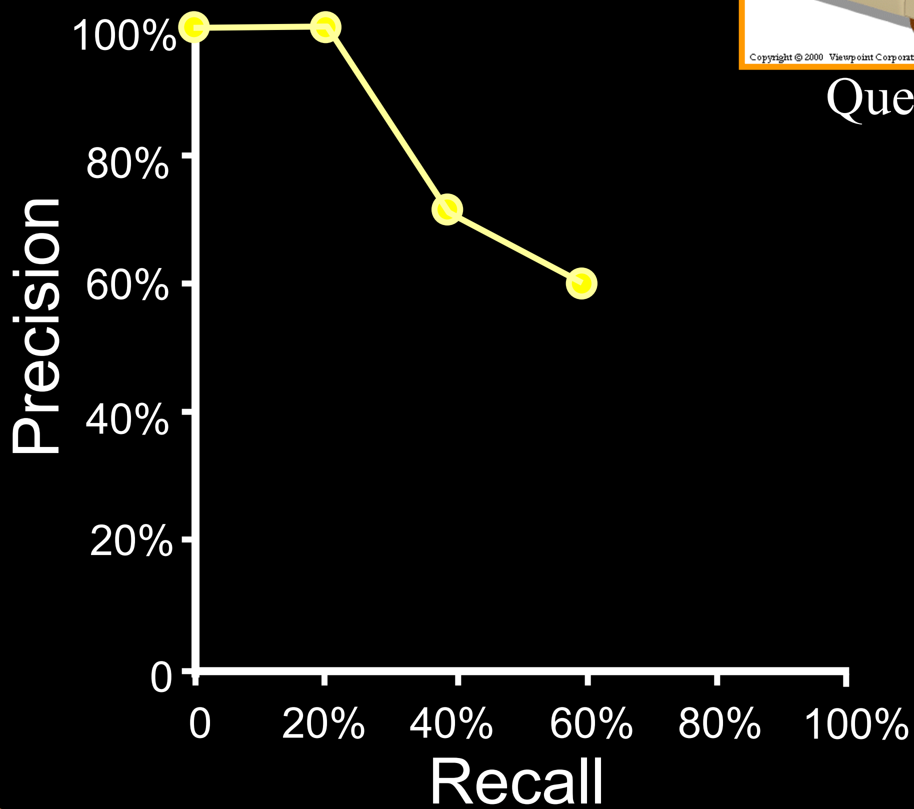
Query

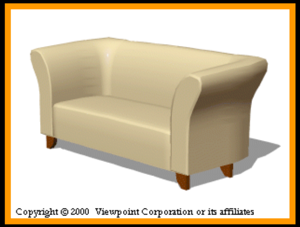
Ranked Matches

Evaluation Methods

Precision-recall curve example

- Precision = 3 / 5
- Recall = 3 / 5





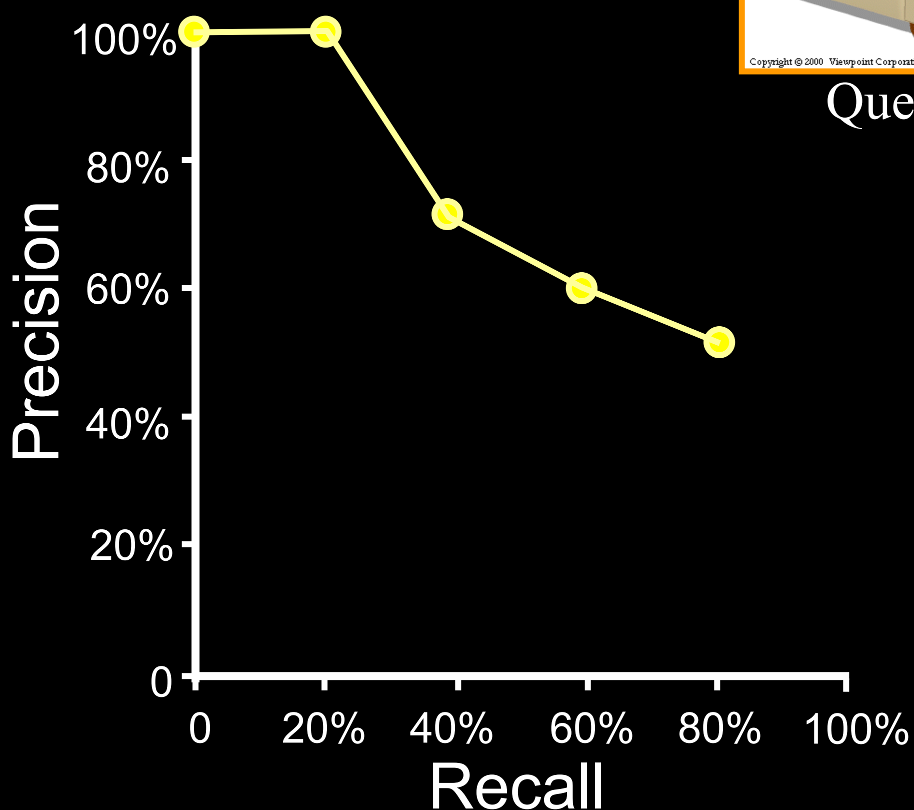
Query

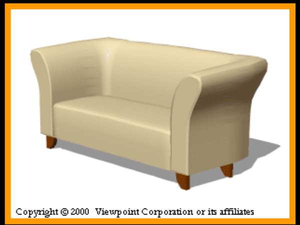
Ranked Matches

Evaluation Methods

Precision-recall curve example

- Precision = 4 / 7
- Recall = 4 / 5





Query

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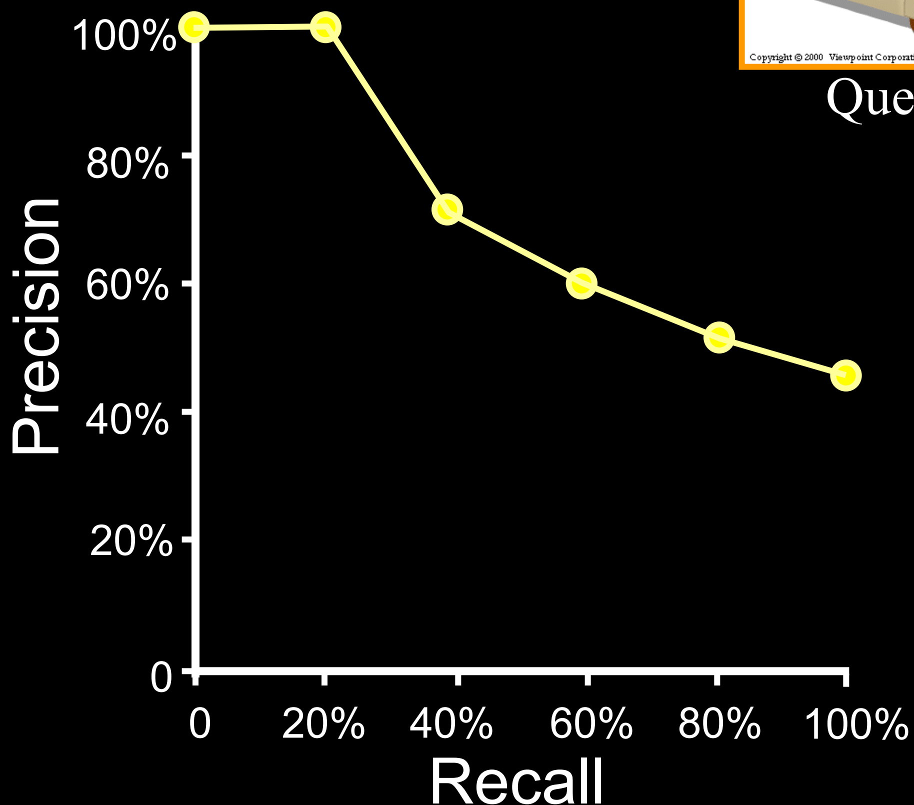
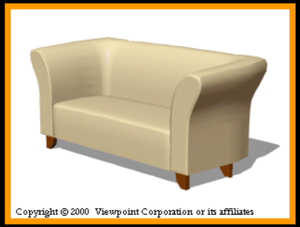
Evaluation Methods

Precision-recall curve example

- Precision = 5 / 9
- Recall = 5 / 5





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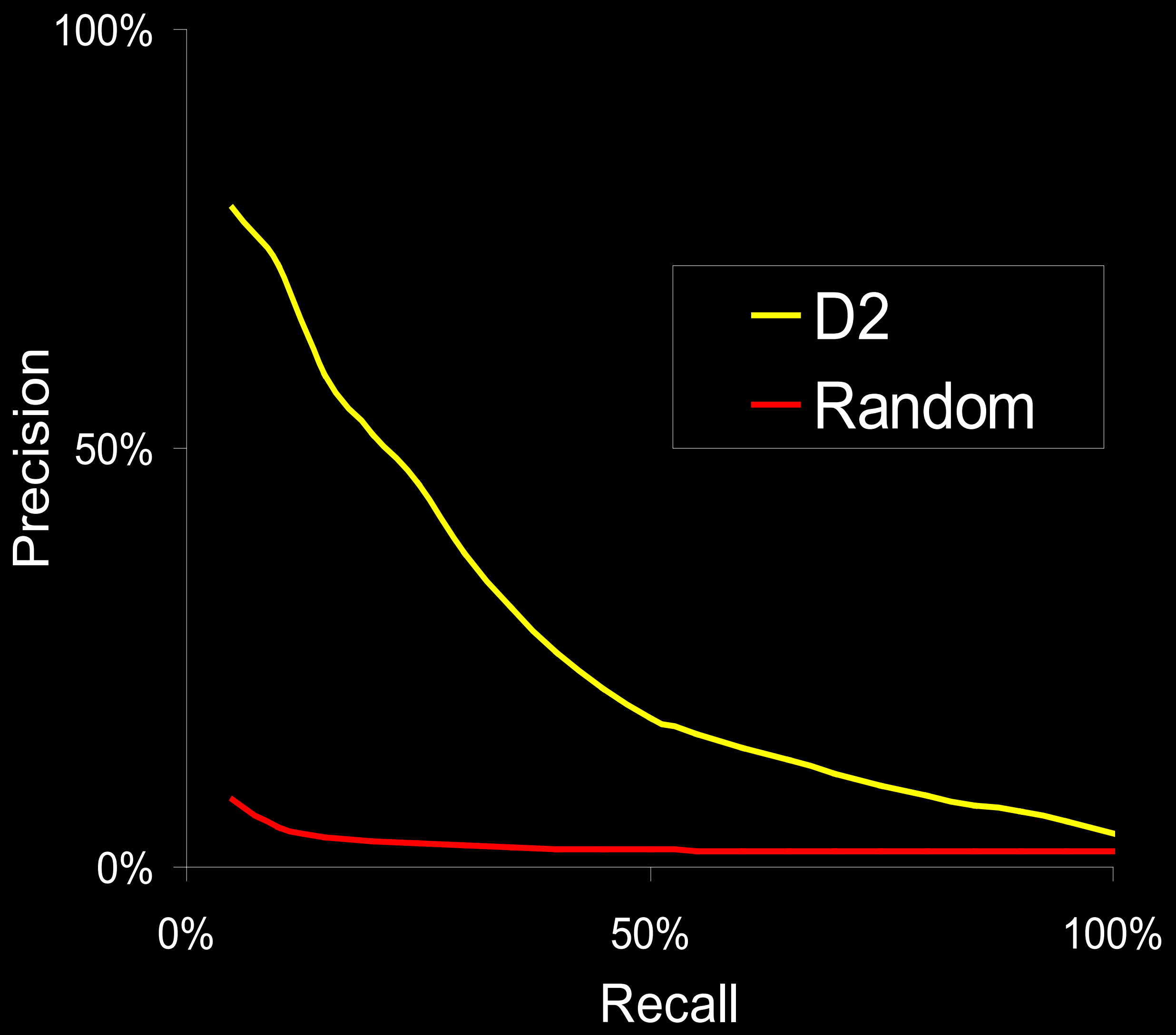
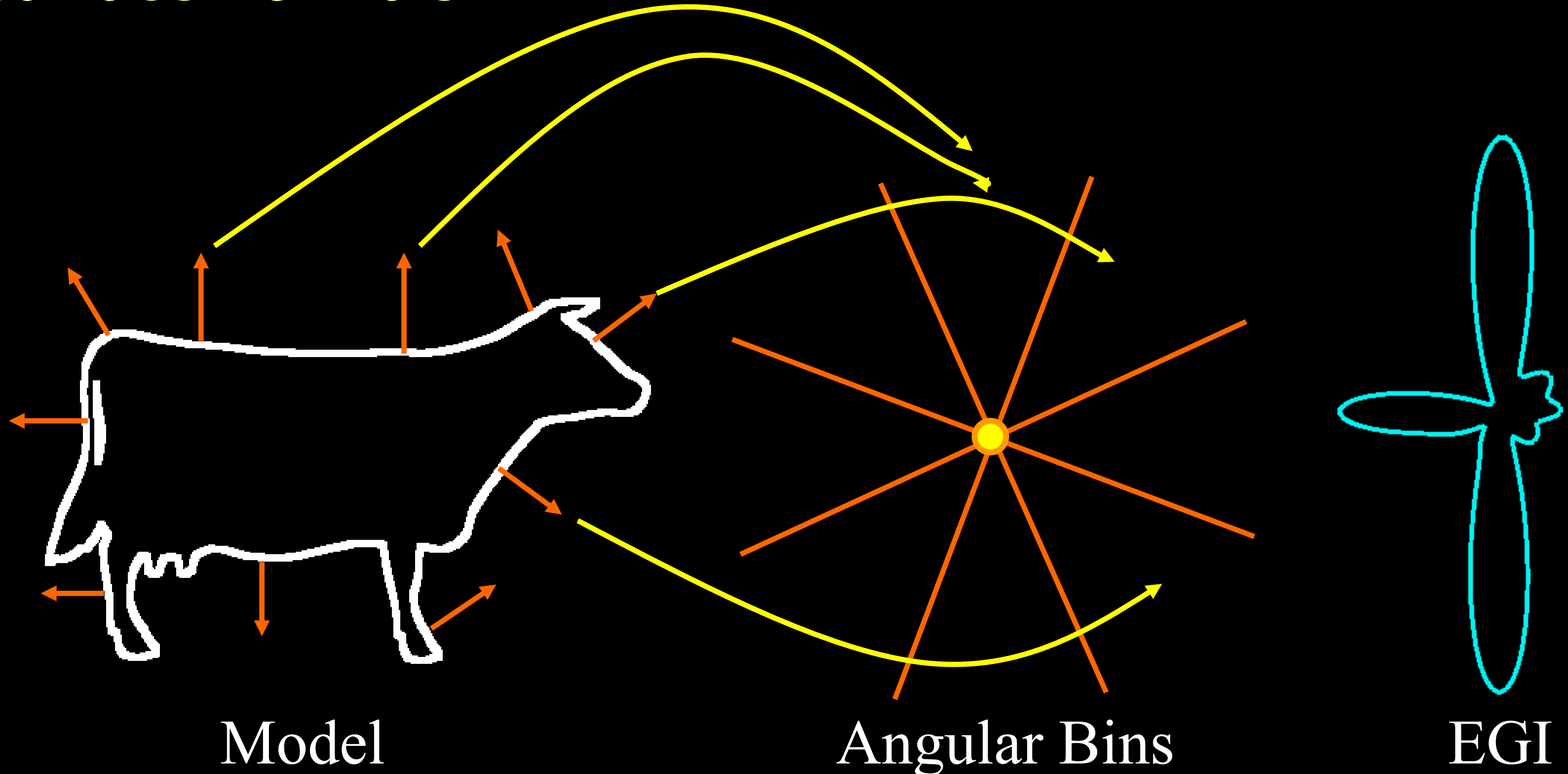


Extended Gaussian Image

[Horn, 1984]

Represent a model by a spherical function by binning surface normals

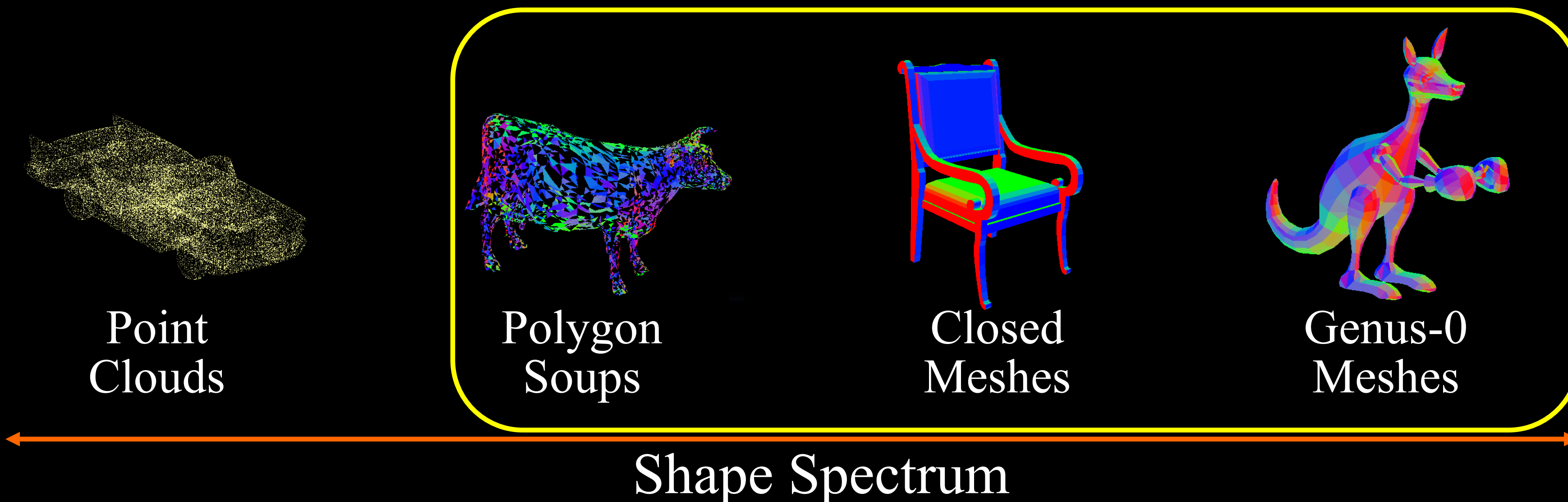


Extended Gaussian Image

[Horn, 1984]

Properties:

- Invertible for convex shapes
- Can be defined for most models
- 2D array of information



Extended Gaussian Image

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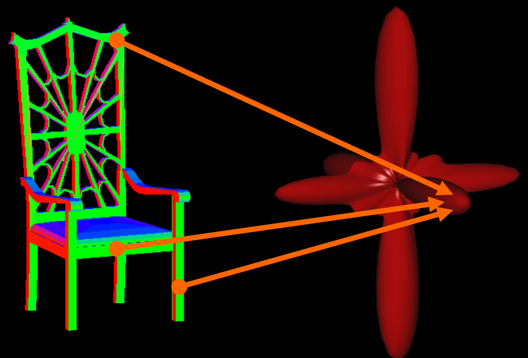
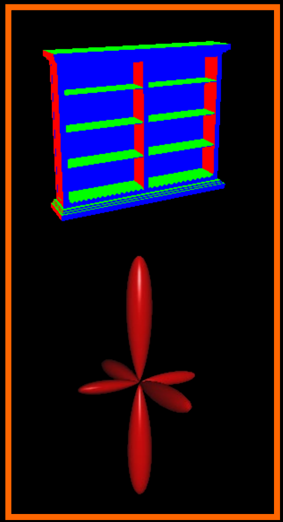
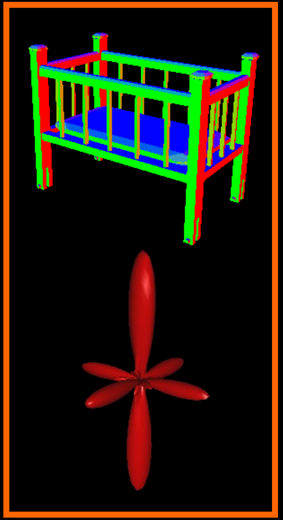
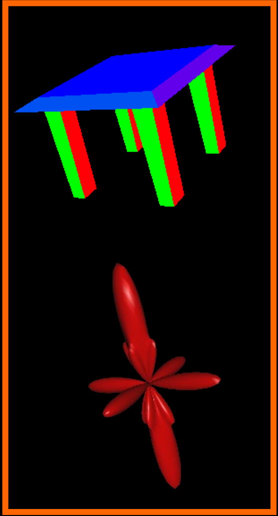
- In general, shapes are not convex
- Normals are sensitive to noise



3D Model EGI







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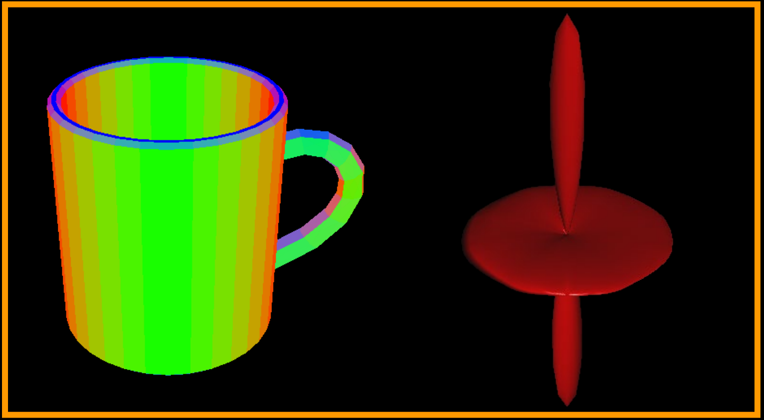
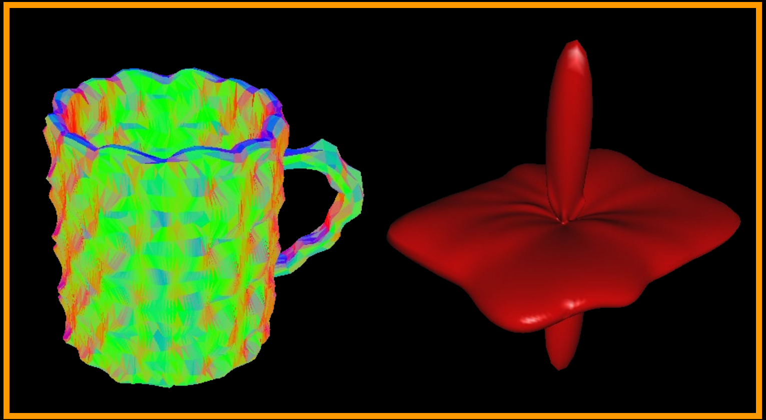


Initial Model

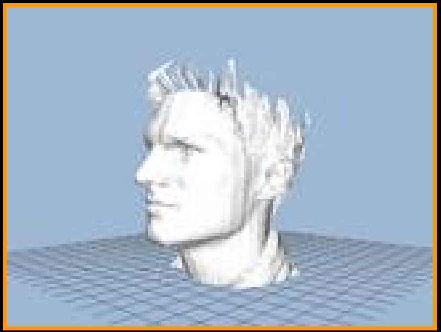


Noisy Model

Retrieval Results

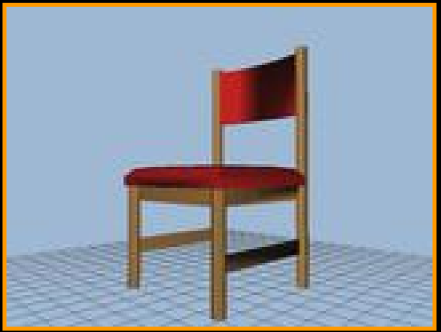
Princeton Shape Benchmark

51 potted plants



33 faces

15 desk chairs



22 dining chairs

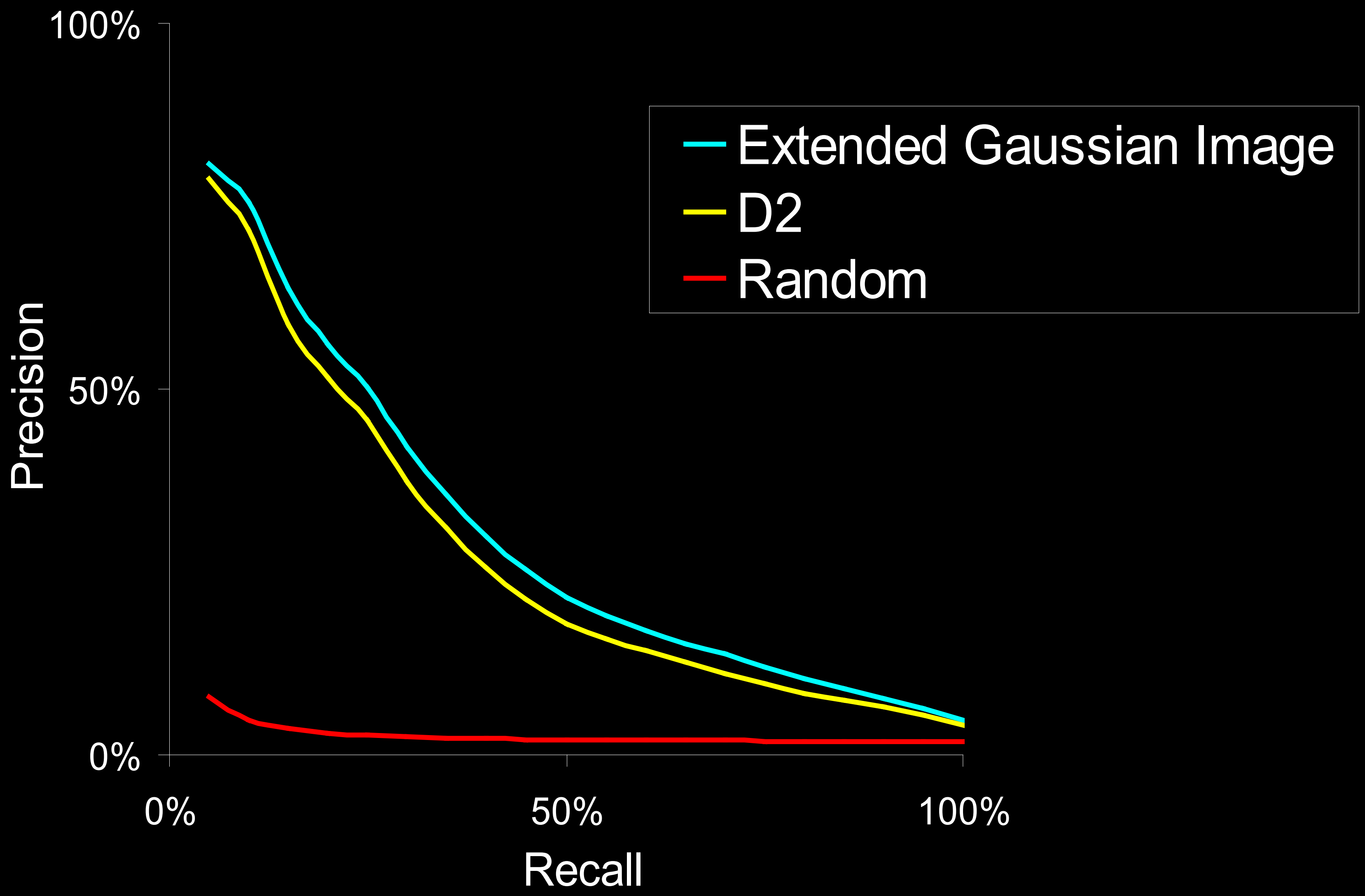
100 humans

28 biplanes

14 flying birds

11 ships

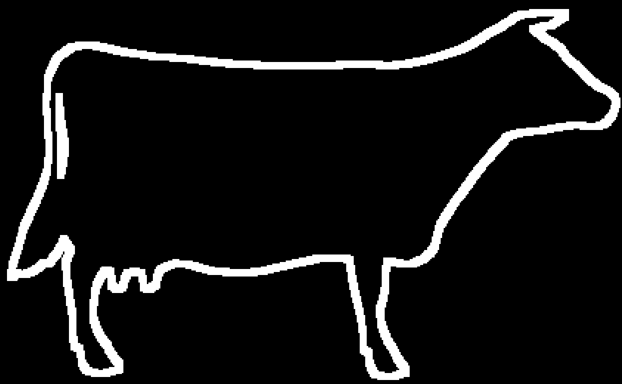
Retrieval Results



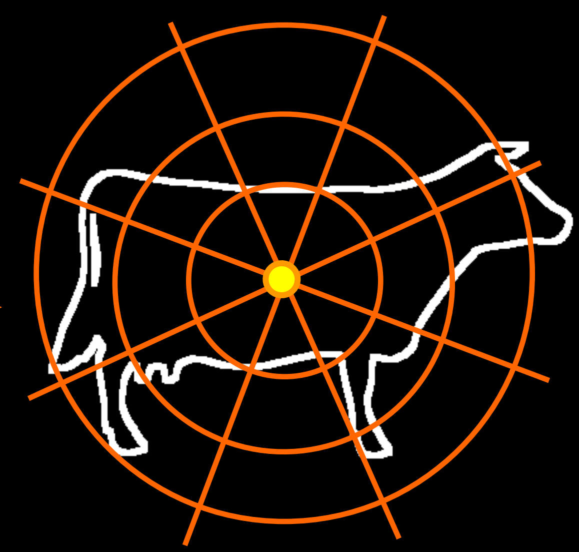
Shape Histograms

[Ankerst *et al.*, 1999]

Shape descriptor stores a histogram of how much surface resides at different bins in space



Model

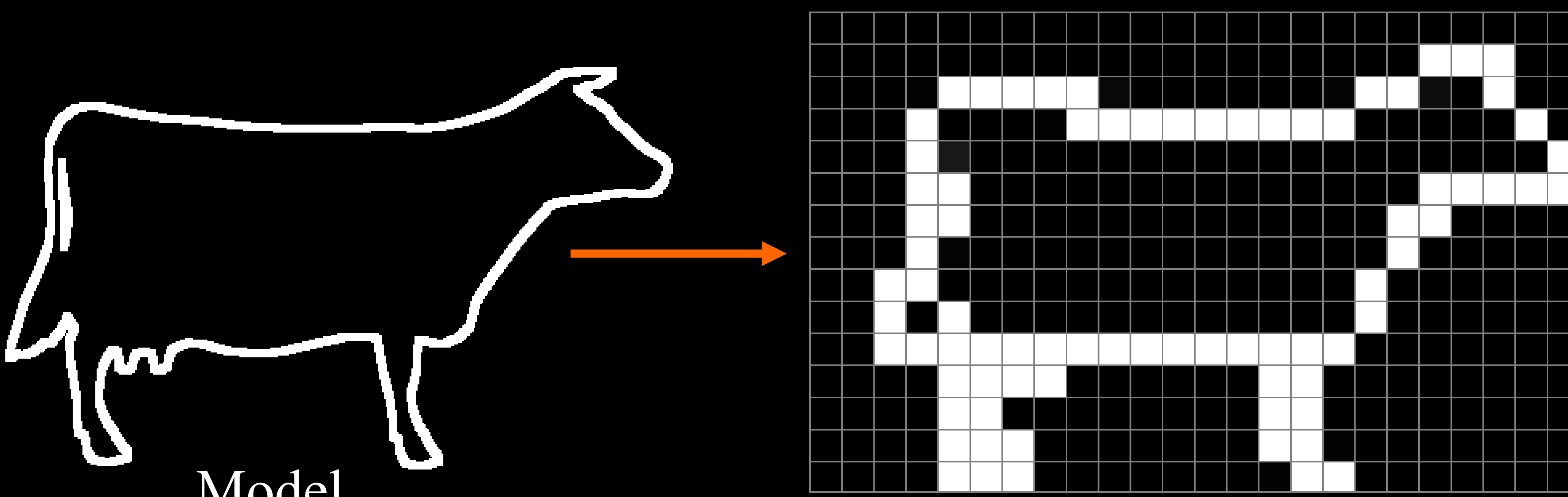


Shape Histogram
(Sectors + Shells)

Boundary Voxel Representation

Represent a model as the (anti-aliased) rasterization of its surface into a regular grid:

- A voxel has value 1 (or area of intersection) if it intersects the boundary
- A voxel has value 0 if it doesn't intersect



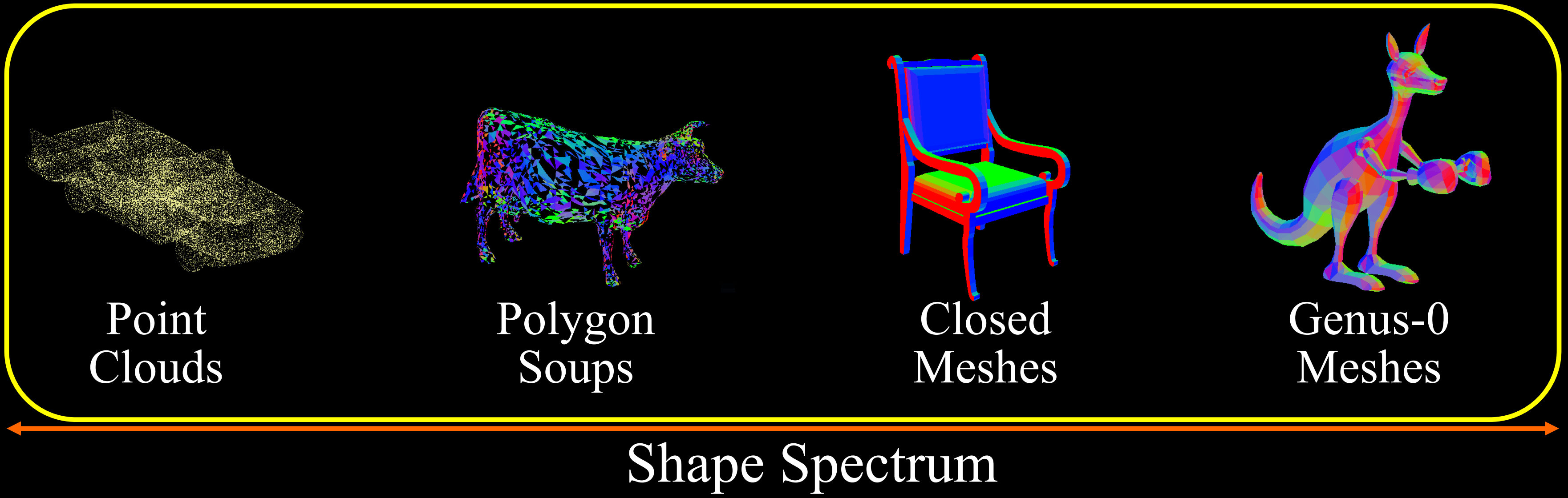
Model

Voxel Grid

Boundary Voxel Representation

Properties:

- Can be defined for any model
- Invertible
- 3D array of information



Boundary Voxel Representation

Properties:

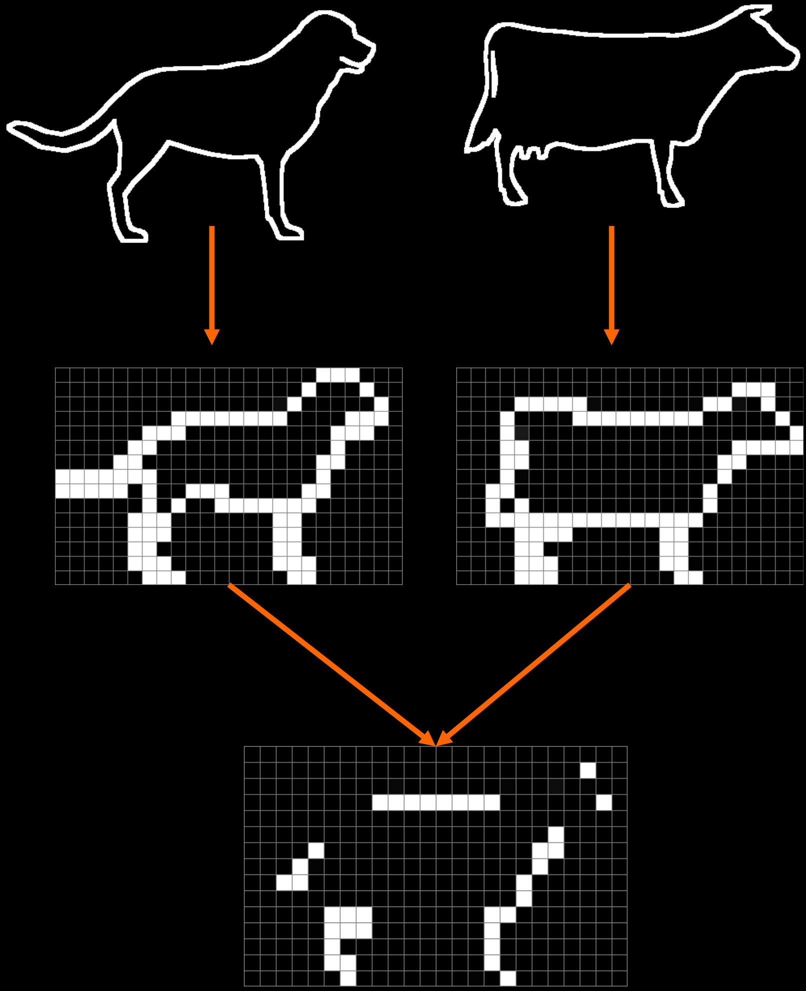
- Can be defined for any model
- Invertible
- 3D array of information

Limitations:

- Difficult to match

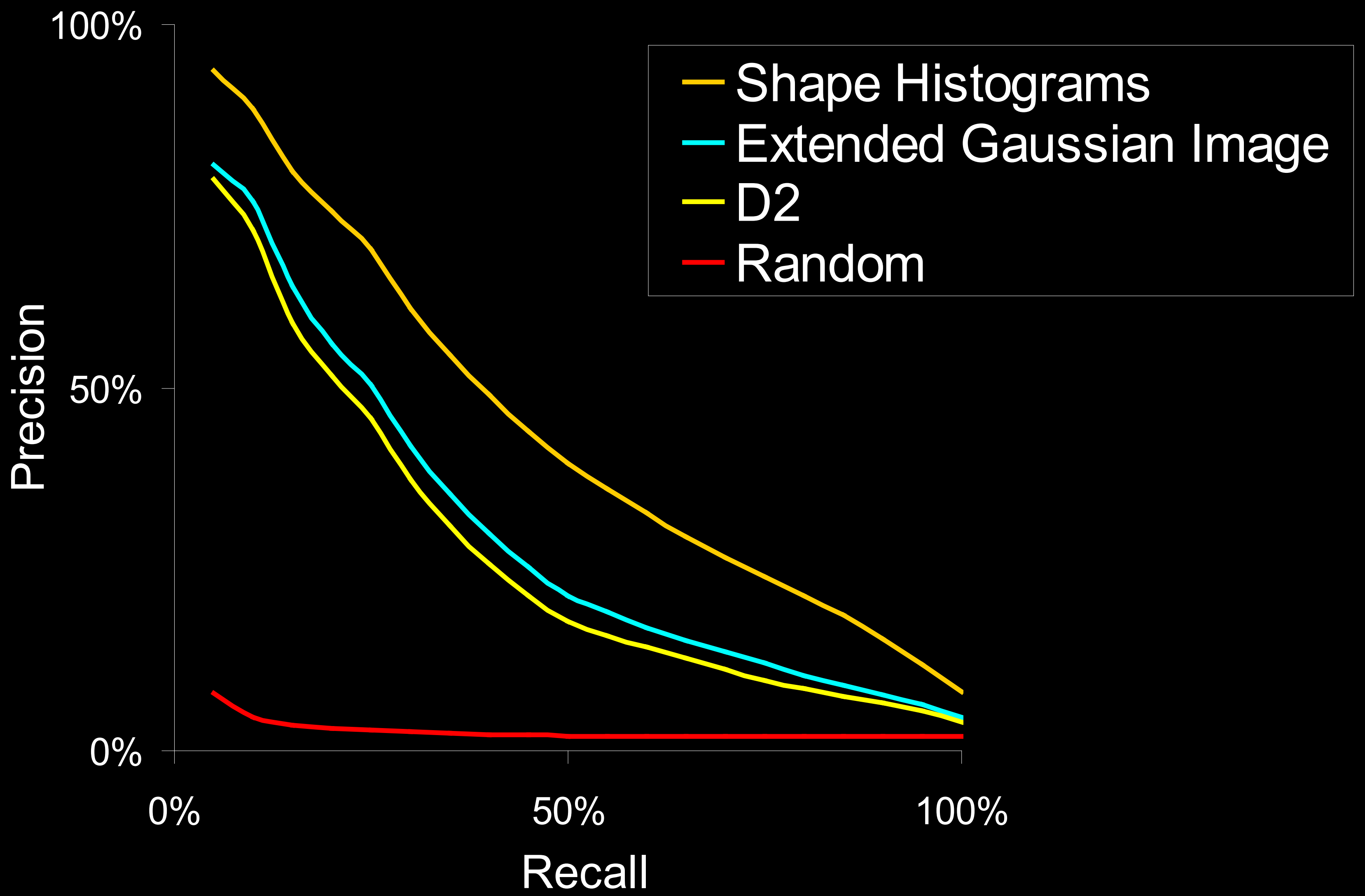
  If the resolution is too high:
  most voxels miss

  If the resolution is too low:
  representation is too coarse



Intersection

Retrieval Results



Histogram Representations

Challenge:

- If shape properties are mapped to nearby bins, they will not be compared

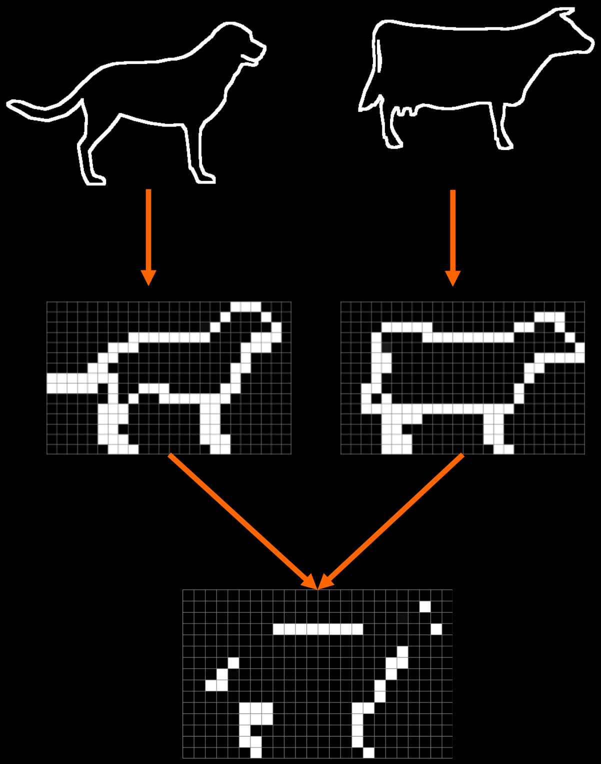
Solutions:

- Match across adjacent bins:

  Earth Mover's Distance

- Low-pass filter:

  Convolution with a Gaussian



Earth Mover's distance

[Rubner *et al.* 1998]

Match by computing the minimal amount of work needed to transform one distribution into the other

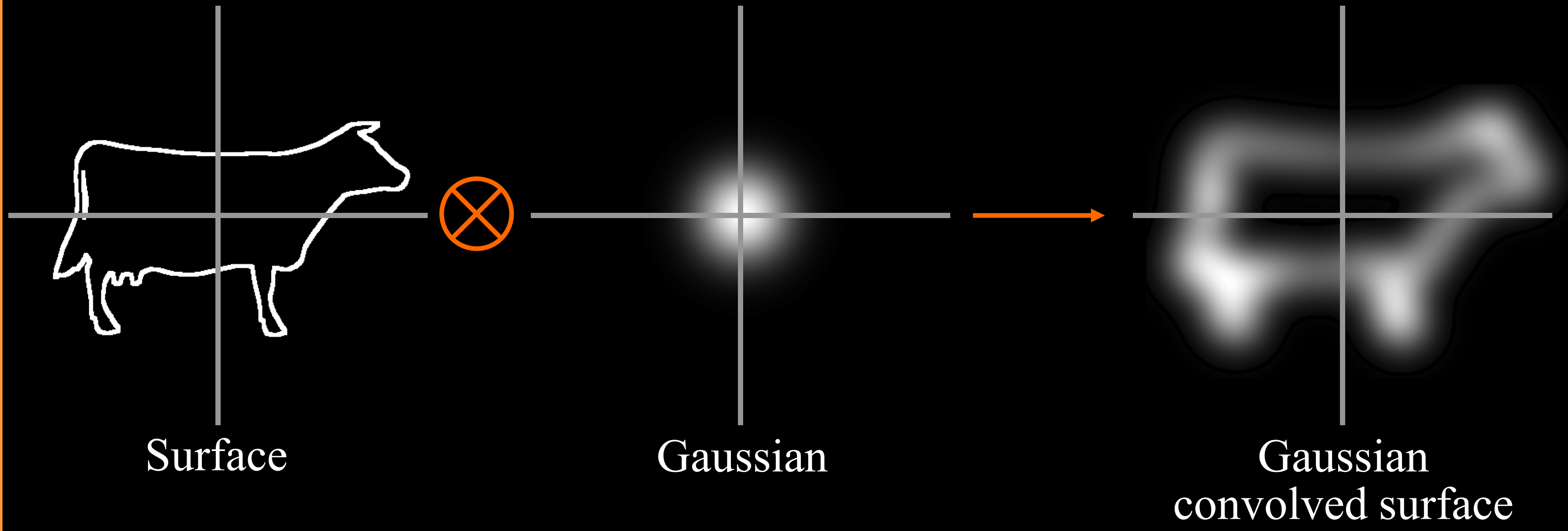
Computing the distance:

- For 1D histograms can use the CDF to compare efficiently
- In general, need to solve the transportation problem which is inefficient for large numbers of bins

Convolving with a Gaussian

The value at a point is obtained by summing Gaussians distributed over the surface of the model

- ✓ Distributes the surface into adjacent bins
- ✗ Blurs the model, loses high frequency information

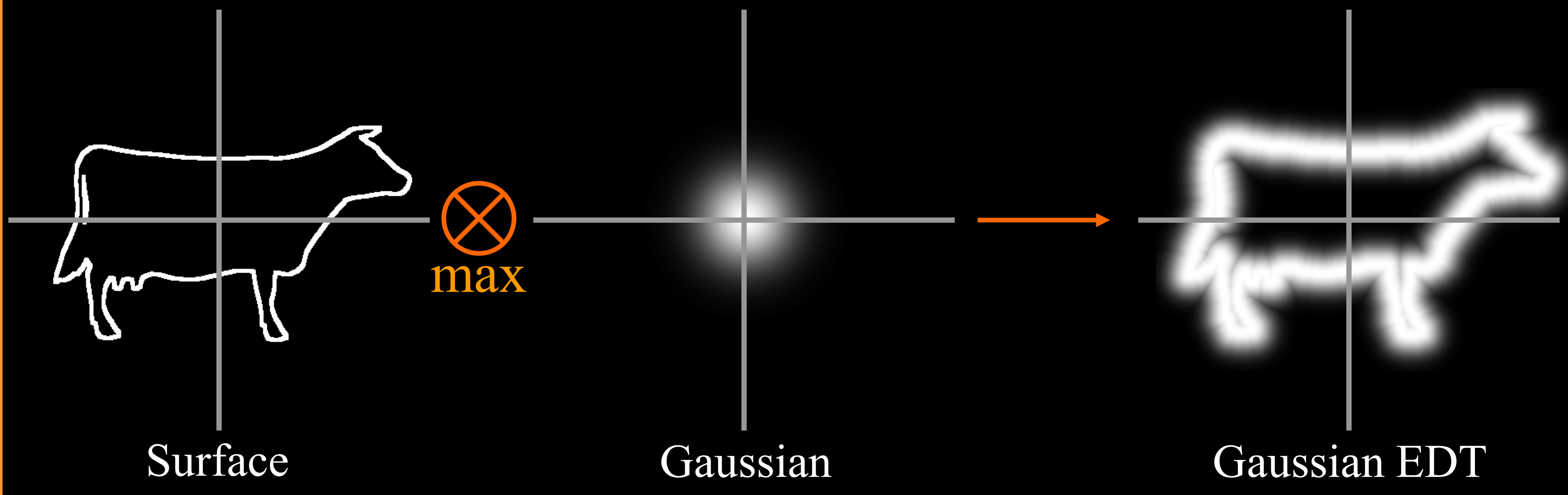


Gaussian EDT

[Kazhdan *et al.*, 2003]

The value at a point is Gaussian applied to distance to closest point on the surface

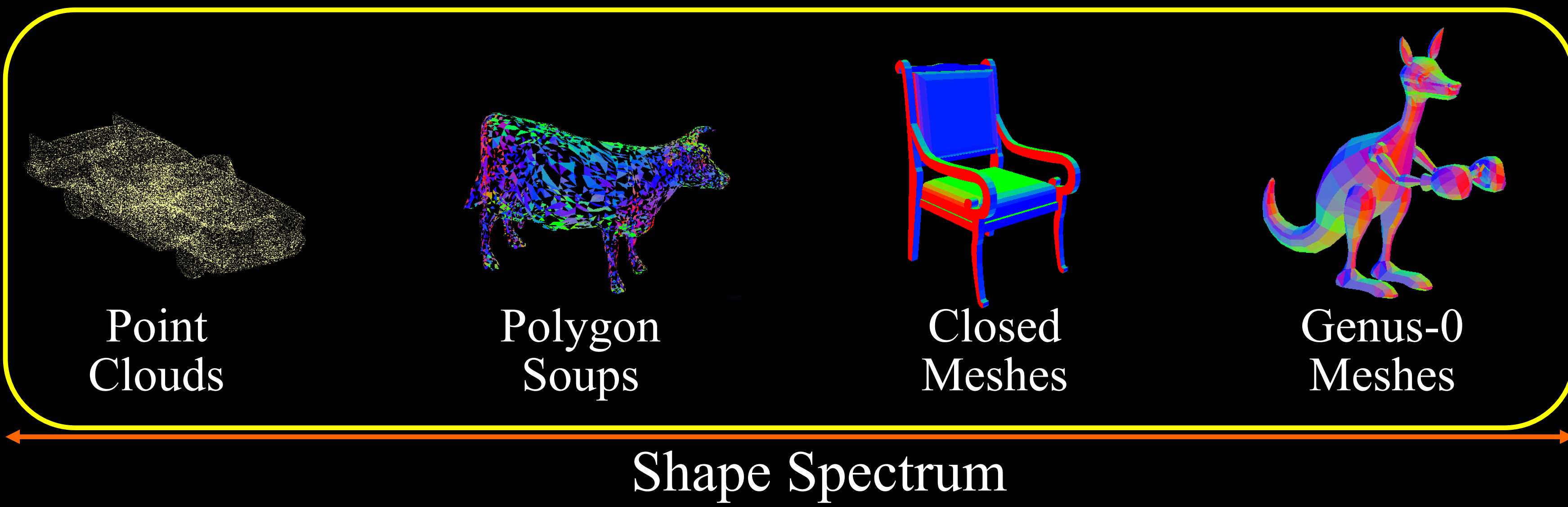
- ✓ Distributes the surface into adjacent bins
- ✓ Maintains high-frequency information



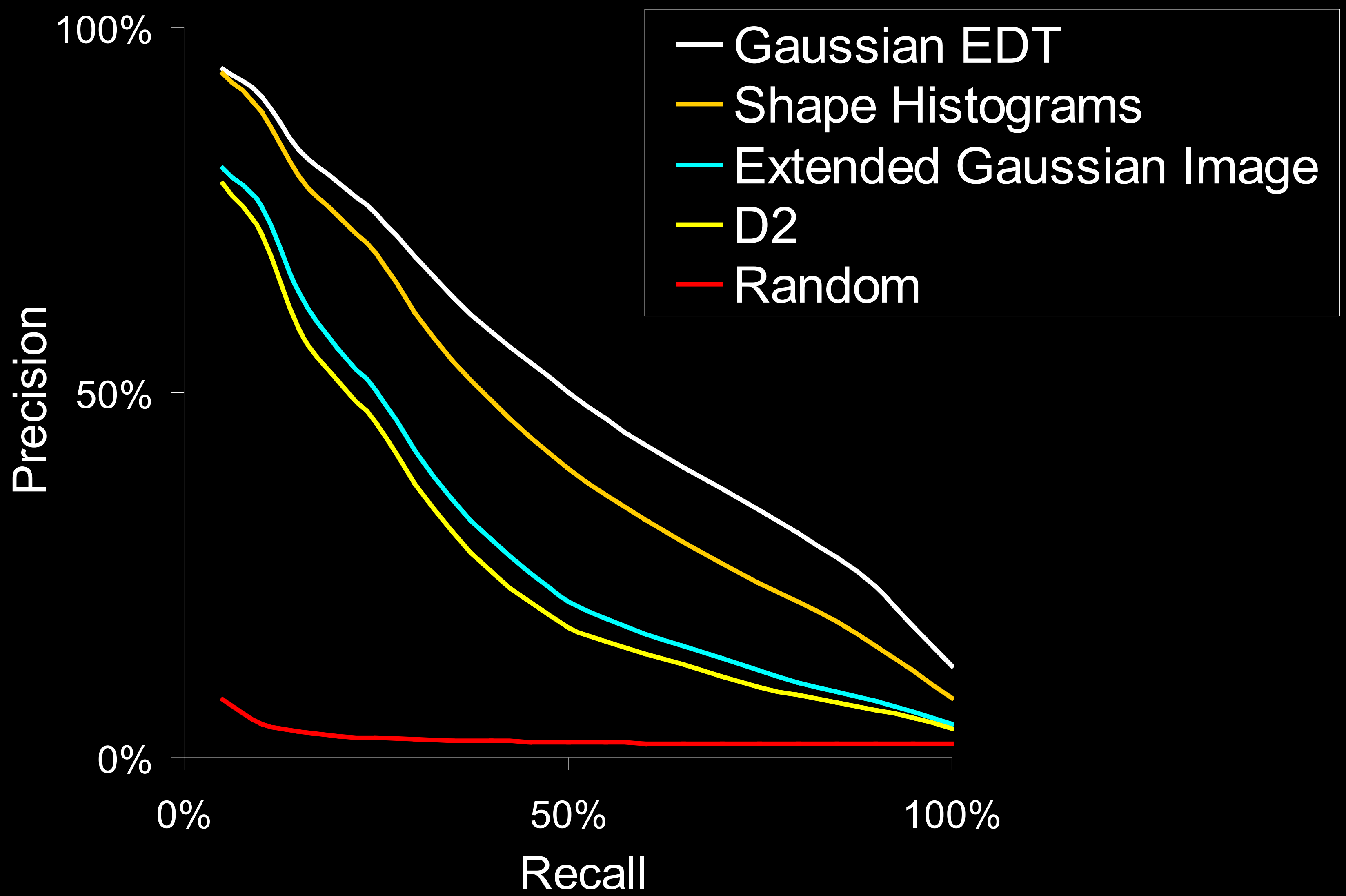
Gaussian EDT

Properties:

- Can be defined for any model
- Invertible
- 3D array of information
- Difference measures proximity between surfaces



Retrieval Results



Handling Transformations

Key difficulty:

locating objects under any rigid-body transformation

Approaches:

- Exhaustive search: try all possibilities
- Invariance: use descriptors that do not change under transformations
- Normalization: align objects to canonical coordinate frame

Exhaustive Search

Search for the best aligning transformation:

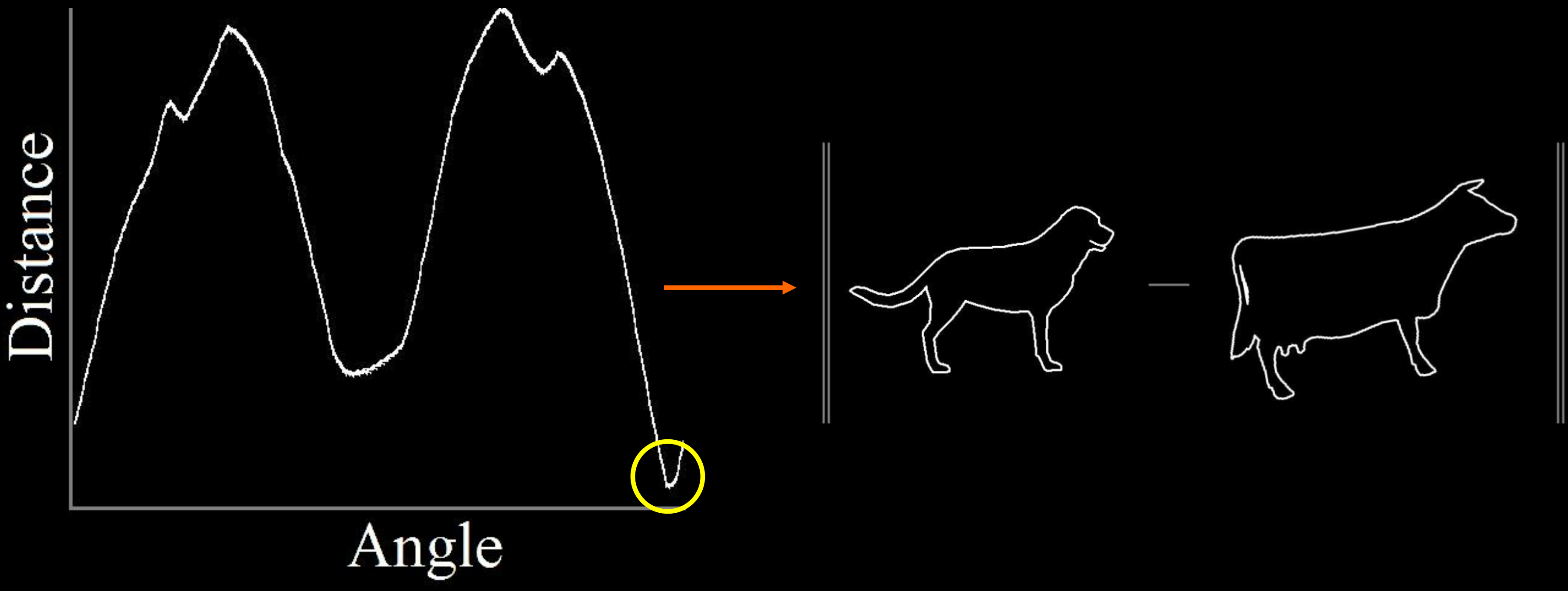
- Compare at all alignments
- Match at the alignment for which models are closest

Exhaustive search for optimal rotation

Exhaustive Search

Search for the best aligning transformation:

- Compare at all alignments
- Match at the alignment for which models are closest



Exhaustive Search

Search for the best aligning transformation:

- Use signal processing for efficient correlation
- Represent model at many different transformations

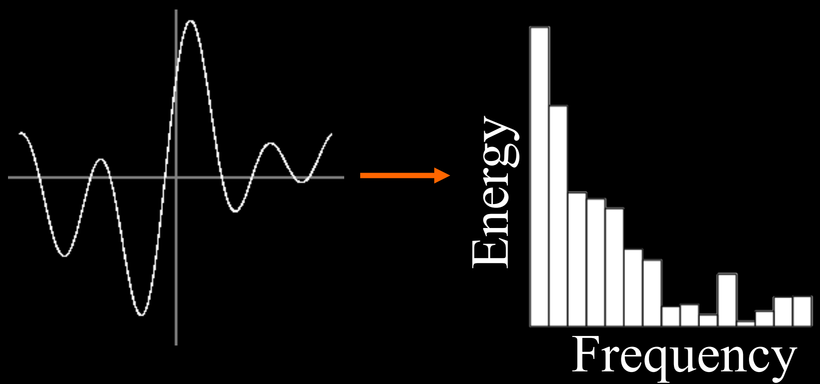
Search for the best aligning transformation:

- Gives the correct answer
- Is hard to do efficiently

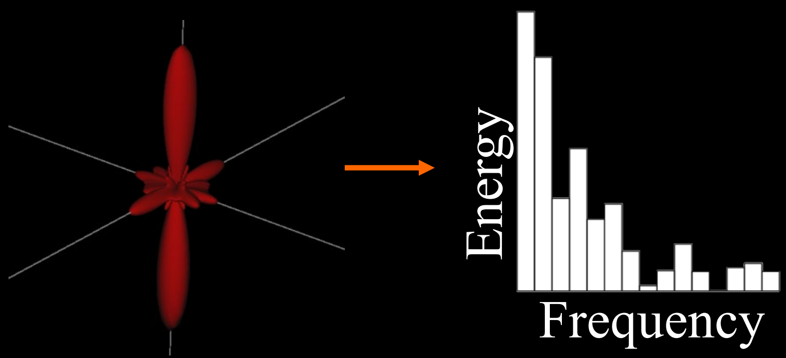
Invariance

Represent a model with information that is independent of the transformation

- Power spectrum representation
  - Fourier Transform for translation and 2D rotations
  - Spherical Harmonic Transform for 3D rotations



Circular Power Spectrum

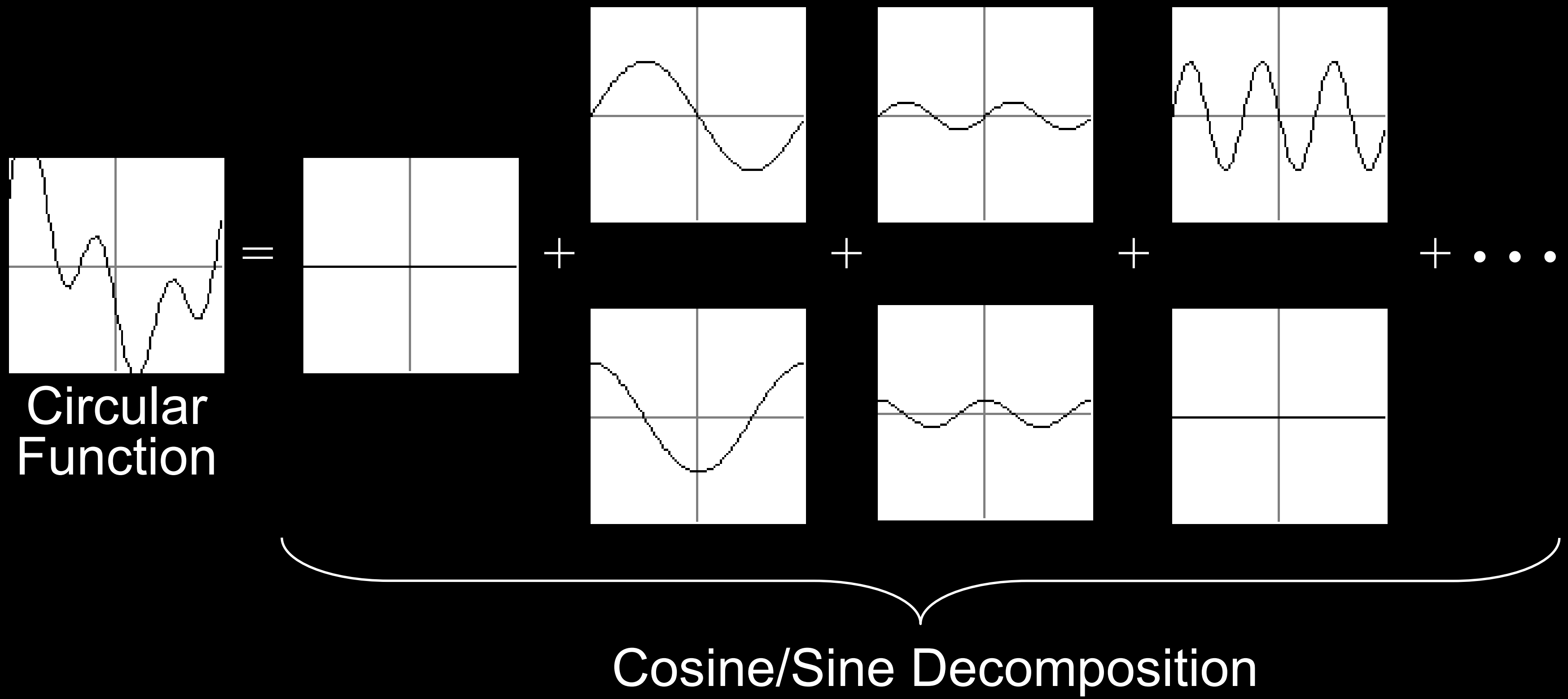


Spherical Power Spectrum

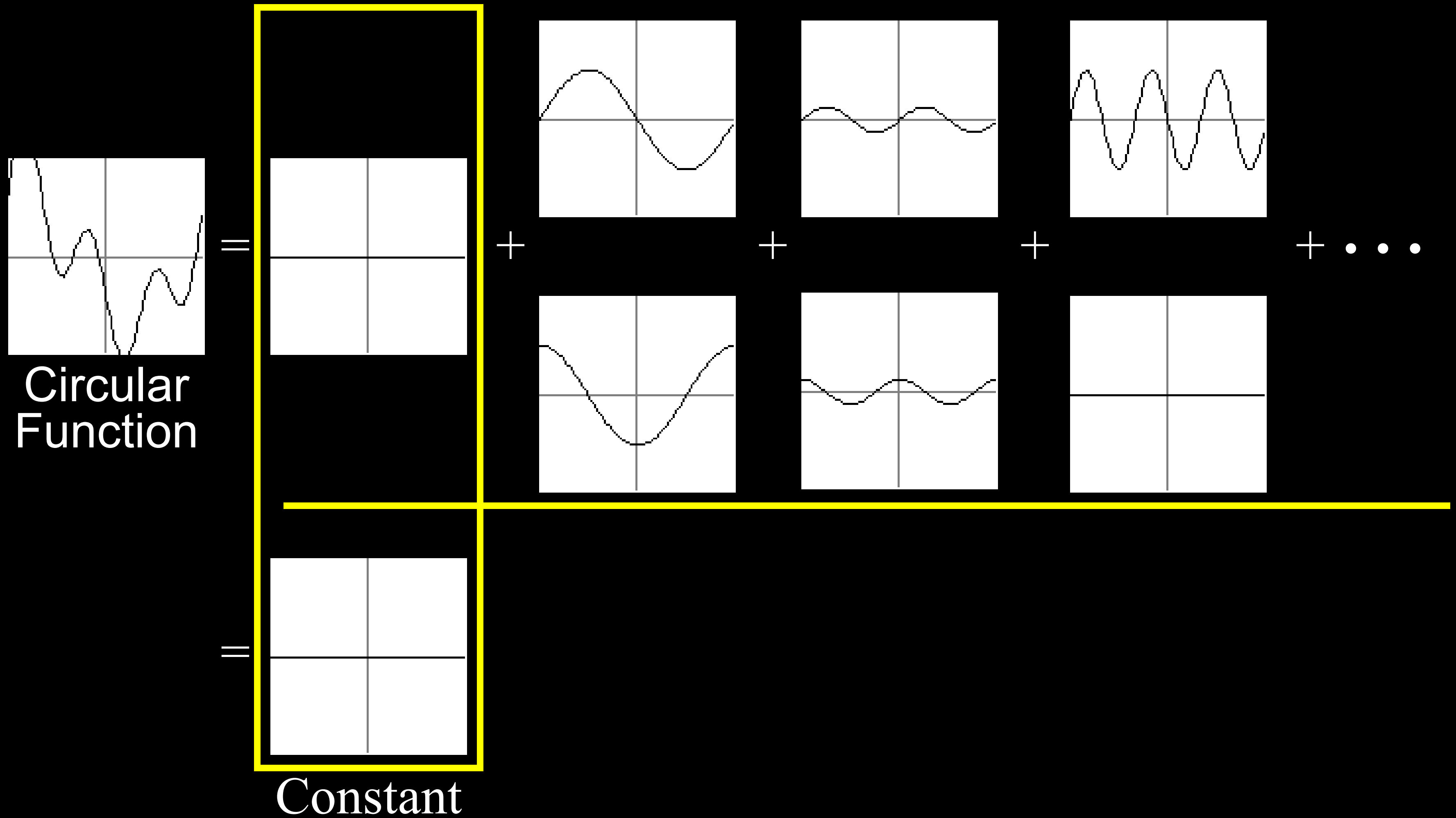
Circular Power Spectrum

Circular
Function

Circular Power Spectrum

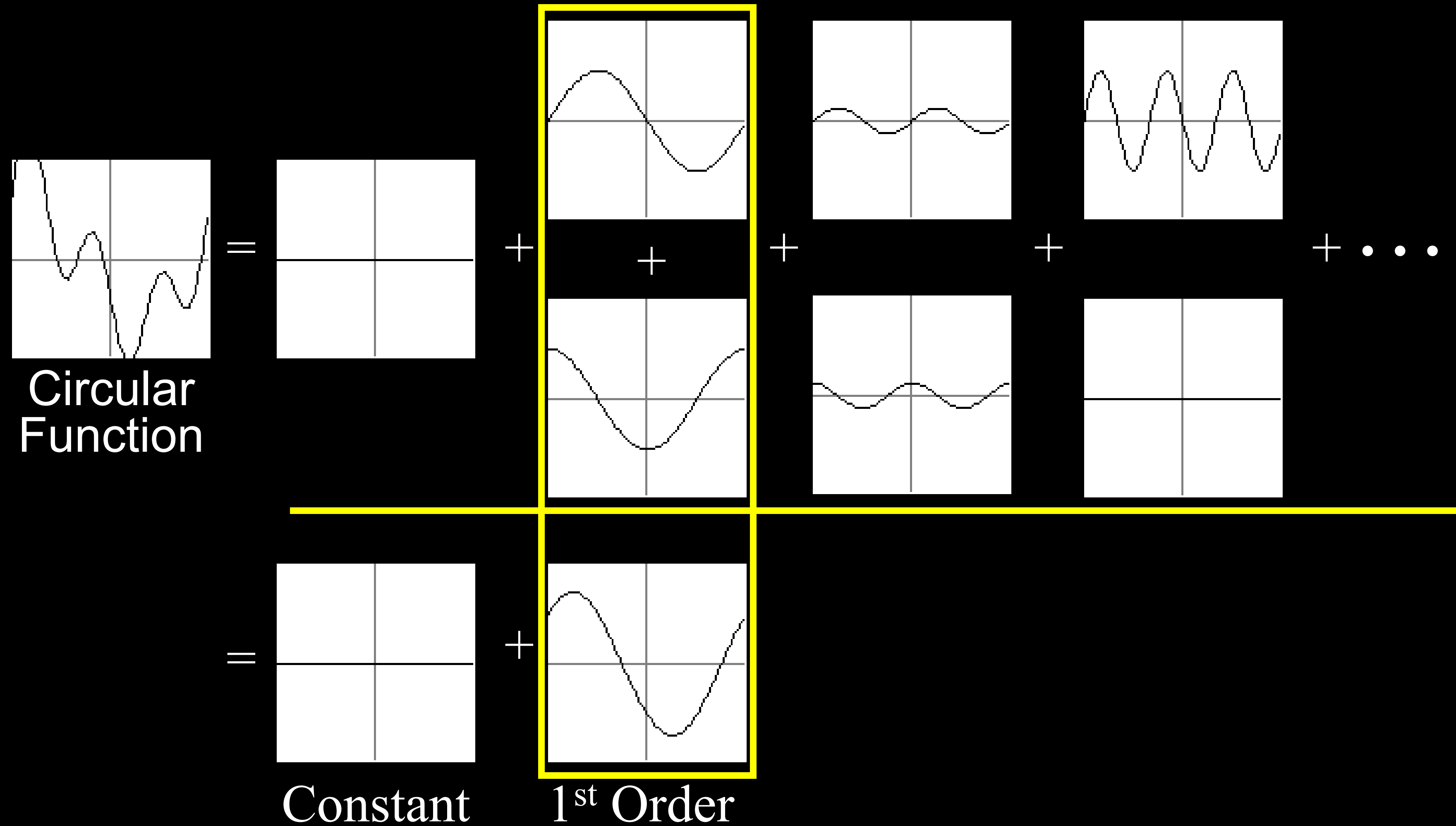


Circular Power Spectrum



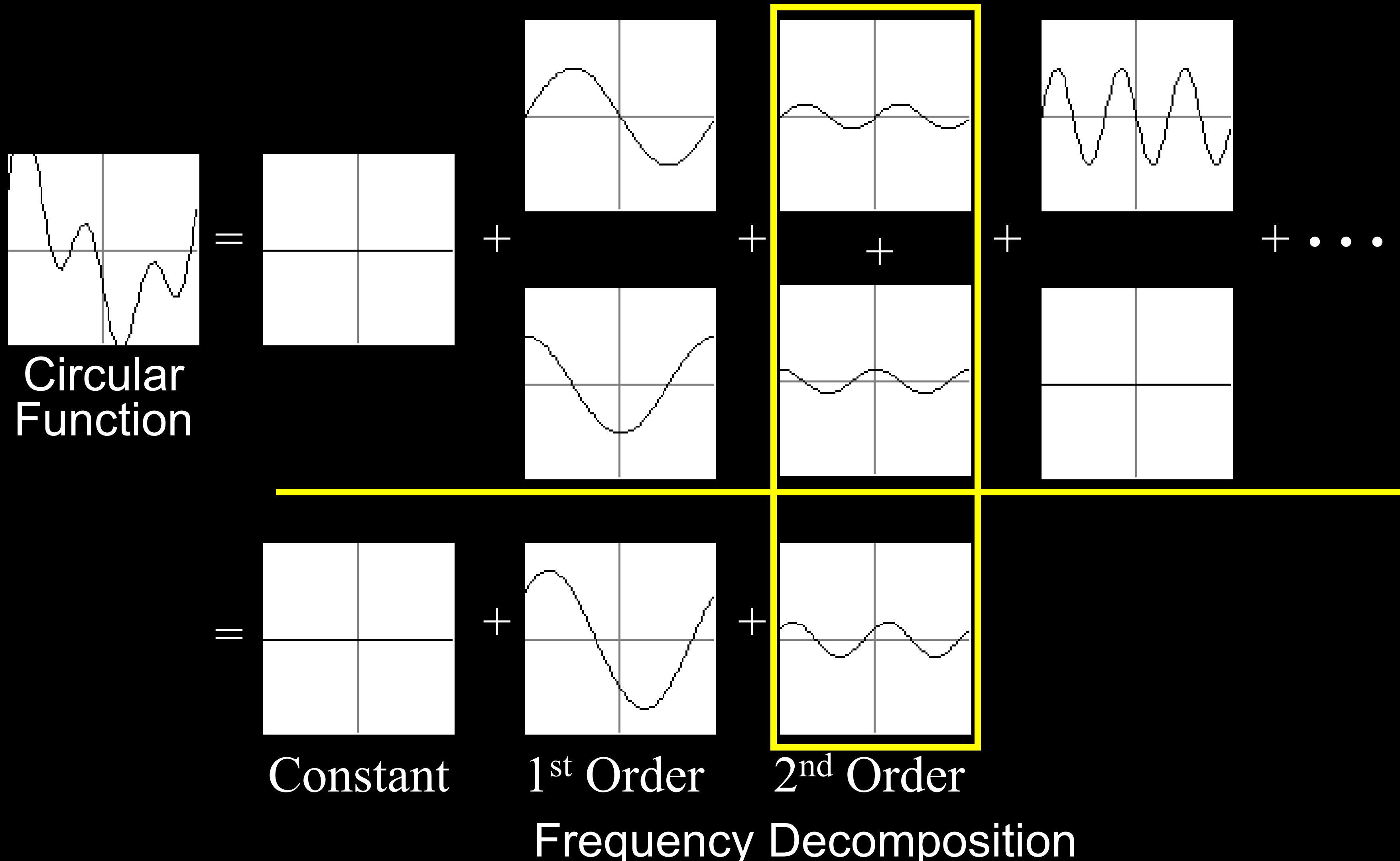
Frequency Decomposition

Circular Power Spectrum

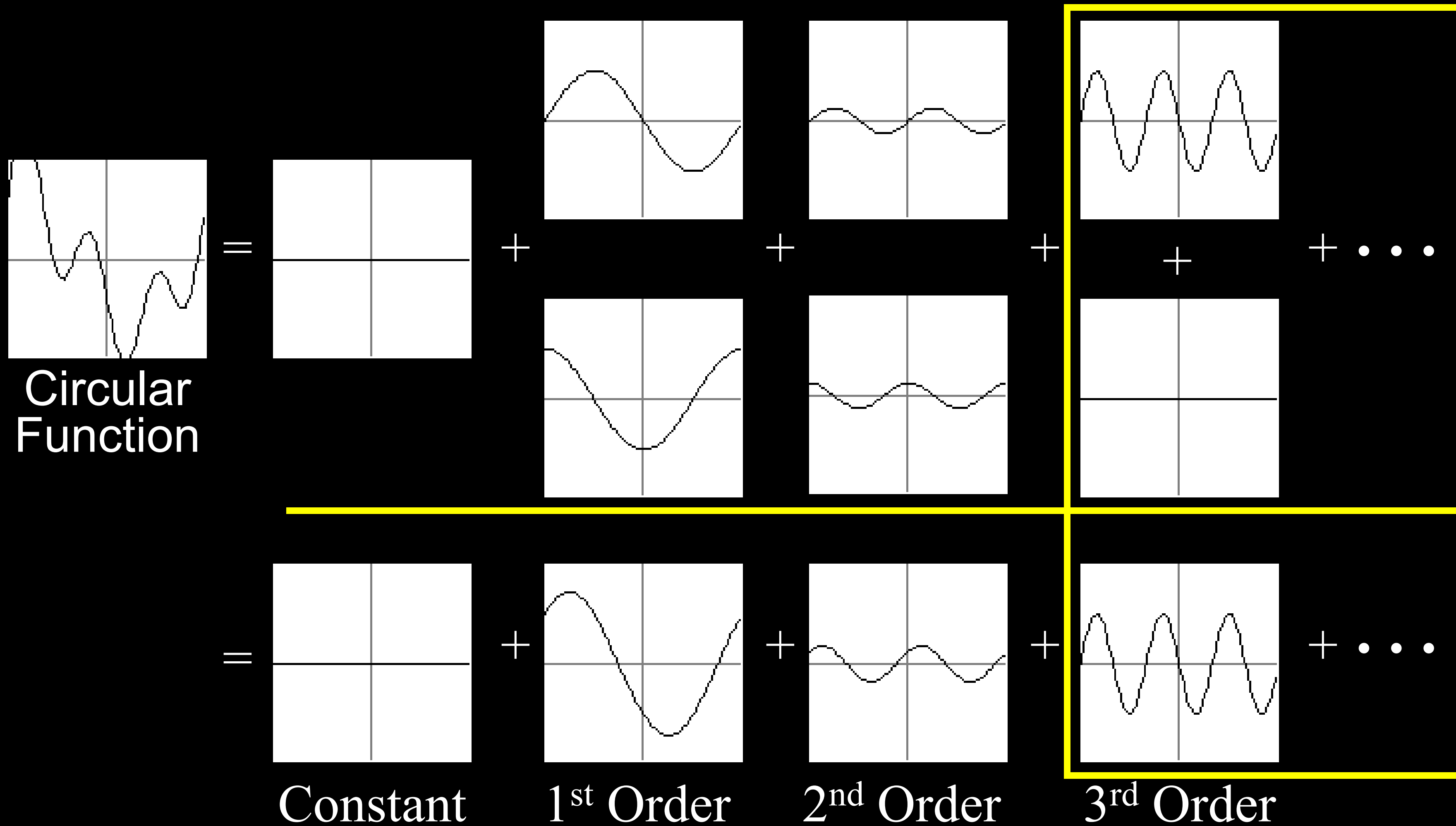


Frequency Decomposition

Circular Power Spectrum

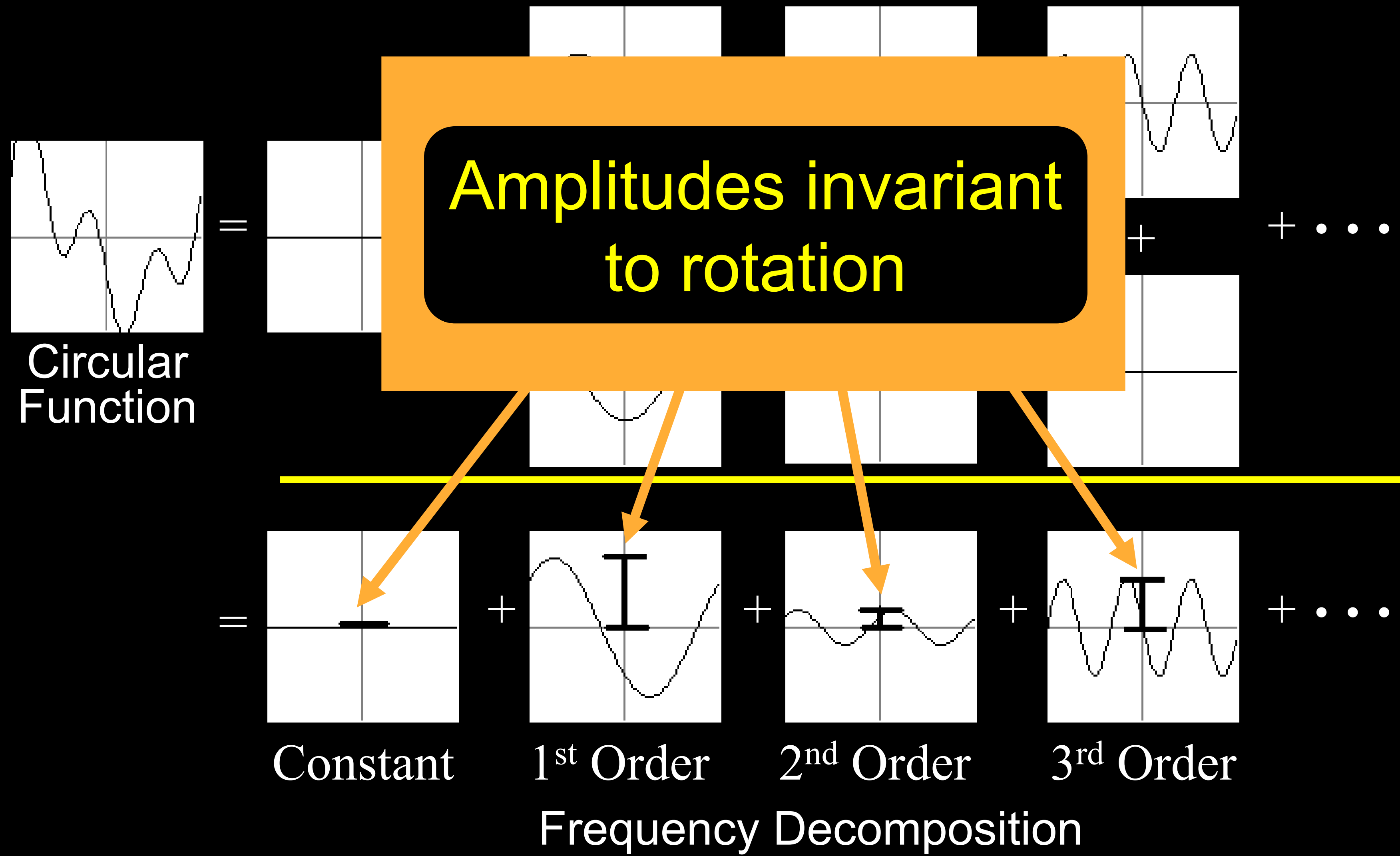


Circular Power Spectrum



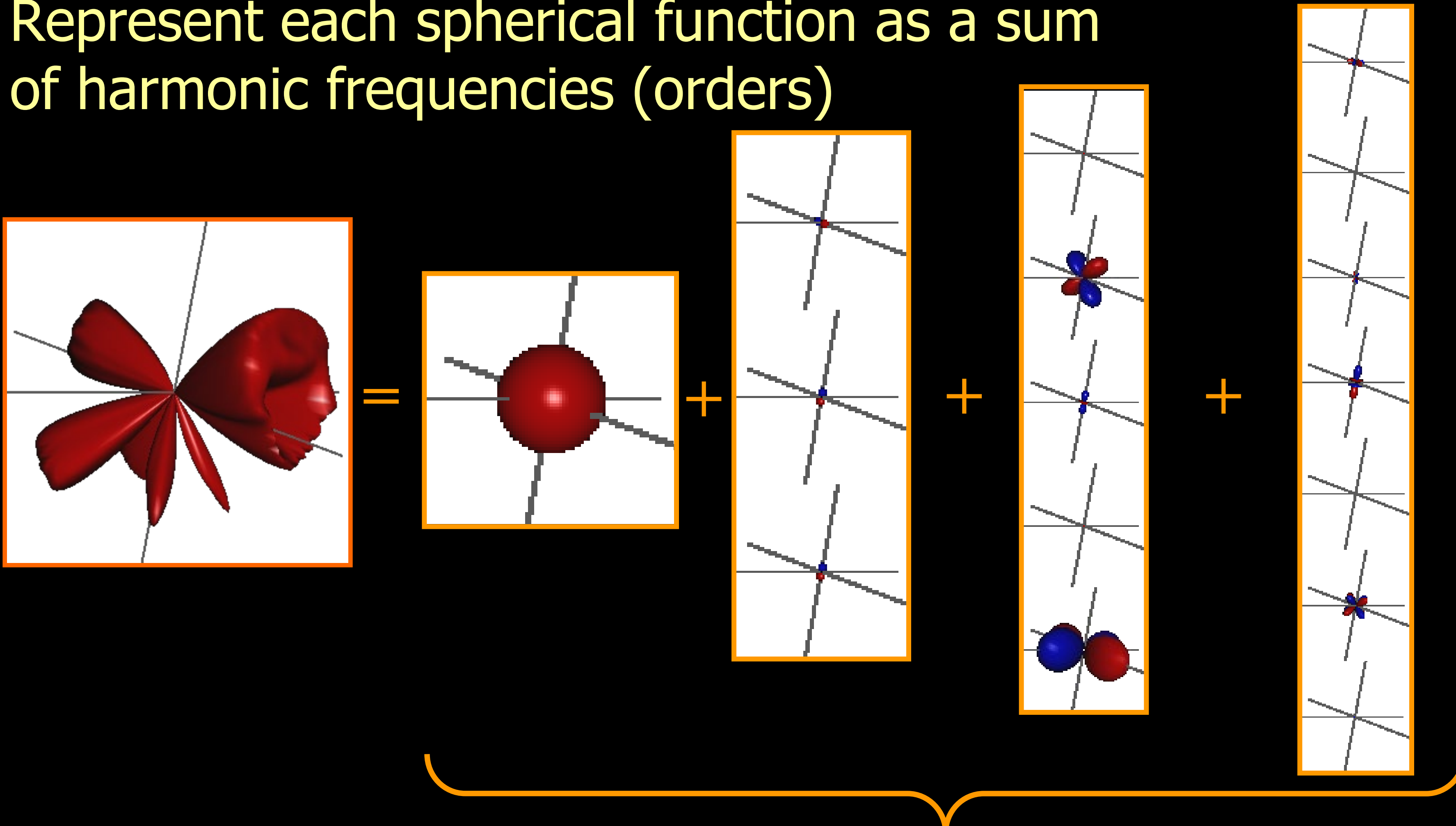
Frequency Decomposition

Circular Power Spectrum



Spherical Power Spectrum

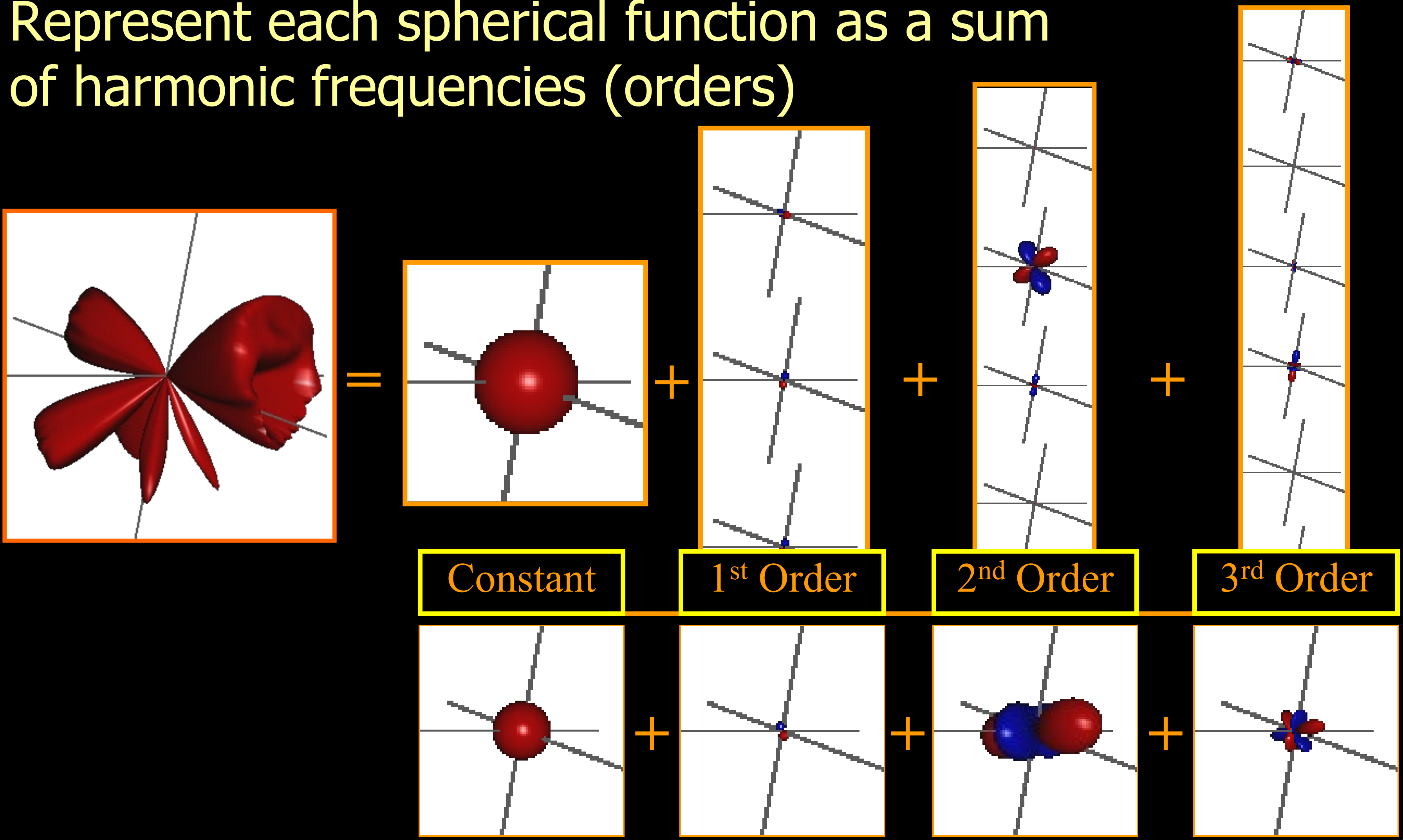
Represent each spherical function as a sum of harmonic frequencies (orders)



Harmonic Decomposition

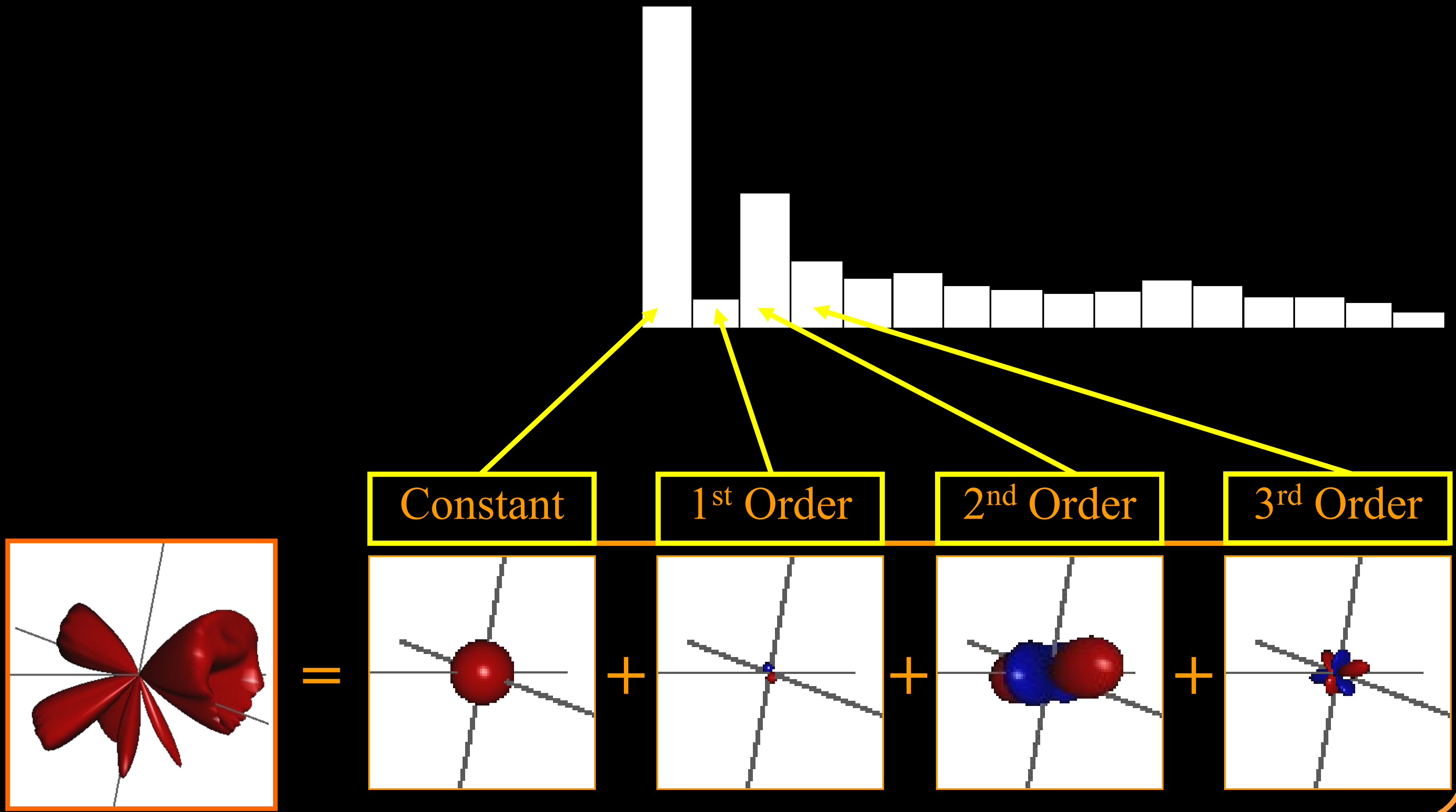
Spherical Power Spectrum

Represent each spherical function as a sum of harmonic frequencies (orders)



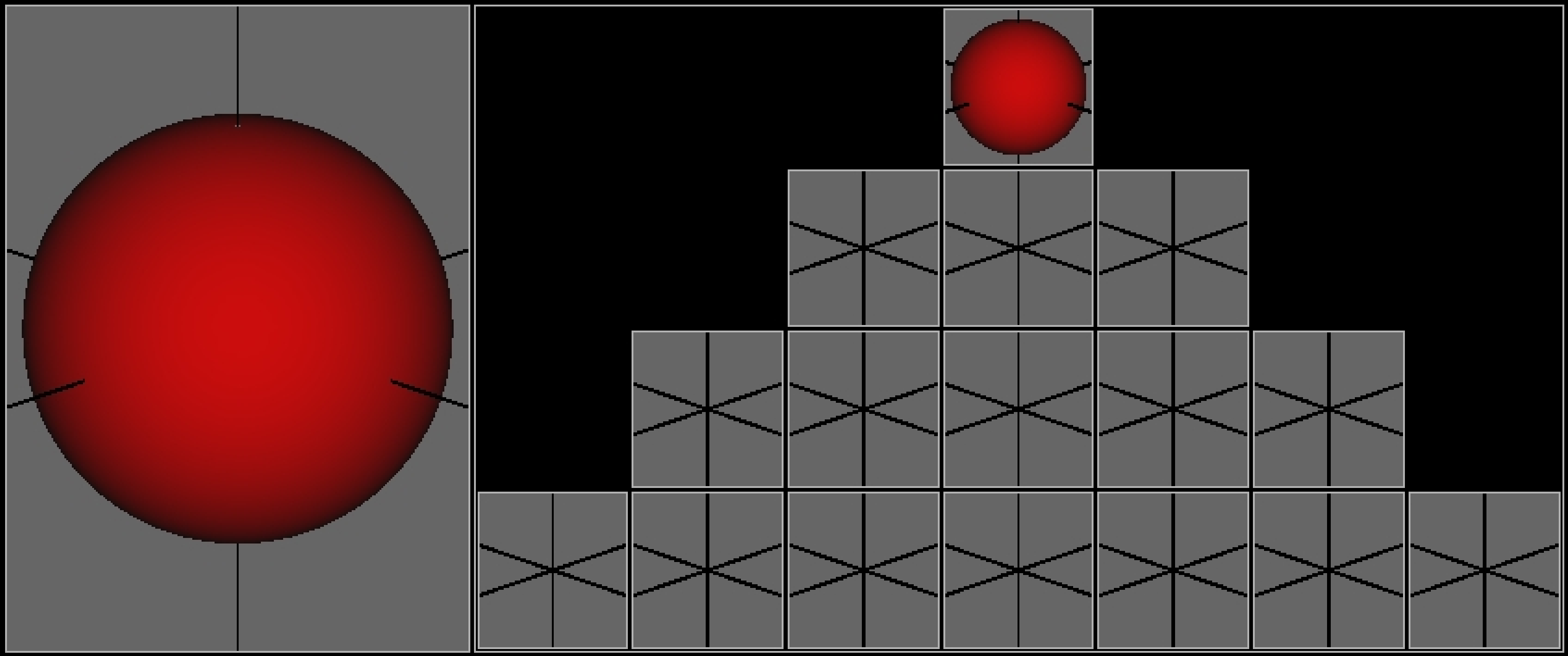
Spherical Power Spectrum

Store "how much" (L_2-norm) of the shape resides at each frequency to get rotation invariant representation



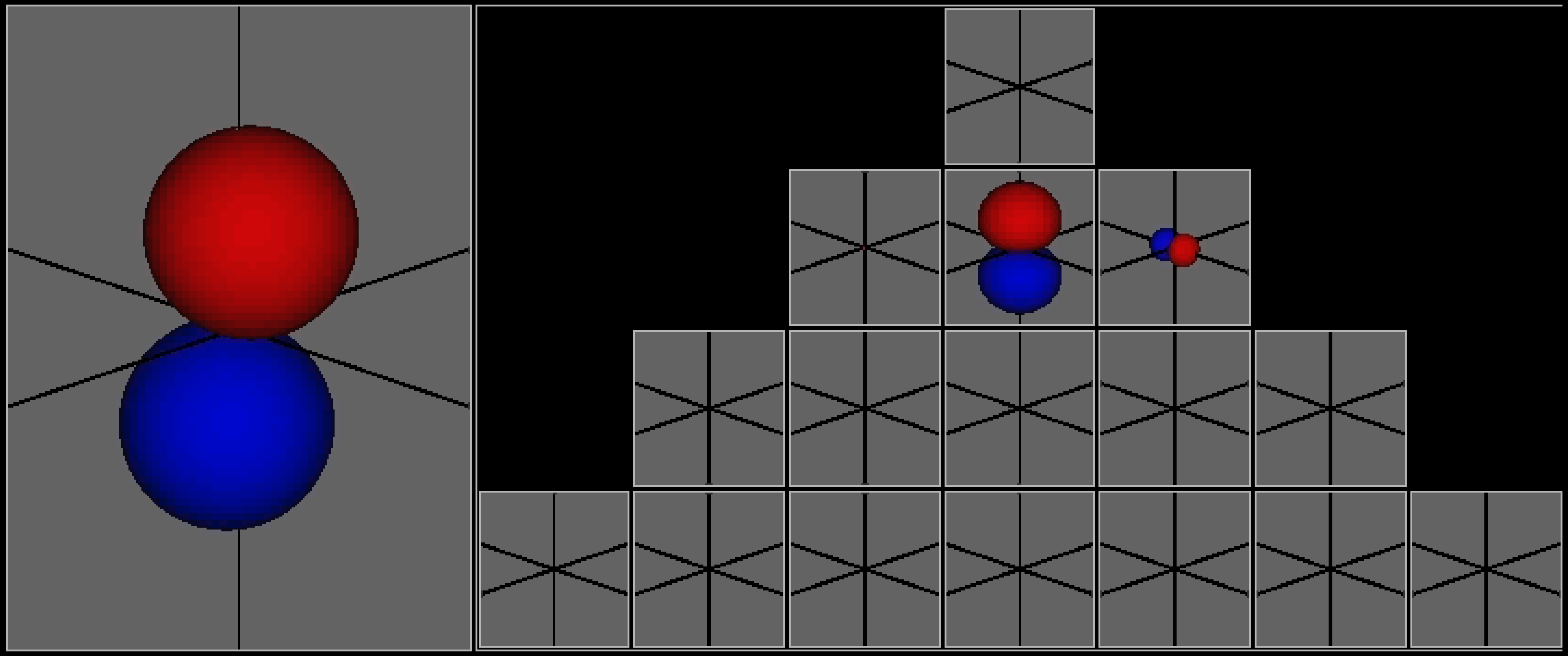
Invariant to transforms?

- Frequency subspaces are fixed by rotations:



Invariant to transforms?

- Frequency subspaces are fixed by rotations:



Invariant to transforms?

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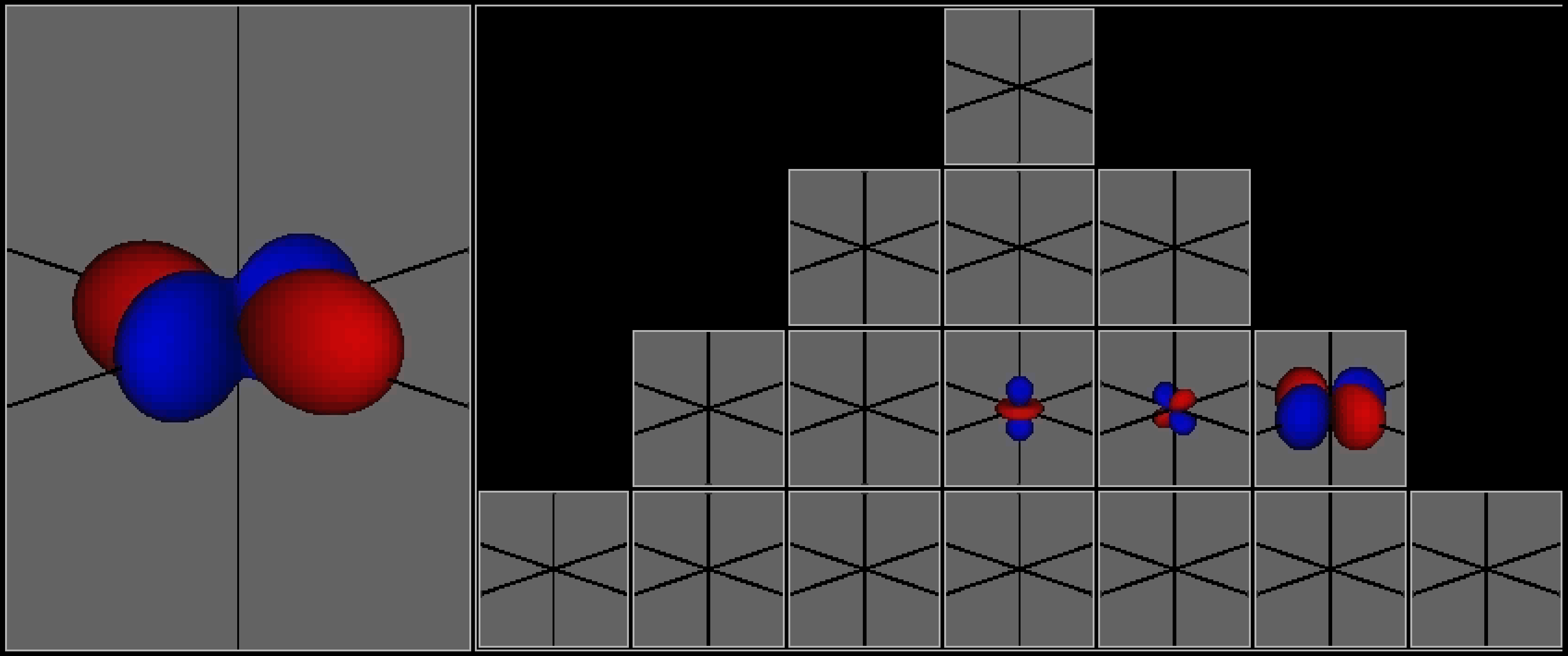

Invariant to transforms?

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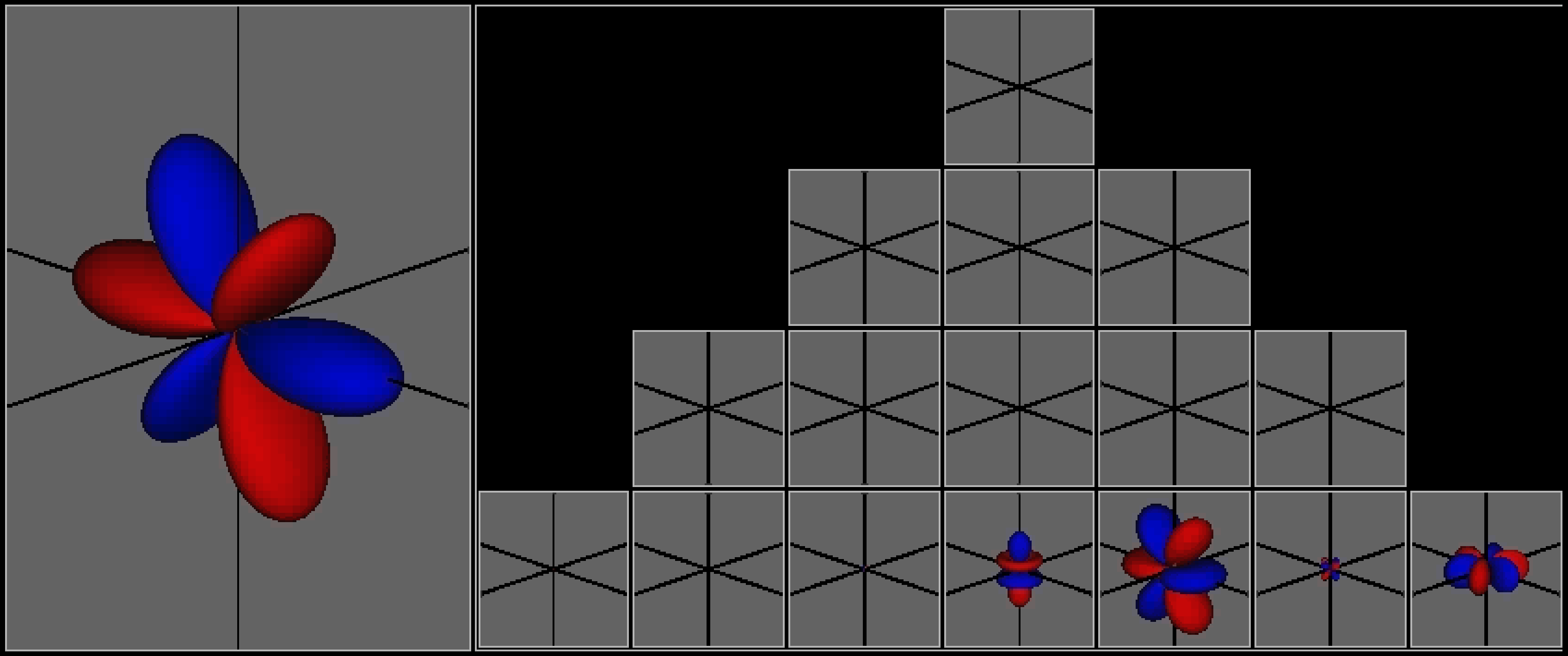
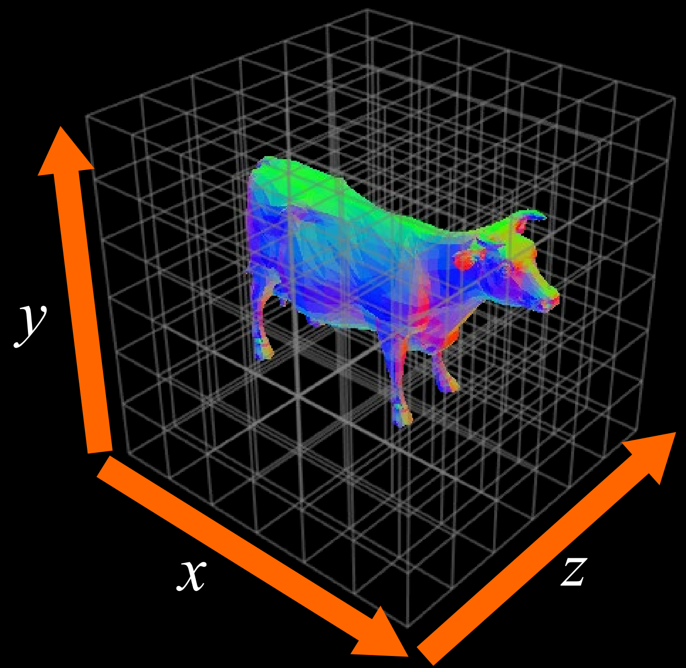


Power Spectrum

Translation-invariance:

- Represent the model in a Cartesian coordinate system
- Compute the 3D Fourier transform
- Store the amplitudes of the frequency components



Cartesian Coordinates

$$f(x,y,z)=\sum_{l,m,n} f_{l,m,n} e^{i(lx+my+zn)}$$

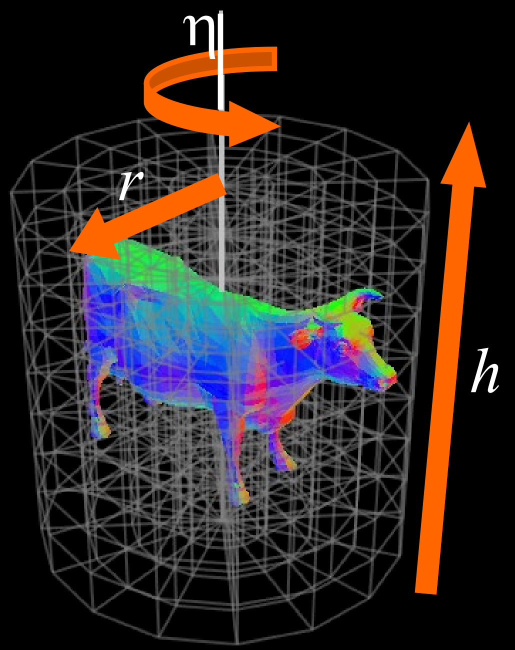
$$\left\{\left\| f_{l,m,n} \right\|\right\}_{l,m,n}$$

Translation Invariant Representation

Power Spectrum

Single axis rotation-invariance:

- Represent the model in a cylindrical coordinate system
- Compute the Fourier transform in the angular direction
- Store the amplitudes of the frequency components



Cylindrical Coordinates

$$f(r,h,\theta)=\sum_k f_k(r,h)e^{i(k\theta)}$$

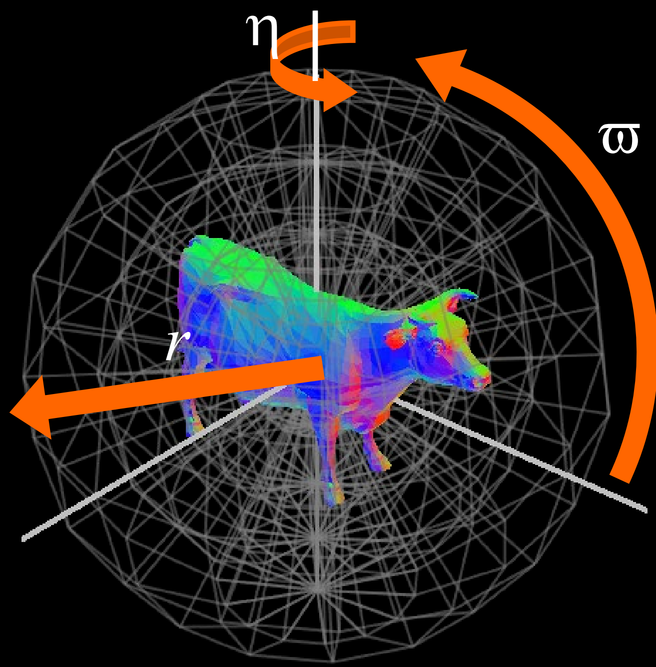
$$\left\{\left\|f_k(r,h)\right\|\right\}_k$$

Rotation Invariant Representation

Power Spectrum

Full rotation-invariance:

- Represent the model in a spherical coordinate system
- Compute the spherical harmonic transform
- Store the amplitudes of the frequency components



Spherical Coordinates

$$f(r,\theta,\phi) = \sum_{l} \sum_{|m|\le l} f_{l,m}(r) Y_l^m(\theta,\phi)$$

$$\left\{ \sqrt{\sum_{|m|\le l} \left\| f_l^m(r) \right\|^2} \right\}_l$$

Rotation Invariant Representation

Power Spectrum

Power spectrum representations

- Are invariant to transformations
- Give a lower bound for the best match
- Tend to discard too much information

Translation invariant: n^3 data -> $n^3/2$ data

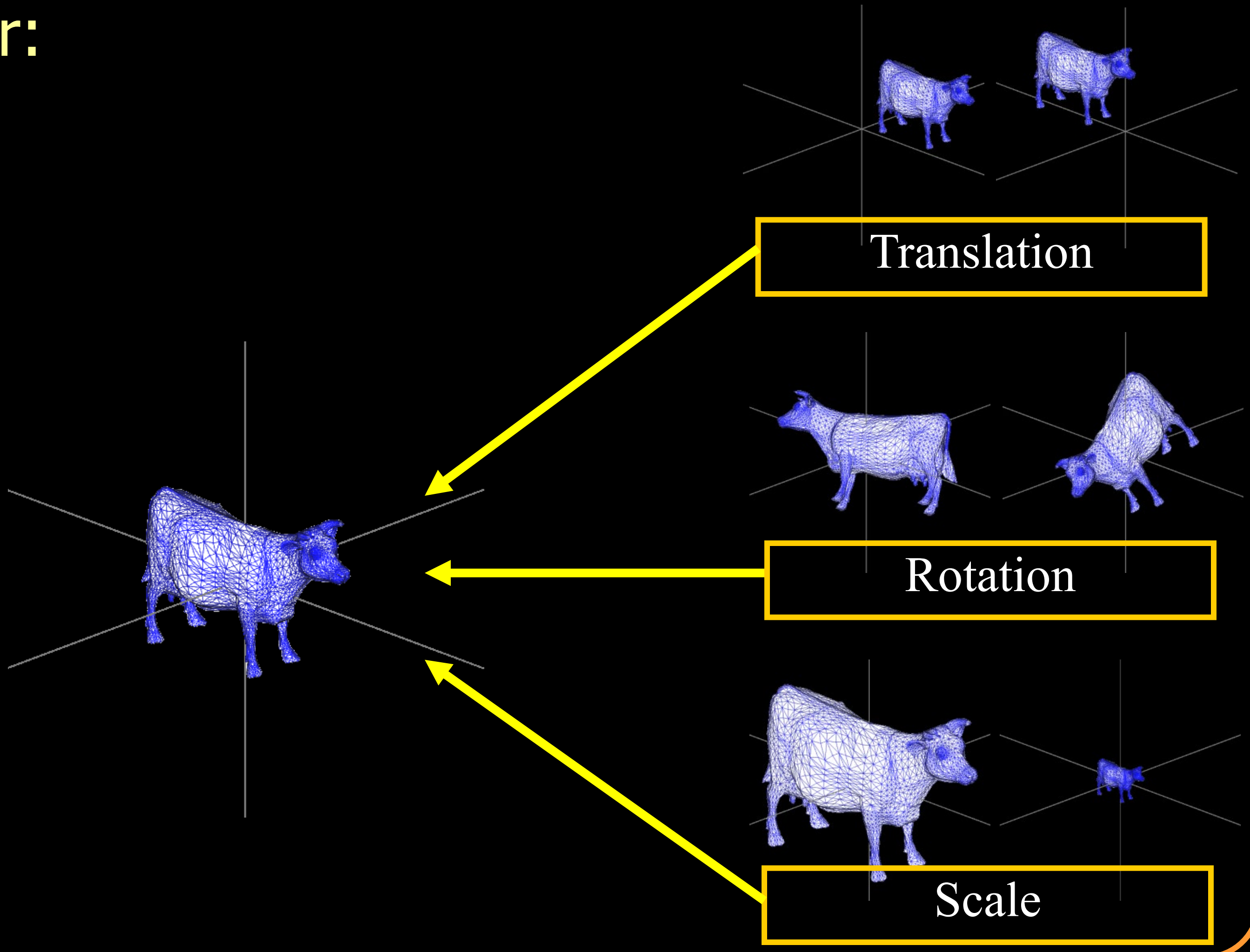
Single-axis rotation invariant: n^3 data -> $n^3/2$ data

Full rotation invariant: n^3 data -> n^2 data

Normalization

Place a model into a canonical coordinate frame by normalizing for:

- translation
- scale
- rotation

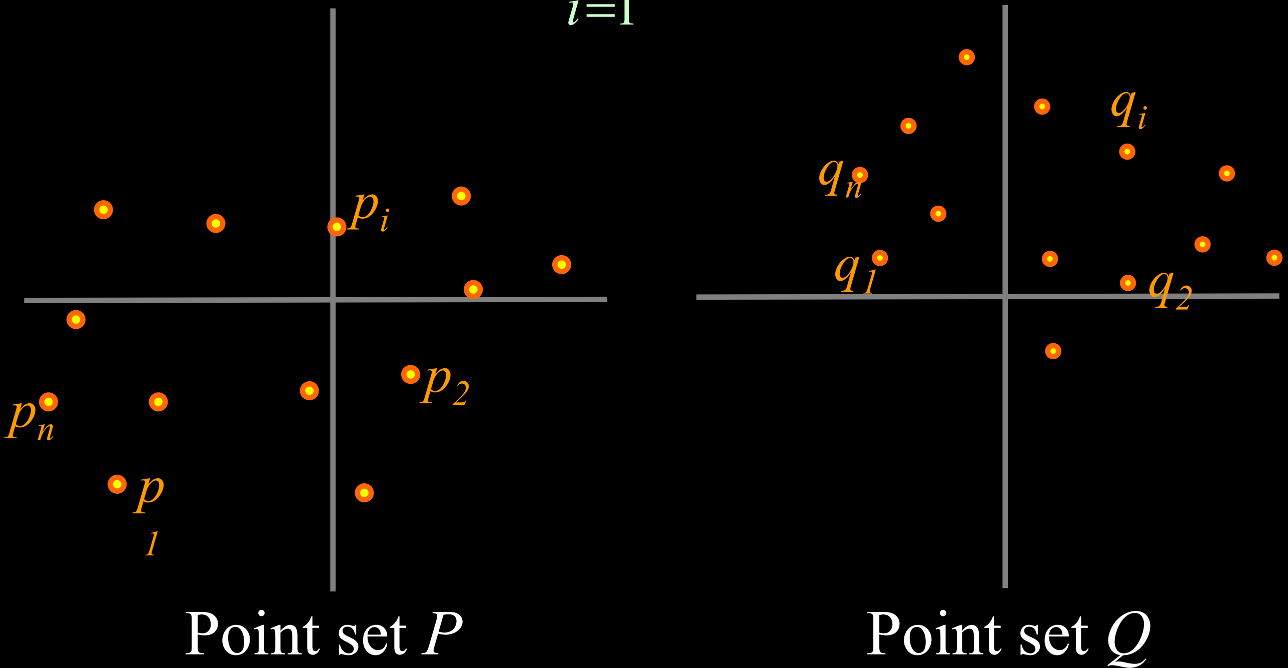


Alignment of Point Sets

[Horn *et al*., 1988]

Given two point sets $P=\{p_1,\ldots,p_n\}$ and $Q=\{q_1,\ldots,q_n\}$, what is the transformation T minimizing the sum of squared distances:

$$d(P,Q)=\sum_{i=1}^{n}\left\|p_i - T(q_i)\right\|^2$$



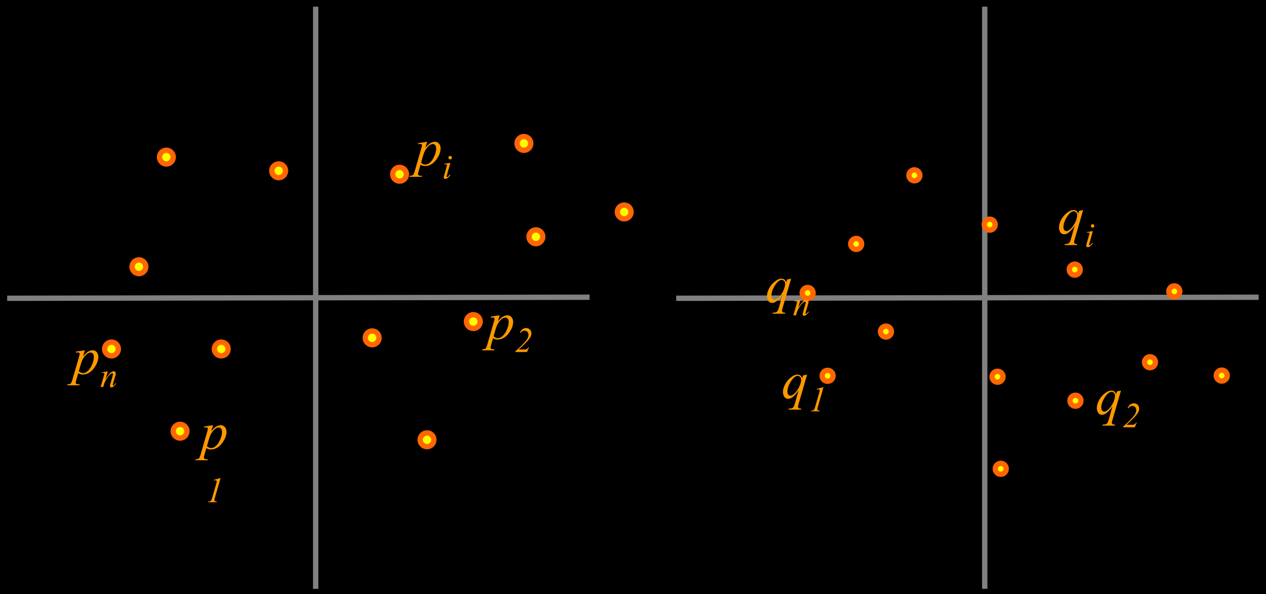
Alignment of Point Sets

[Horn *et al.*, 1988]

Translation

- Align the models so that their center of mass is at the origin.

$$\sum_{i=1}^{n} p_i = 0 \quad \text{and} \quad \sum_{i=1}^{n} q_i = 0$$



Translated point set P Translated point set Q

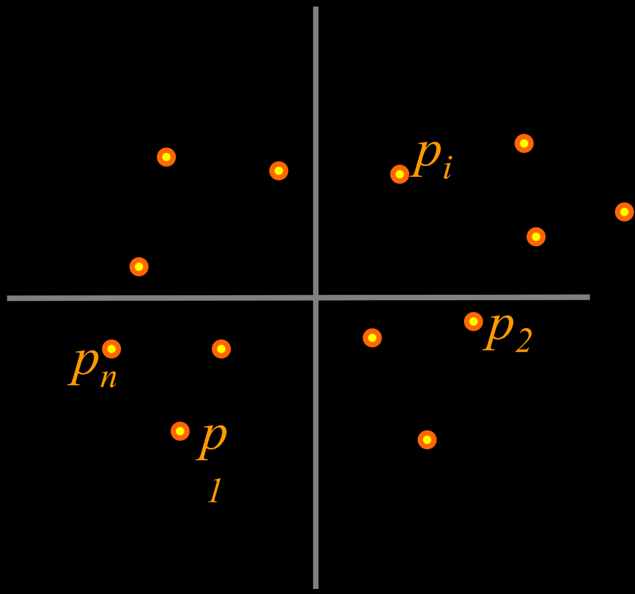
Alignment of Point Sets

[Horn *et al.*, 1988]

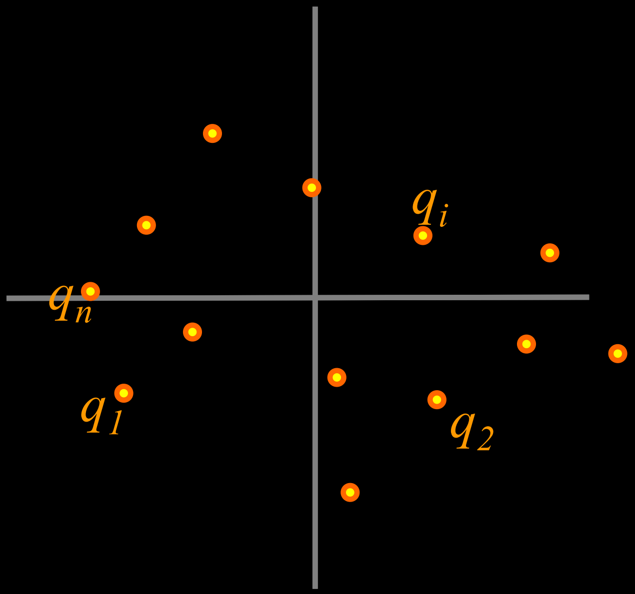
Scale

- Align the models so that their mean variance is 1.

$$\sum_{i=1}^{n} \|p_i\|^2 = 1 \quad \text{and} \quad \sum_{i=1}^{n} \|q_i\|^2 = 1$$



Scaled point set *P*



Scaled point set *Q*

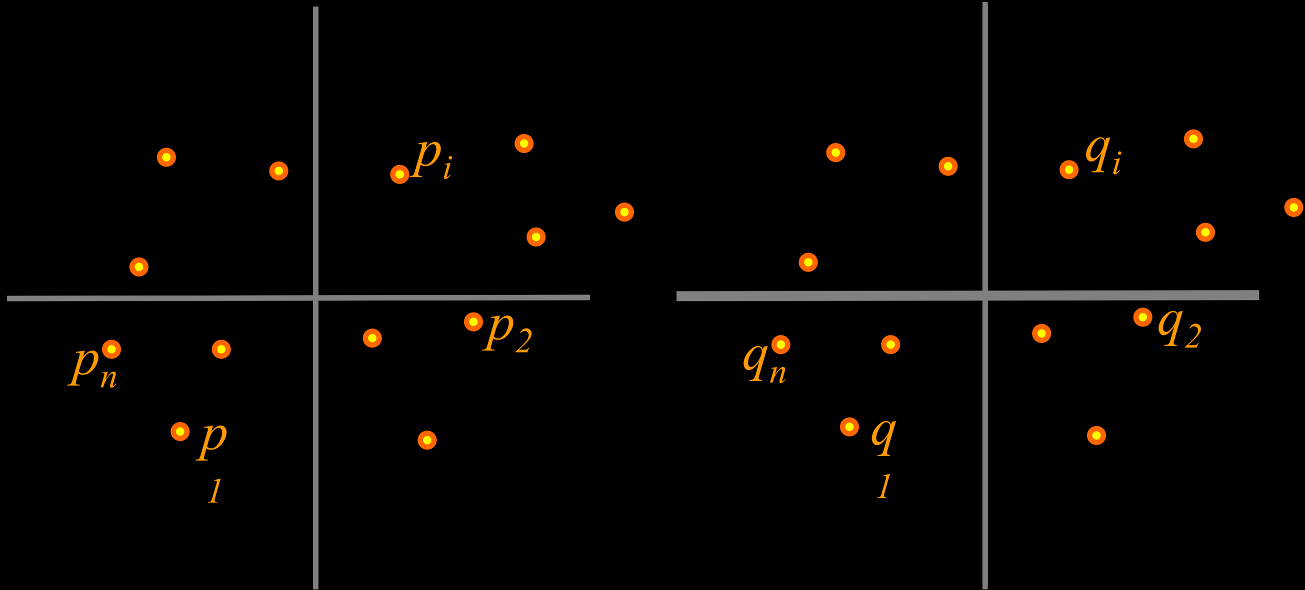
Alignment of Point Sets

[Horn *et al.*, 1988]

Rotation

- SVD on cross covariance matrix:

$$M = \left(p_1 \middle| \ldots \middle| p_n\right) \cdot \left(q_1 \middle| \ldots \middle| q_n\right)^T$$



Rotationally aligned point sets P and Q

Normalization

Place a model into a canonical coordinate frame:

- Translation: center of mass

$$\sum_{i=1}^{n} p_i = 0 \qquad \text{and} \qquad \sum_{i=1}^{n} q_i = 0$$

Can be done on a per-model basis

- Scale: mean variance

$$\sum_{i=1}^{n} \|p_i\|^2 = 1 \qquad \text{and} \qquad \sum_{i=1}^{n} \|q_i\|^2 = 1$$

Can be done on a per-model basis

- Rotation: SVD on cross covariance matrix

$$M = \left(p_1 | \ldots | p_n\right) \cdot \left(q_1 | \ldots | q_n\right)^t$$

Need to know the correspondences between models

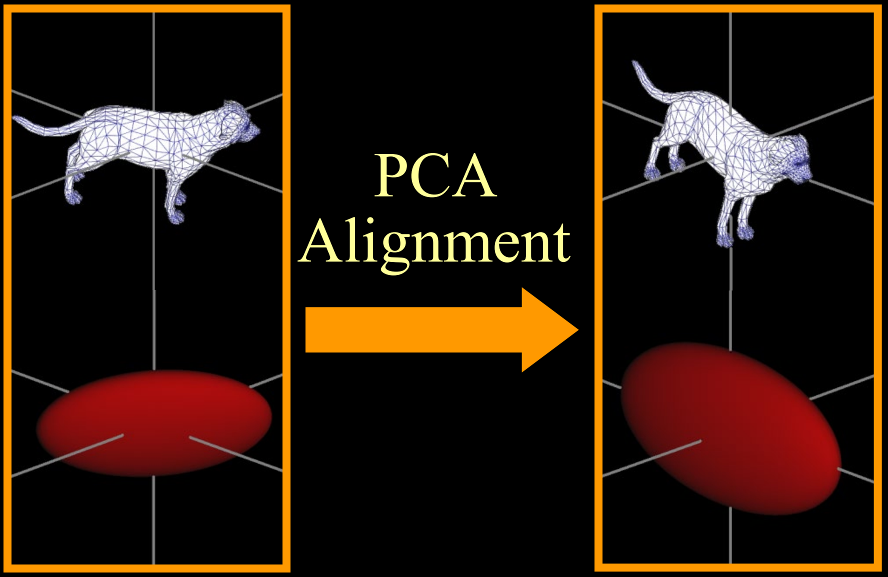
Rotation

Challenge:

- We want to normalize for rotation on a per-model basis

Solution:

- Align the model so that the principal axes align with the coordinate axes



Rotation

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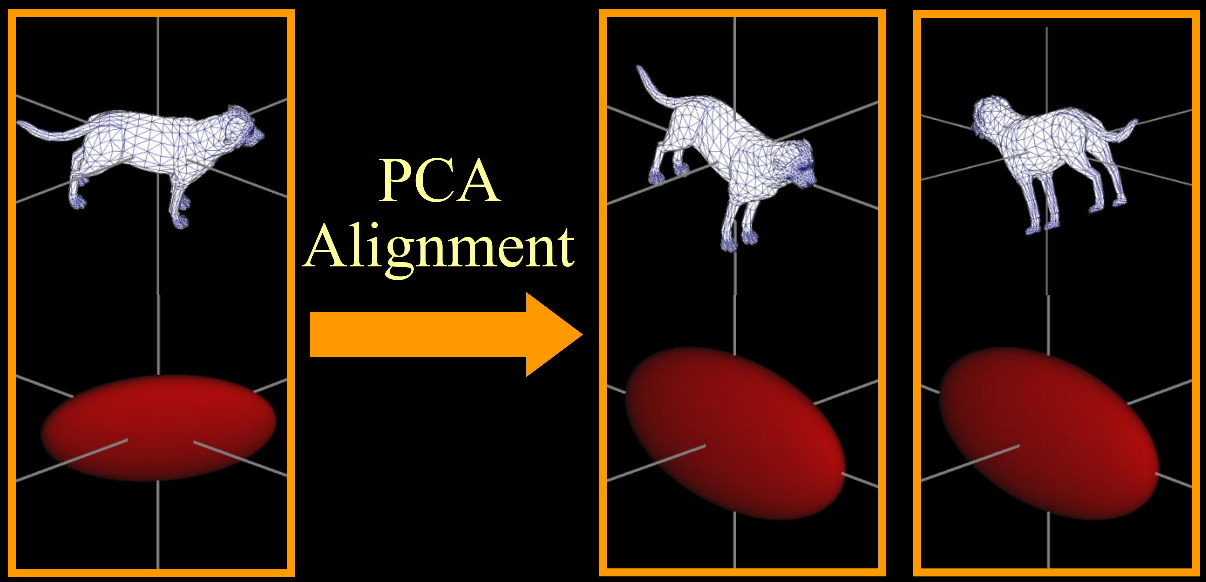


Directions of the axes are ambiguous

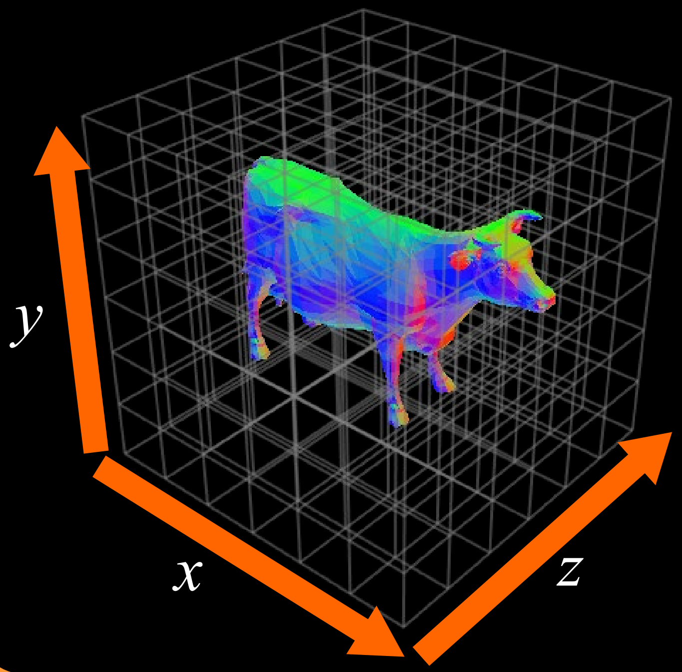
Normalization (PCA)

PCA defines a coordinate frame up to reflection in the coordinate axes.

- Make descriptor invariant to the eight reflections

  Reflections fix the cosine term

  Reflections multiply the sine term by -1



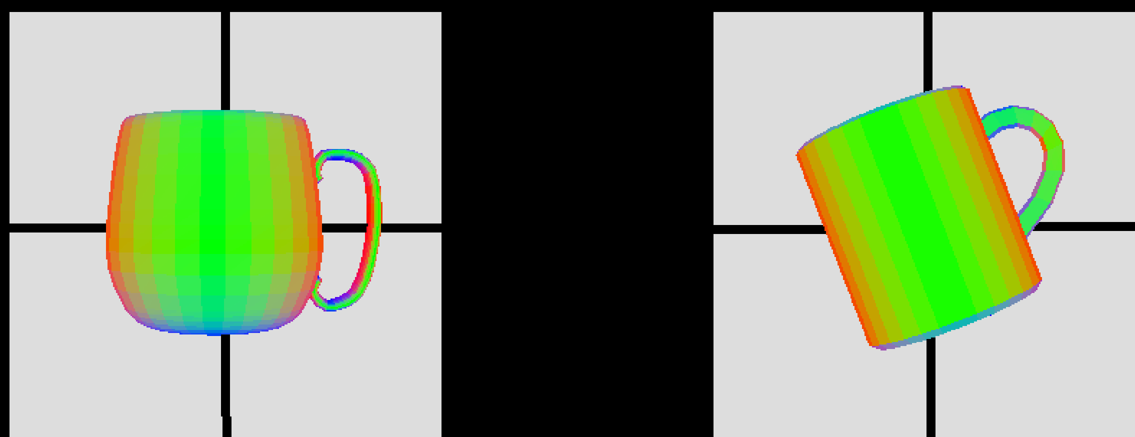
$$f(\theta) = \sum_k a_k \cos(k\theta) + b_k \sin(k\theta)$$

$$\left\{a_k, |b_k|\right\}_k$$

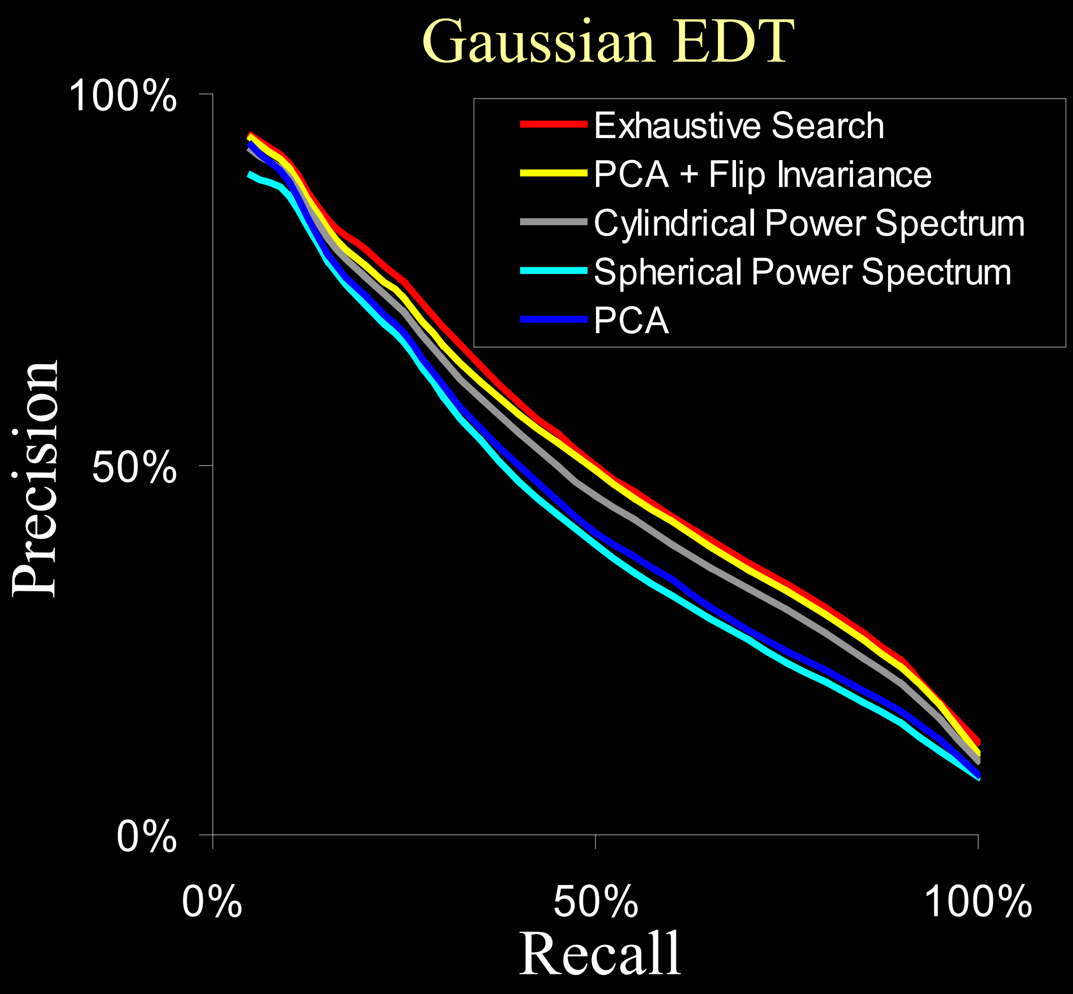
Translation Invariant Representation

Problem with PCA-Based Alignment

If singular values are close, axes unstable



Retrieval Results (Rotation)



Size:

Method	Floats
Exhaustive Search	8192
PCA + Flip Invariance	8192
PCA	8192
Cylindrical PS	4352
Spherical PS	512

Time:

Method	Secs.
Exhaustive Search	20.59
PCA + Flip Invariance	.67
PCA	.67
Cylindrical PS	.32
Spherical PS	.03