Lecture 12
Tracking

COS 429: Computer Vision

PRINCETON UNIVERSITY

Slides credit:
Many slides adapted from James Hays, Derek Hoeim, Lana Lazebnik, Silvio Saverse, who in turn adapted slides from Steve Seitz, Rick Szeliski, Martial Hebert, Mark Pollefeys, and others

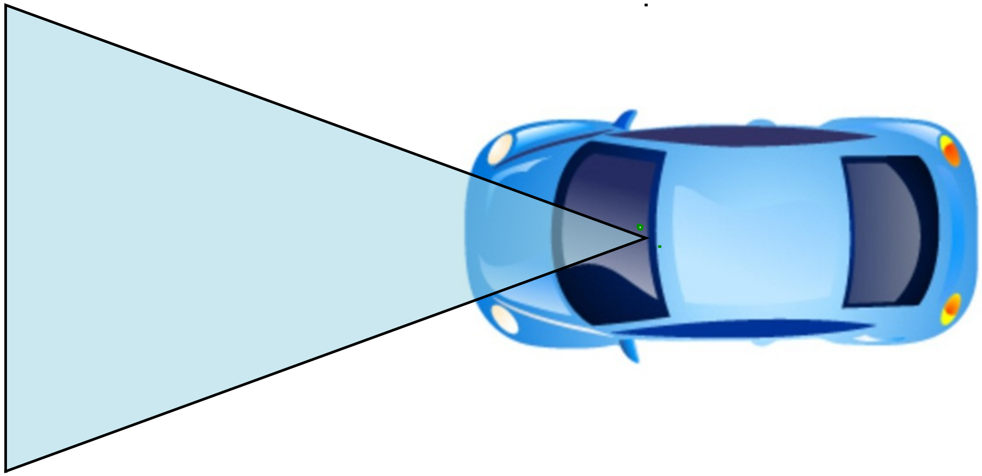
Motivation: Mobileye

Camera-based Driver Assistance System

Safety Application based on single forward looking camera:

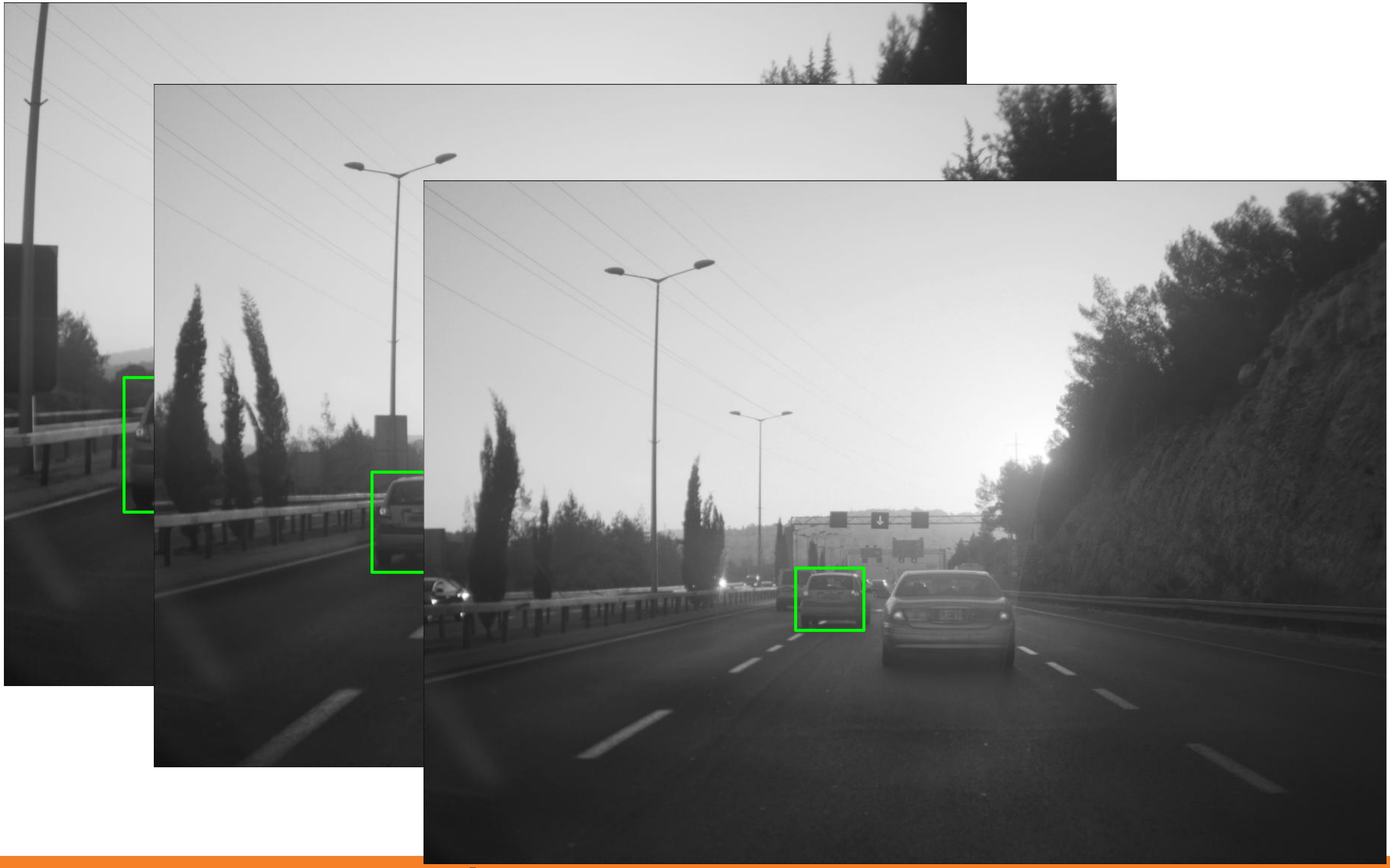
- **Lane Detection**
  - Lane Departure Warning (LDW)
  - Lane Keeping and Support
- **Vehicle Detection**
  - Forward Collision Warning (FCW)
  - Headway Monitoring and Warning
  - Adaptive Cruise Control (ACC)
  - Traffic Jam Assistant
  - Emergency Braking (AEB)
- **Pedestrian Detection**
  - Pedestrian Collision Warning (PCW)
  - Pedestrian Emergency Braking

For Videos, visit
www.mobileye.com



Slide Credit: Mobileye

Detect… Detect … Detect…



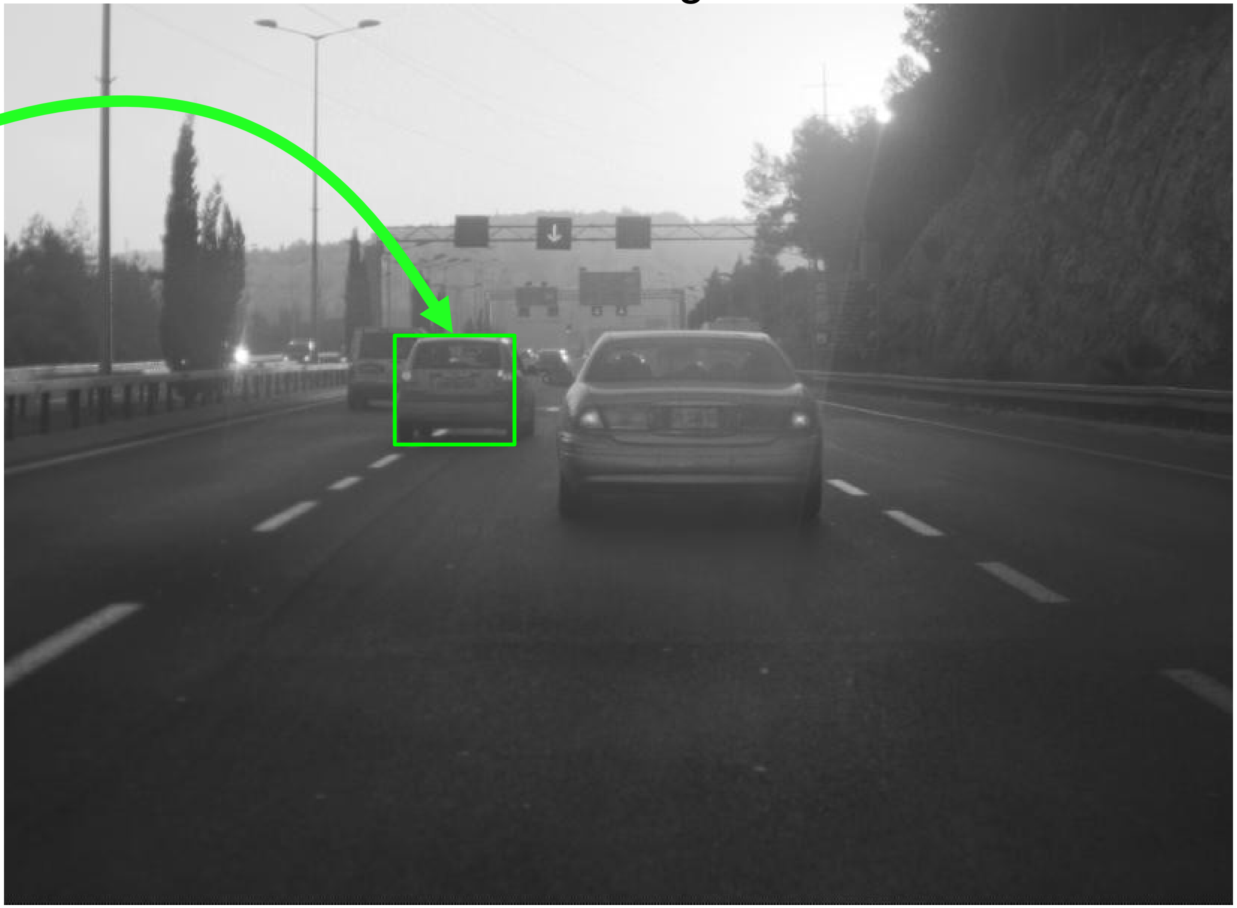
Slide Credit:

Or Track?

Once target has been located, and we “learn” what it looks like, should be easier to find in later frames... this is object tracking.

Template

Future Image Frame



Slide Credit:

Approaches to Object Tracking

- Motion model (translation, translation+scale, affine, non-rigid, …)
- Image representation (gray/color pixel, edge image, histogram, HOG, wavelet...)
- Distance metric (L1, L2, normalized correlation, Chi-Squared, …)
- Method of optimization (gradient descent, naive search, combinatoric search...)
- What is tracked: whole object or selected features

Template

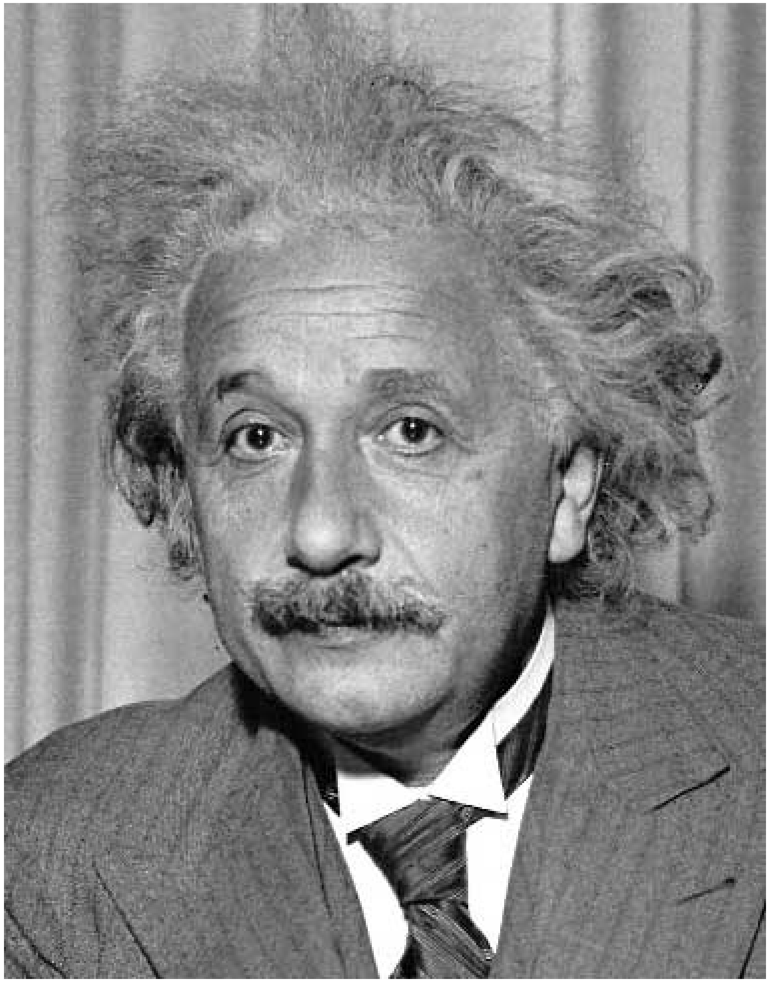
Slide Credit:

Distance Metric

- Goal: find  in image,
  assume translation only: no scale change or rotation,

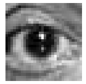
using search (scanning the image)

- What is a good similarity or distance measure between two patches?
  - Correlation
  - Zero-mean correlation
  - Sum Square Difference
  - Normalized Cross Correlation



Slide Credit:

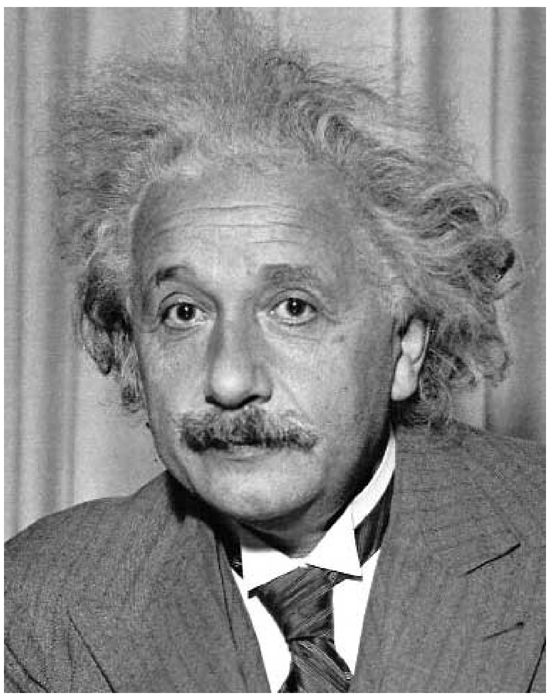
Matching with filters

Goal: find  in image

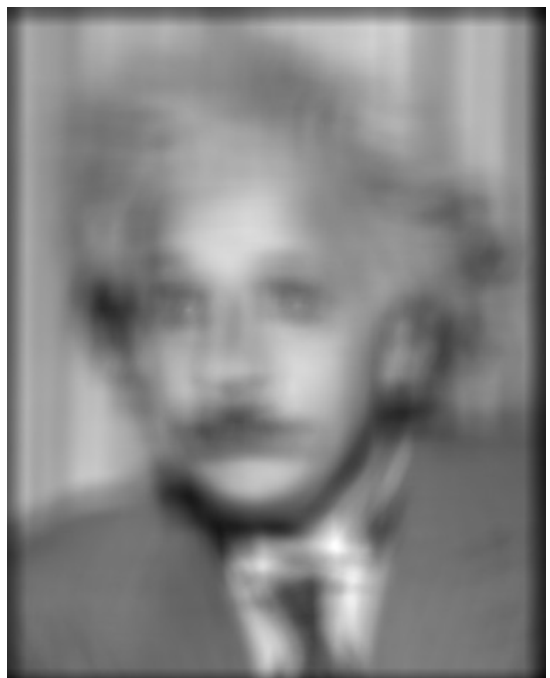
- Method 0: filter the image with eye patch

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

f = image
g = filter



Input



Filtered Image

What went wrong?

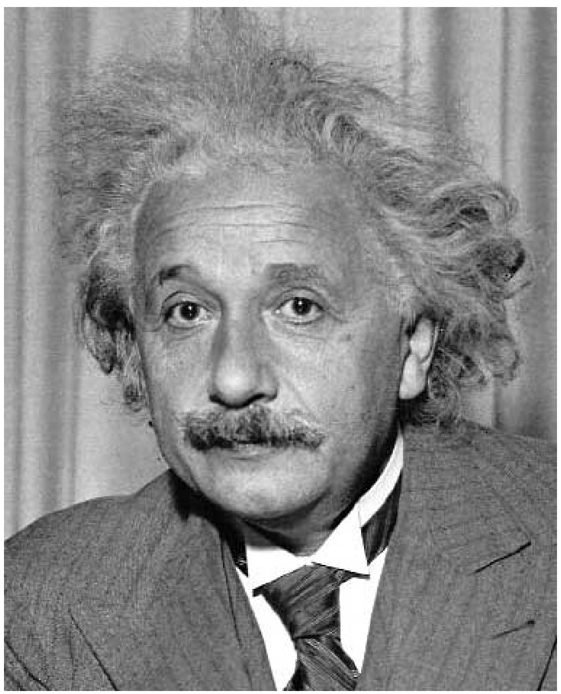
response is stronger for higher intensity

Slide Credit:

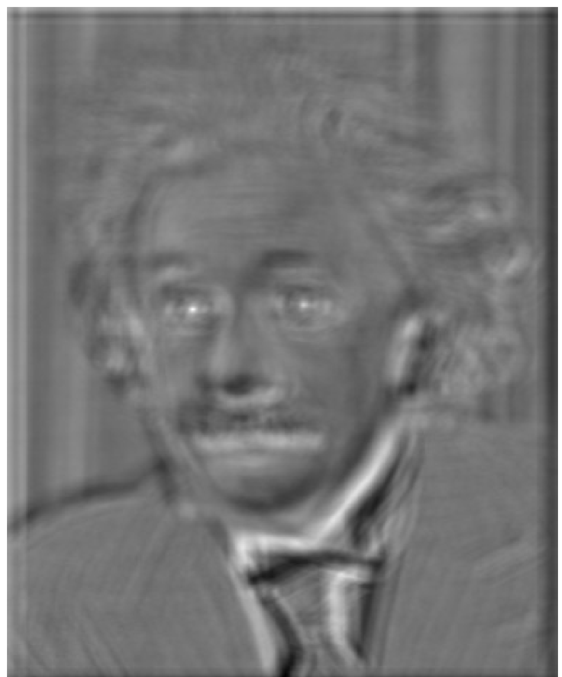
0-mean filter

- Goal: find  image
- Method 1: filter the image with zero-mean eye

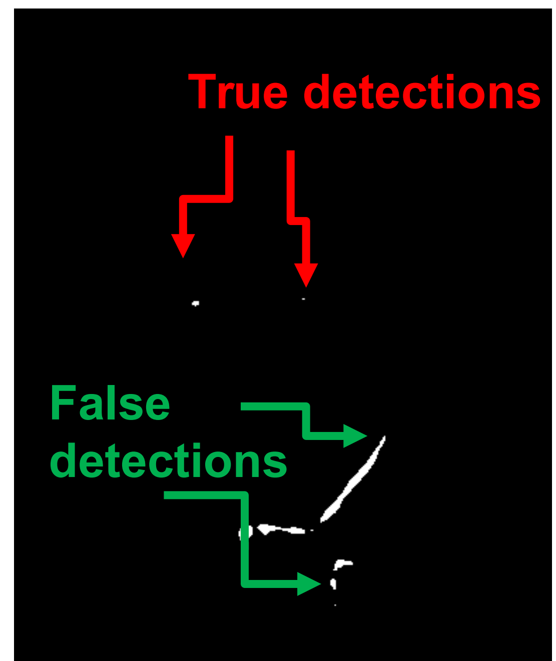
$$h[m,n]=\sum_{k,l}(f[k,l]-\underbrace{\bar{f}}_{\text{mean of f}})\,(g[m+k,n+l])$$



Input



Filtered Image (scaled)



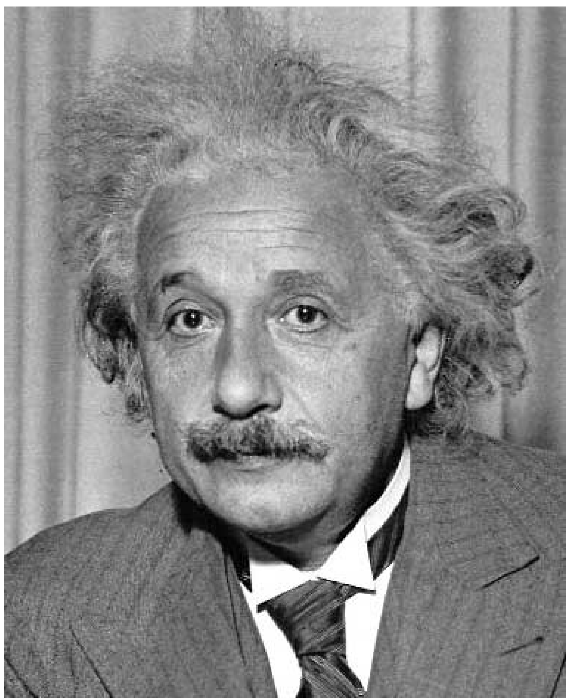
Thresholded Image

Slide Credit:

Sum of Squared error (L2)

- Goal: find  in image
- Method 2: SSD

$$h[m,n]=\sum_{k,l}(g[k,l]-f[m+k,n+l])^2$$



Input

1- sqrt(SSD)

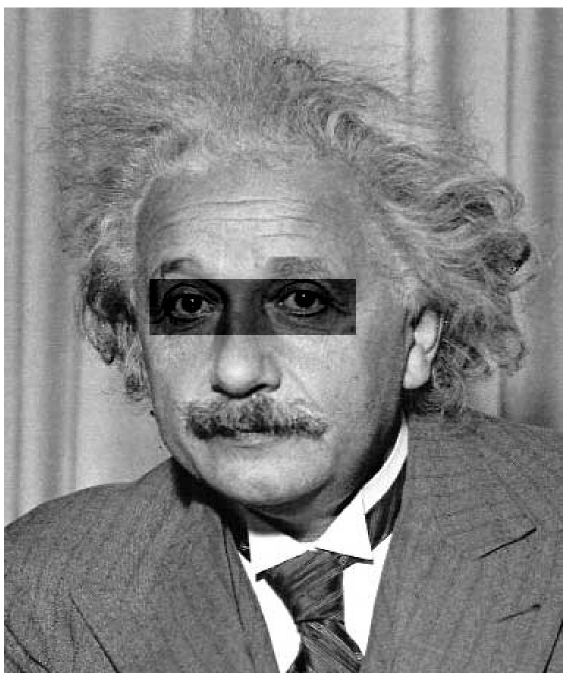
Thresholded Image

Slide Credit:

Sum of Squared error (L2)

- Goal: find  in image
- Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$



Input

1- sqrt(SSD)

One potential downside of SSD:

Brightness Constancy Assumption

Slide Credit:

Normalized Cross-Correlation

- Goal: find  in image
- Method 3: Normalized cross-correlation
  (= angle between zero-mean vectors)

Template mean

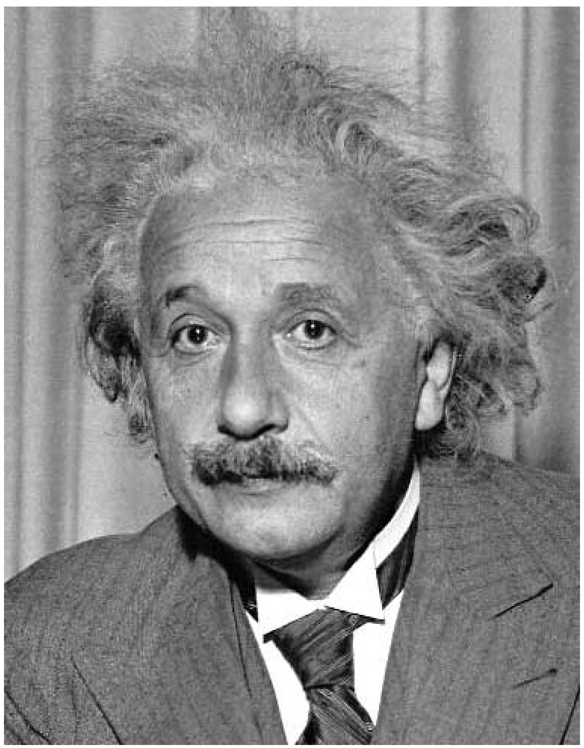
image patch mean

$$h[m,n] = \frac{\sum_{k,l}(g[k,l]-\bar{g})(f[m-k,n-l]-\bar{f}_{m,n})}{\left[\sum_{k,l}(g[k,l]-\bar{g})^2 \sum_{k,l}(f[m-k,n-l]-\bar{f}_{m,n})^2\right]^{0.5}}$$

Matlab: `normxcorr2(template, im)`

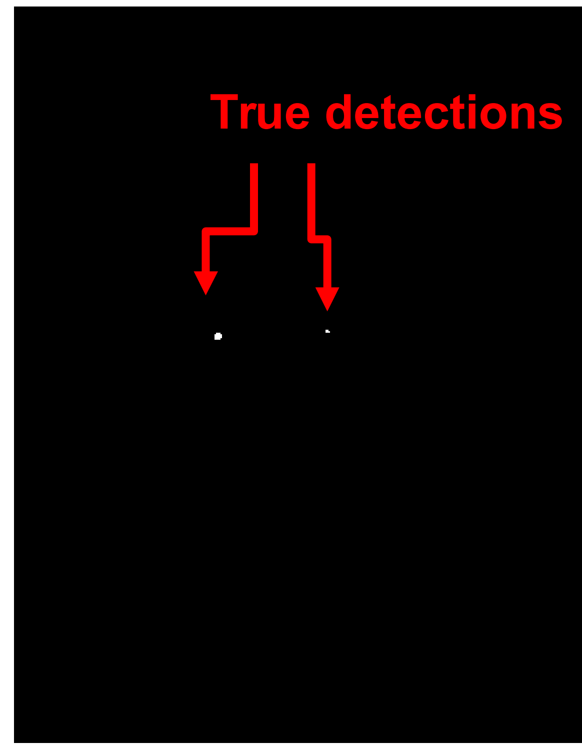
Normalized Cross-Correlation

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

Normalized X-Correlation

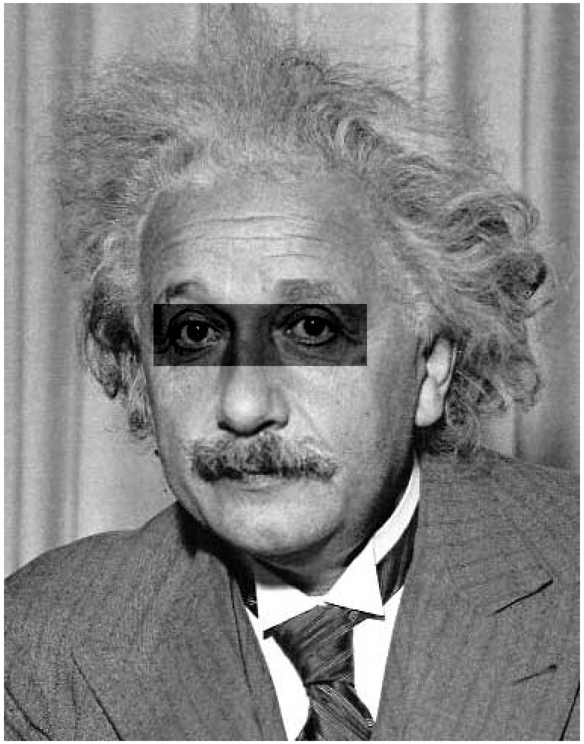


Thresholded Image

Slide Credit:

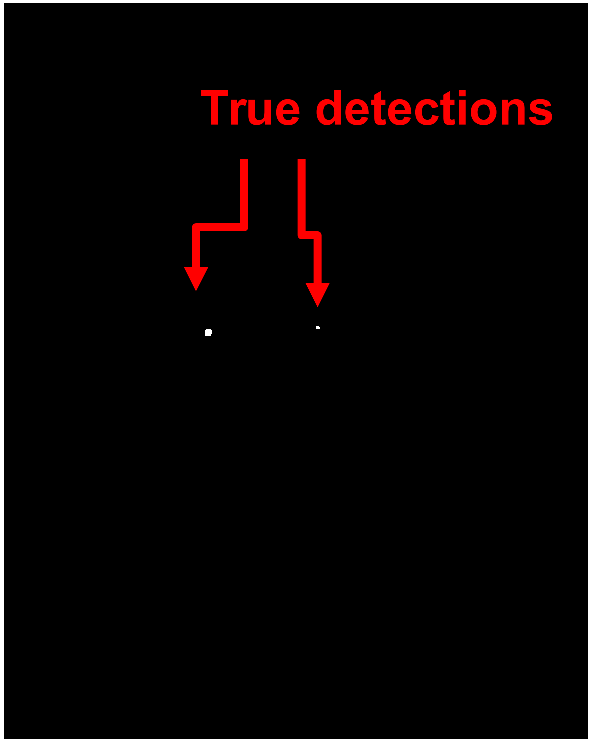
Normalized Cross-Correlation

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

Normalized X-Correlation



Thresholded Image

Slide Credit:

Search vs. Gradient Descent

- Search:
  - Pros: Free choice of representation, distance metric; no need for good initial guess
  - Cons: expensive when searching over complex motion models (scale, rotation, affine)

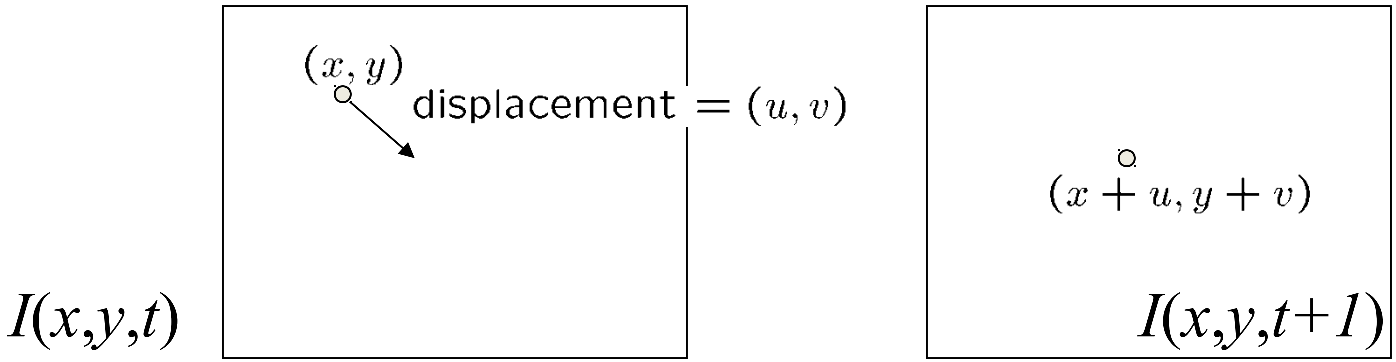
- If we have a good guess, can we do something cheaper?
  - Gradient Descent

Lucas-Kanade Object Tracker

- Key assumptions:
  - **Brightness constancy:** projection of the same point looks the same in every frame (uses SSD as metric)
  - **Small motion:** points do not move very far (from guessed location)
  - **Spatial coherence:** points move in some coherent way (according to some parametric motion model)
    - For this example, assume whole object just translates in (u,v)

Slide Credit:

The brightness constancy constraint



- Brightness Constancy Equation:

$$I(x,y,t) = I(x+u,y+v,t+1)$$

Take Taylor expansion of $I(x+u, y+v, t+1)$ at (x,y,t) to linearize the right side:

Image derivative along x Difference over frames

$$I(x+u, y+v, t+1) \approx I(x, y, t) + \boxed{I_x} \cdot u + I_y \cdot v + \boxed{I_t}$$

$$I(x+u, y+v, t+1) - I(x, y, t) = + I_x \cdot u + I_y \cdot v + I_t$$

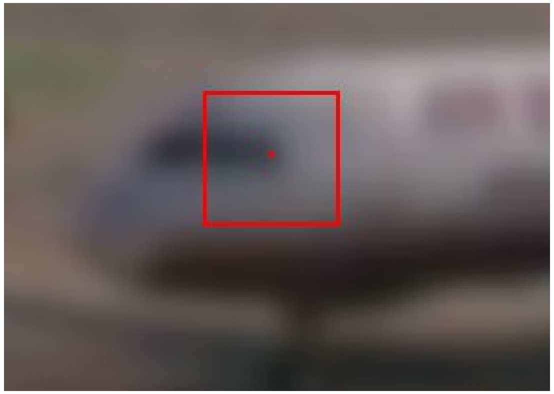
Hence,

$$I_x \cdot u + I_y \cdot v + I_t \approx 0 \quad \rightarrow \nabla I \cdot [u \;\; v]^T + I_t = 0$$

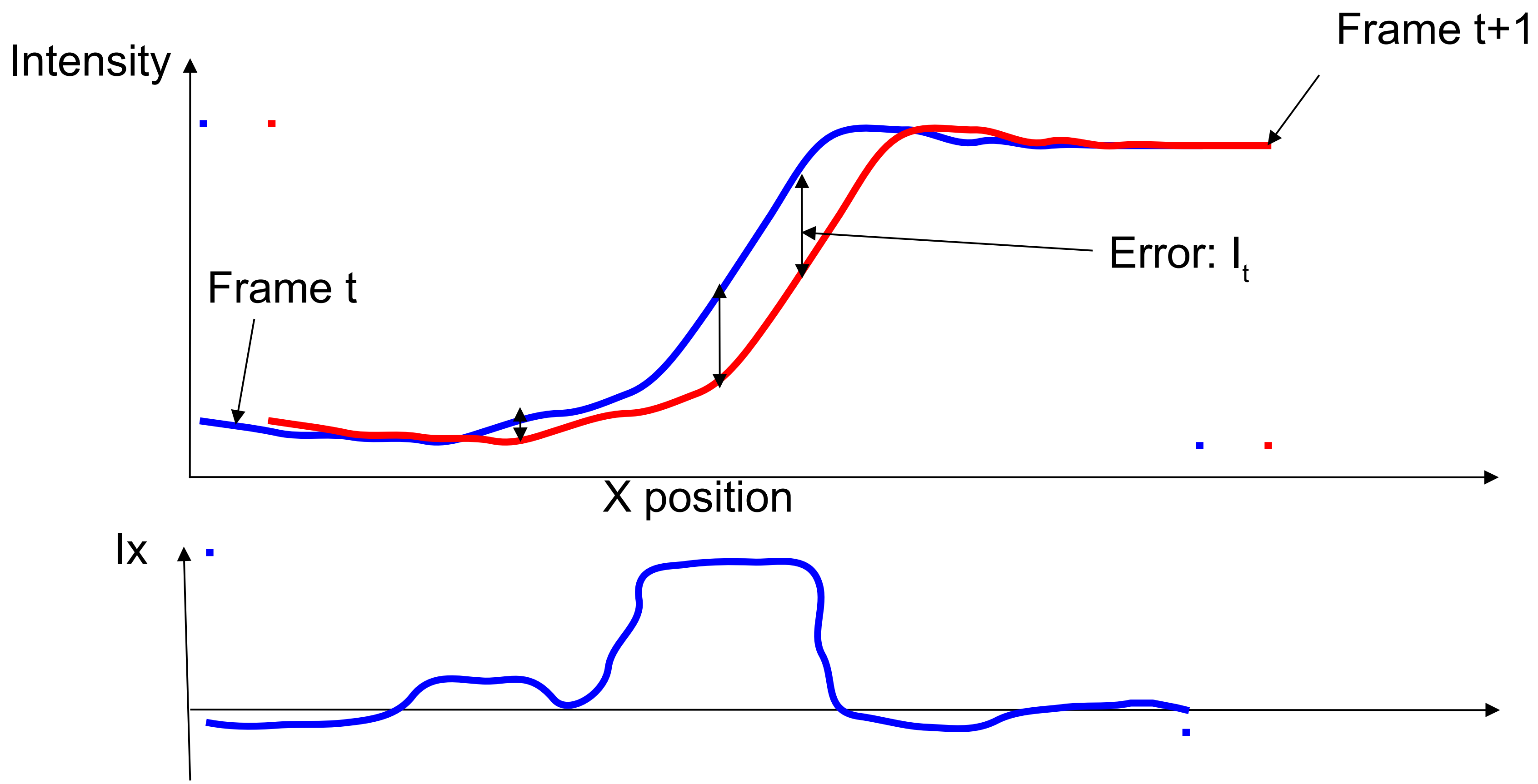
How does this make sense?

$$\nabla I \cdot [u\ v]^T + I_t = 0$$

- What do the static image gradients have to do with motion estimation?

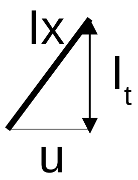


Intuition in 1-D



Solve for u in:

$$I_x \cdot u + I_t \approx 0$$



Slide Credit:

The brightness constancy constraint

Can we use this equation to recover image motion (u,v) at each pixel?

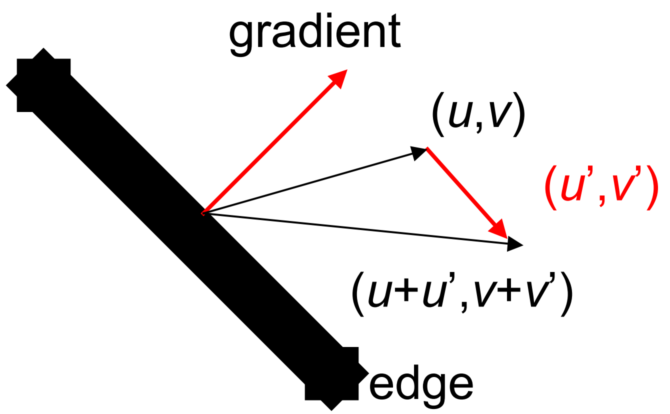
$$\nabla I \cdot [u \; v]^T + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation (this is a scalar equation!), two unknowns (u,v)

The component of the motion perpendicular to the gradient (i.e., parallel to the edge) cannot be measured

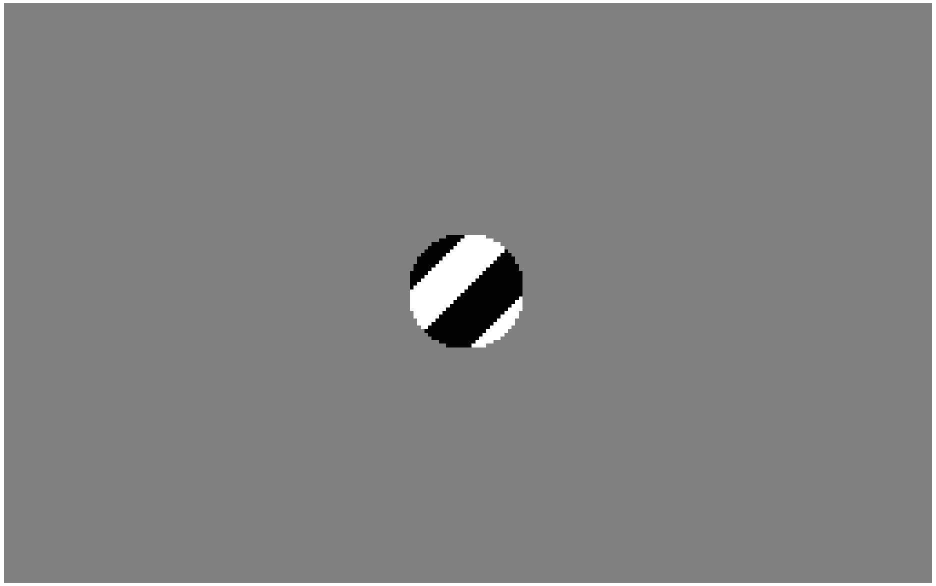
If (u, v) satisfies the equation, so does ($u+u'$, $v+v'$) if

$$\nabla I \cdot [u' \; v']^T = 0$$



Slide Credit:

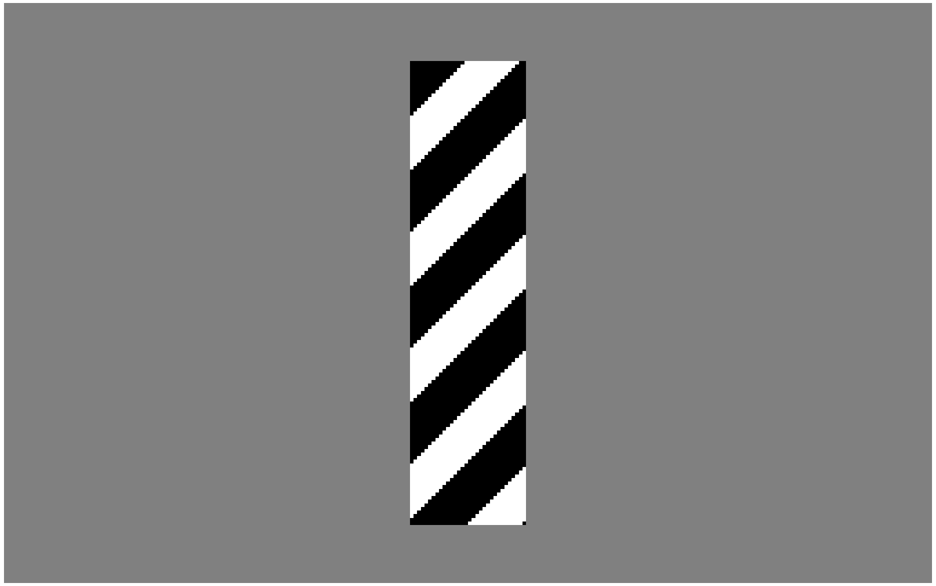
The barber pole illusion



http://en.wikipedia.org/wiki/Barberpole_illusion

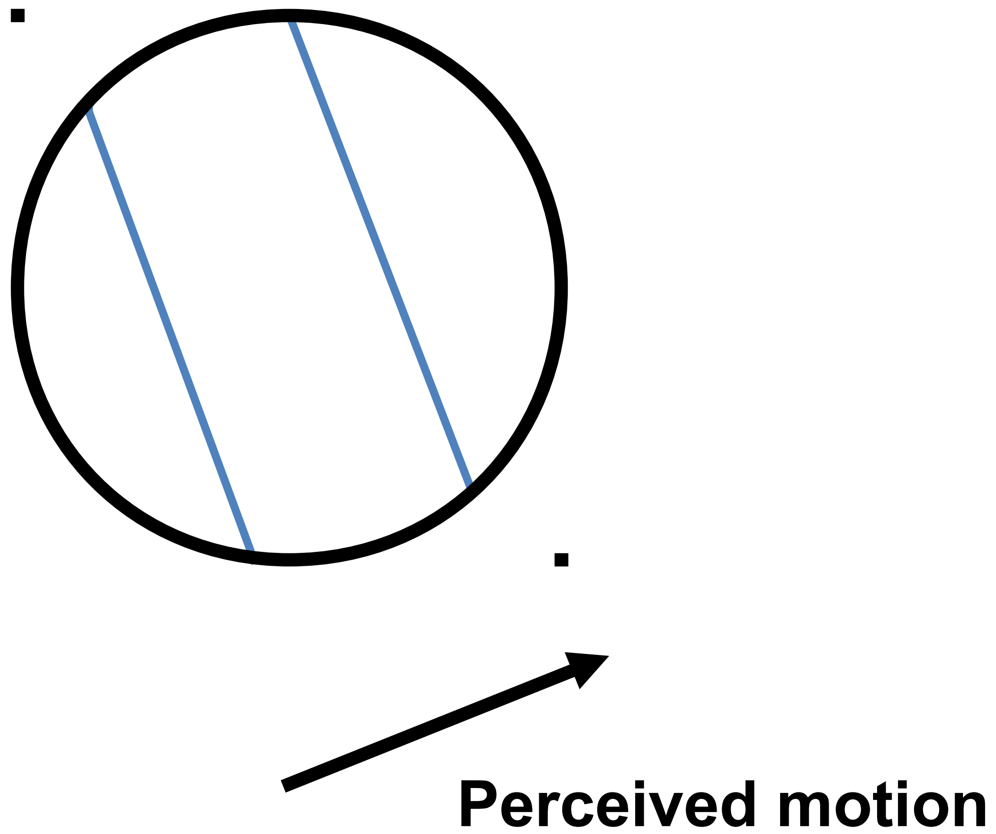
Slide Credit:

The barber pole illusion

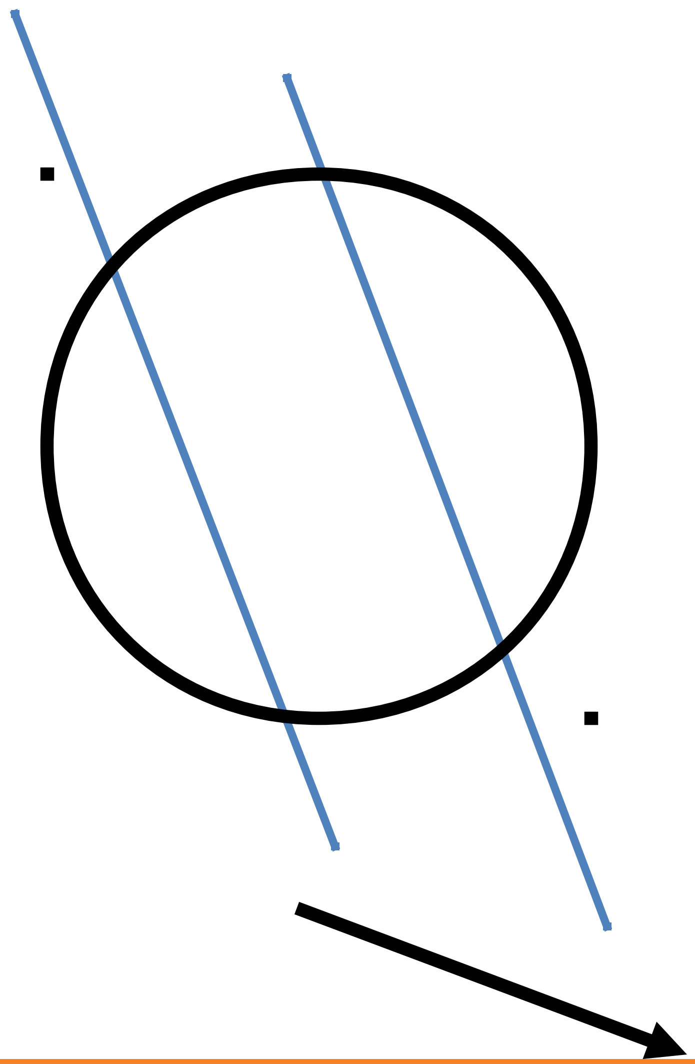


http://en.wikipedia.org/wiki/Barberpole_illusion

The aperture problem



The aperture problem



Actual motion

Solving the ambiguity…

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

- Spatial coherence constraint: solve for many pixels and assume they all have the same motion
- In our case, if the object fits in a 5x5 pixel patch, this gives us 25 equations:

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix}$$

 Slide Credit:

Solving the ambiguity…

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \qquad \underset{25\text{x}2}{A} \; \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$

Slide Credit:

Matching patches across images

- Over-constrained linear system

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \qquad A \quad d = b$$

Least squares solution for *d* given by $(A^T A)\ d = A^T b$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

The summations are over all pixels in the K x K window

Slide Credit:

Dealing with larger movements: Iterative refinement

Original (x,y) position

$I_t = I(x', y', t+1) - I(x, y, t)$

1. Initialize (x',y') = (x,y)
2. Compute (u,v) by

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

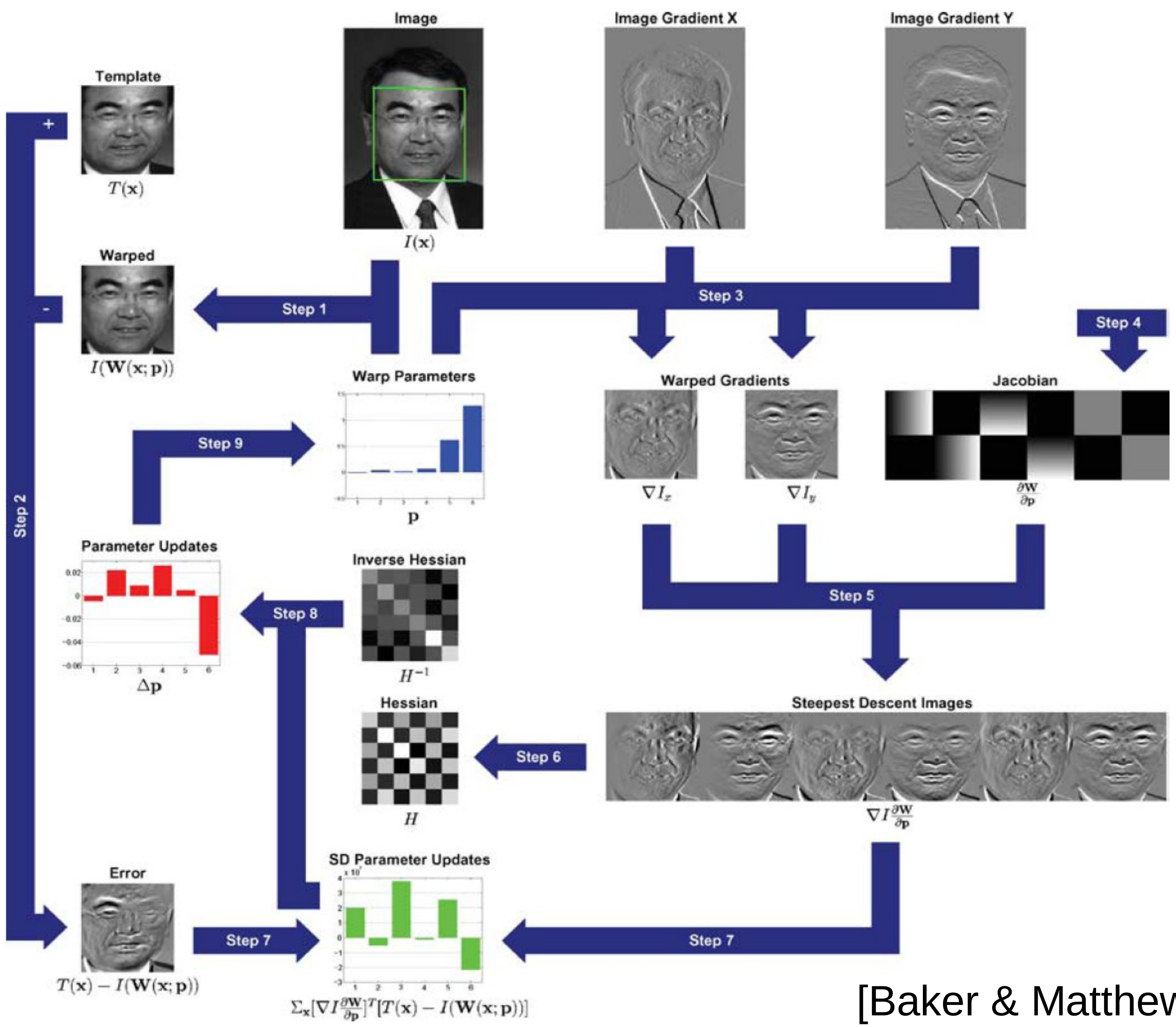
2nd moment matrix for feature patch in first image

displacement

1. Shift window by (u, v): `x'=x'+u; y'=y'+v;`
2. Recalculate I_t
3. Repeat steps 2-4 until small change
   - Use interpolation to warp by subpixel values

Slide Credit:

Schematic of Lucas-Kanade

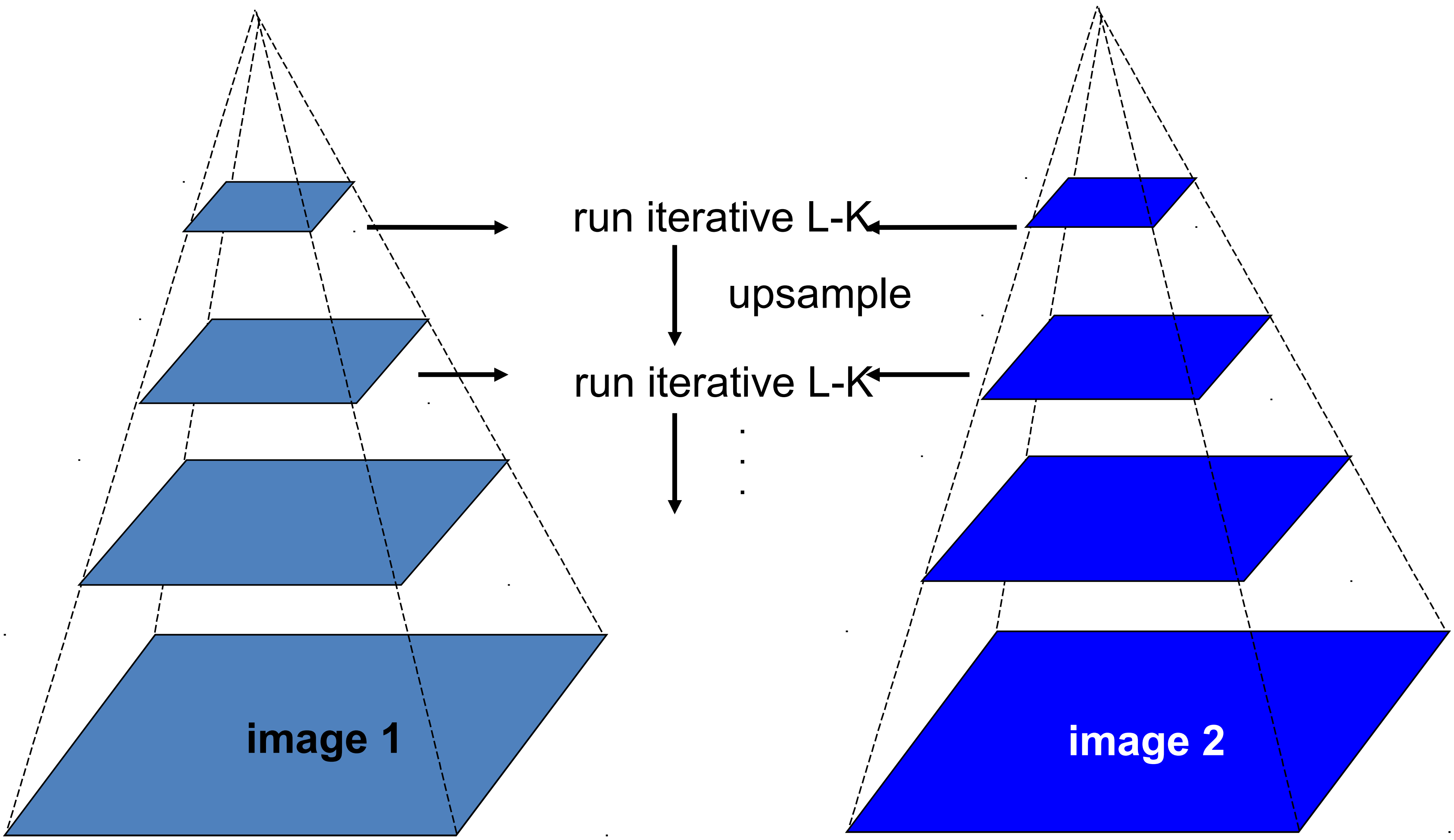


[Baker & Matthews, 2003]

Dealing with larger movements

- How to deal with cases where the initial guess is not within a few pixels of the solution?

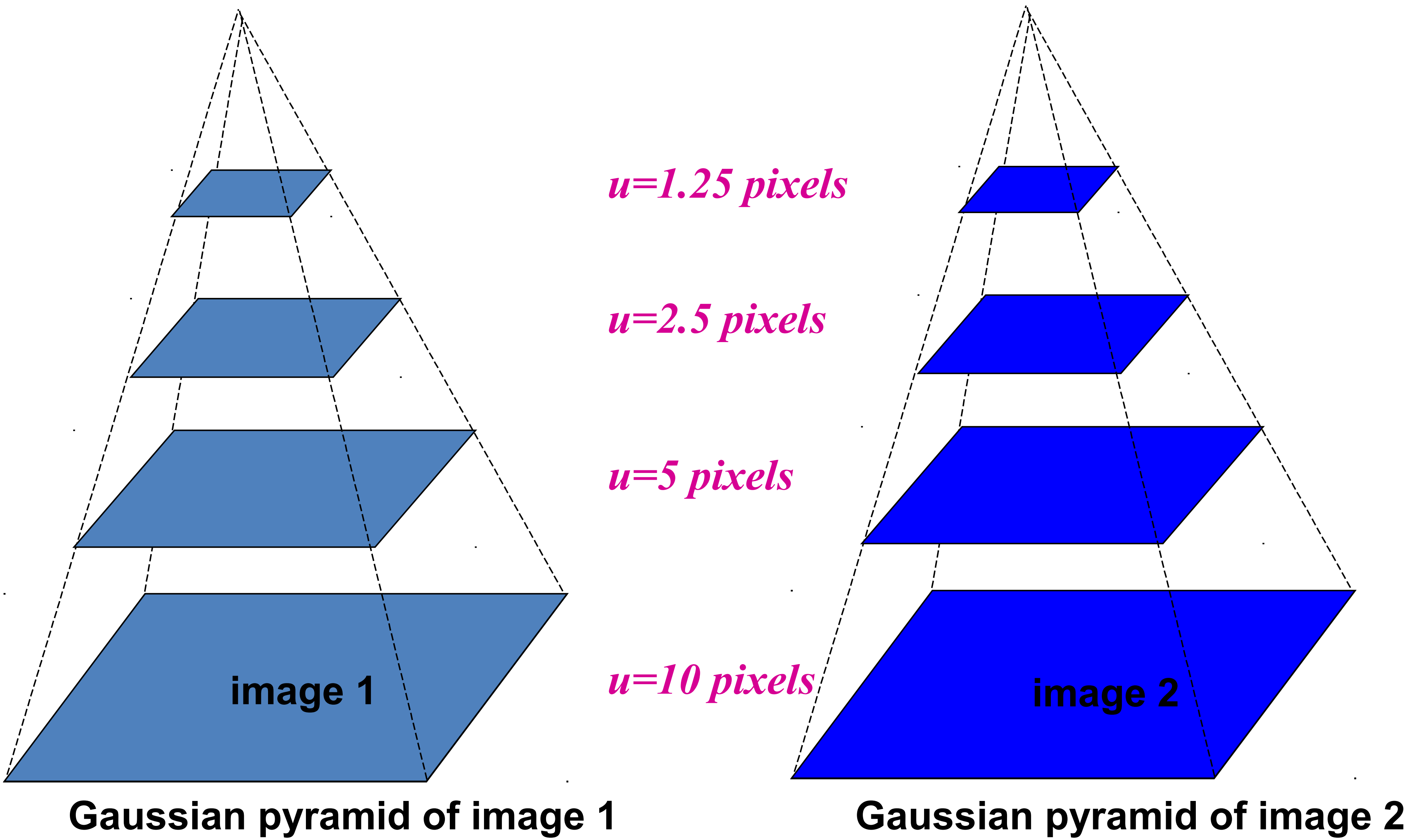
Dealing with larger movements: coarse-to-fine registration



Gaussian pyramid of image 1 (t) **Gaussian pyramid of image 2 (t+1)**

 Slide Credit:

Coarse-to-fine optical flow estimation



Slide Credit:

Summary

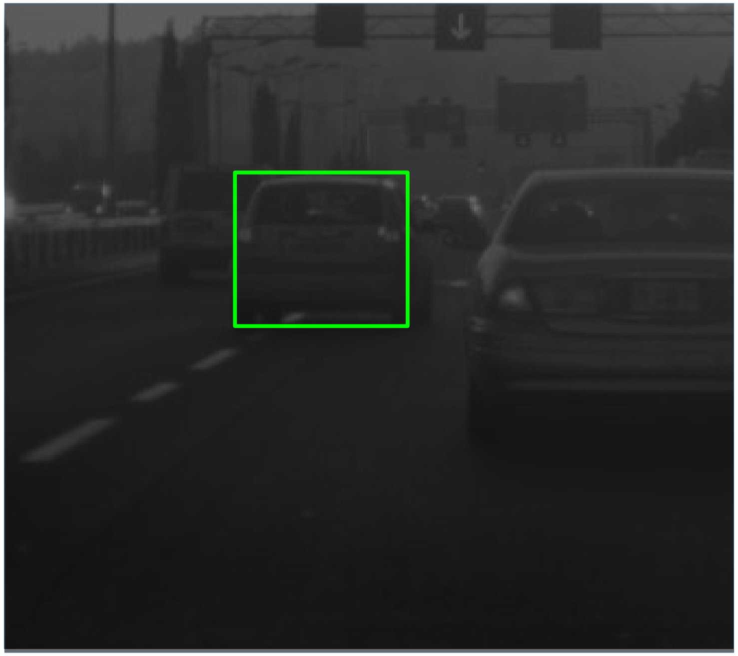
- L-K works well when:
  - Have a good initial guess
  - L2 (SSD) is a good metric
  - Can handle more degrees of freedom in motion model (scale, rotation, affine, etc.), which are too expensive for search
- But has problems with:
  - Changes in brightness
  - ...

 Slide Credit:

LK Problem: Change in Brightness

Possible Solutions:

- Subtract mean intensity (based on current estimate before iteration)
- Transform gray values into some features that are not effected by brightness
  - Any filter that is zero-mean
  - Example: vertical, horizontal edge filters
  - Example: Non-parametric filters (Rank & Census Transforms)



Slide Credit:

More Problems

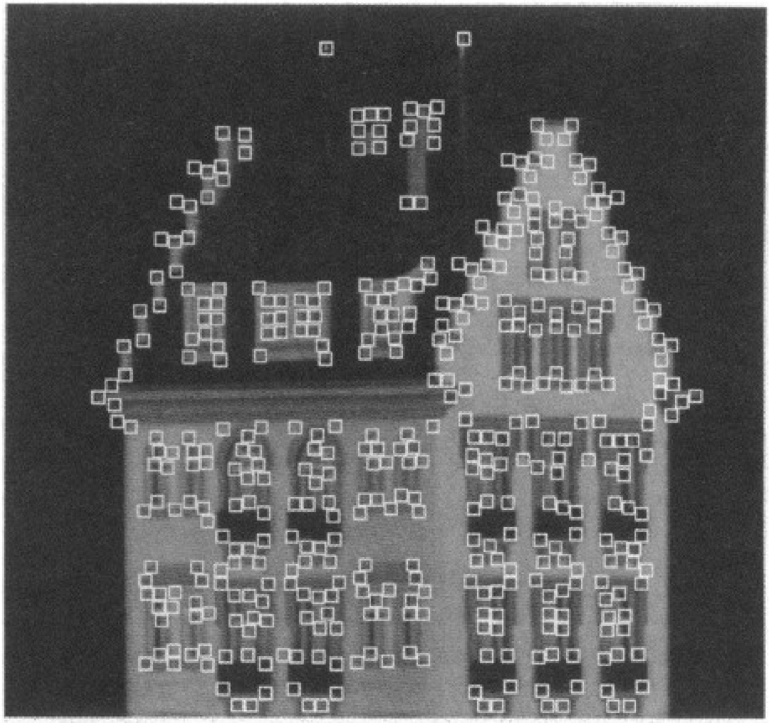
- Outliers: bright strong features that are wrong

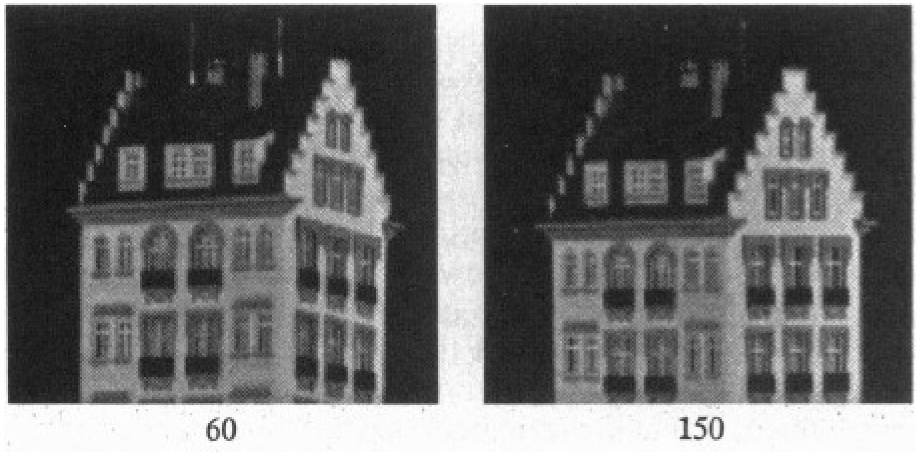

- Complex, high dimensional, or non-rigid motion


Slide Credit:

Feature Tracking

- Similar to feature matching, but track instead of match:
  - Track small, good features using translation only (u,v)
  - Use RANSAC to solve more complex motion model (Scale, Rotation, Similarity, Affine, Homography, ... Articulated, non-rigid)





Slide Credit:

Conditions for solvability

Optimal (u, v) satisfies Lucas-Kanade equation

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$$A^T A \qquad\qquad\qquad\qquad A^T b$$

When is this solvable? I.e., what are good points to track?

- **A^{T}A** should be invertible
- **A^{T}A** should not be too small due to noise
  - eigenvalues λ_1 and λ_2 of **A^{T}A** should not be too small
- **A^{T}A** should be well-conditioned
  - λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)

Recall: This is the Harris Corner Detector!

Slide Credit:

Low-texture region

$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

Slide Credit:

Edge

$$\sum \nabla I (\nabla I)^T$$

– gradients very large or very small

– large λ_1, small λ_2

Slide Credit:

High-texture region

$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

Slide Credit:

Feature Point tracking

- Find a good point to track (harris corner)
- Track small patches (5x5 to 31x31) (e.g. using Lucas-Kanade)
- For rigid objects with affine motion: solve motion model parameters by robust estimation (RANSAC)

[Kanade, Lucas, Tamasi] Slide Credit:

Implementation issues

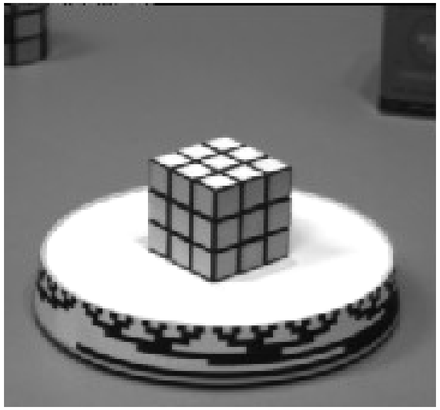
- Window size
  - Small window more sensitive to noise and may miss larger motions (without pyramid)
  - Large window more likely to cross an occlusion boundary (and it's slower)
  - 15x15 to 31x31 seems typical

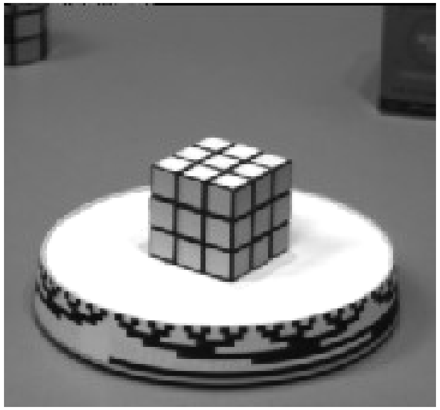
- Weighting the window
  - Common to apply weights so that center matters more (e.g., with Gaussian)

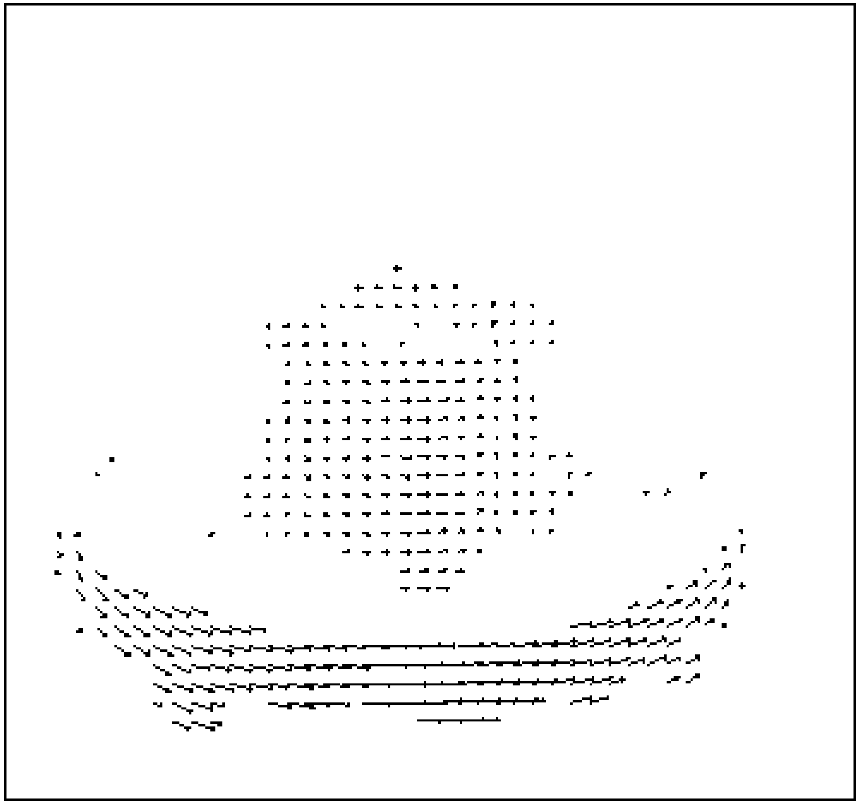
 Slide Credit:

Dense Motion field

- The motion field is the projection of the 3D scene motion into the image







What would the motion field of a non-rotating ball moving towards the camera look like?

Slide Credit:

Optical flow

- Definition: optical flow is the *apparent* motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field
- Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  - Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

Lucas-Kanade Optical Flow

- Same as Lucas-Kanade feature tracking, but densely for each pixel
  - As we saw, works better for textured pixels
- Operations can be done one frame at a time, rather than pixel by pixel
  - Efficient

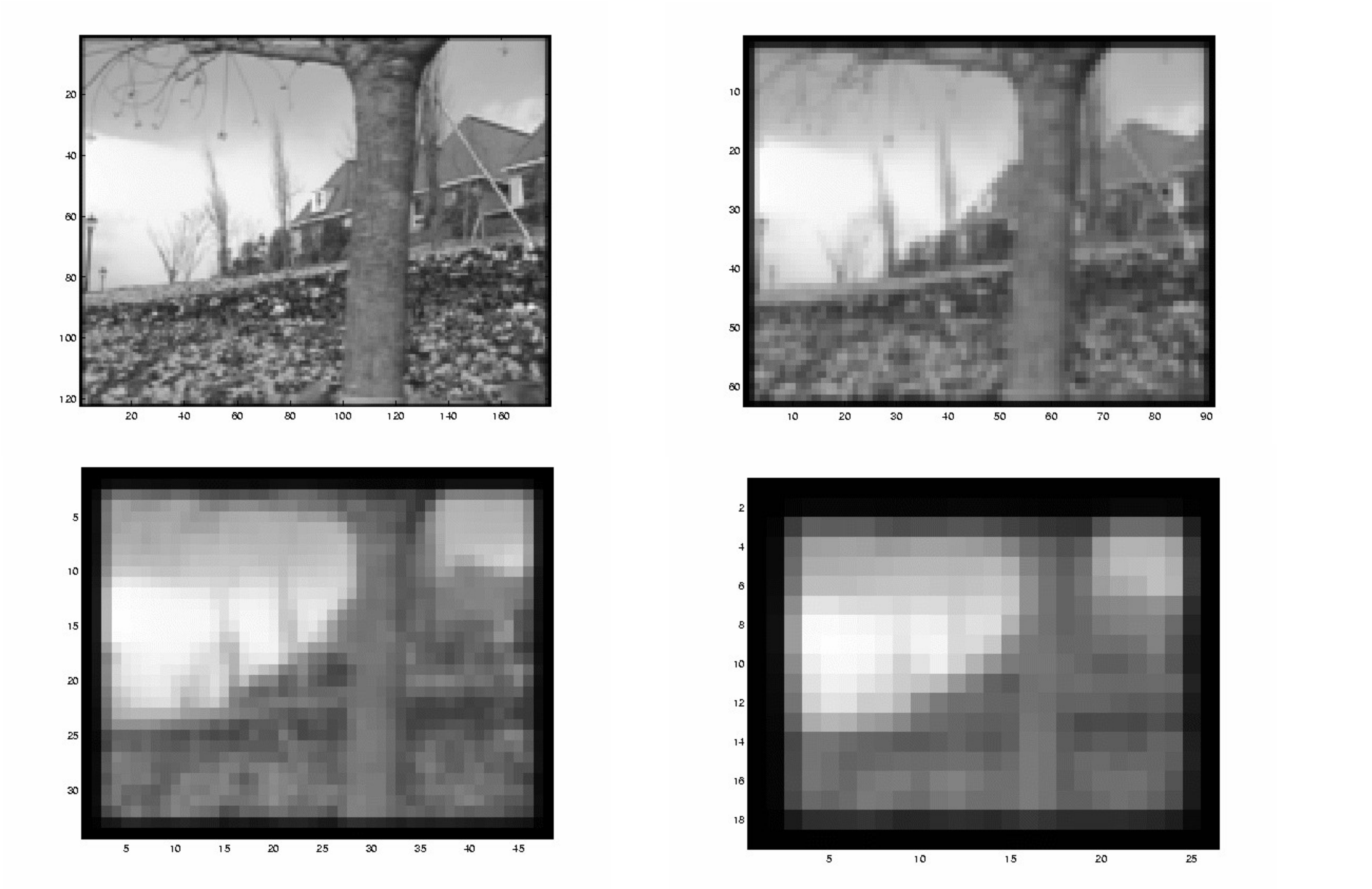
Slide Credit:

Example

* From Khurram Hassan-Shafique CAP5415 Computer Vision 20

Slide Credit:

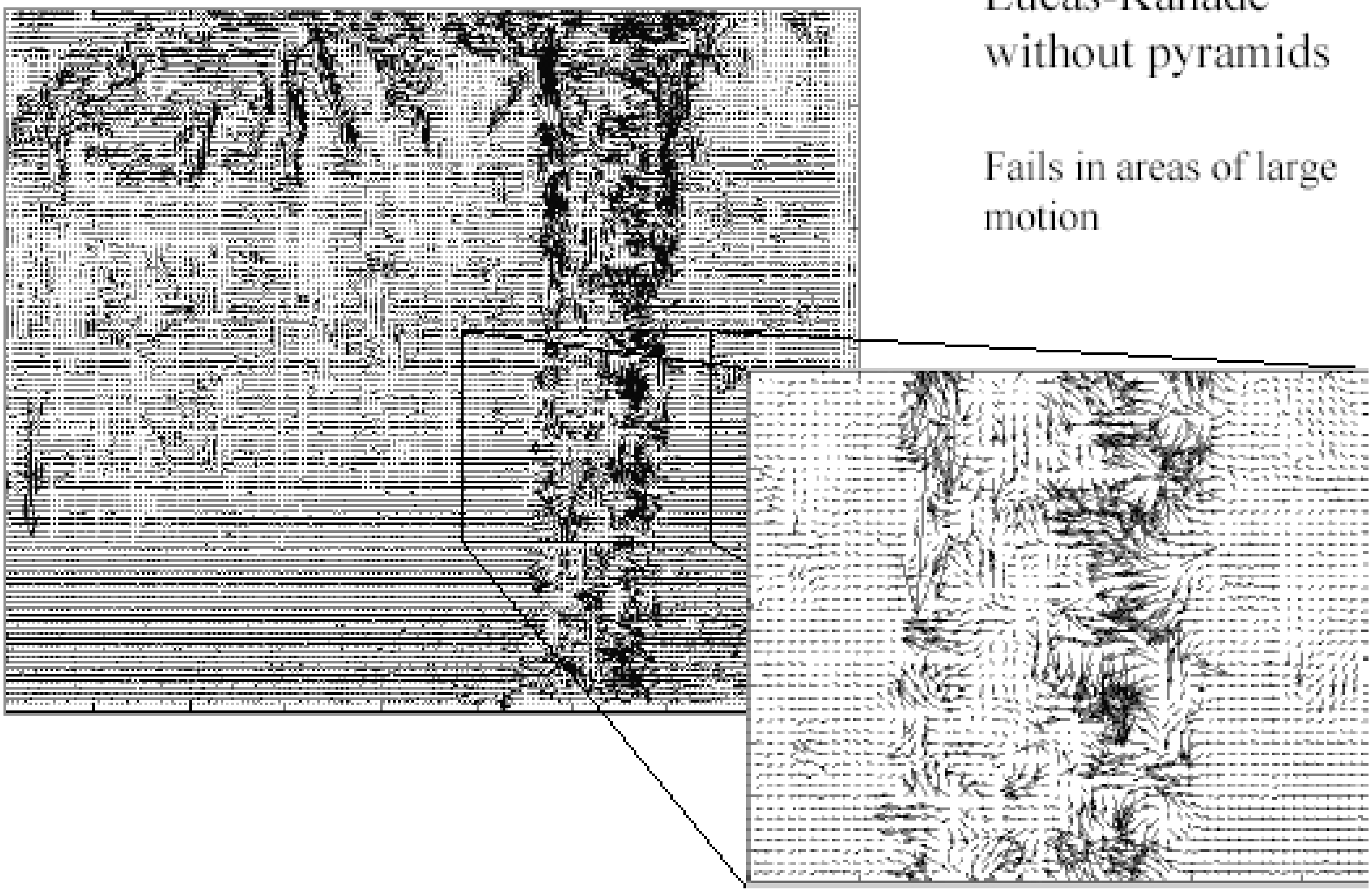
Multi-resolution registration



* From Khurram Hassan-Shafique CAP5415 Computer Vision 20

Slide Credit:

Optical Flow Results



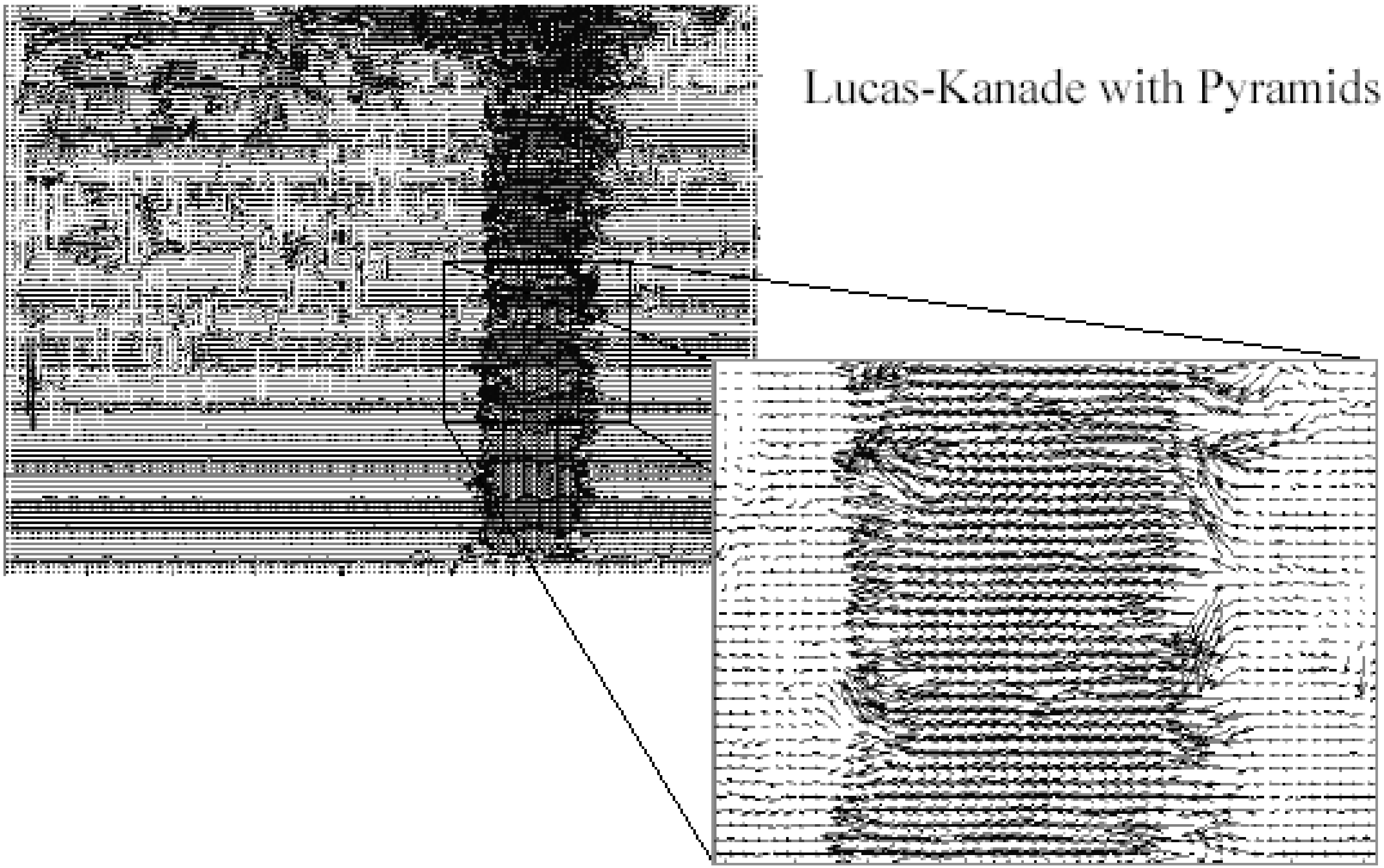
Lucas-Kanade
without pyramids

Fails in areas of large motion

* From Khurram Hassan-Shafique CAP5415 Computer Vision 20

Slide Credit:

Optical Flow Results



Slide Credit: * From Khurram Hassan-Shafique CAP5415 Computer Vision 20

Errors in Lucas-Kanade

- The motion is large
  - Possible Fix: Keypoint matching, coarse search, multiresolution
- A point does not move like its neighbors
  - Possible Fix: Region-based matching
- Brightness constancy does not hold
  - Possible Fix: Gradient constancy

Slide Credit: