Shape from Shading and Texture

Lambertian Reflectance Model

- Diffuse surfaces appear equally bright from all directions

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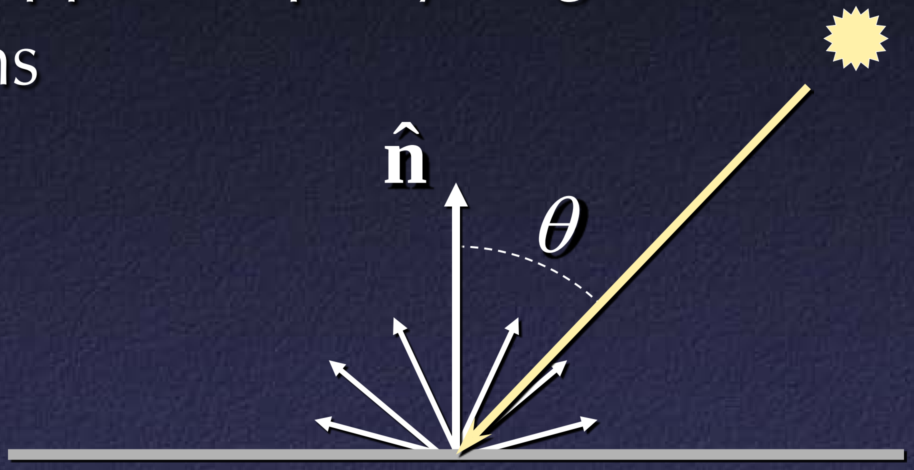


- For illumination coming from a single direction, brightness proportional to cos θ

Lambertian Reflectance Model

- Therefore, for a constant-colored object with distant illumination, we can write

$$E = L \rho \, l \cdot n$$

E = observed brightness

L = brightness of light source

ρ = reflectance (albedo) of surface

l = direction to light source

n = surface normal

Shape from Shading

- The above equation contains some information about shape, and in some cases is enough to recover shape completely (in theory) if L, ρ, and l are known
- Similar to integration (surface normal is like a derivative), but only know a part of derivative
- Have to assume surface continuity

Shape from Shading

- Assume surface is given by Z(x,y)
- Let

$$p = \frac{\partial Z}{\partial x}, \quad q = \frac{\partial Z}{\partial y}$$

- In this case, surface normal is

$$\mathbf{n} = \frac{1}{\sqrt{1 + p^2 + q^2}} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix}$$

Shape from Shading

- So, write

$$E = \frac{L\rho}{\sqrt{1+p^2+q^2}} \begin{pmatrix} l_x & l_y & l_z \end{pmatrix} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix}$$

- Discretize: end up with one equation per pixel
- But this is *p* equations in 2*p* unknowns…

Shape from Shading

- Integrability constraint:

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial^2 Z}{\partial y \partial x} \quad \Rightarrow \quad \frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$

- Wind up with system of $2p$ (nonlinear) differential equations
- No solution in presence of noise or depth discontinuities

Estimating Illumination and Albedo

- Need to know surface reflectance and Illumination brightness and direction
- In general, can't compute from single image
- Certain assumptions permit estimating these
  - Assume uniform distribution of normals, look at distribution of intensities in image
  - Insert known reference object into image
  - Slightly specular (shiny) object: estimate lighting from specular highlights, then discard pixels in highlights

Variational Shape from Shading

- Approach: energy minimization
- Given observed E(x,y), find shape Z(x,y) that minimizes energy

$$\mathcal{E} = \iint \left(\left(E(x,y) - L\rho \mathbf{l} \cdot \mathbf{n}(x,y) \right)^2 + \lambda \left({p_x}^2 + {p_y}^2 + {q_x}^2 + {q_y}^2 \right) \right) dx\, dy$$

- Regularization: minimize combination of disparity w. data, surface curvature

Variational Shape from Shading

- Solve by techniques from calculus of variations
- Use Euler-Lagrange equations to get a PDE, solve numerically
  - Unlike with snakes, "greedy" methods tend not to work well

$$\frac{\partial E}{\partial p} - \frac{d}{dx}\frac{\partial E}{\partial p_x} - \frac{d}{dy}\frac{\partial E}{\partial p_y} = 0$$

$$\frac{\partial E}{\partial q} - \frac{d}{dx}\frac{\partial E}{\partial q_x} - \frac{d}{dy}\frac{\partial E}{\partial q_y} = 0$$

Enforcing Integrability

- Let f_Z be the Fourier transform of Z, f_p and f_q be Fourier transforms of p and q
- Then

$$f_Z = \frac{f_p}{i\omega_x} = \frac{f_q}{i\omega_y}$$

- For nonintegrable p and q these aren′t equal

Enforcing Integrability

- Construct

$$f_{Z'} = \frac{\omega_x f_p + \omega_y f_q}{i\left(\omega_x^2 + \omega_y^2\right)}$$

and recompute

$$f_{p'} = i\omega_x f_{Z'}, \quad f_{q'} = i\omega_y f_{Z'}$$

- The new p' and q' are the integrable equations *closest* to the original p and q

Difficulties with Shape from Shading

- Robust estimation of L, ρ, l?
- Shadows
- Non-Lambertian surfaces
- More than 1 light, or "diffuse illumination"
- Interreflections

Shape from Shading Results

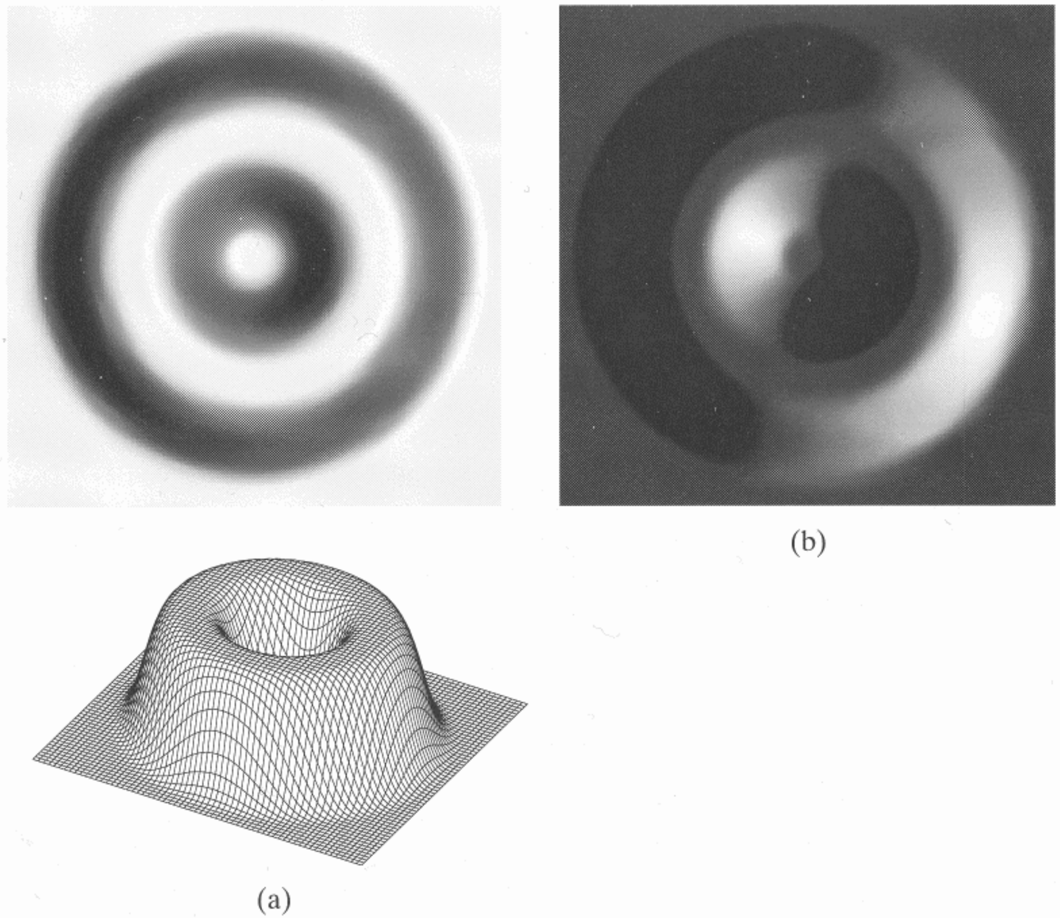


Figure 9.2 Two images of the same Lambertian surface seen from above but illuminated from different directions and 3-D rendering of the surface. Practically all the points in the top left image receive direct illumination ($\mathbf{i} = [0.20, 0, 0.98]^\top$); some regions of the top right image are in the dark due to self-shadowing effects ($\mathbf{i} = [0.94, 0.31, 0.16]^\top$).

[Trucco & Verri]

Shape from Shading Results

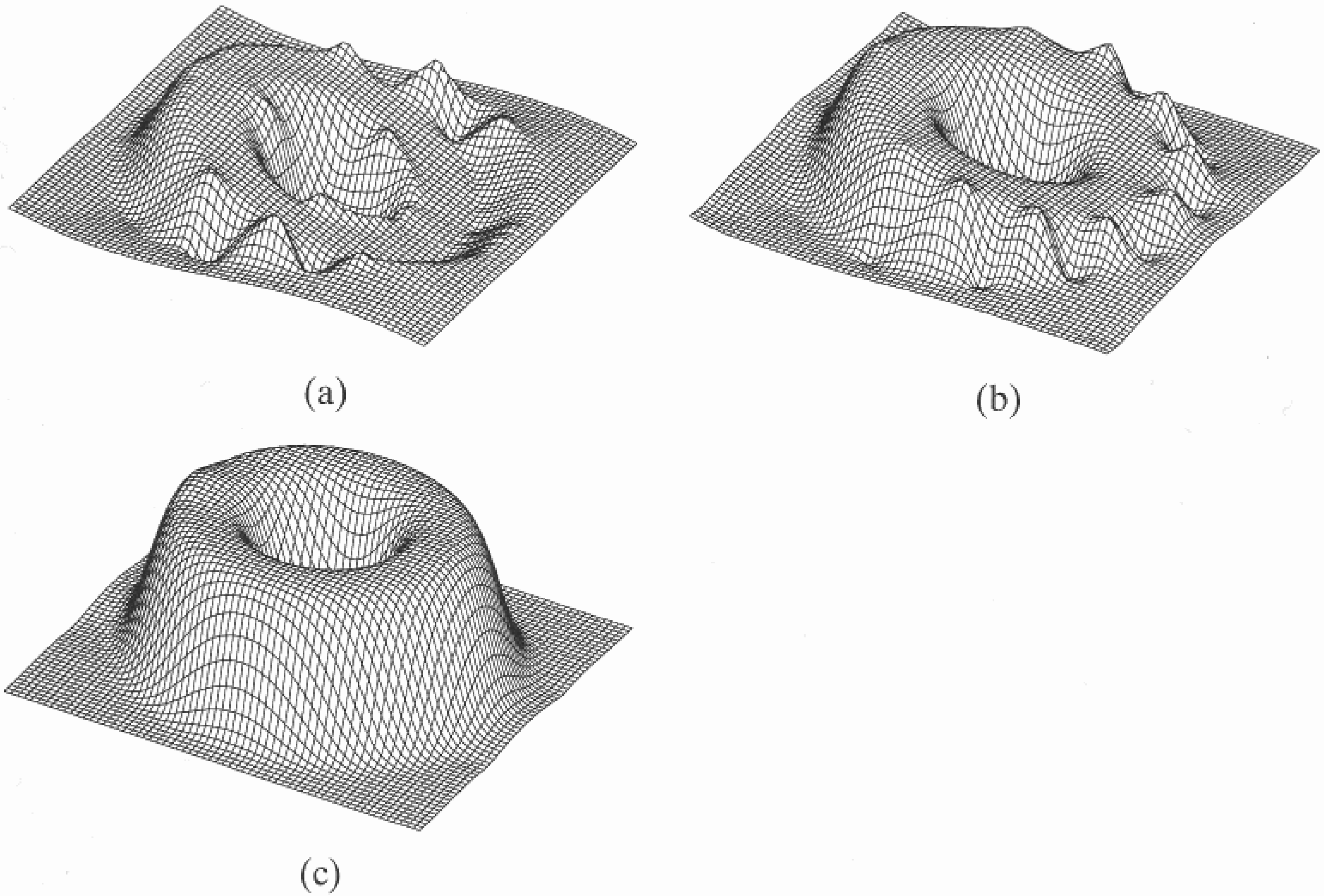
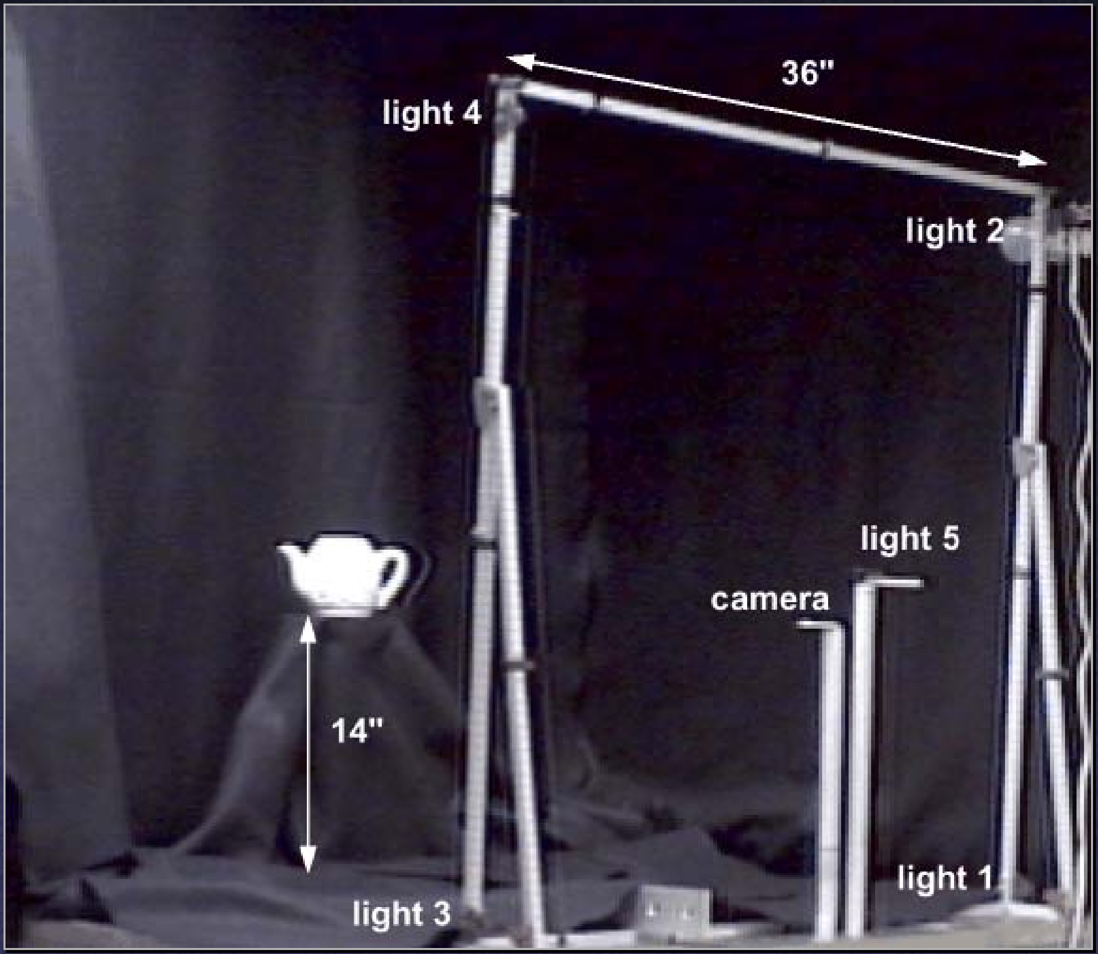


Figure 9.4 Reconstructions of the surface in Figure 9.2 after 100 (a), 1000 (b) and 2000 (c) iterations. The initial surface was a plane of constant height. The asymmetry of the first two reconstruction is due to the illuminant direction.

Active Shape from Shading

- Idea: several (user-controlled) light sources
- More data
  - Allows determining surface normal directly
  - Allows spatially-varying reflectance
  - Redundant measurements: discard shadows and specular highlights
- Often called "photometric stereo"

Photometric Stereo Setup



[Rushmeier et al., 1997]

Photometric Stereo Math

- For each point *p*, can write

$$\rho_p \begin{bmatrix} l_{1,x} & l_{1,y} & l_{1,z} \\ l_{2,x} & l_{2,y} & l_{2,z} \\ l_{3,x} & l_{3,y} & l_{3,z} \end{bmatrix} \begin{bmatrix} n_{p,x} \\ n_{p,y} \\ n_{p,z} \end{bmatrix} = \alpha \begin{bmatrix} E_{p,1} \\ E_{p,2} \\ E_{p,3} \end{bmatrix}$$

- Constant α incorporates light source brightness, camera sensitivity, etc.

Photometric Stereo Math

- Solving above equation gives (ρ / α) n
- n must be unit-length $\Rightarrow$ uniquely determined
- Determine ρ up to global constant
- With more than 3 light sources:
  - Discard highest and lowest measurements
  - If still more, solve by least squares

Photometric Stereo Results

Input images

Recovered normals (re-lit)

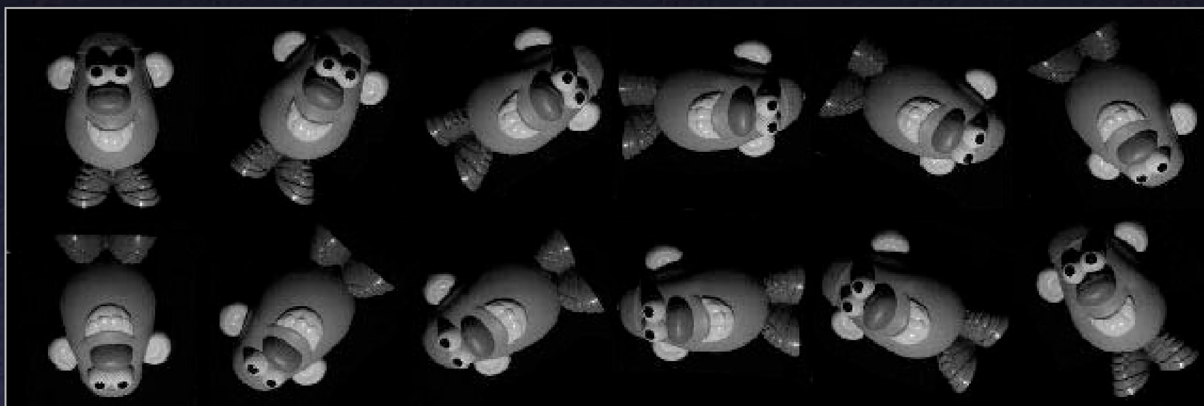
Recovered color

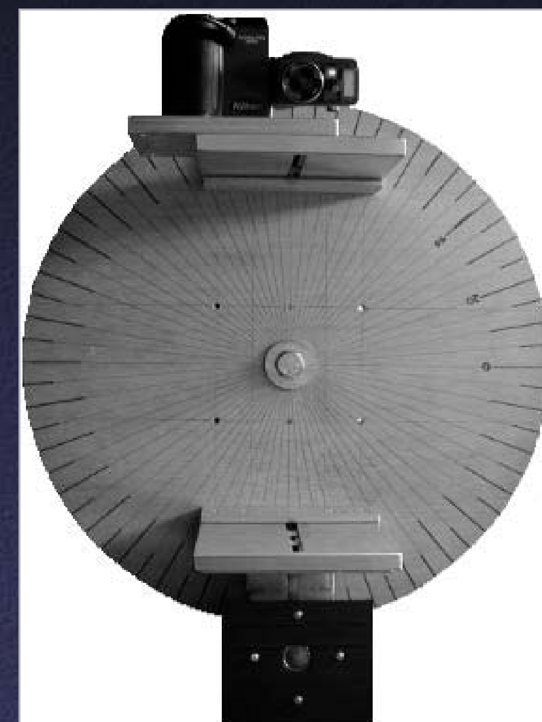
[Rushmeier et al., 1997]

Helmholtz Stereopsis

- Based on Helmholtz reciprocity: surface reflectance is the same under interchange of light, viewer
- So, take pairs of observations with viewer, light interchanged
- Ratio of the observations in a pair is independent of surface material

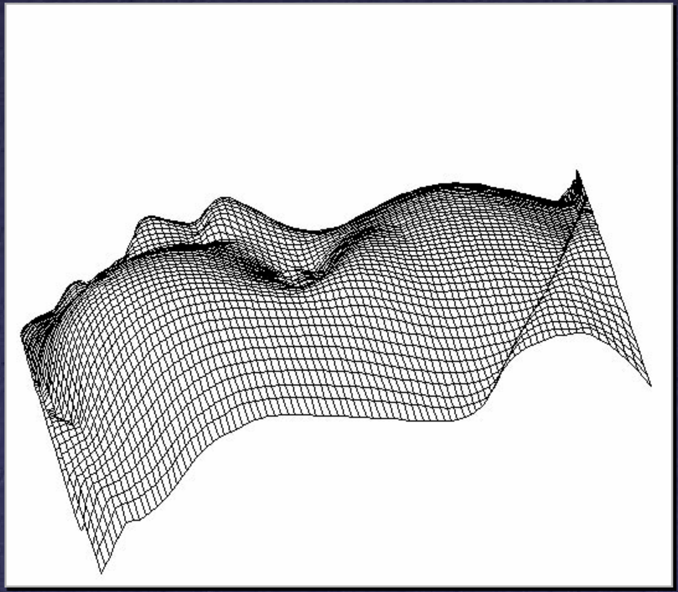
Helmholtz Stereopsis

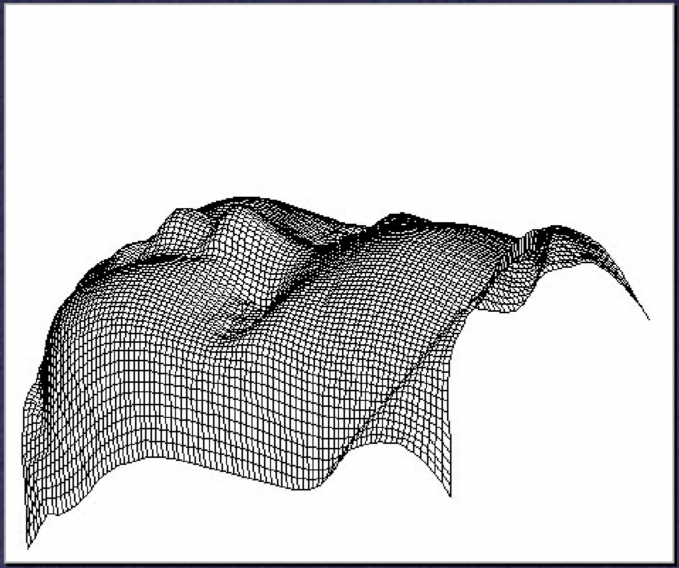




[Zickler, Belhumeur, & Kriegman]

Helmholtz Stereopsis





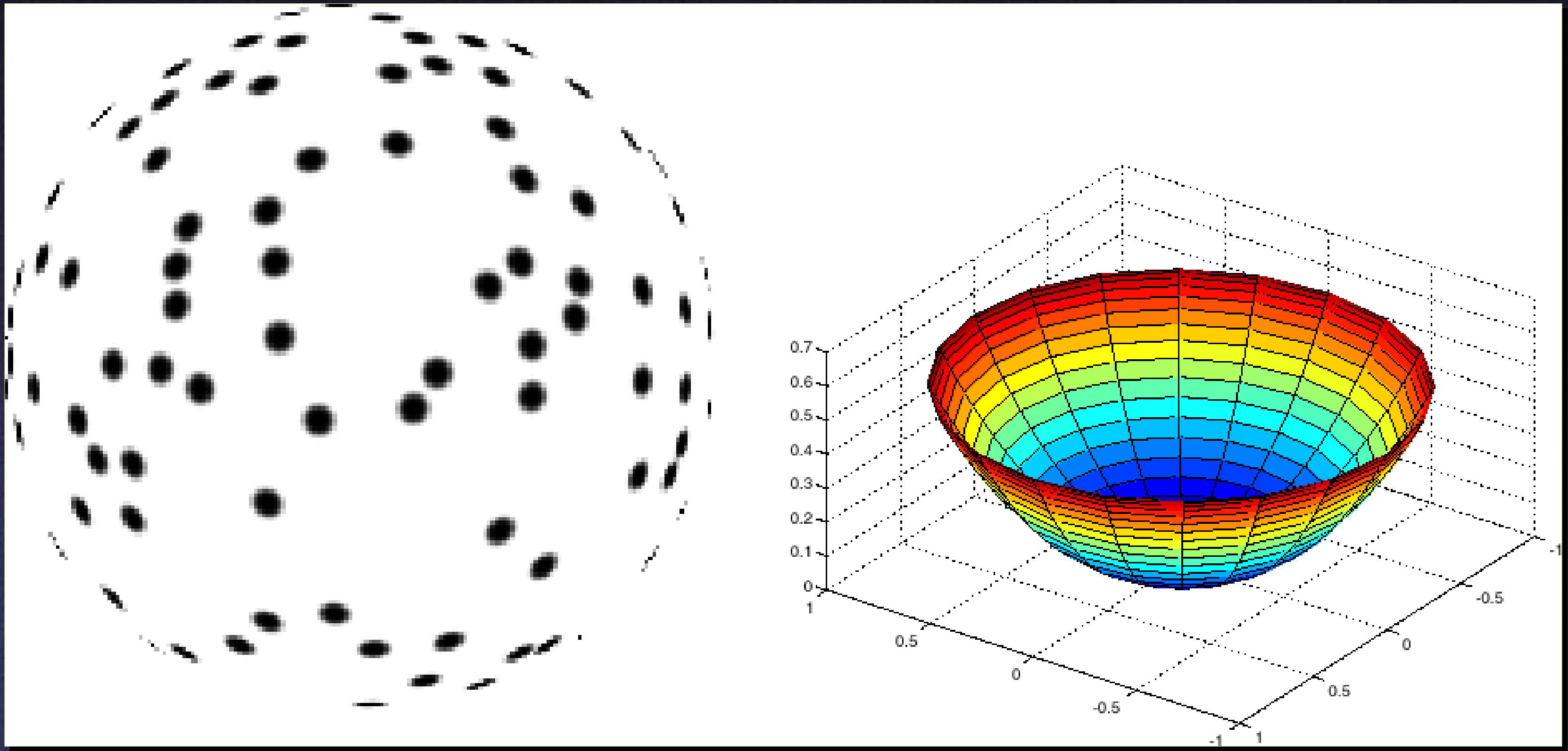
Texture

- Texture: repeated pattern on a surface
- Elements (“textons”) either identical or come from some statistical distribution
- Shape from texture comes from looking at deformation of individual textons or from distribution of textons on a surface

Shape from Texture

- Much the same as shape from shading, but have more information
  - Foreshortening: gives surface normal (not just one component, as in shape from shading)
  - Perspective distortion: gives information about depth directly
- Sparse depth information (only at textons)
  - About the same as shape from shading, because of smoothness term in energy eqn.

Shape from Texture Results



[Forsyth]