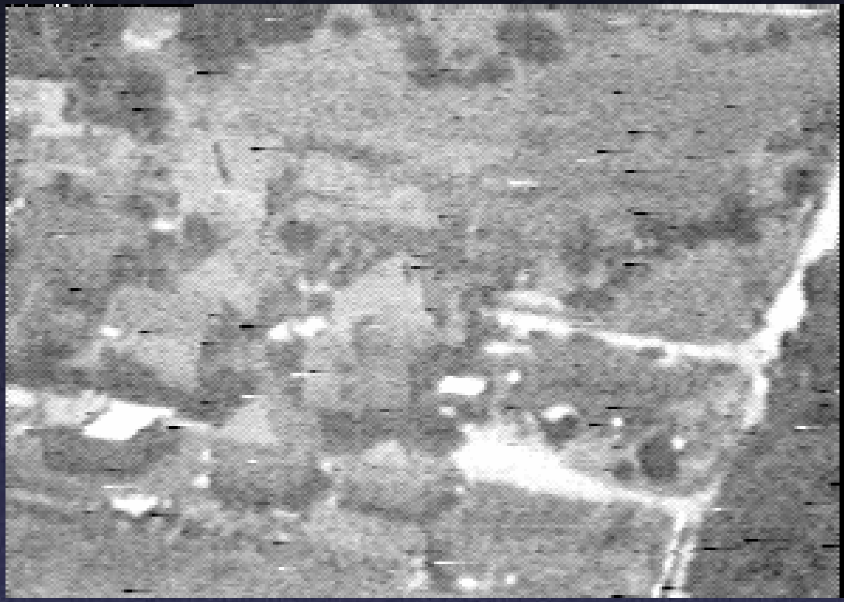
Image Alignment and Mosaicing

Image Alignment Applications

- Local alignment:
  - Tracking
  - Stereo
- Global alignment:
  - Camera jitter elimination
  - Image enhancement
  - Panoramic mosaicing

Image Enhancement



Original

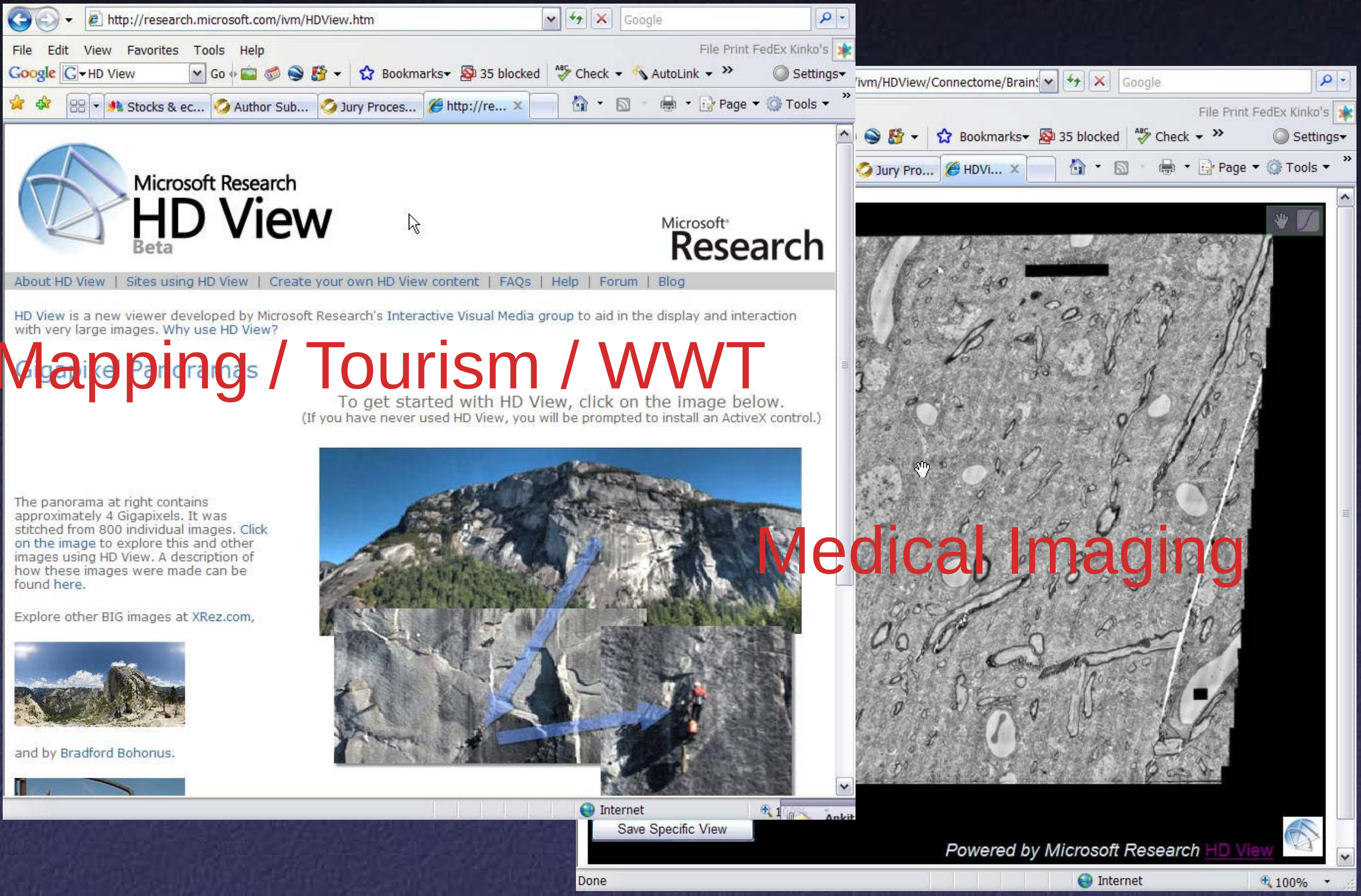
Enhanced

Anandan

Panoramic Mosaicing

Anandan

Gigapixel panoramas & images



[Szeliski]

Panoramic Mosaicing

1. Align images
2. Merge overlapping regions

Anandan

Correspondence Approaches

- Optical flow
- Correlation
- Correlation + optical flow
- Any of the above, iterated (e.g. Lucas-Kanade)
- Any of the above, coarse-to-fine
- Feature matching + RANSAC

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Optical Flow for Image Registration

- Compute local matches
- Least-squares fit to motion model
- Problem: outliers

Outlier Rejection

- Robust estimation: tolerant of outliers
- In general, methods based on absolute value rather than square:

$$\text{minimize } \Sigma|x_i - f|, \text{ not } \Sigma(x_i - f)^2$$

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Correlation / Search Methods

- Assume translation only
- Given images I_1, I_2
- For each translation (t_x, t_y) compute

$$c(I_1, I_2, \mathbf{t}) = \sum_i \sum_j \psi\big(I_1(i,j), I_2(i-t_x, j-t_y)\big)$$

- Select translation that maximizes c
- Depending on window size, local or global

Cross-Correlation

- Statistical definition of correlation:

$$\psi(u,v) = uv$$

- Disadvantage: sensitive to local variations in image brightness

Normalized Cross-Correlation

- Normalize to eliminate brightness sensitivity:

$$\psi(u,v) = \frac{(u-\bar{u})(v-\bar{v})}{\sigma_u \sigma_v}$$

where

$$\bar{u} = \text{average}(u)$$

$$\sigma_u = \text{standard deviation}(u)$$

Sum of Squared Differences

- More intuitive measure:

$$\psi(u,v) = -(u-v)^2$$

- Negative: higher values $\rightarrow$ greater similarity
- Expand:

$$-(u-v)^2 = -u^2 - v^2 + 2uv$$

Local vs. Global

- Correlation with local windows not too expensive
- High cost if window size = whole image

- But computation looks like convolution
  - FFT to the rescue!

Fourier Transform with Translation

$$\mathcal{F}\Big(f(x+\Delta x, y+\Delta y)\Big) = \mathcal{F}\Big(f(x,y)\Big)e^{i(\omega_x \Delta x + \omega_y \Delta y)}$$

Fourier Transform with Translation

- Therefore, if I_1 and I_2 differ by translation,

$$\mathcal{F}\left(I_1(x,y)\right) = \mathcal{F}\left(I_2(x,y)\right) e^{i(\omega_x \Delta x + \omega_y \Delta y)}$$

$$\frac{F_1}{F_2} = e^{i(\omega_x \Delta x + \omega_y \Delta y)}$$

- So, $\mathcal{F}^{-1}(F_1/F_2)$ will have a peak at $(\Delta x, \Delta y)$

Phase Correlation

- In practice, use cross power spectrum

$$\frac{F_1 F_2^*}{\left| F_1 F_2^* \right|}$$

- Compute inverse FFT, look for peaks
- [Kuglin & Hines 1975]

Phase Correlation

- Advantages
  - Fast computation
  - Low sensitivity to global brightness changes (since equally sensitive to all frequencies)

Phase Correlation

- Disadvantages
  - Sensitive to white noise
  - No robust version
  - Translation only
    - Extensions to rotation, scale
    - But not local motion
    - Not too bad in practice with small local motions

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Correlation plus Optical Flow

- Use e.g. phase correlation to find average translation (may be large)
- Use optical flow to find local motions

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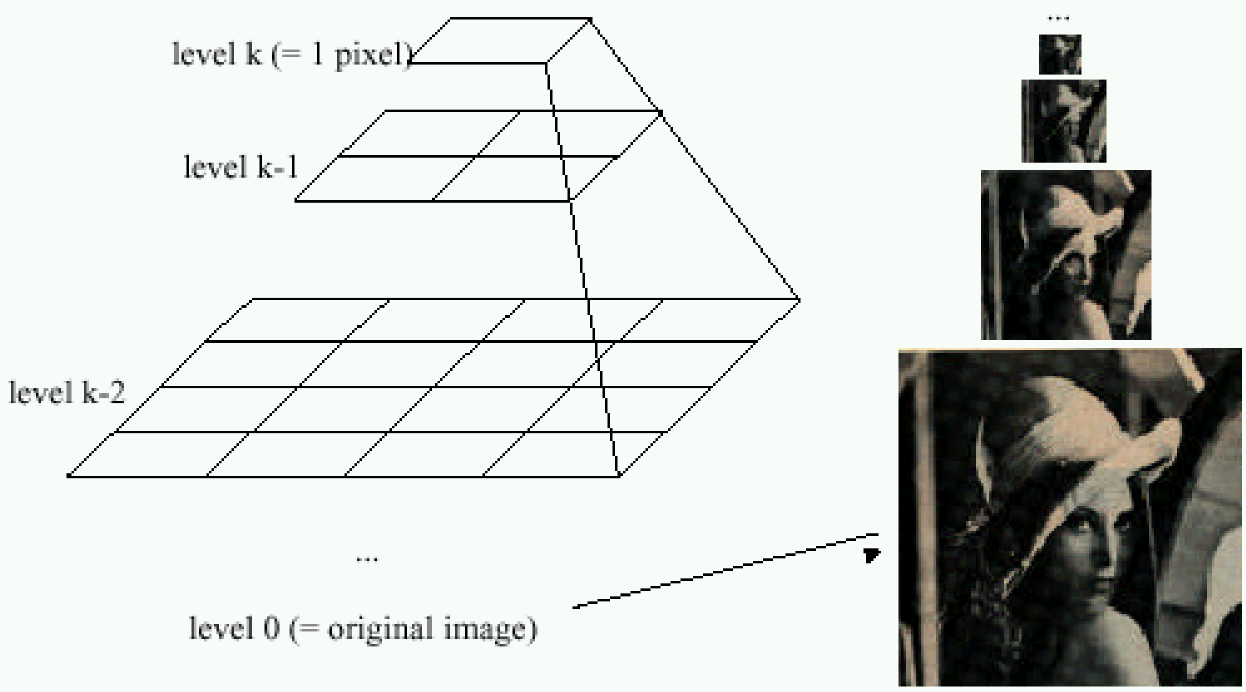
Image Pyramids

- Pre-filter images to collect information at different scales
- More efficient computation, allows larger motions

Image Pyramids

Idea: Represent NxN image as a “pyramid” of 1×1, 2×2, 4×4,…, $2^k \times 2^k$ images (assuming $N=2^k$)



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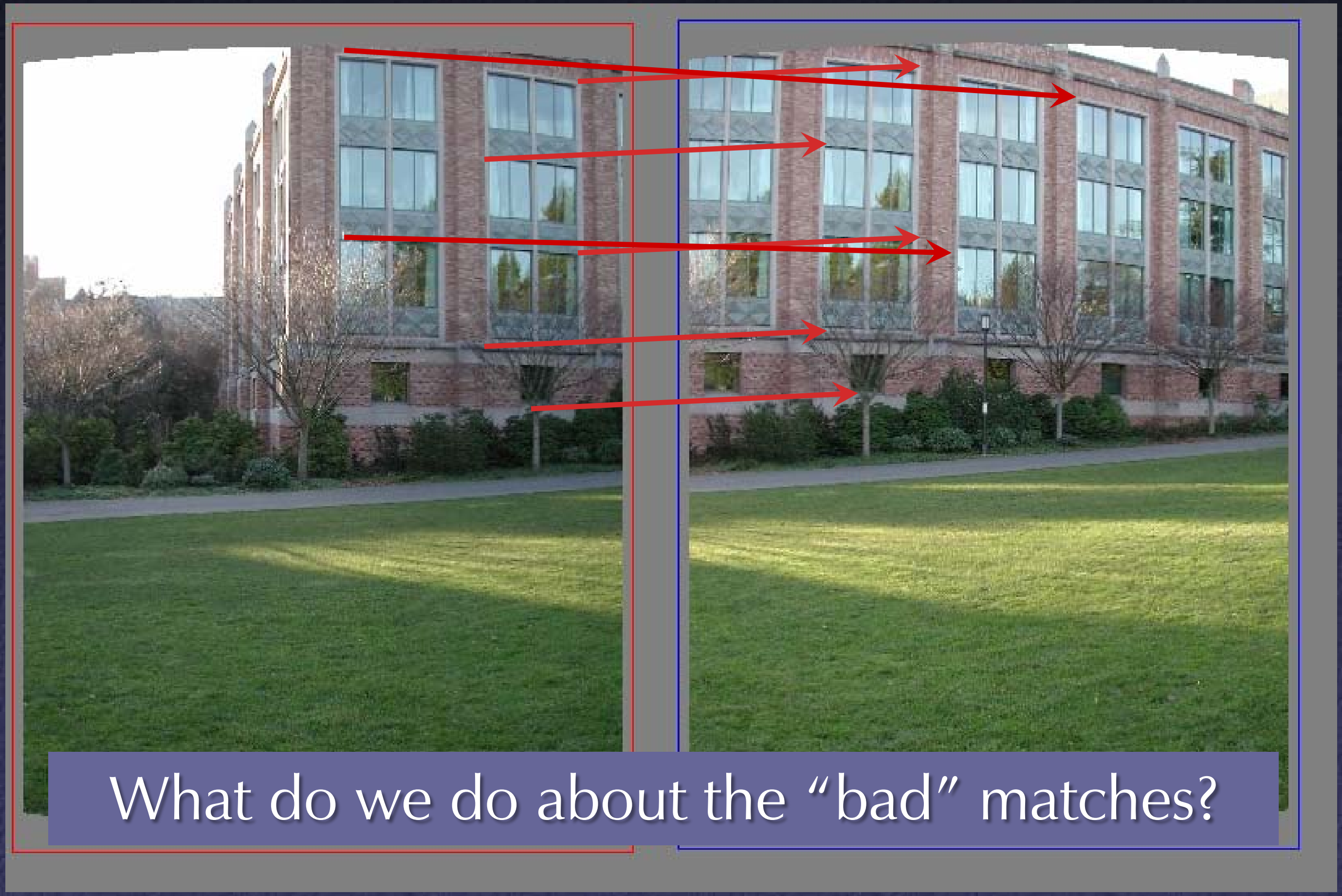
Matching features



What do we do about the “bad” matches?

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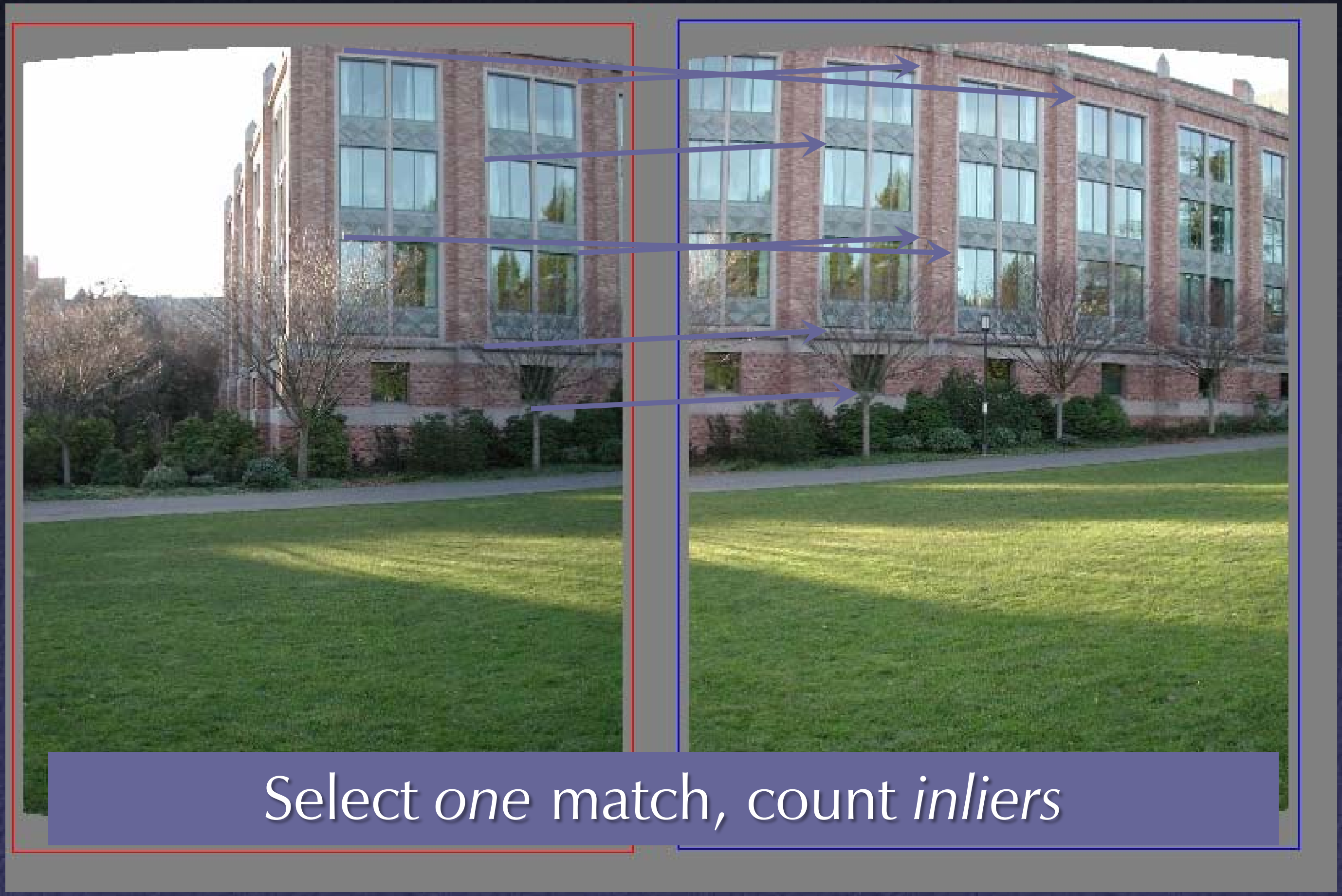
RAndom SAmple Consensus



Select *one* match, count *inliers*

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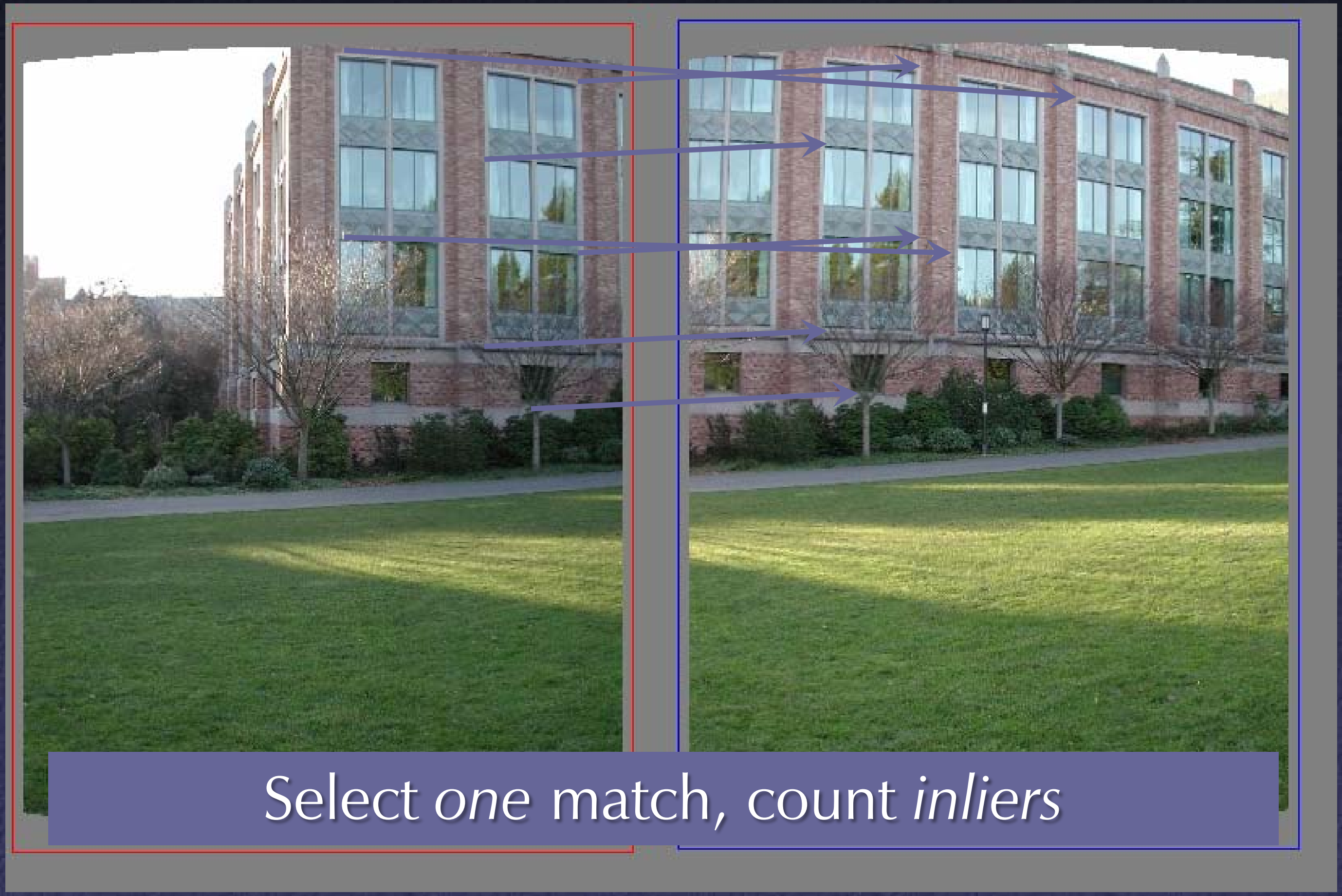
RAndom SAmple Consensus



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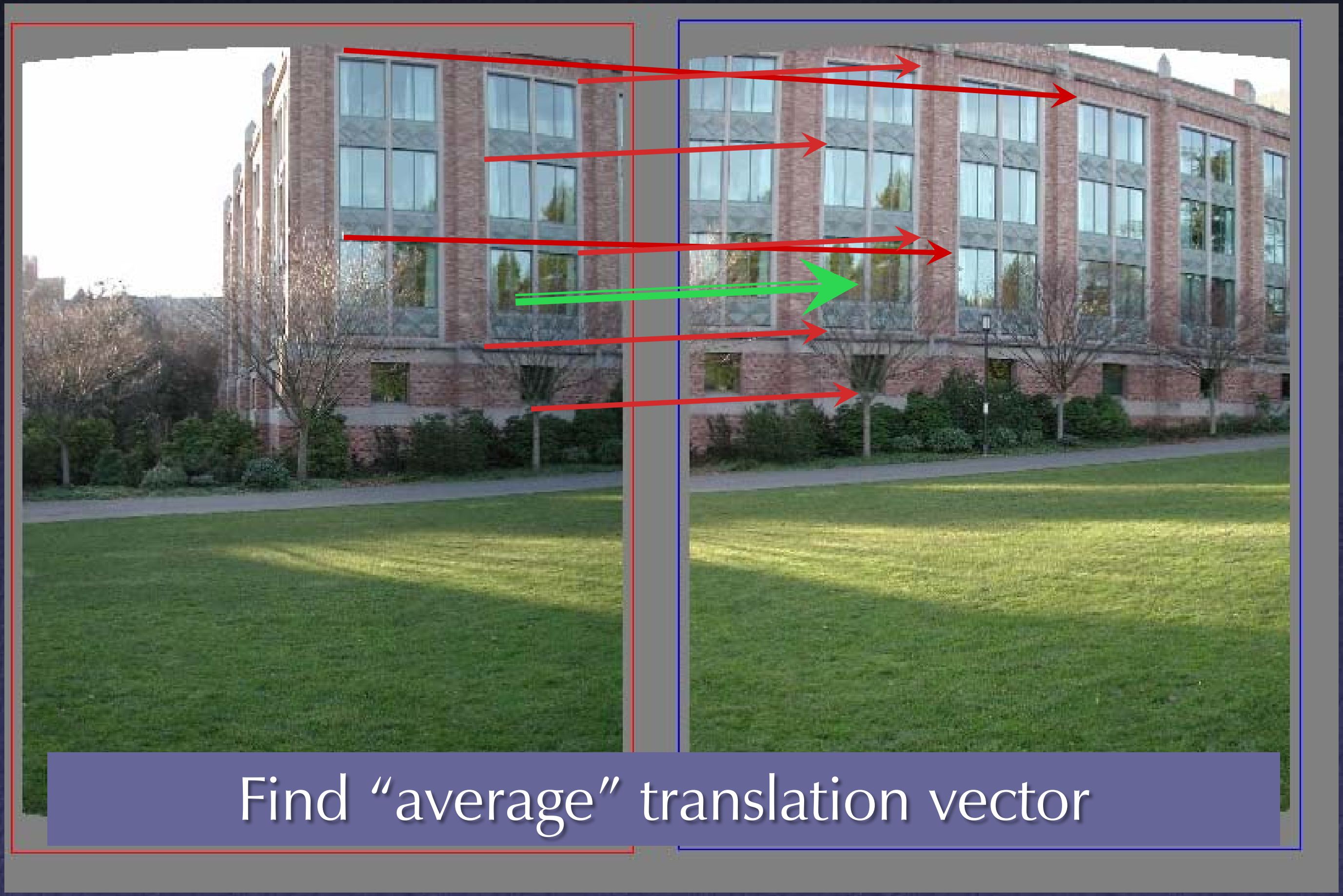
Least squares fit



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Panoramic Mosaicing

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Anandan

Blending

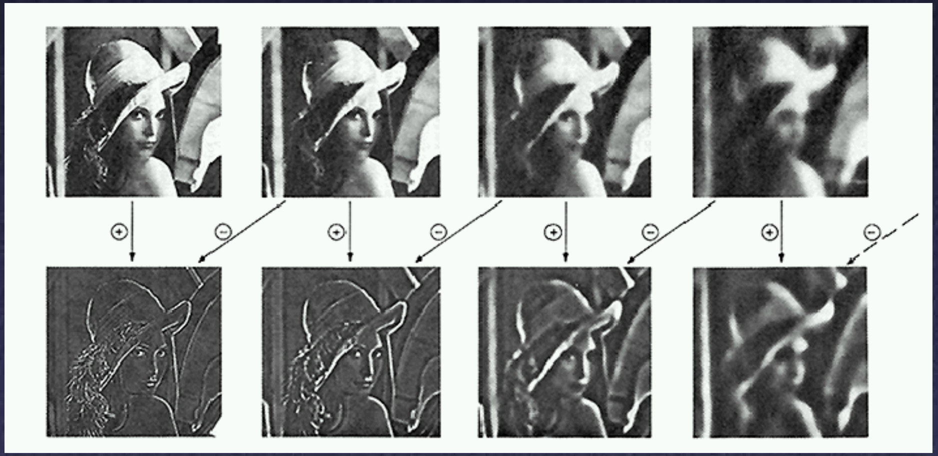
- Blend over too small a region: seams
- Blend over too large a region: ghosting

Multiresolution Blending

- Different blending regions for different levels in a pyramid [Burt & Adelson]
  - Blend low frequencies over large regions (minimize seams due to brightness variations)
  - Blend high frequencies over small regions (minimize ghosting)

Pyramid Creation

- "Gaussian" Pyramid
- "Laplacian" Pyramid
  - Created from Gaussian pyramid by subtraction $L_i = G_i - \text{expand}(G_{i+1})$



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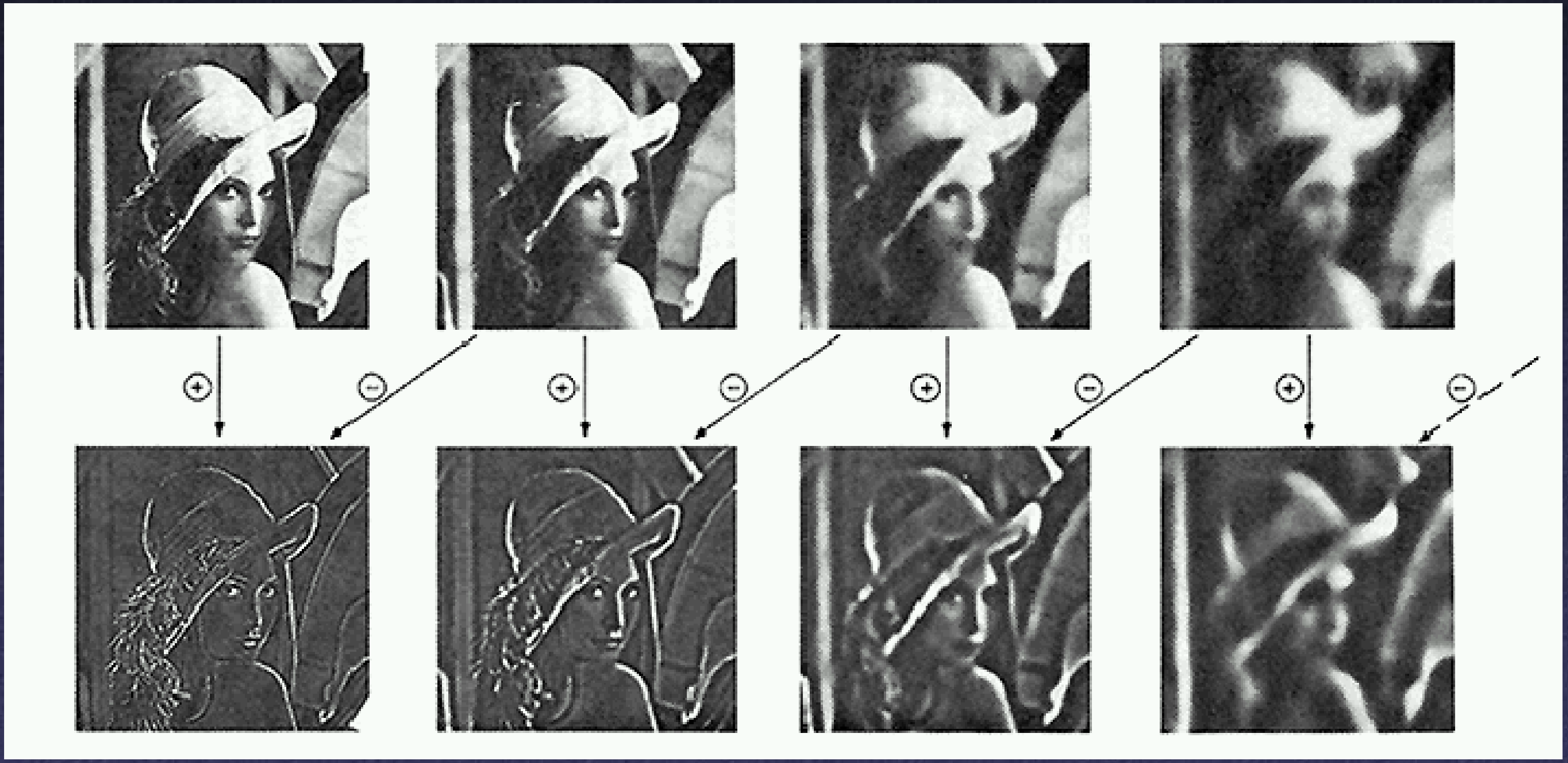
Octaves in the Spatial Domain

Lowpass Images



Bandpass Images

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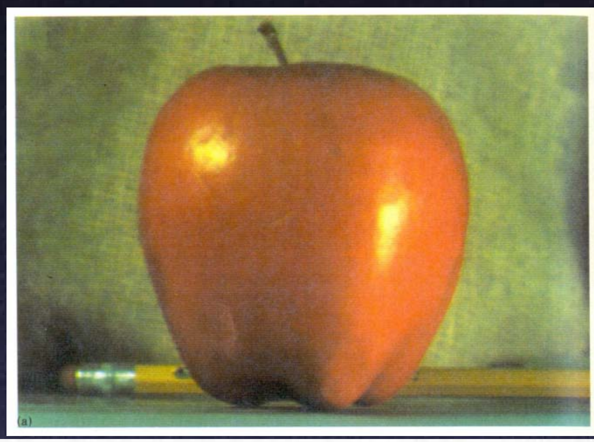
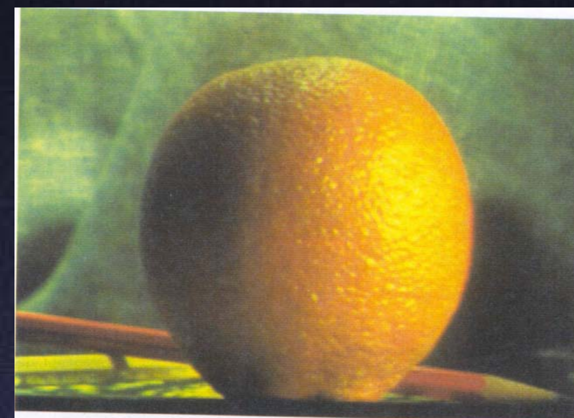
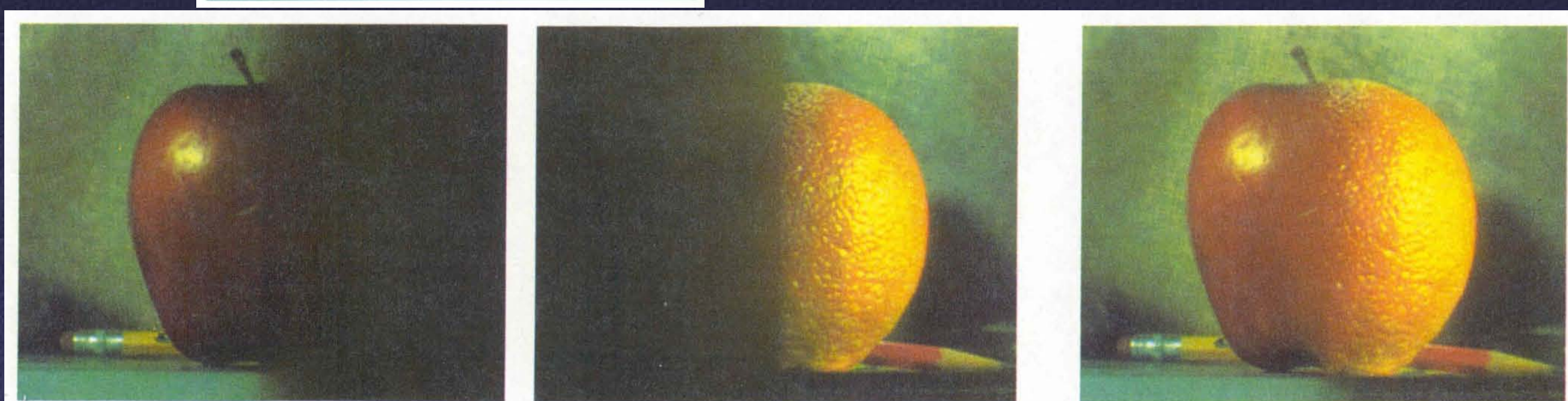
Pyramid Blending







(d) (h) (l)

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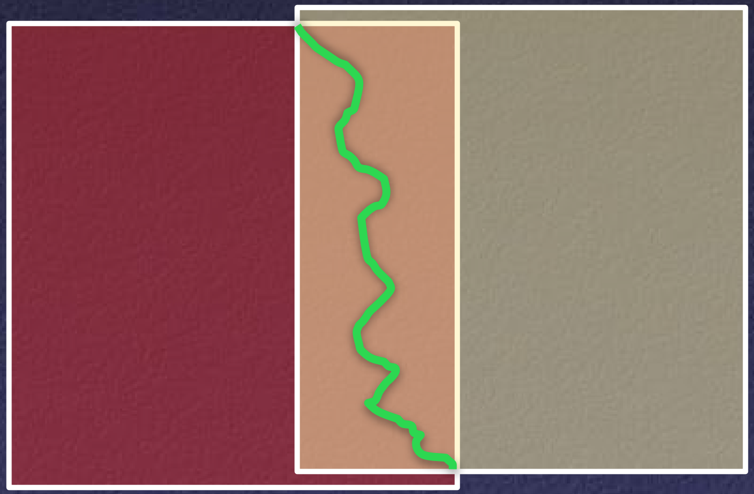
Minimum-Cost Cuts

- Instead of blending high frequencies along a straight line, blend along line of minimum differences in image intensities



Minimum-Cost Cuts

Moving object, simple blending → blur

[Davis 98]

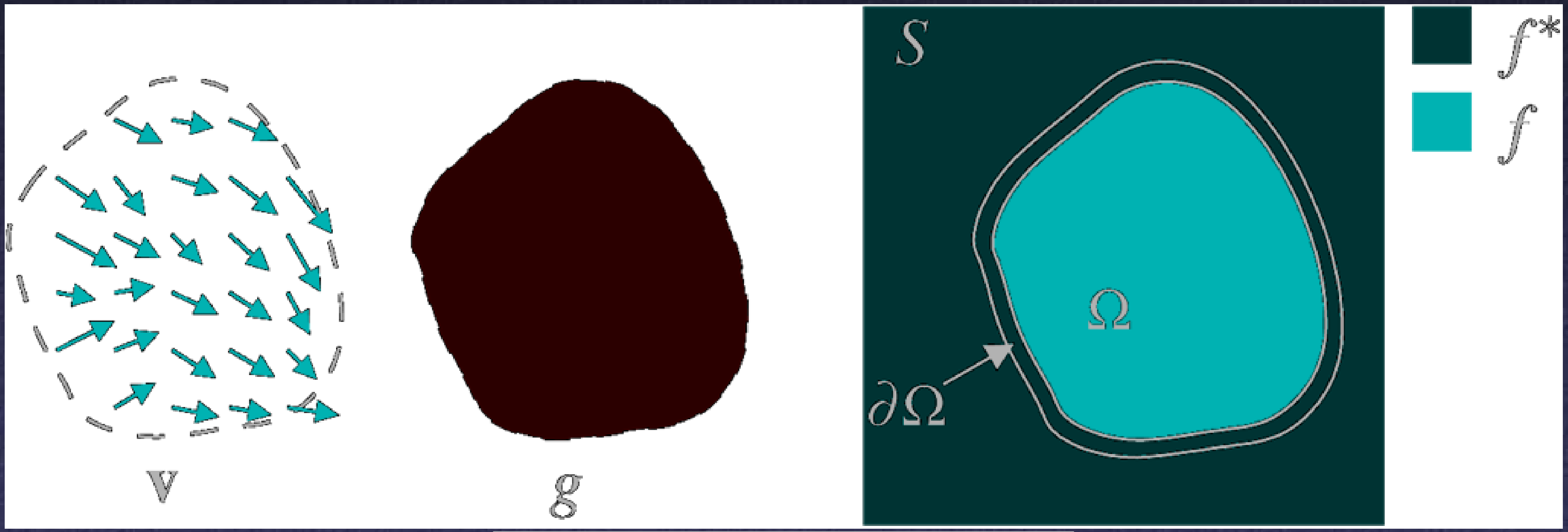
Minimum-Cost Cuts

Minimum-cost cut → no blur

[Davis 98]

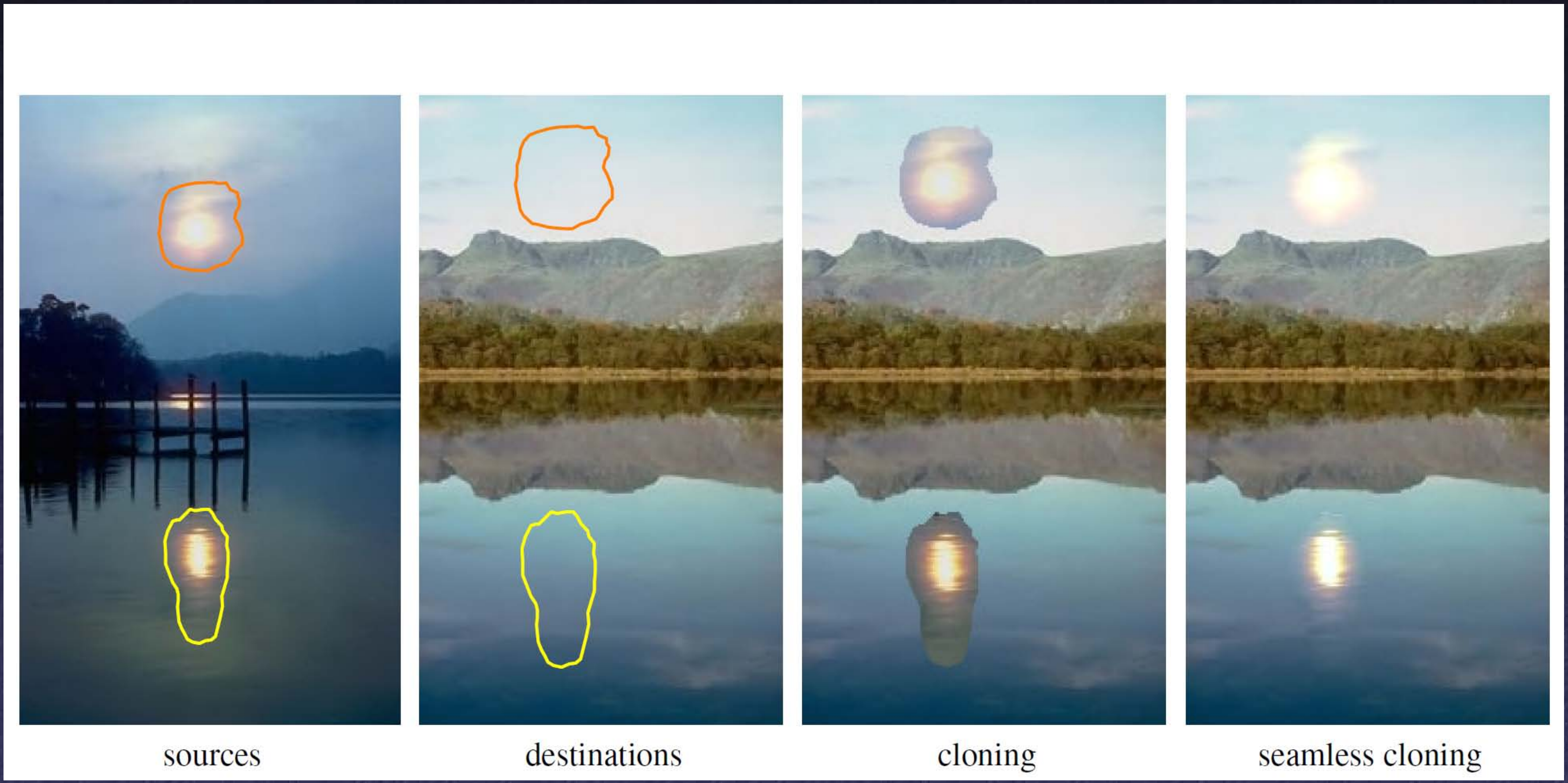
Poisson Image Blending

- Follow gradients of source subject to boundary conditions imposed by dest



$$\begin{cases} \nabla^2 f = \nabla \cdot \mathbf{v} \\ f|_{\partial\Omega} = f^*|_{\partial\Omega} \end{cases}$$

Poisson Image Blending



Poisson Image Blending