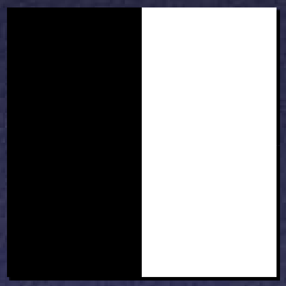
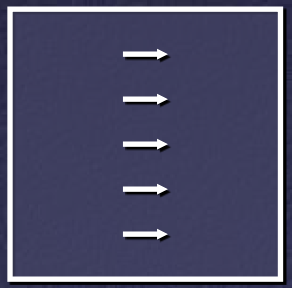
Feature Detectors and Descriptors: Corners, Lines, etc.

Edges vs. Corners

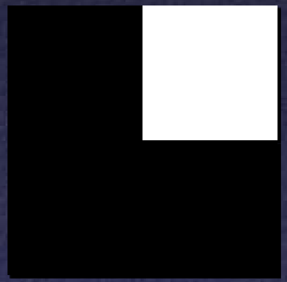
- Edges = maxima in intensity gradient

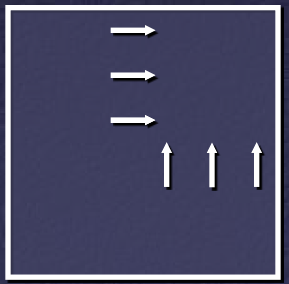




Edges vs. Corners

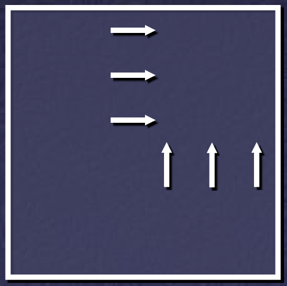
- Corners = lots of variation in direction of gradient in a small neighborhood





Detecting Corners

- How to detect this variation?
- Not enough to check average $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$



Detecting Corners

- Claim: the following covariance matrix summarizes the statistics of the gradient

$$C = \begin{bmatrix} \sum f_x^2 & \sum f_x f_y \\ \sum f_x f_y & \sum f_y^2 \end{bmatrix} \qquad f_x = \frac{\partial f}{\partial x}, \; f_y = \frac{\partial f}{\partial y}$$

Summations over local neighborhoods

- Can have spatially-varying weights (Gaussian, etc.)

Detecting Corners

- Examine behavior of C by testing its effect in simple cases
- Case #1: Single edge in local neighborhood

Case#1: Single Edge

- Let (a,b) be gradient along edge
- Compute $C \cdot (a,b)$:

$$C \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum f_x^{\ 2} & \sum f_x f_y \\ \sum f_x f_y & \sum f_y^{\ 2} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \sum (\nabla f)(\nabla f)^{\mathrm{T}} \begin{bmatrix} a \\ b \end{bmatrix}$$

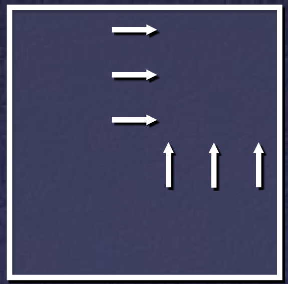
$$= \sum (\nabla f) \left(\nabla f \cdot \begin{bmatrix} a \\ b \end{bmatrix} \right)$$

Case #1: Single Edge

- However, in this simple case, the only nonzero terms are those where $\nabla f = (a,b)$
- So, $C \cdot (a,b)$ is just some multiple of (a,b)

Case #2: Corner

- Assume there is a corner, with perpendicular gradients (a,b) and (c,d)



Case #2: Corner

- What is $C \cdot (a,b)$?
  - Since $(a,b) \cdot (c,d) = 0$, the only nonzero terms are those where $\nabla f = (a,b)$
  - So, $C \cdot (a,b)$ is again just a multiple of (a,b)
- What is $C \cdot (c,d)$?
  - Since $(a,b) \cdot (c,d) = 0$, the only nonzero terms are those where $\nabla f = (c,d)$
  - So, $C \cdot (c,d)$ is a multiple of (c,d)

Corner Detection

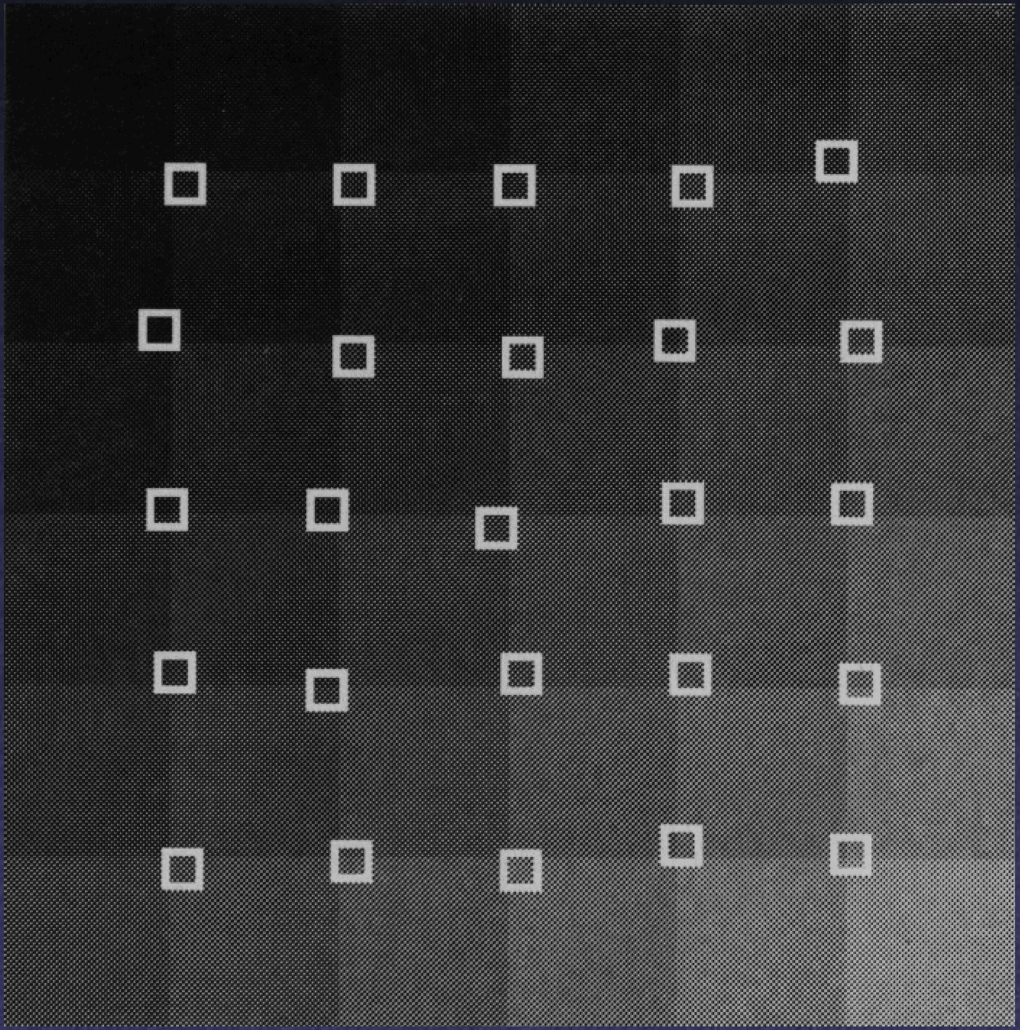
- Matrix times vector = multiple of vector
- Eigenvectors and eigenvalues!
- In particular, if C has **one** large eigenvalue, there's an edge
- If C has **two** large eigenvalues, have corner
- Tomasi-Kanade corner detector
  - Variants you may hear about: Förstner 1986, Harris & Stephens 1988, Shi & Tomasi 1994

Corner Detection Implementation

1. Compute image gradient
2. For each $m \times m$ neighborhood, compute matrix C (optionally using weighted sum)
3. If smaller eigenvalue λ_2 is larger than threshold τ, record a corner
4. Nonmaximum suppression: only keep strongest corner in each $m \times m$ window

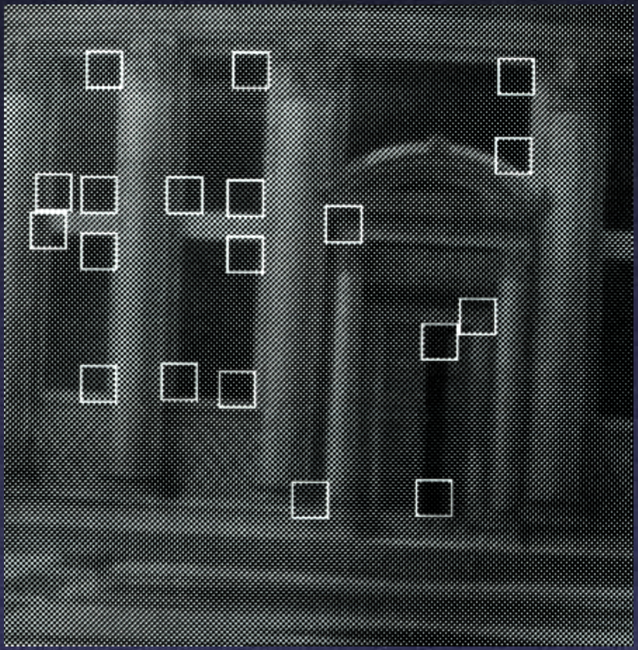
Corner Detection Results

- Checkerboard with noise

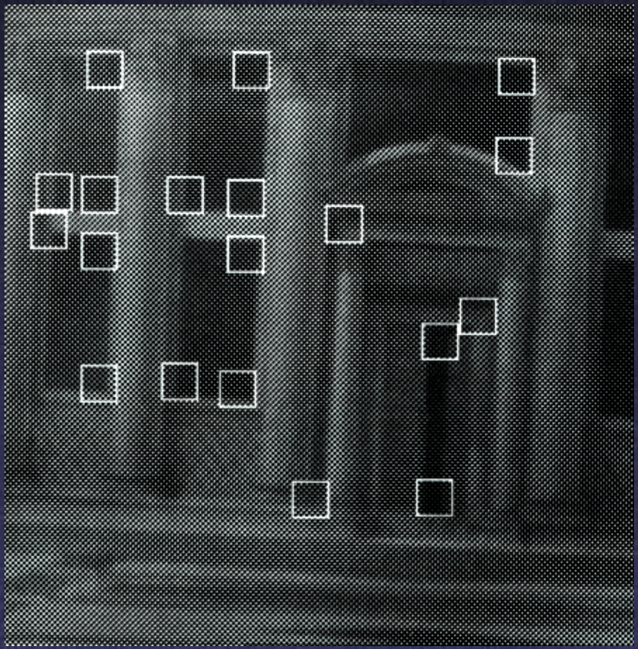


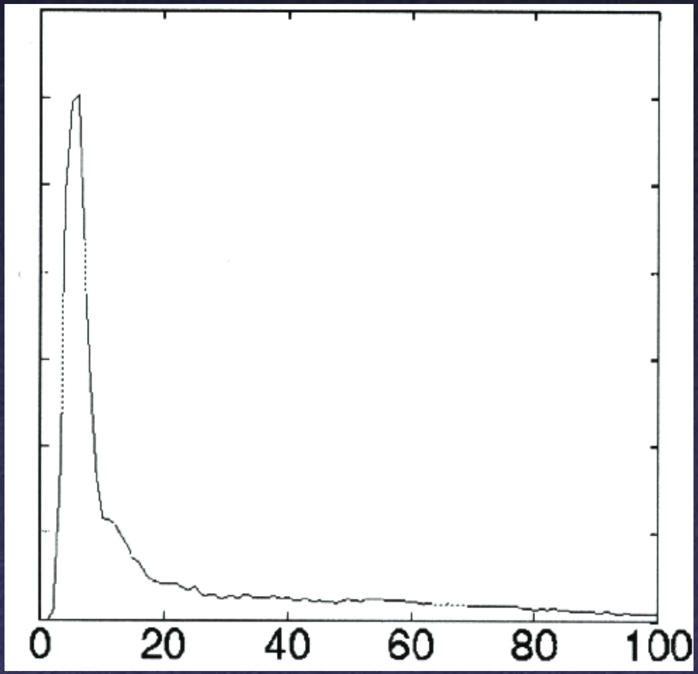
Trucco & Verri

Corner Detection Results



Corner Detection Results





Histogram of λ_2 (smaller eigenvalue)

Corner Detection

- Application: good features for tracking, correspondence, etc.
  - Why are corners better than edges for tracking?
- Other corner detectors
  - Look for curvature in edge detector output
  - Perform color segmentation on neighborhoods
  - Others…

Invariance

- Suppose you rotate the image by some angle
  - Will you still find the same corners?
- What if you change the brightness?
- Scale?

[Seitz]

Scale-Invariant Feature Detection

- Key idea: compute some function *f* over different scales, find extremum
  - Common definition of *f*: LoG or DoG
  - Find local minima or maxima over position and scale

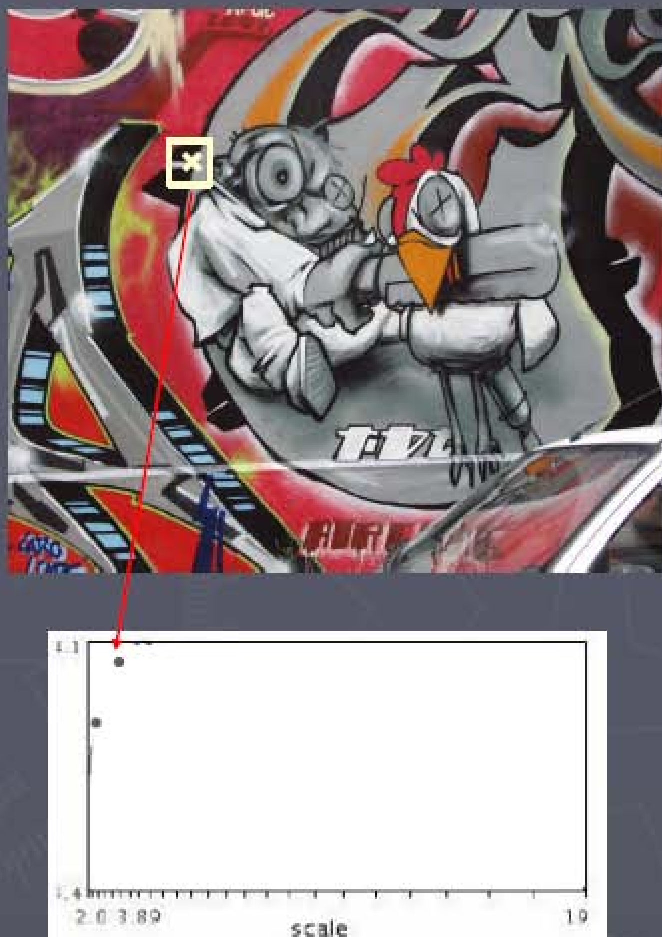
Automatic scale selection

Lindeberg et al., 1996

$f(I_{i_1 \ldots i_m}(x,\sigma))$

Slide from Tinne Tuytelaars

Automatic scale selection

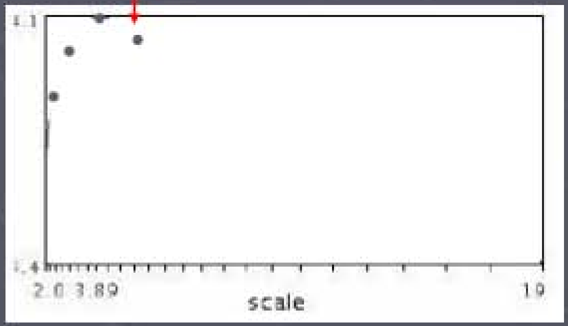


$f(I_{i_1 \dots i_m}(x,\sigma))$

Automatic scale selection

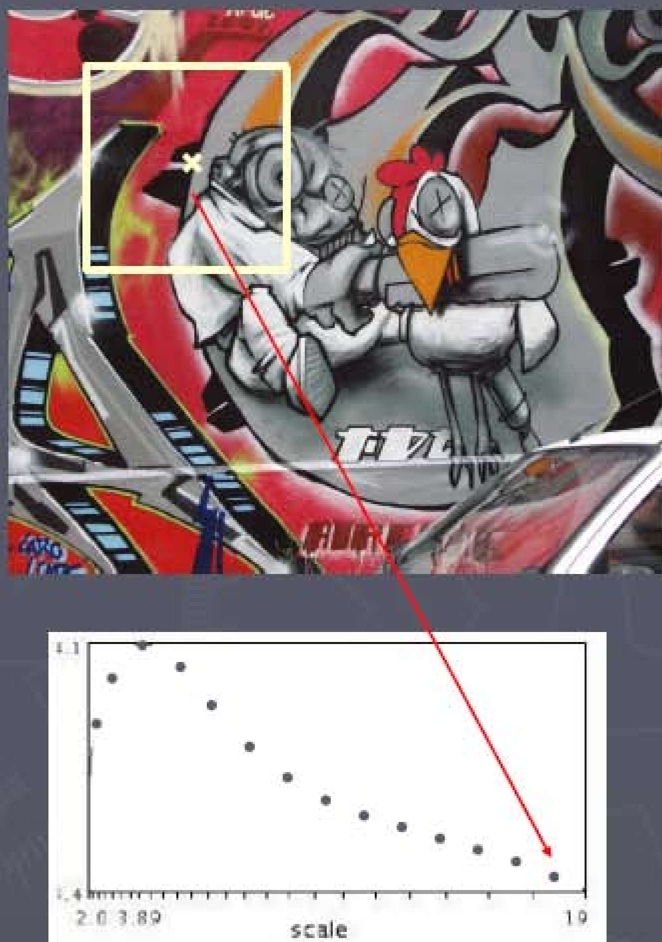
$f(I_{i_1 \ldots i_m}(x,\sigma))$

Automatic scale selection



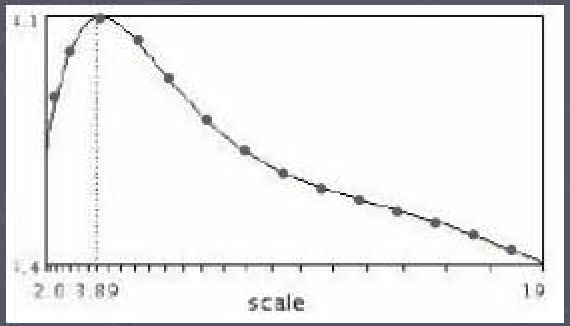
$f(I_{i_1 \ldots i_m}(x,\sigma))$

Automatic scale selection



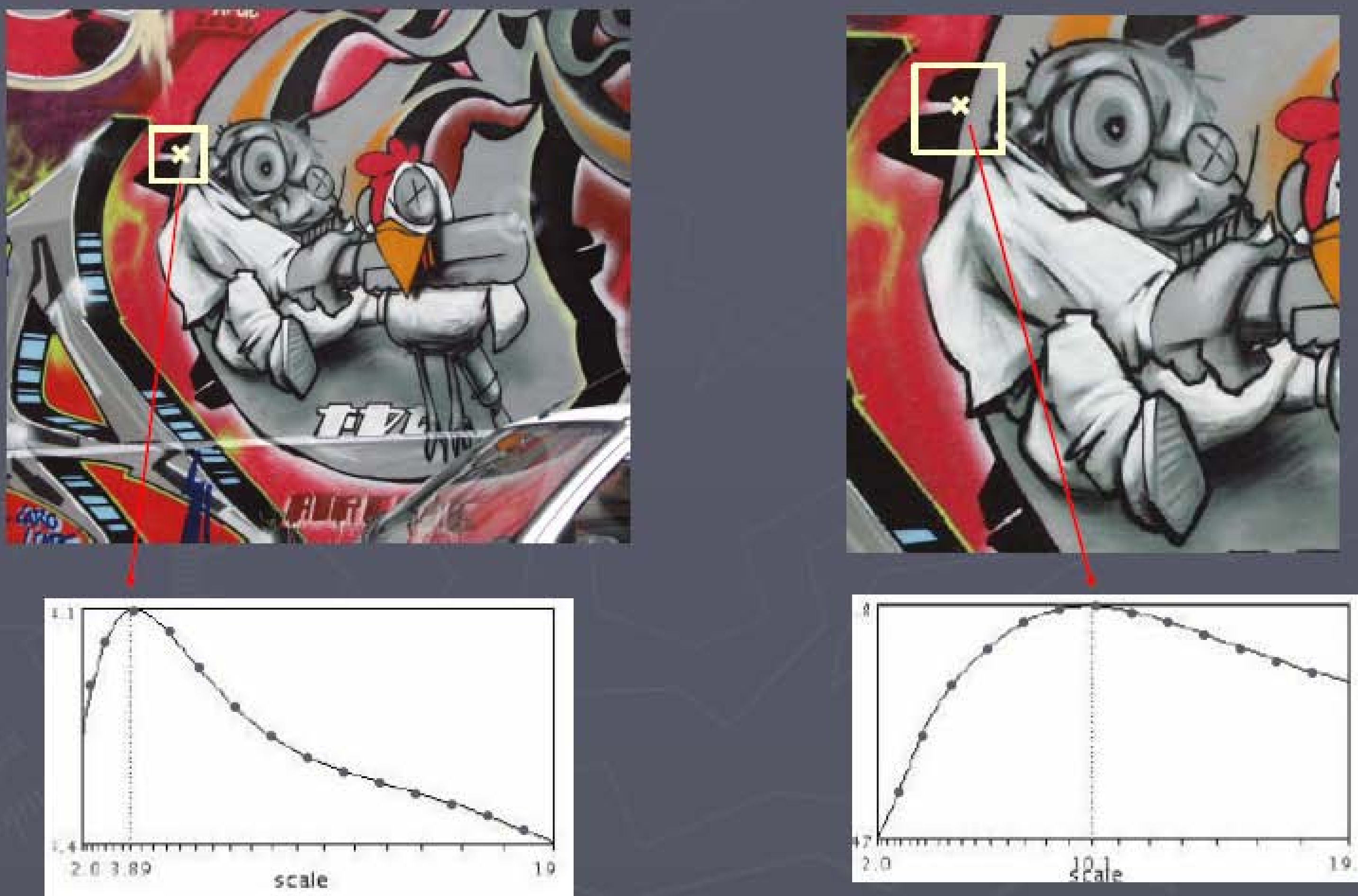
$f(I_{i_1 \ldots i_m}(x, \sigma))$

Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$

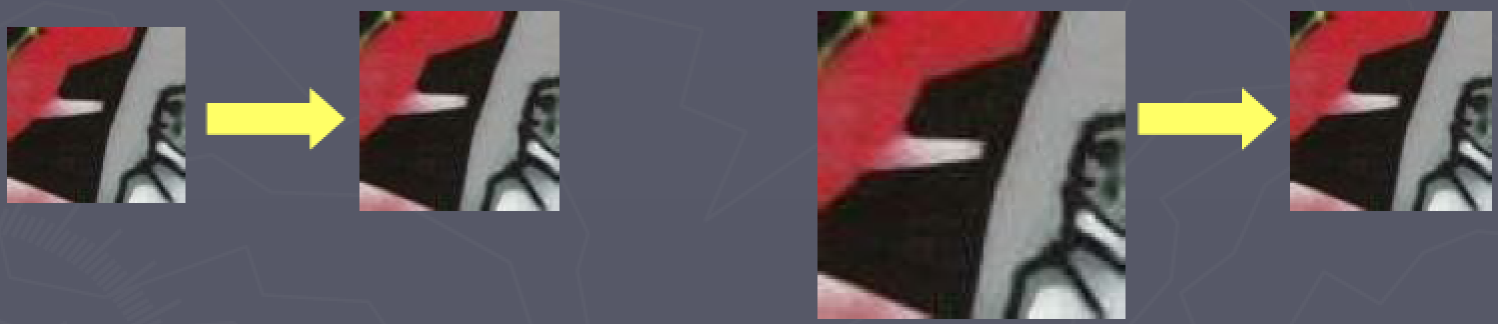
Automatic scale selection



$f(I_{i_1 \dots i_m}(x,\sigma))$ $f(I_{i_1 \dots i_m}(x',\sigma'))$

Automatic scale selection

Normalize: rescale to fixed size



Fitting and Matching

- We've seen low-level *detectors*
- Next step: using output for higher-level tasks
  - Detection/fitting of more complex primitives
  - Matching

Detecting Lines

- What is the difference between line detection and edge detection?
  - Edges = local
  - Lines = nonlocal
- Line detection usually performed on the output of an edge detector

Detecting Lines

- Possible approaches:
  - Brute force: enumerate all lines, check if present
  - Hough transform: vote for lines to which detected edges might belong
  - Fitting: given guess for approximate location, refine it
- Second method *efficient* for finding unknown lines, but not always *accurate*

Hough Transform

- General idea: transform from image coordinates to parameter space of feature
  - Need parameterized model of features
  - For each pixel, determine all parameter values that might have given rise to that pixel; vote
  - At end, look for peaks in parameter space

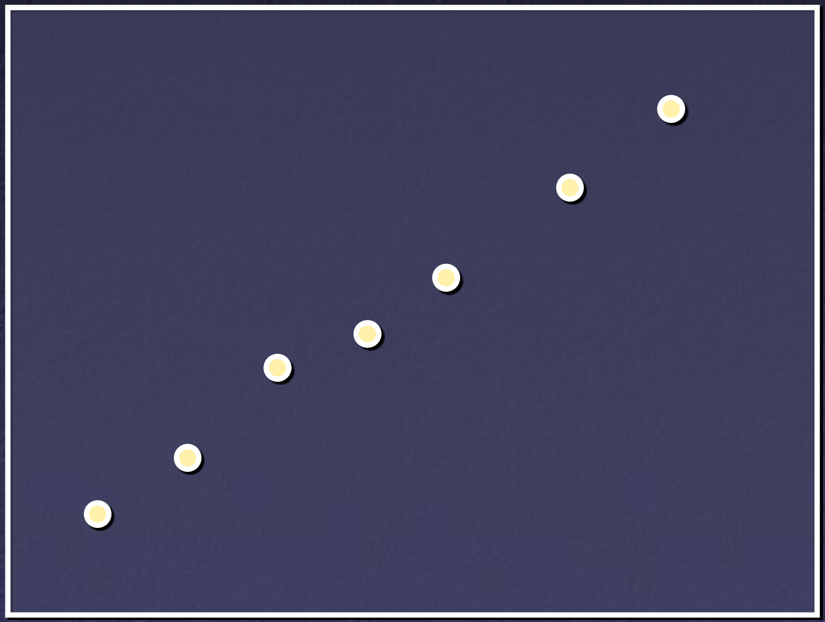
Hough Transform for Lines

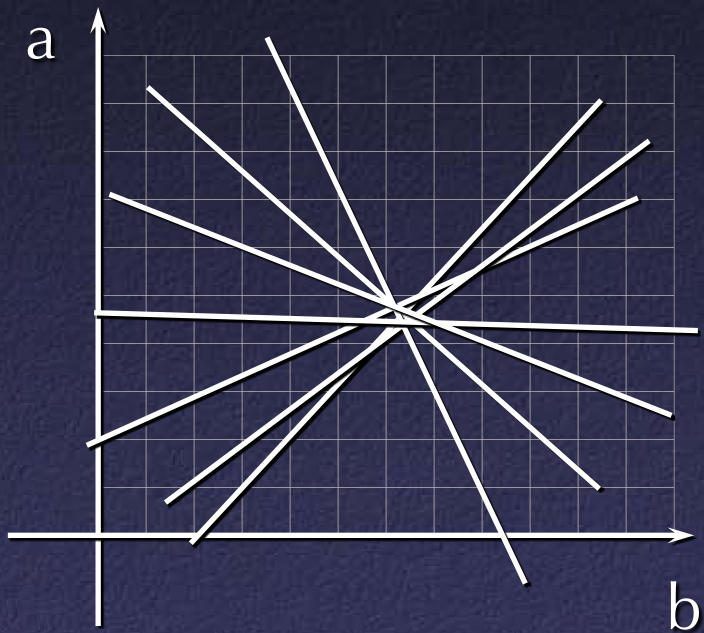
- Generic line: $y = ax+b$
- Parameters: a and b

Hough Transform for Lines

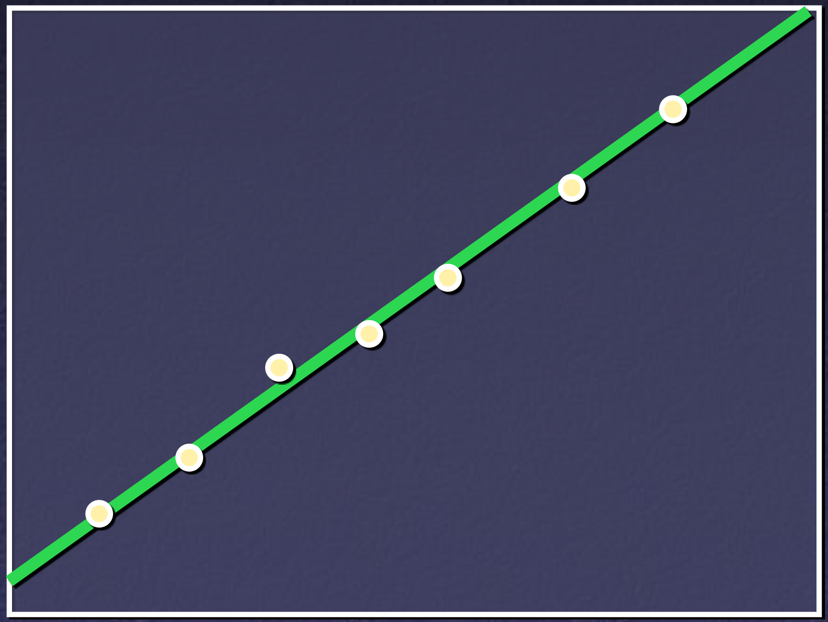
1. Initialize table of *buckets*, indexed by *a* and *b*, to zero
2. For each detected edge pixel (x,y):
   a. Determine all (a,b) such that $y = ax+b$
   b. Increment bucket (a,b)
3. Buckets with many votes indicate probable lines

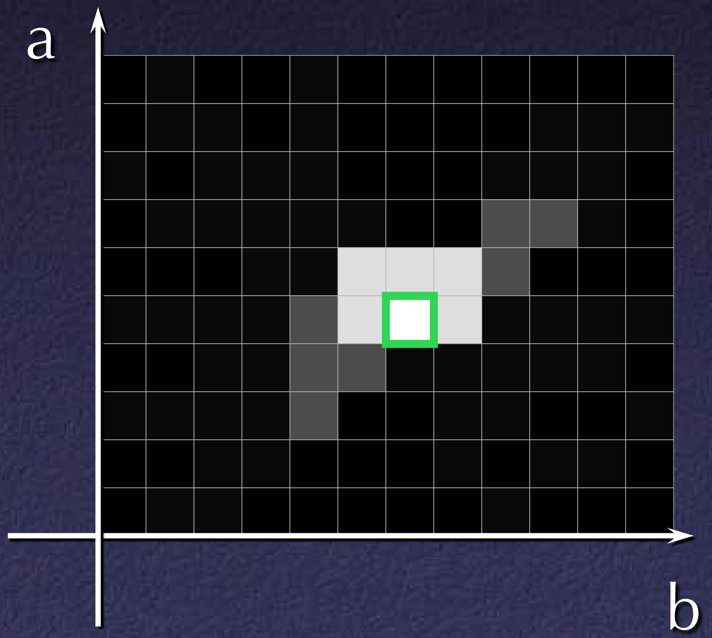
Hough Transform for Lines





Hough Transform for Lines



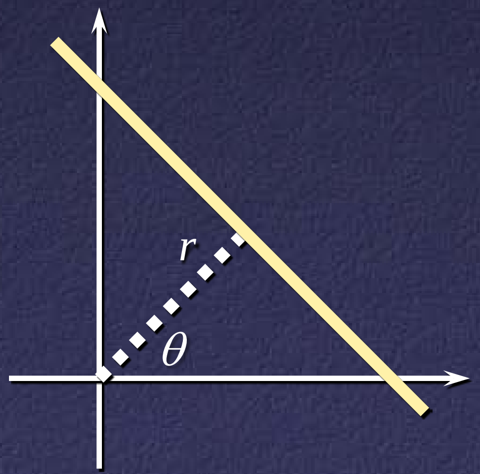


Bucket Selection

- How to select bucket size?
  - Too small: poor performance on noisy data
  - Too large: poor accuracy, long running times, possibility of false positives
- Large buckets + verification and refinement
  - Problems distinguishing nearby lines
- Be smarter at selecting buckets
  - Use gradient information to select subset of buckets
  - More sensitive to noise

Difficulties with Hough Transform for Lines

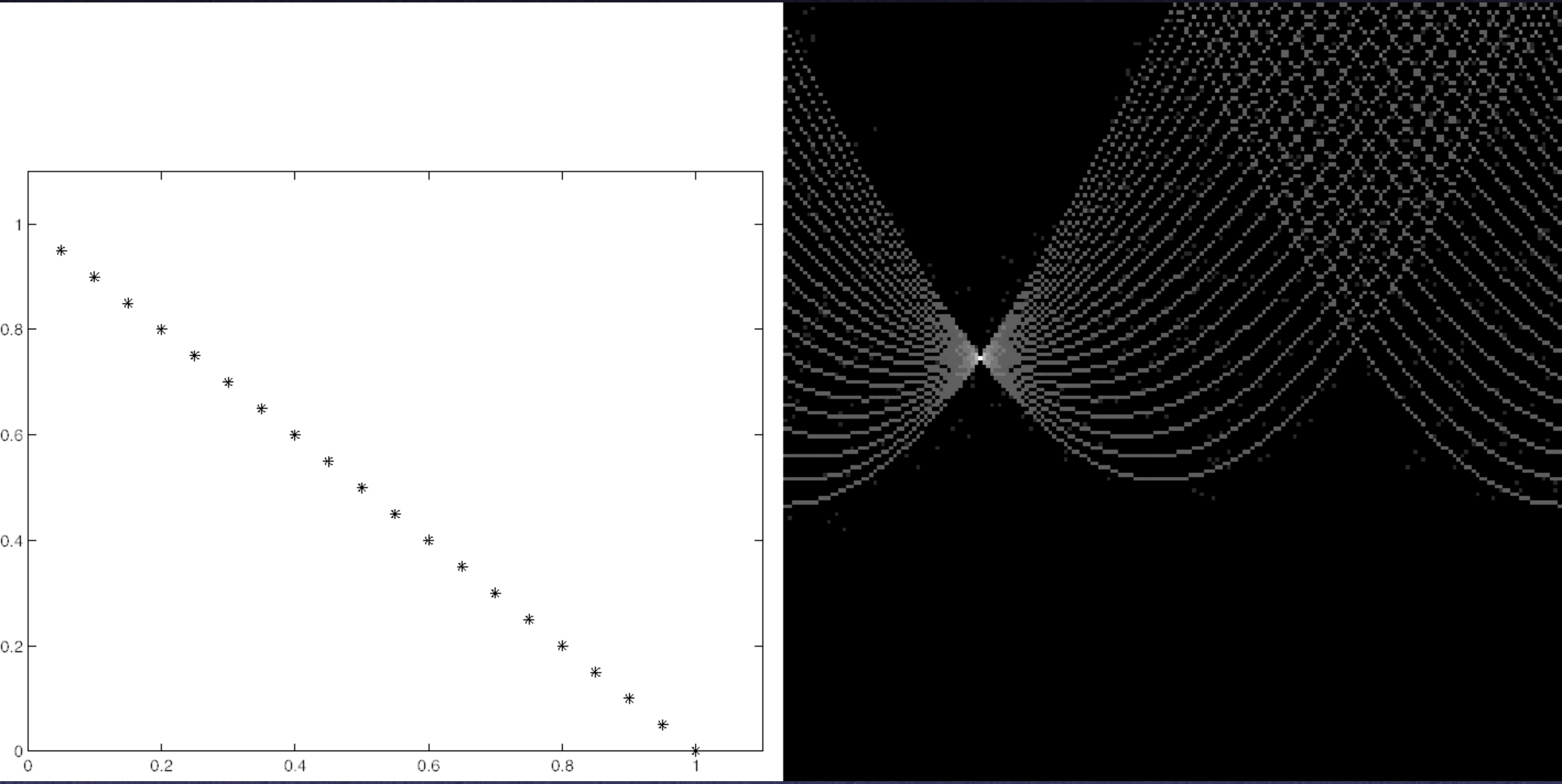
- Slope / intercept parameterization not ideal
  - Non-uniform sampling of directions
  - Can't represent vertical lines
- Angle / distance parameterization
  - Line represented as (r, θ) where
    $x \cos\theta + y \sin\theta = r$



Angle / Distance Parameterization

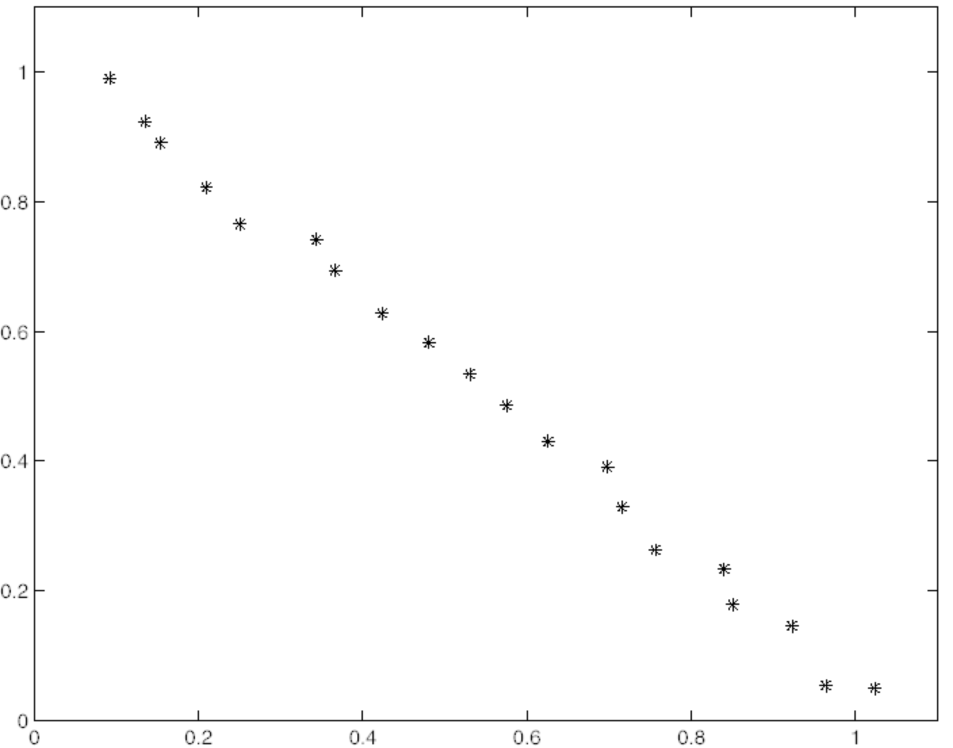
- Advantage: uniform parameterization of directions
- Disadvantage: space of all lines passing through a point becomes a sinusoid in (r,θ) space

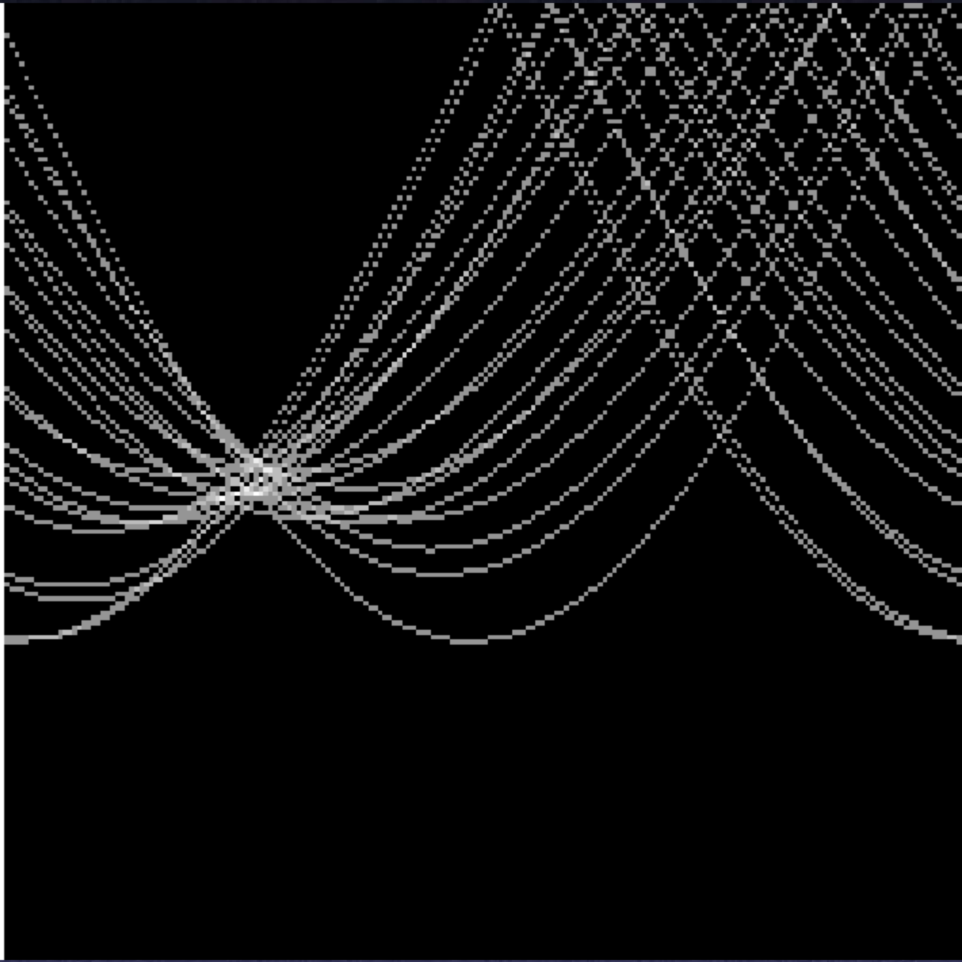
Hough Transform Results



Forsyth & Ponce

Hough Transform Results





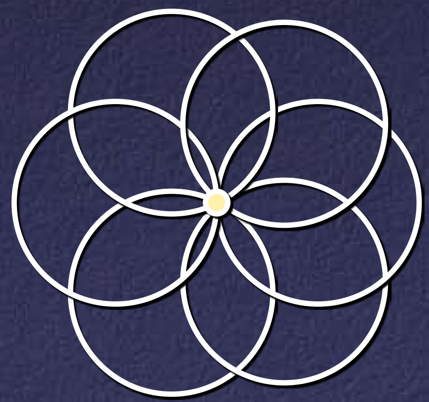
Forsyth & Ponce

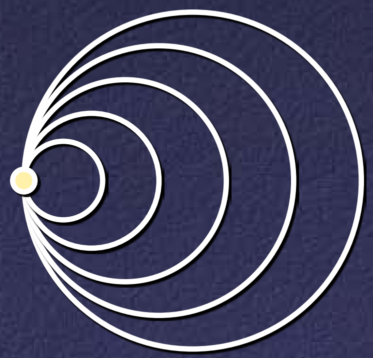
Hough Transform

- What else can be detected using Hough transform?
- Anything, but *dimensionality* is key

Hough Transform for Circles

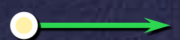
- Space of circles has a 3-dimensional parameter space: position (2-d) and radius
- So, each pixel gives rise to 2-d sheet of values in 3-d space





Hough Transform for Circles

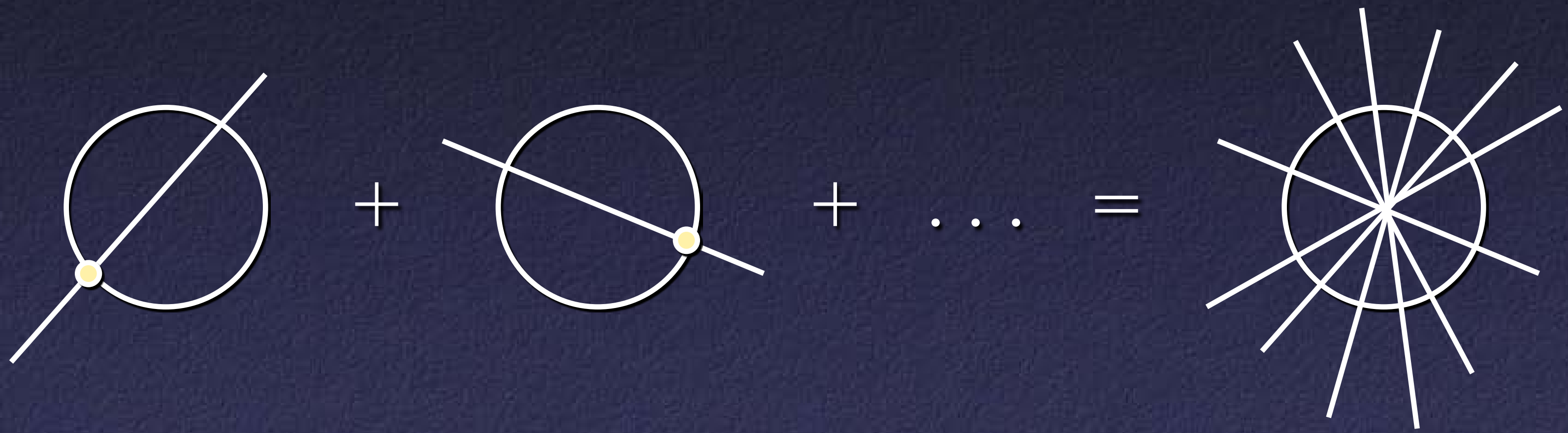
- In many cases, can simplify problem by using more information
- Example: using gradient information



- Still need 3-d bucket space, but each pixel only votes for 1-d subset

Hough Transform for Circles – Secants

- 2-D bucket space: vote only for center, not *r*



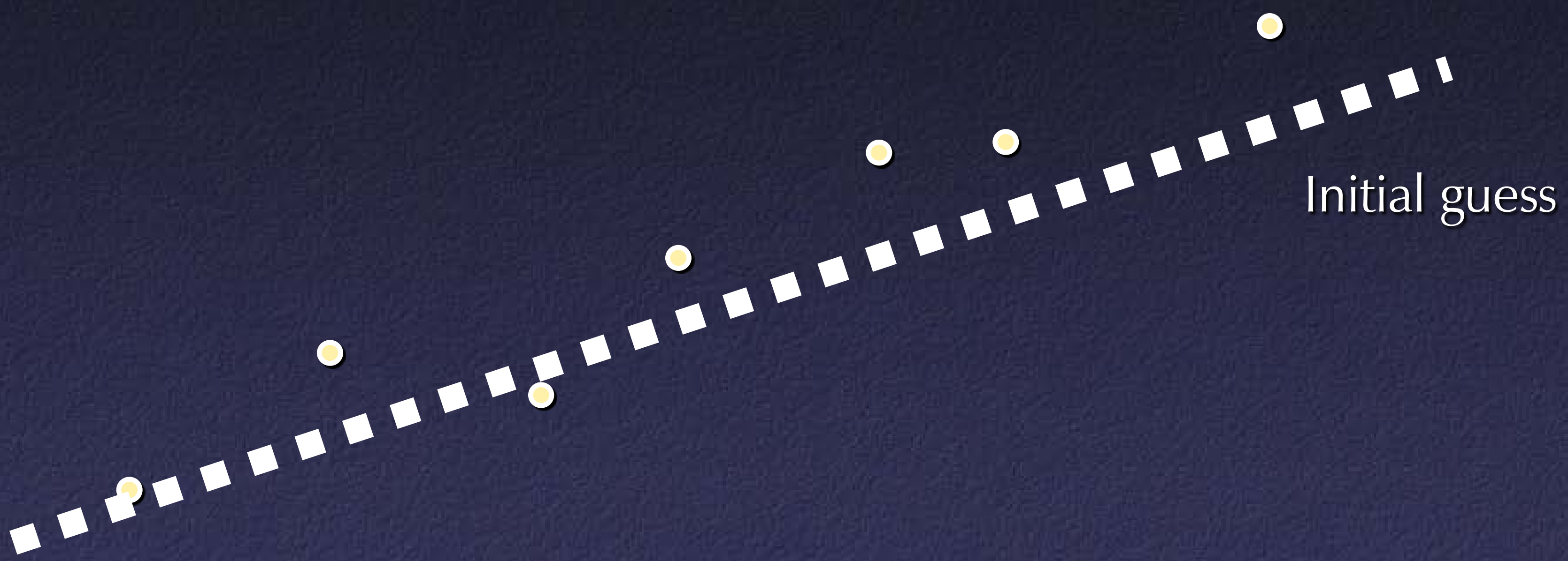
Simplifying Hough Transforms

- Another trick: use prior information
  - For example, if looking for circles of a particular size, reduce votes even further

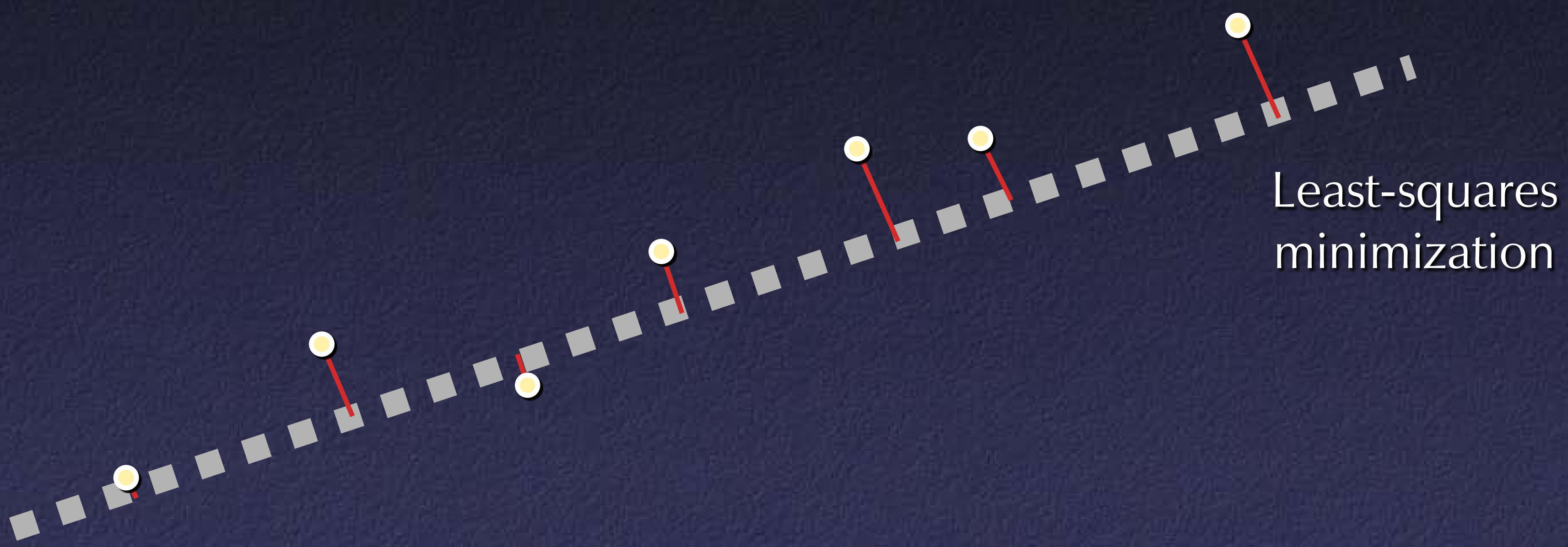
Fitting

- Output of Hough transform often not accurate enough
- Use as initial guess for fitting

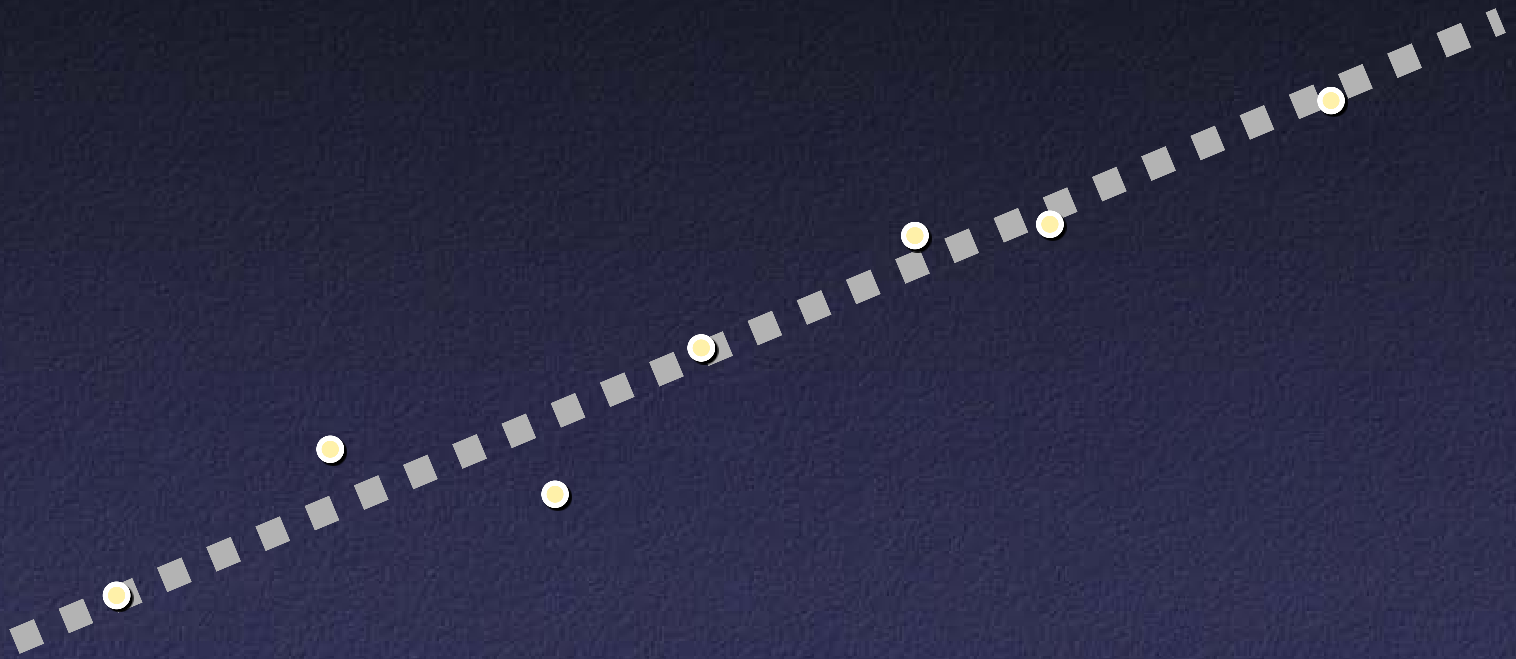
Fitting Lines



Fitting Lines



Fitting Lines

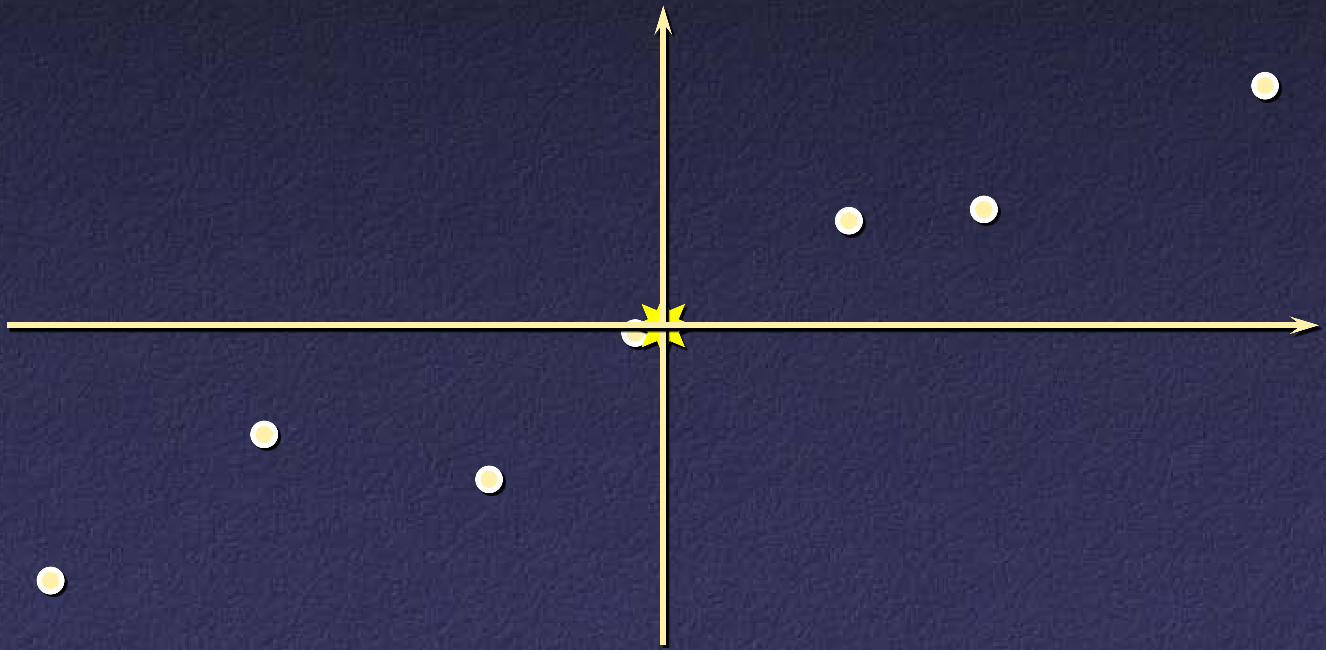


Fitting Lines

- As before, have to be careful about parameterization
- Simplest line fitting formulas minimize *vertical* (not perpendicular) point-to-line distance
- Closed-form solution for point-to-line distance, not necessarily true for other curves

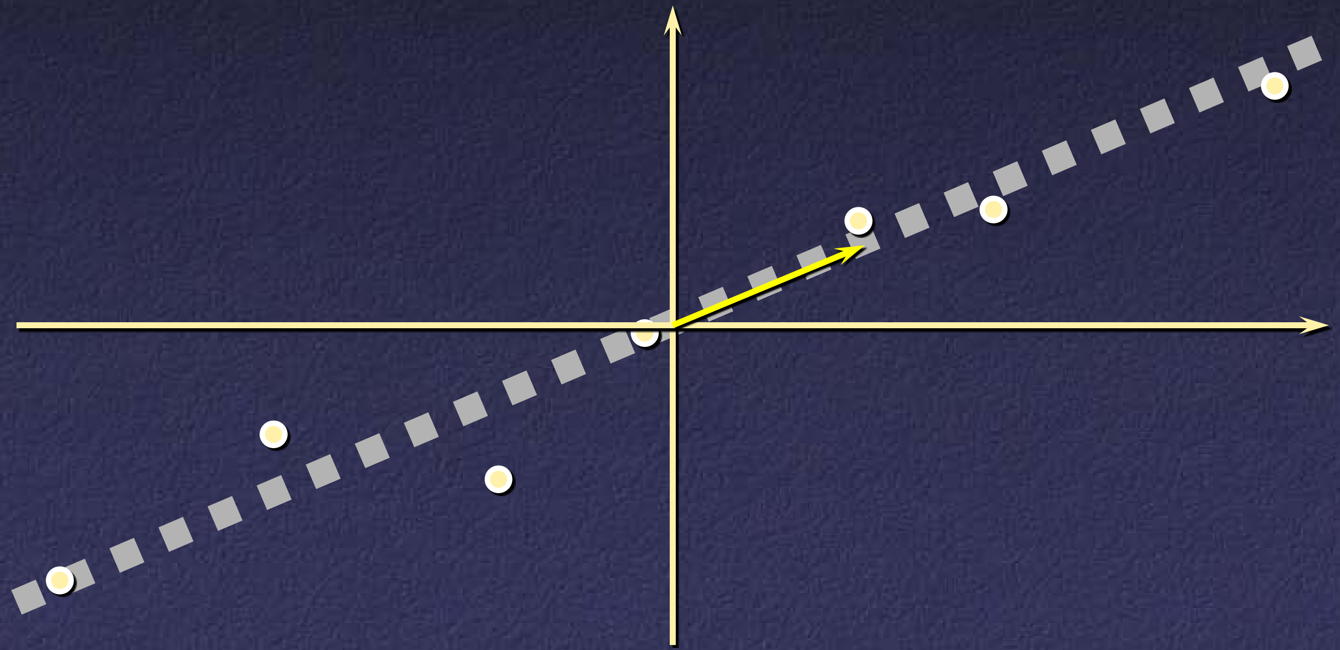
Total Least Squares

1. Translate center of mass to origin



Total Least Squares

2. Compute covariance matrix,
   find eigenvector w. largest eigenvalue



Outliers

- Least squares assumes Gaussian errors
- Outliers: points with extremely low probability of occurrence (according to Gaussian statistics)
  - Can be result of *data association* problems
- Can have strong influence on least squares

Robust Estimation

- Goal: develop parameter estimation methods insensitive to *small* numbers of *large* errors
- General approach: try to give large deviations less weight
- M-estimators: minimize some function other than $(y - f(x,a,b,\ldots))^2$

Least Absolute Value Fitting

- Minimize $\sum_i \left| y_i - f(x_i, a, b, \ldots) \right|$

  instead of $\sum_i \left(y_i - f(x_i, a, b, \ldots) \right)^2$

- Points far away from trend get comparatively less influence

Example: Constant

- For constant function $y = a$, minimizing $\Sigma(y-a)^2$ gives $a =$ mean
- Minimizing $\Sigma|y-a|$ gives $a =$ median

Doing Robust Fitting

- In general case, nasty function: discontinuous derivative
- Numerical methods (e.g. Nelder-Mead simplex) sometimes work

Iteratively Reweighted Least Squares

- Sometimes-used approximation: convert to iterated *weighted* least squares

$$\sum_i \left| y_i - f(x_i, a, b, \ldots) \right|$$

$$= \sum_i \frac{1}{\left| y_i - f(x_i, a, b, \ldots) \right|} \left(y_i - f(x_i, a, b, \ldots) \right)^2$$

$$= \sum_i w_i \left(y_i - f(x_i, a, b, \ldots) \right)^2$$

with w_i based on previous iteration

Iteratively Reweighted Least Squares

- Different options for weights
  - Avoid problems with infinities
  - Give even less weight to outliers

$$w_i = \frac{1}{\left|y_i - f(x_i, a, b, \ldots)\right|}$$

$$w_i = \frac{1}{k + \left|y_i - f(x_i, a, b, \ldots)\right|}$$

$$w_i = \frac{1}{k + \left(y_i - f(x_i, a, b, \ldots)\right)^2}$$

$$w_i = e^{-k\left(y_i - f(x_i, a, b, \ldots)\right)^2}$$

Outlier Detection and Rejection

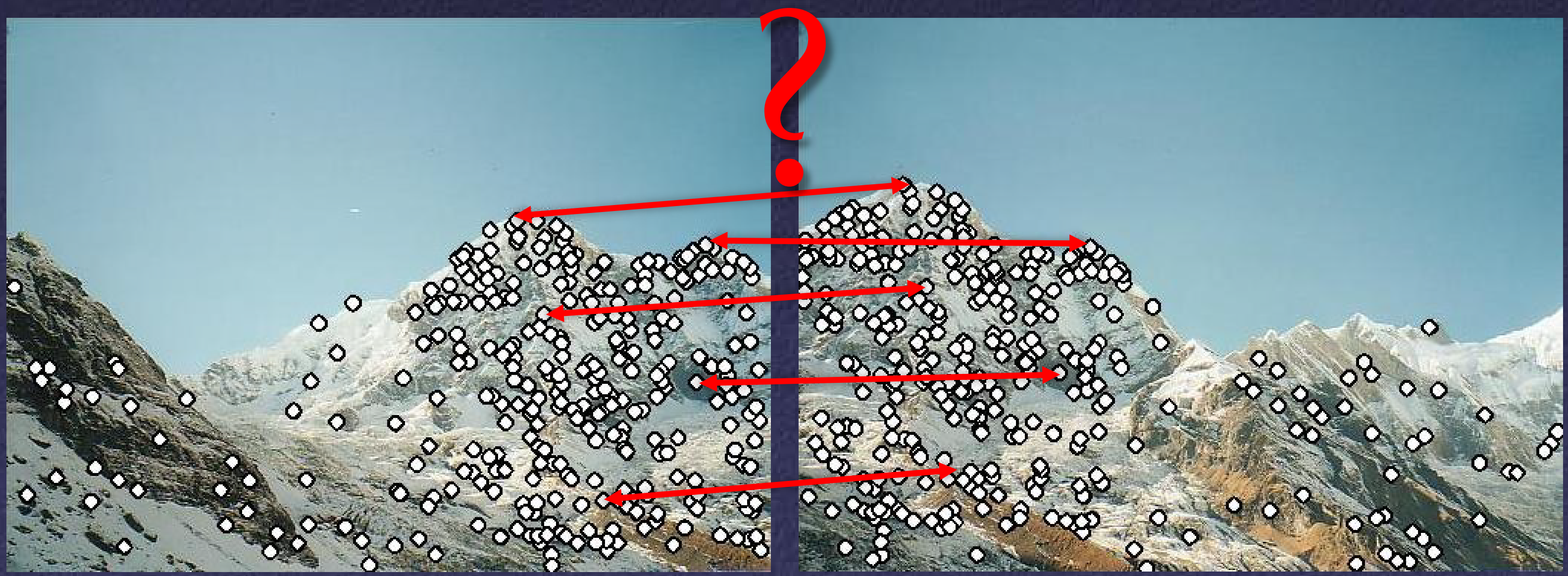
- Special case of IRWLS: set weight = 0 if outlier, 1 otherwise
- Detecting outliers: $(y_i - f(x_i))^2 >$ threshold
  - One choice: multiple of mean squared difference
  - Better choice: multiple of *median* squared difference
  - Can iterate…
  - As before, not guaranteed to do anything reasonable, tends to work OK if only a few outliers

RANSAC

- RANdom SAmple Consensus: desgined for bad data (in best case, up to 50% outliers)
- Take many *minimal* random subsets of data
  - Compute least squares fit for each sample
  - See how many points agree: $(y_i - f(x_i))^2 <$ threshold
  - Threshold user-specified or estimated from more trials
- At end, use fit that agreed with most points
  - Can do one final least squares with all inliers

Feature Descriptors

- Feature matching useful for:
  Image alignment (e.g., mosaics), 3D reconstruction, motion tracking, object recognition, indexing and database retrieval, robot navigation, etc.



[Seitz]

Properties of Feature Descriptors

- Easily computed
- Easily compared (compact, fixed-dimensional)
- Invariant
  - Translation
  - Rotation
  - Scale
  - Change in image brightness
  - Change in perspective?

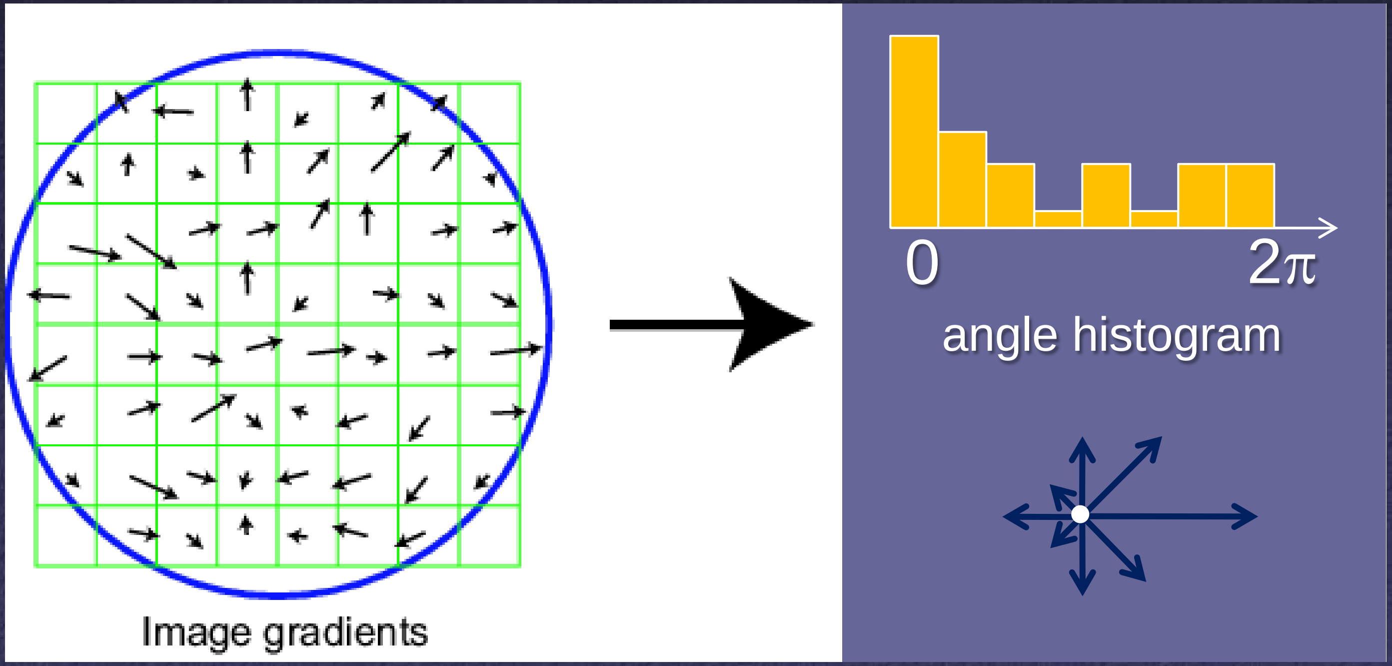
Rotation Invariance for Feature Descriptors

- Rotate window according to dominant orientation
  - Eigenvector of C corresponding to maximum eigenvalue

[Matthew Brown]

Scale Invariant Feature Transform

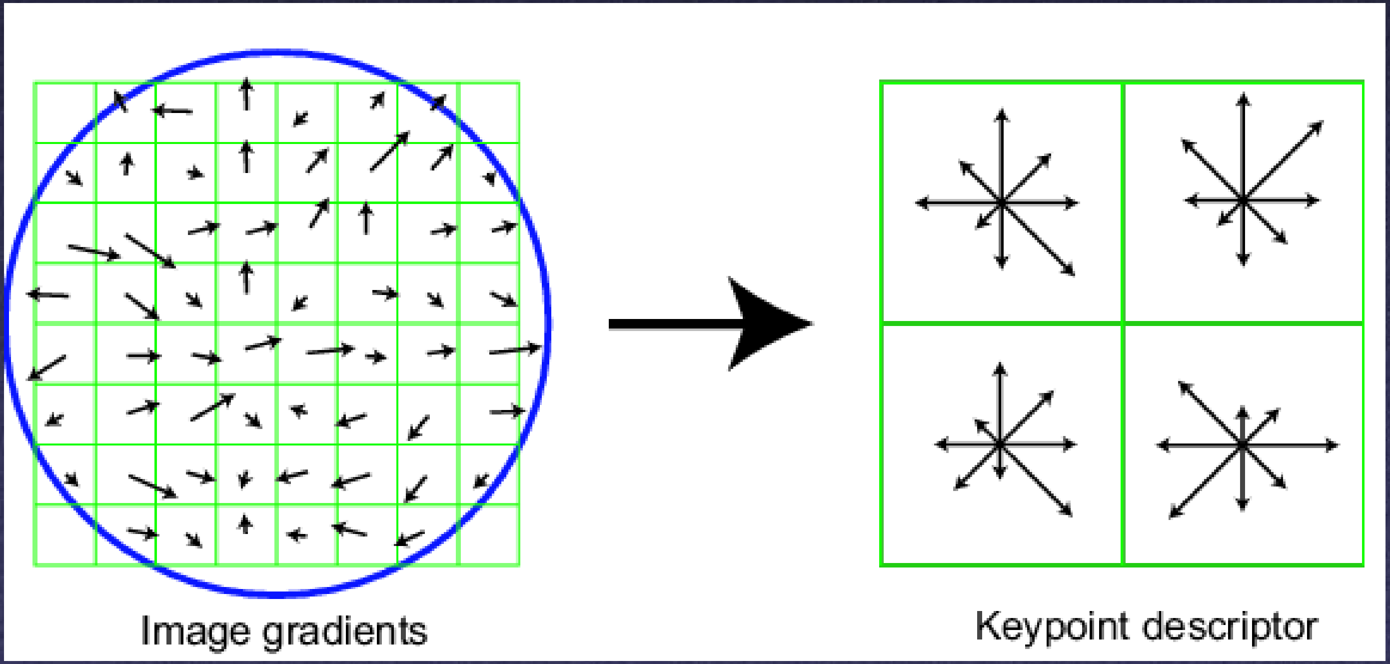
- Take 16×16 window around detected feature
- Create histogram of thresholded edge orientations



[Seitz / Lowe]

Full SIFT Descriptor

- Divide 16×16 window into 4×4 grid of cells
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128-dimensional descriptor



[Seitz / Lowe]

Properties of SIFT

- Fast (real-time) and robust descriptor for matching
  - Handles changes in viewpoint (~60° out of plane rotation)
  - Handles significant changes in illumination
  - Lots of code available

[Seitz]