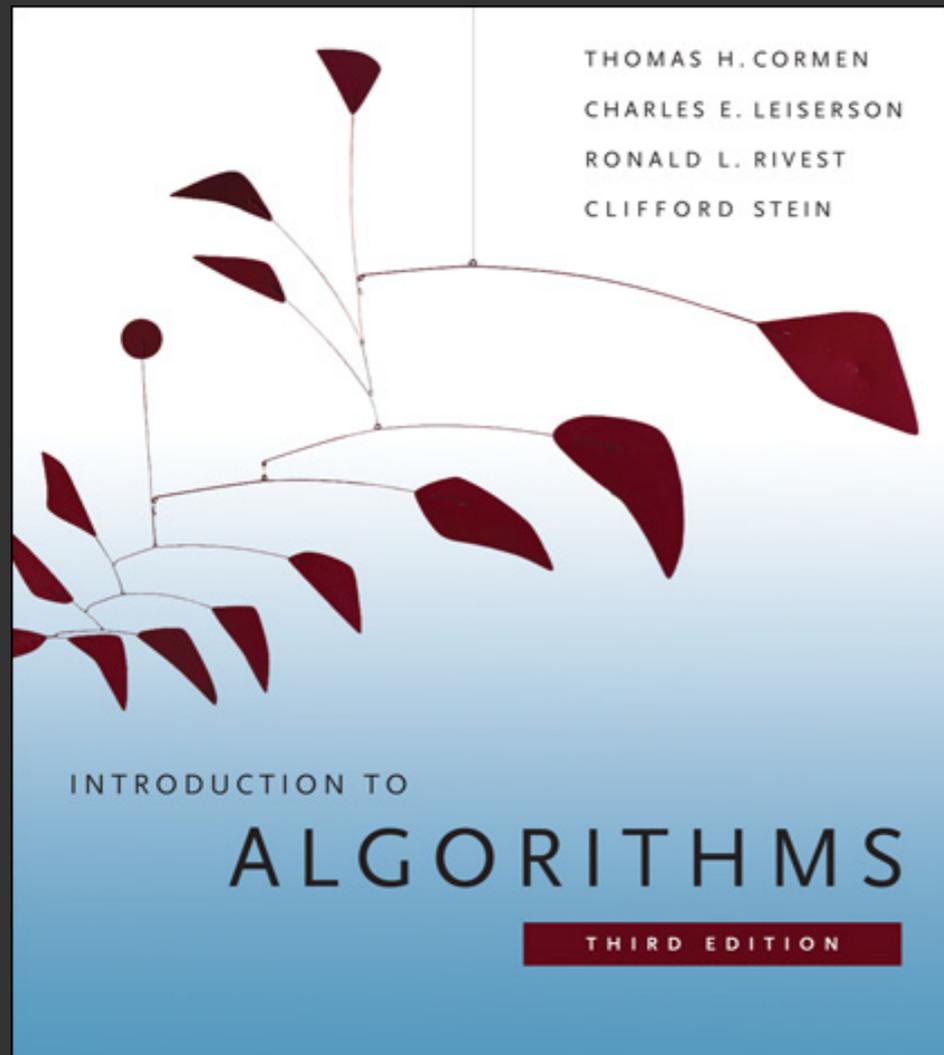


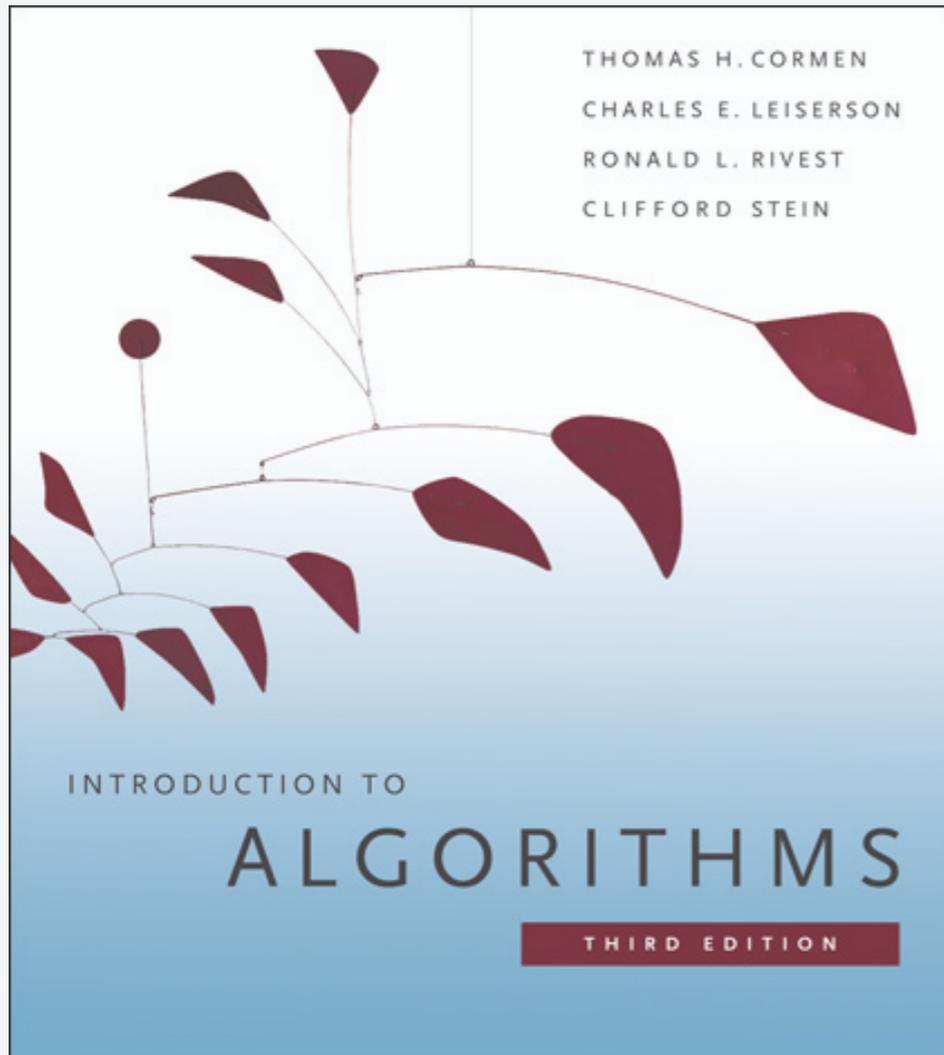
AMORTIZED ANALYSIS

- ▶ *dynamic table demo: insert*
- ▶ *dynamic table demo: insert and delete*



Lecture slides by Kevin Wayne

<http://www.cs.princeton.edu/~wayne/kleinberg-tardos>



CHAPTER 17

AMORTIZED ANALYSIS

- ▶ *dynamic table demo: insert*
- ▶ *dynamic table demo: insert and delete*

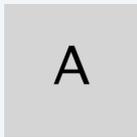
Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert A

capacity = 1



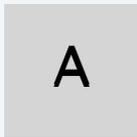
Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert B

capacity = 1



← destroy extra credit
(not needed)

capacity = 2



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

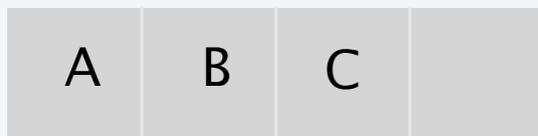
Invariant. 2 credits with each item in right half of table; none in left half.

insert C

capacity = 2



capacity = 4



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert D

capacity = 4



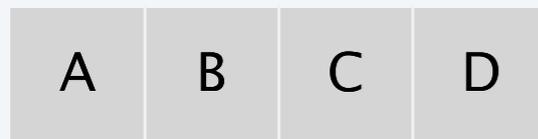
Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert E

capacity = 4



capacity = 8



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert F

capacity = 8



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert G

capacity = 8



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert H

capacity = 8



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert I

capacity = 8



capacity = 16



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert J

capacity = 16



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert K

capacity = 16



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert L

capacity = 16



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert M

capacity = 16



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert N

capacity = 16



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

insert O

capacity = 16



Dynamic table demo: insert only (accounting method)

Insert. Charge 3 credits (use 1 credit to insert; save 2 with new item).

Invariant. 2 credits with each item in right half of table; none in left half.

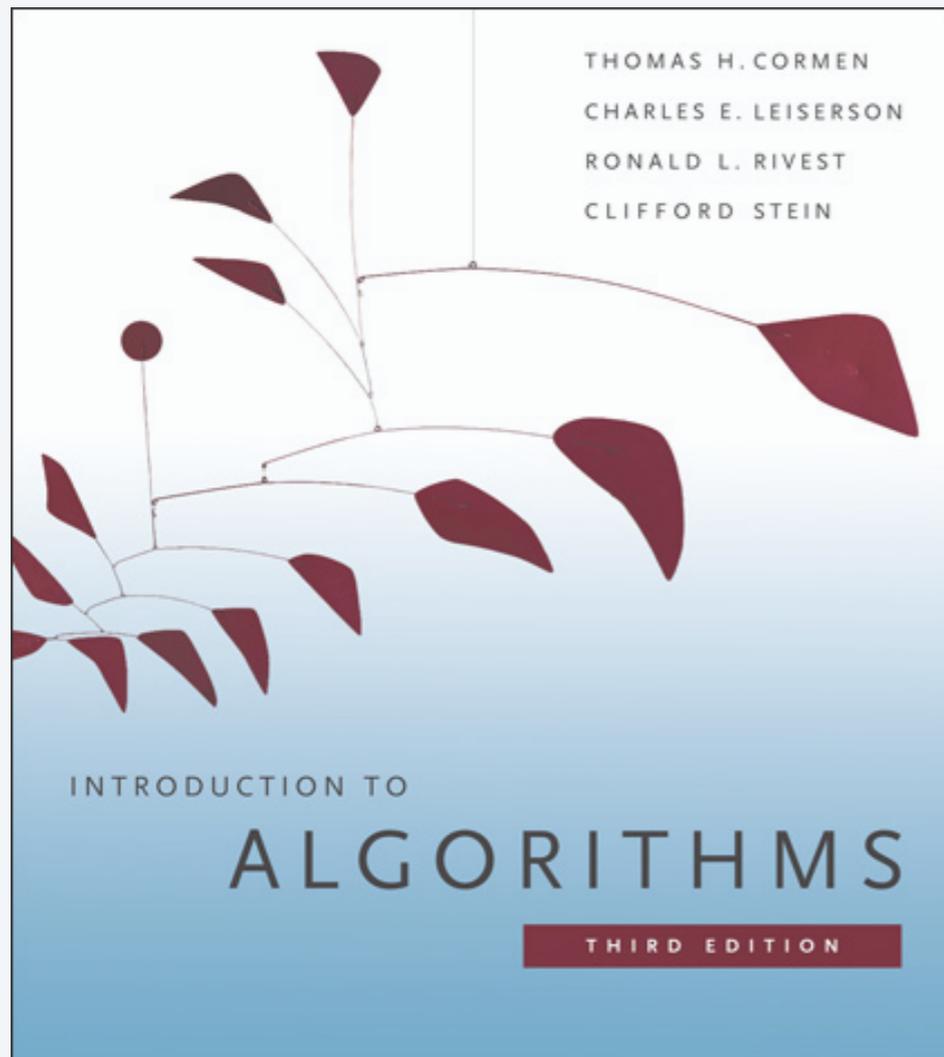
insert P

capacity = 16



AMORTIZED ANALYSIS

- ▶ *dynamic table demo: insert*
- ▶ *dynamic table demo: insert and delete*



CHAPTER 17

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

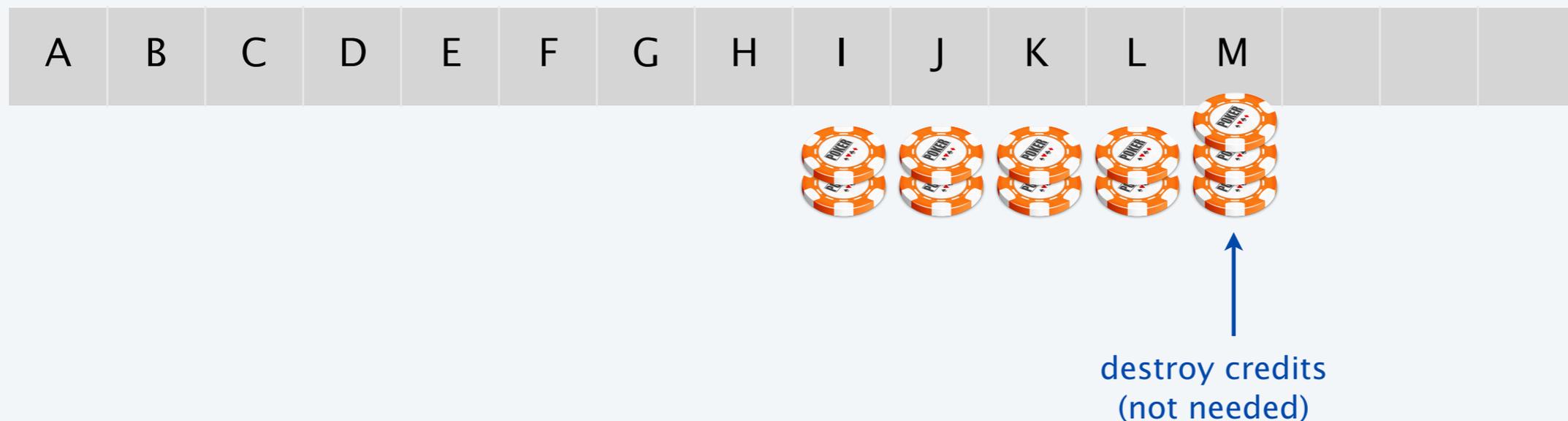
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete M

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

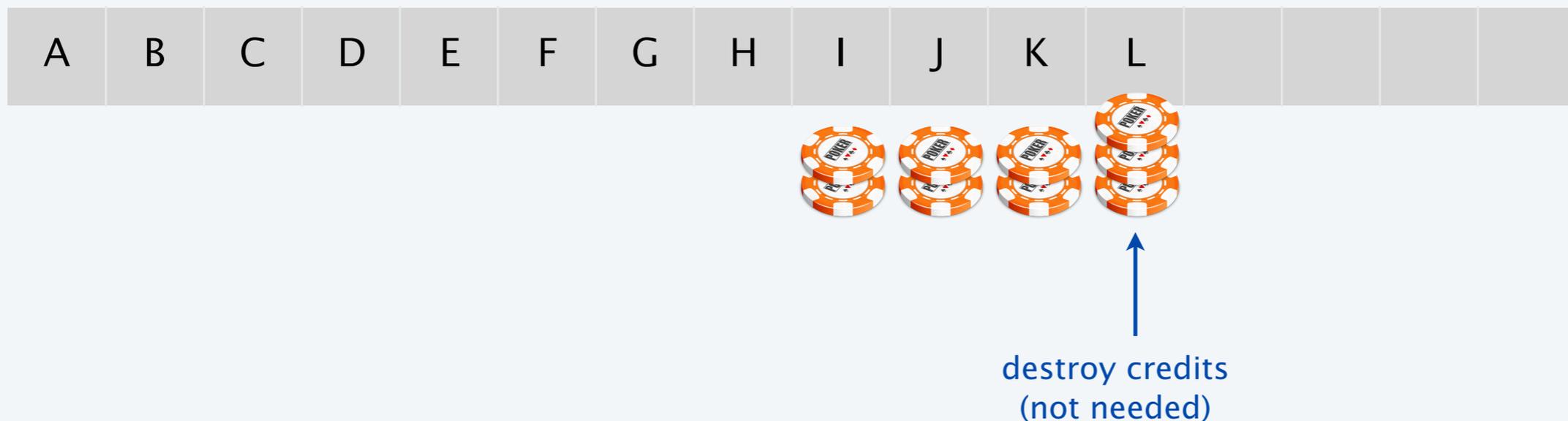
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete L

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

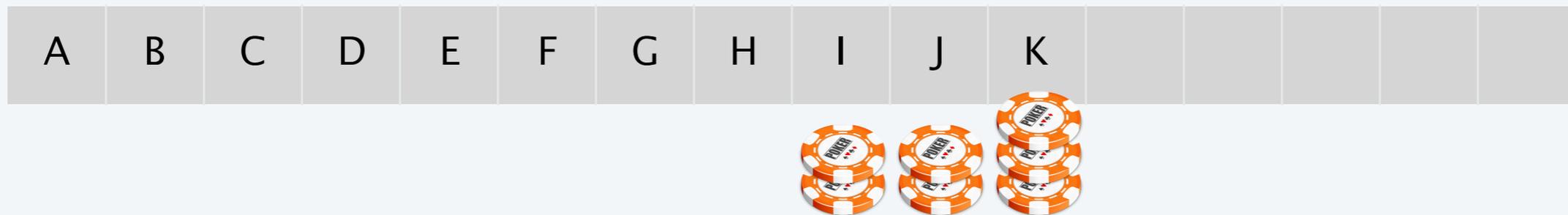
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete K

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

insert N

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete N

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete J

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete I

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete H

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

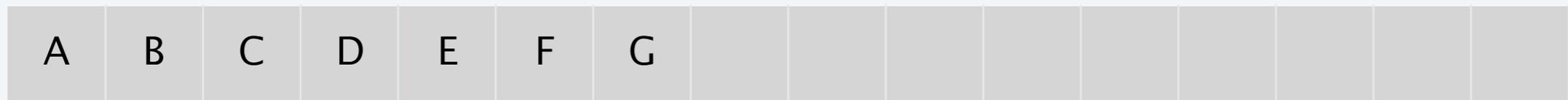
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete G

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

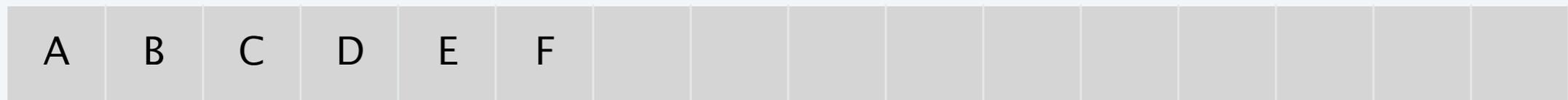
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete F

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

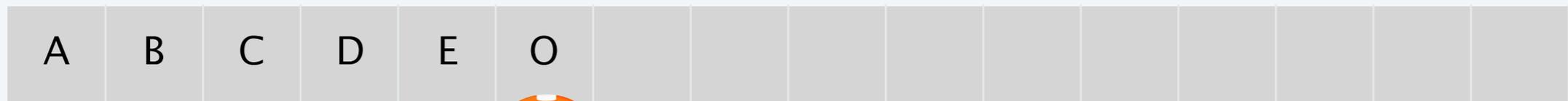
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

insert O

capacity = 16



↑
destroy extra credit
(not needed)

Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

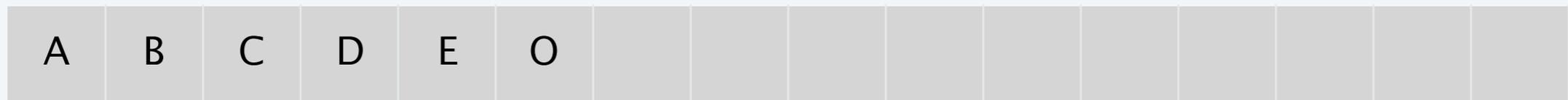
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete O

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

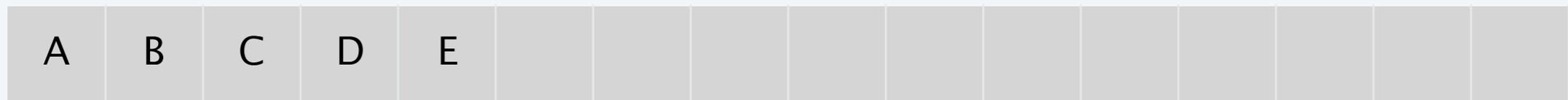
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

delete E

capacity = 16



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

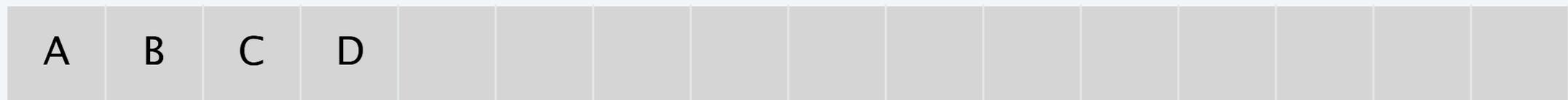
Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

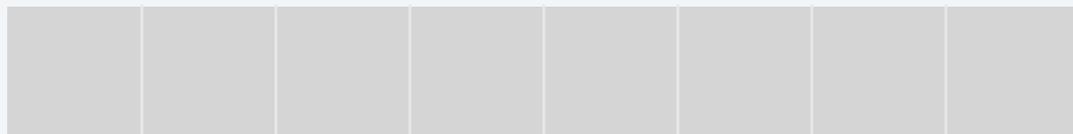
Invariant 2. 1 credit with each empty slot in left half of table.

delete D

capacity = 16



capacity = 8



Dynamic table demo: insert and delete (accounting method)

Insert. Charge 3 credits (1 to insert; save 2 with item if in right half).

Delete. Charge 2 credits (1 to delete; save 1 in empty slot if in left half).

Invariant 1. 2 credits with each item in right half of table.

Invariant 2. 1 credit with each empty slot in left half of table.

capacity = 8

