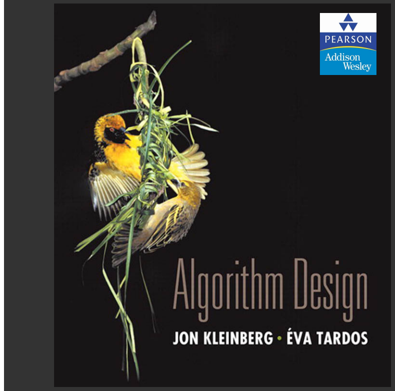


5. Divide And Conquer I

- 3-way partitioning demo
- randomized quickselect demo

Lecture slides by Kevin Wayne

http://www.cs.princeton.edu/~wayne/kleinberg-tardos

Last updated on 3/5/18 9:47 AM

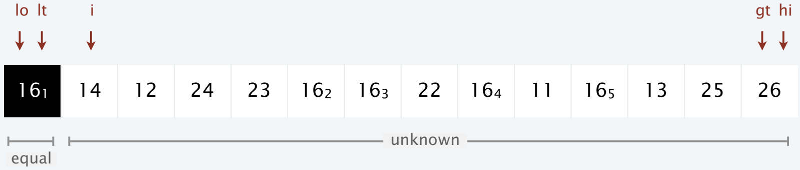
Section 2.3

5. Divide and Conquer

- 3-way partitioning demo
- randomized quickselect demo

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



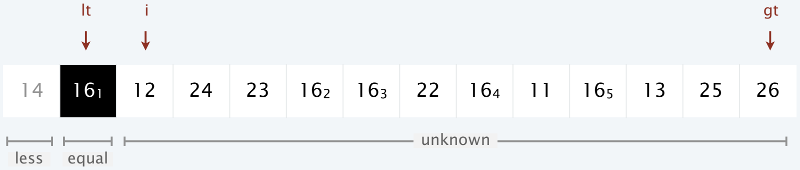
lo lt i gt hi

16_1	14	12	24	23	16_2	16_3	22	16_4	11	16_5	13	25	26

equal unknown

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



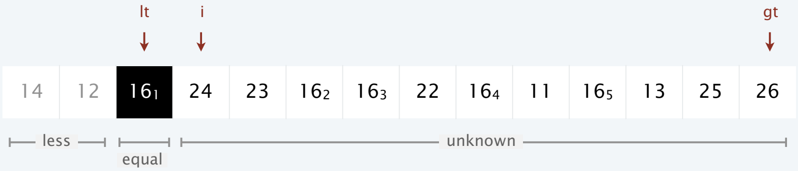
lt i gt

14	16_1	12	24	23	16_2	16_3	22	16_4	11	16_5	13	25	26

less equal unknown

Dijkstra 3-way partitioning demo

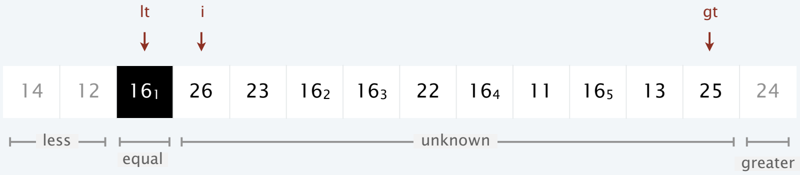
- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



		lt	i										gt
14	12	16_1	24	23	16_2	16_3	22	16_4	11	16_5	13	25	26

Dijkstra 3-way partitioning demo

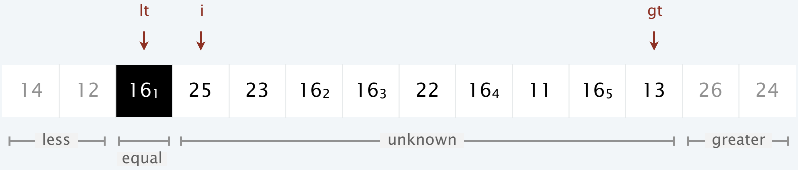
- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



		lt	i									gt	
14	12	16_1	26	23	16_2	16_3	22	16_4	11	16_5	13	25	24

Dijkstra 3-way partitioning demo

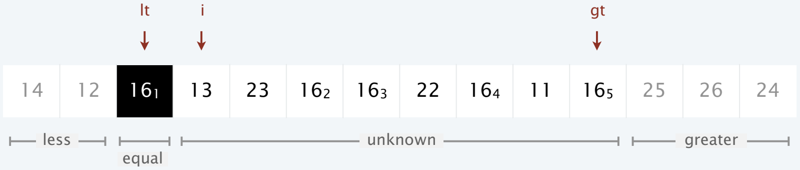
- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



		lt	i								gt		
14	12	16_1	25	23	16_2	16_3	22	16_4	11	16_5	13	26	24

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



		lt	i							gt			
14	12	16_1	13	23	16_2	16_3	22	16_4	11	16_5	25	26	24

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
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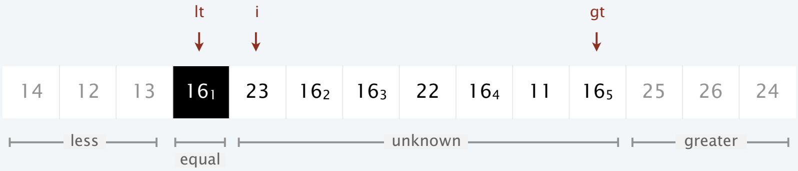


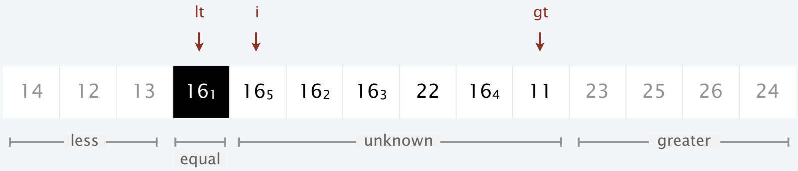
lt i gt

14	12	13	16_1	23	16_2	16_3	22	16_4	11	16_5	25	26	24

less equal unknown greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



lt i gt

14	12	13	16_1	16_5	16_2	16_3	22	16_4	11	23	25	26	24

less equal unknown greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
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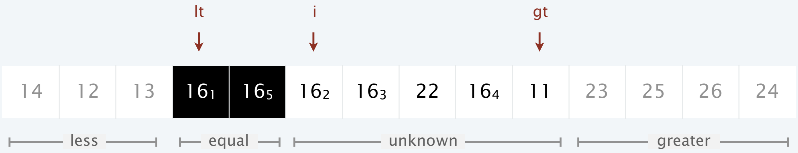

lt i gt

14	12	13	16_1	16_5	16_2	16_3	22	16_4	11	23	25	26	24

less equal unknown greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
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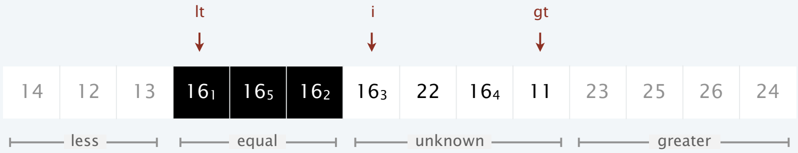


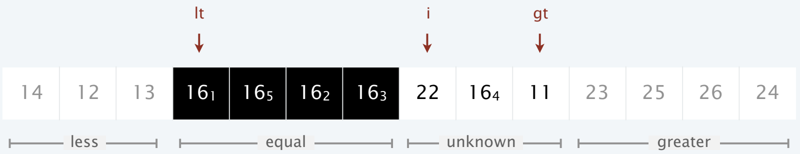
lt i gt

14	12	13	16_1	16_5	16_2	16_3	22	16_4	11	23	25	26	24

less equal unknown greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



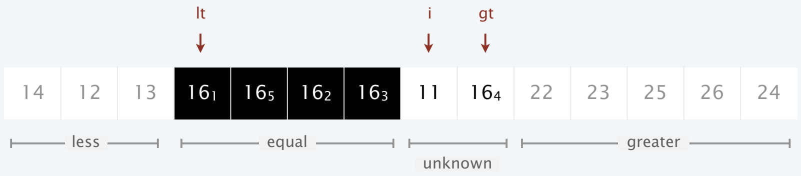
lt i gt

14 12 13 16_1 16_5 16_2 16_3 22 16_4 11 23 25 26 24

less equal unknown greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



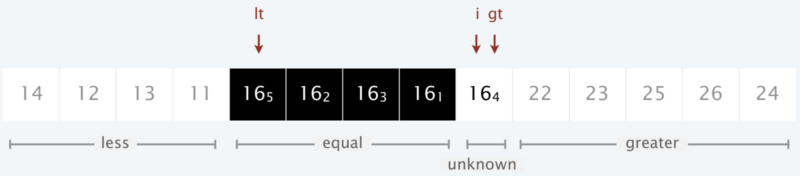
lt i gt

14 12 13 16_1 16_5 16_2 16_3 11 16_4 22 23 25 26 24

less equal unknown greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i



lt i gt

14 12 13 11 16_5 16_2 16_3 16_1 16_4 22 23 25 26 24

less equal unknown greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
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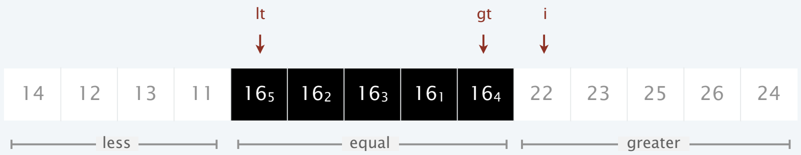
  - $(A[i] = p)$: increment i



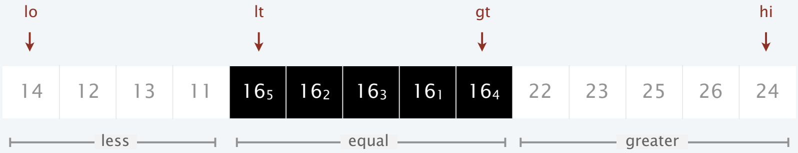
lt gt i

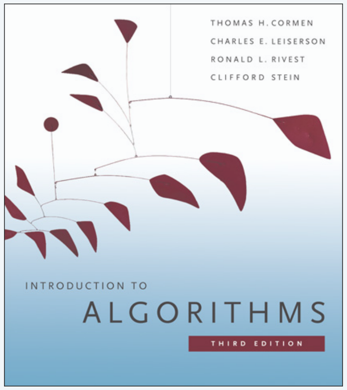
14 12 13 11 16_5 16_2 16_3 16_1 16_4 22 23 25 26 24

less equal greater

Dijkstra 3-way partitioning demo

- Let p be pivot item.
- Swap p to index lo.
- Scan i from left to right.
  - $(A[i] < p)$: exchange $A[lt]$ with $A[i]$; increment both lt and i
  - $(A[i] > p)$: exchange $A[gt]$ with $A[i]$; decrement gt
  - $(A[i] = p)$: increment i





SECTION 7.1–7.3

5. Divide and Conquer

- 3-way partitioning demo
- *randomized quickselect demo*

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the k^{th} smallest element.

select the k = 8th smallest

65	28	59	33	21	56	22	95	50	12	90	53	28	77	39

k = 8th smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the k^{th} smallest element.

choose a pivot element at random and partition

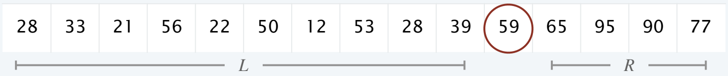
65	28	59	33	21	56	22	95	50	12	90	53	28	77	39

k = 8th smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the kth smallest element.

partitioned array



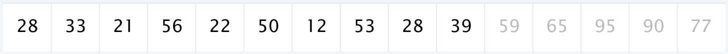
28	33	21	56	22	50	12	53	28	39	59	65	95	90	77

k = 8th smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the kth smallest element.

recursively select 8th smallest element in left subarray



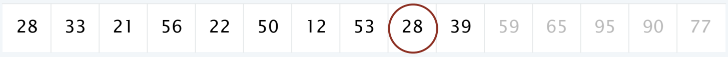
28	33	21	56	22	50	12	53	28	39	59	65	95	90	77

k = 8th smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the kth smallest element.

choose a pivot element at random and partition



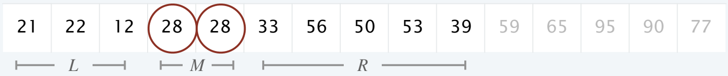
28	33	21	56	22	50	12	53	28	39	59	65	95	90	77

k = 8th smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the kth smallest element.

partitioned array



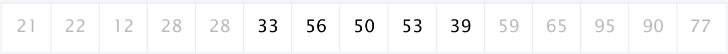
21	22	12	28	28	33	56	50	53	39	59	65	95	90	77

k = 8th smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the k^{th} smallest element.

recursively select the 3rd smallest element in right subarray



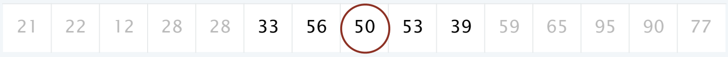
21	22	12	28	28	33	56	50	53	39	59	65	95	90	77

k = 3rd smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the k^{th} smallest element.

choose a pivot element at random and partition



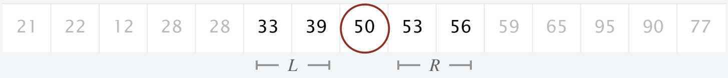
21	22	12	28	28	33	56	50	53	39	59	65	95	90	77

k = 3rd smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the k^{th} smallest element.

partitioned array



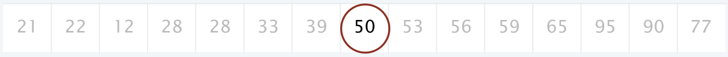
21	22	12	28	28	33	39	50	53	56	59	65	95	90	77

k = 3rd smallest

Quickselect demo

- Pick a random pivot element $p \in A$.
- 3-way partition the array into L, M, and R.
- Recur in one subarray—the one containing the k^{th} smallest element.

stop: desired element is in middle subarray



21	22	12	28	28	33	39	50	53	56	59	65	95	90	77