5. **Divide and Conquer I**

- *merge demo*
- *merge-and-count demo*
5. **Divide and Conquer**

- *merge demo*
- *merge-and-count demo*
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$. 

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 7 10 14 18</td>
<td>2 11 16 20 23</td>
</tr>
</tbody>
</table>
Merge demo

Given two sorted lists \( A \) and \( B \), merge into sorted list \( C \).

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3  7  10  14  18</td>
<td>2  11  16  20  23</td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 2

sorted list C
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

sorted list $A$

| 3 | 7 | 10 | 14 | 18 |

sorted list $B$

| 2 | 11 | 16 | 20 | 23 |

compare minimum entry in each list: copy 3

sorted list $C$

| 2 |


Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

**sorted list A**

| 3 | 7 | 10 | 14 | 18 |

**sorted list B**

| 2 | 11 | 16 | 20 | 23 |

compare minimum entry in each list: copy 7

**sorted list C**

| 2 | 3 |
Merge demo

Given two sorted lists \( A \) and \( B \), merge into sorted list \( C \).

sorted list A

\[
\begin{array}{c}
3 \\
7 \\
10 \\
14 \\
18 \\
\end{array}
\]

sorted list B

\[
\begin{array}{c}
2 \\
11 \\
16 \\
20 \\
23 \\
\end{array}
\]

compare minimum entry in each list: copy 10

sorted list C

\[
\begin{array}{c}
2 \\
3 \\
7 \\
\end{array}
\]
Given two sorted lists $A$ and $B$, merge into sorted list $C$.

**Sorted list A**

3 7 10 14 18

**Sorted list B**

2 11 16 20 23

**Sorted list C**

2 3 7 10

Compare minimum entry in each list: copy 11
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

sorted list $A$

3 7 10 14 18

sorted list $B$

2 11 16 20 23

compare minimum entry in each list: copy 14

sorted list $C$

2 3 7 10 11
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

**sorted list A**

3  7  10  14  18

**sorted list B**

2  11  16  20  23

compare minimum entry in each list: copy 16

**sorted list C**

2  3  7  10  11  14
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

**sorted list A**

| 3 | 7 | 10 | 14 | 18 |

**sorted list B**

| 2 | 11 | 16 | 20 | 23 |

compare minimum entry in each list: copy 18

**sorted list C**

| 2 | 3 | 7 | 10 | 11 | 14 | 16 |
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

**sorted list A**

| 3 | 7 | 10 | 14 | 18 |

**sorted list B**

| 2 | 11 | 16 | 20 | 23 |

list A exhausted: copy 20

**sorted list C**

| 2 | 3 | 7 | 10 | 11 | 14 | 16 | 18 |
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

sorted list A

| 3 | 7 | 10 | 14 | 18 |

sorted list B

| 2 | 11 | 16 | 20 | 23 |

list A exhausted: copy 23

sorted list C

| 2 | 3 | 7 | 10 | 11 | 14 | 16 | 18 | 20 |
Merge demo

Given two sorted lists $A$ and $B$, merge into sorted list $C$.

sorted list A

| 3 | 7 | 10 | 14 | 18 |

sorted list B

| 2 | 11 | 16 | 20 | 23 |

done

sorted list C

| 2 | 3 | 7 | 10 | 11 | 14 | 16 | 18 | 20 | 23 |
5. **DIVIDE AND CONQUER**

- *merge demo*
- *merge-and-count demo*
Merge-and-count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3  7  10  14  18</td>
<td>2  11  16  20  23</td>
</tr>
</tbody>
</table>
Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

### Merge-and-count demo

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 7 10 14 18</td>
<td>2 11 16 20 23</td>
</tr>
</tbody>
</table>

**compare minimum entry in each list:** copy 2 and add $x$ to inversion count

<table>
<thead>
<tr>
<th>sorted list C</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 5</td>
</tr>
<tr>
<td>inversions = 0</td>
</tr>
</tbody>
</table>

x = 5 ← number of elements remaining in $A$
Merge-and-count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**sorted list $A$**

| 3 | 7 | 10 | 14 | 18 |

**sorted list $B$**

| 2 | 11 | 16 | 20 | 23 |

$x = 5$

inversions $= 5$
### Merge-and-count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>sorted list B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 7 10 14 18</td>
<td>2 11 16 20 23</td>
</tr>
</tbody>
</table>

**compare minimum entry in each list: copy 7 and decrement $x$**

<table>
<thead>
<tr>
<th>sorted list C</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2 3</td>
<td></td>
</tr>
</tbody>
</table>

$x = 4$

inversions = 5
Merge-and-count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sorted list B</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11</td>
<td>16</td>
<td>20</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 10 and decrement $x$

<table>
<thead>
<tr>
<th>sorted list C</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

x = 3
inversions = 5
Merge-and-count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**sorted list A**

| 3 | 7 | 10 | 14 | 18 |

**sorted list B**

| 2 | 11 | 16 | 20 | 23 |

Compare minimum entry in each list: copy 11 and add $x$ to increment count

**sorted list C**

| 2 | 3 | 7 | 10 |

$x = 2$

inversions $= 5$
Merge-and-count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

### sorted list A

| 3 | 7 | 10 | 14 | 18 |

### sorted list B

| 2 | 11 | 16 | 20 | 23 |

compare minimum entry in each list: copy 14 and decrement $x$

### sorted list C

| 2 | 3 | 7 | 10 | 11 |

$x = 2$

inversions $= 7$
Merge-and-count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list $A$</th>
<th>sorted list $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 7 10 14 18</td>
<td>2 11 16 20 23</td>
</tr>
</tbody>
</table>

compare minimum entry in each list: copy 16 and add $x$ to increment count

<table>
<thead>
<tr>
<th>sorted list $C$</th>
<th>x = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 3 7 10 11 14</td>
<td></td>
</tr>
</tbody>
</table>

inversions = 7
Merge-and-count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

### Sorted List A

| 3 | 7 | 10 | 14 | 18 |

### Sorted List B

| 2 | 11 | 16 | 20 | 23 |

---

**compare minimum entry in each list:** copy 18 and decrement $x$

### Sorted List C

| 2 | 3 | 7 | 10 | 11 | 14 | 16 |

---

$x = 1$

inversions = 8
Merge-and-count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

<table>
<thead>
<tr>
<th>sorted list A</th>
<th>3</th>
<th>7</th>
<th>10</th>
<th>14</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>sorted list B</td>
<td>2</td>
<td>11</td>
<td>16</td>
<td>20</td>
<td>23</td>
</tr>
</tbody>
</table>

list A exhausted: copy 20

| sorted list C | 2 | 3 | 7 | 10 | 11 | 14 | 16 | 18 |

$x = 0$

inversions = 8
Merge-and-count demo

Given two sorted lists $A$ and $B$,

- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

sorted list $A$

| 3 | 7 | 10 | 14 | 18 |

sorted list $B$

| 2 | 11 | 16 | 20 | 23 |

list $A$ exhausted: copy 23

sorted list $C$

| 2 | 3 | 7 | 10 | 11 | 14 | 16 | 18 | 20 |

$x = 0$

inversions = 8
Merge-and-count demo

Given two sorted lists $A$ and $B$,
- Count number of inversions $(a, b)$ with $a \in A$ and $b \in B$.
- Merge $A$ and $B$ into sorted list $C$.

**sorted list A**

3  7  10  14  18

**sorted list B**

2  11  16  20  23

**sorted list C**

2  3  7  10  11  14  16  18  20  23

$x = 0$

inversions = 8

*done: return 8 inversions*