String Searching

Reference: Chapter 19, *Algorithms in C* by R. Sedgewick.
Addison Wesley, 1990.

Princeton University • COS 423 • Theory of Algorithms • Spring 2002 • Kevin Wayne

Strings

String.
- Sequence of characters over some alphabet.
  - binary { 0, 1 }
  - ASCII, UNICODE

Some applications.
- Word processors.
- Virus scanning.
- Text information retrieval systems. (Lexis, Nexis)
- Digital libraries.
- Natural language processing.
- Specialized databases.
- Computational molecular biology.
- Web search engines.

String Searching

Search Text																				
n	n	e	e	n	l	e	d	e	n	e	e	n	e	e	d	l	e	n	l	d

Search Pattern					
n	e	e	d	l	e

Successful Search																				
n	n	e	e	n	l	e	d	e	n	e	e	**n**	**e**	**e**	**d**	**l**	**e**	n	l	d

Parameters.
- N = # characters in text.
- M = # characters in pattern.
- Typically, N >> M.
  - e.g., N = 1 million, M = 1 hundred

Brute Force

Brute force.
- Check for pattern starting at every text position.

Brute Force String Search

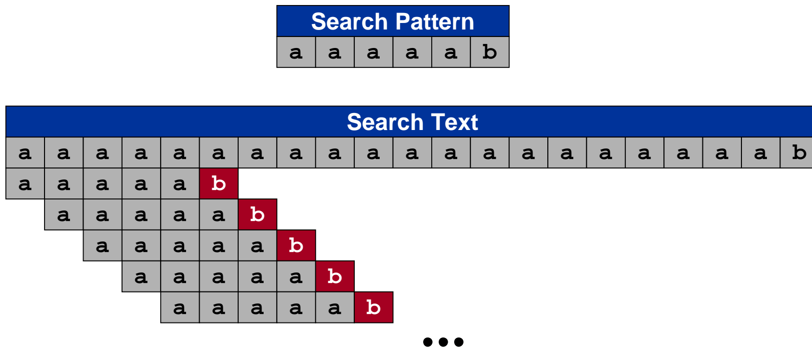
```
int brutesearch(char p[], char t[]) {
   int i, j;
   int M = strlen(p);          // pattern length
   int N = strlen(t);          // text length

   for (i = 0; i < N; i++) {
       for (j = 0; j < M; j++) {
          if (t[i+j] != p[j]) break;
       }
       if (j == M) return i;   // found at offset i
   }
   return -1;                  // not found
}
```

Analysis of Brute Force

Analysis of brute force.

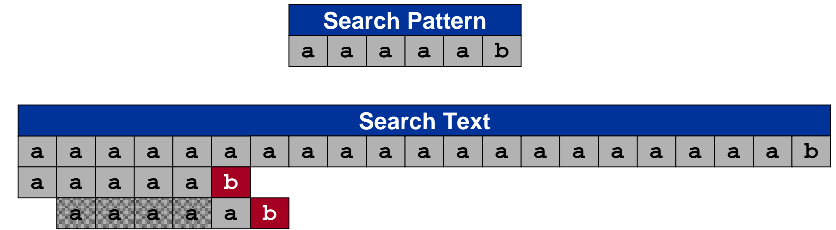
- Running time depends on pattern and text.
  - can be slow when strings repeat themselves
- Worst case: MN comparisons.
  - too slow when M and N are large



How To Save Comparisons

How to avoid recomputation?

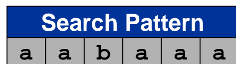
- Pre-analyze search pattern.
- Ex: suppose that first 5 characters of pattern are all a's.
  - If `t[0..4]` matches `p[0..4]` then `t[1..4]` matches `p[0..3]`.
  - no need to check i = 1, j = 0, 1, 2, 3
  - saves 4 comparisons
- Need better ideas in general.

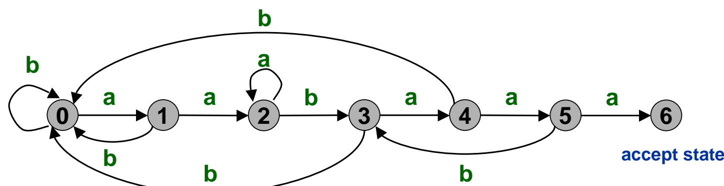


Knuth-Morris-Pratt

KMP algorithm.

- Use knowledge of how search pattern repeats itself.
- Build FSA from pattern.
- Run FSA on text.
- O(M + N) worst-case running time.





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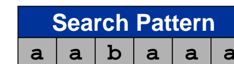

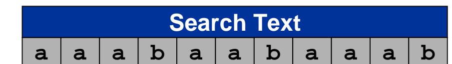

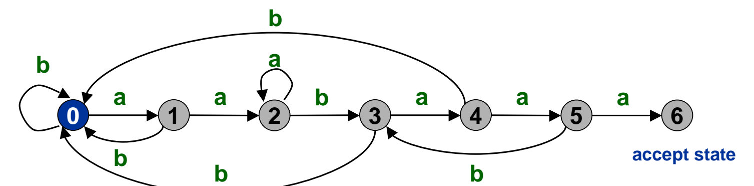

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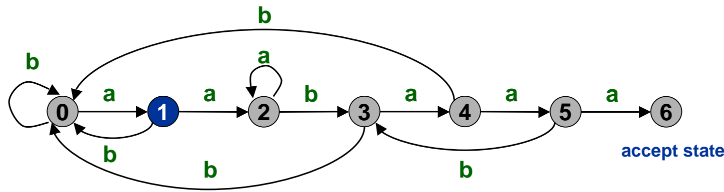
Search Pattern					
a	a	b	a	a	a

Search Text										
a	a	a	b	a	a	b	a	a	a	b



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Search Text										
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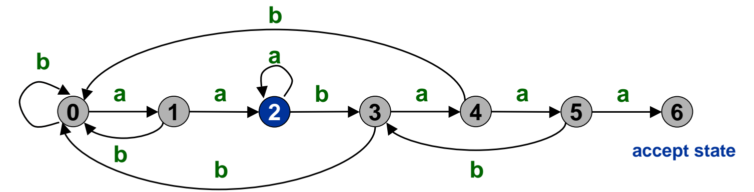

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Search Text										
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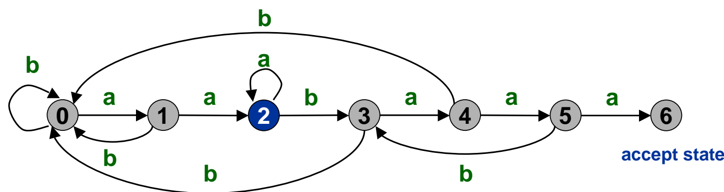

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Search Text										
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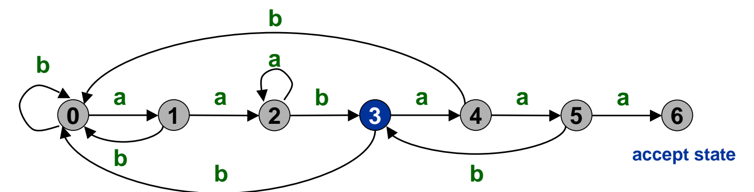

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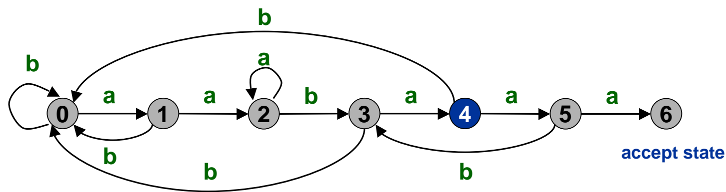

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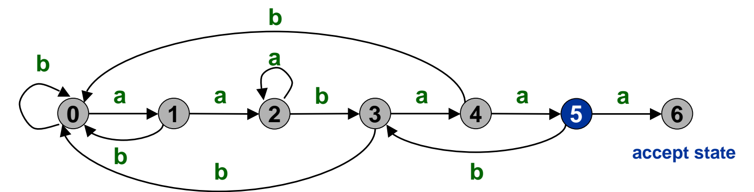
Search Text										
a	a	a	b	a	a	b	a	a	a	b



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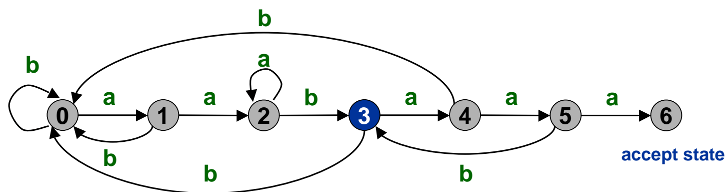
Search Text										
a	a	a	b	a	a	b	a	a	a	b



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Search Text										
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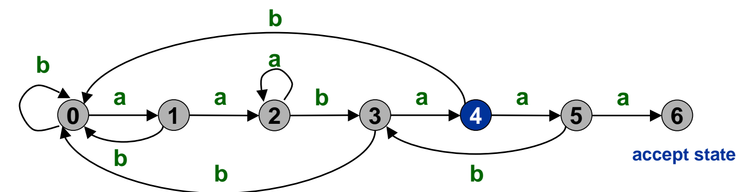

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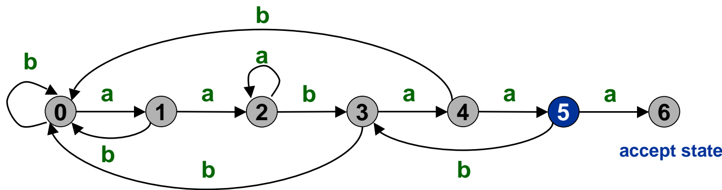

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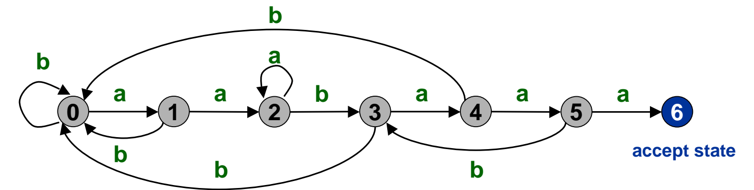
Search Text										
a	a	a	b	a	a	b	a	a	a	b



Knuth-Morris-Pratt

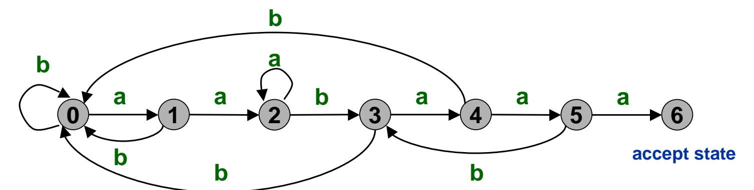
KMP algorithm.
- Use knowledge of how search pattern repeats itself.
- Build FSA from pattern.
- Run FSA on text.
- ➡ O(M + N) worst-case running time.
  - FSA simulation takes O(N) time
  - can build FSA in O(M) time with cleverness

FSA Representation

FSA used in KMP has special property.
- Upon character match, go forward one state.
- Only need to keep track of where to go upon character mismatch.
  - go to state `next[j]` if character mismatches in state `j`

Search Pattern					
a	a	b	a	a	a

	0	1	2	3	4	5
a	1	2	2	4	5	6
b	0	0	3	0	0	3
next	0	0	2	0	0	3



KMP Algorithm

Given the FSA, string search is easy.

- The array `next[]` contains next FSA state if character mismatches.

KMP String Search

```
int kmpsearch(char p[], char t[], int next[]) {
   int i, j = 0;
   int M = strlen(p);                // pattern length
   int N = strlen(t);                // text length

   for (i = 0; i < N; i++) {
       if (t[i] == p[j]) j++;        // char match
       else j = next[j];             // char mismatch
       if (j == M) return i - M + 1; // found
   }

   return -1;                        // not found
}
```

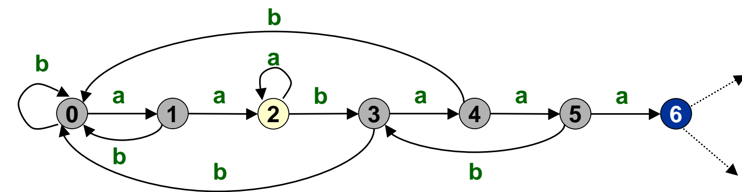

FSA Construction for KMP

FSA construction for KMP.

- FSA builds itself!

Example. Building FSA for `aabaaabb`.

- State 6. `p[0..5] = aabaaa`
  - assume you know state for `p[1..5] = abaaa` X = 2
  - if next char is `b` (match): go forward 6 + 1 = 7
  - if next char is `a` (mismatch): go to state for `abaaaa` X + 'a' = 2
  - update X to state for `p[1..6] = abaaab` X + 'b' = 3



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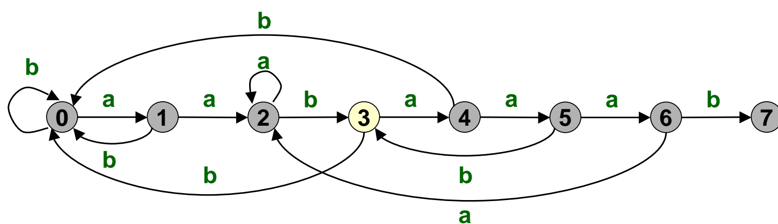

FSA Construction for KMP

FSA construction for KMP.

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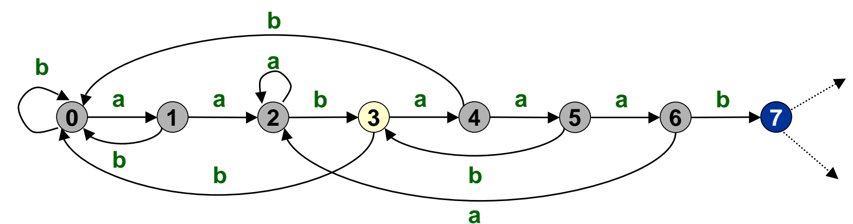
Example. Building FSA for `aabaaabb`.

- State 7. `p[0..6] = aabaaab`
  - assume you know state for `p[1..6] = abaaab` X = 3
  - if next char is `b` (match): go forward 7 + 1 = 8
  - next char is `a` (mismatch): go to state for `abaaaba` X + 'a' = 4
  - update X to state for `p[1..7] = abaaabb` X + 'b' = 0



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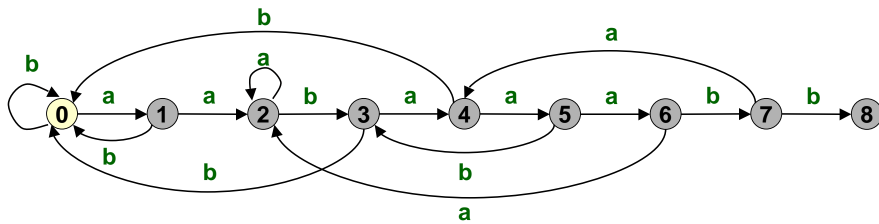


FSA Construction for KMP

FSA construction for KMP.

- FSA builds itself!

Crucial insight.

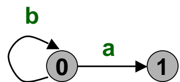
- To compute transitions for state n of FSA, suffices to have:
  - FSA for states 0 to n-1
  - state X that FSA ends up in with input `p[1..n-1]`
- To compute state X' that FSA ends up in with input `p[1..n]`, it suffices to have:
  - FSA for states 0 to n-1
  - state X that FSA ends up in with input `p[1..n-1]`

FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

j	pattern[1..j]	X

a
b

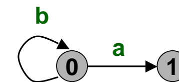


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

j	pattern[1..j]	X	next
0		0	0

	0
a	1
b	0

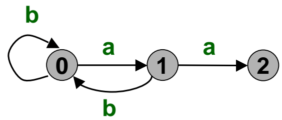


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

	0	1
a	1	2
b	0	0

j	pattern[1..j]							X	next
0								0	0
1	a							1	0

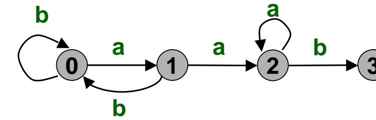


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

	0	1	2
a	1	2	2
b	0	0	3

j	pattern[1..j]							X	next
0								0	0
1	a							1	0
2	a	b						0	2

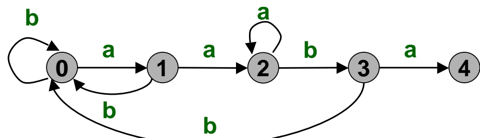


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

	0	1	2	3
a	1	2	2	4
b	0	0	3	0

j	pattern[1..j]							X	next
0								0	0
1	a							1	0
2	a	b						0	2
3	a	b	a					1	0

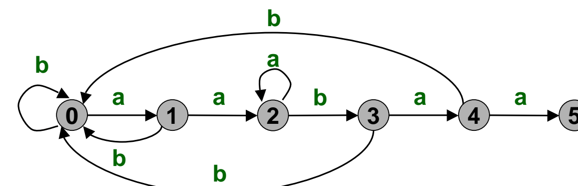


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

	0	1	2	3	4
a	1	2	2	4	5
b	0	0	3	0	0

j	pattern[1..j]							X	next
0								0	0
1	a							1	0
2	a	b						0	2
3	a	b	a					1	0
4	a	b	a	a				2	0

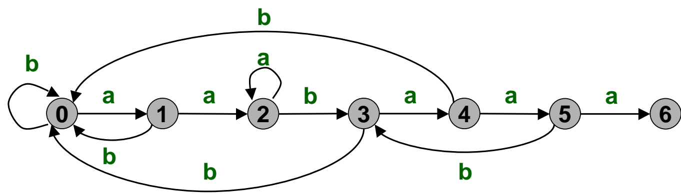


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

	0	1	2	3	4	5
a	1	2	2	4	5	6
b	0	0	3	0	0	3

j	pattern[1..j]							X	next
0								0	0
1	a							1	0
2	a	b						0	2
3	a	b	a					1	0
4	a	b	a	a				2	0
5	a	b	a	a	a			2	3

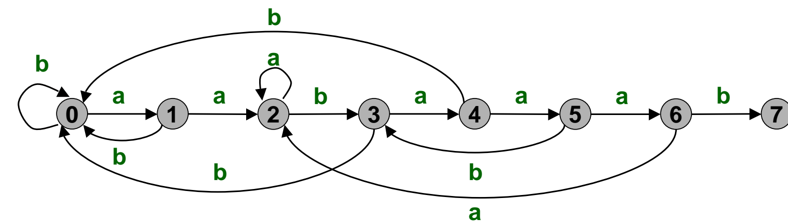


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

	0	1	2	3	4	5	6
a	1	2	2	4	5	6	2
b	0	0	3	0	0	3	7

j	pattern[1..j]							X	next
0								0	0
1	a							1	0
2	a	b						0	2
3	a	b	a					1	0
4	a	b	a	a				2	0
5	a	b	a	a	a			2	3
6	a	b	a	a	a	b		3	2

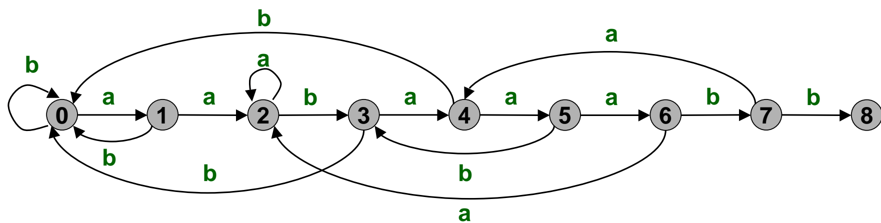


FSA Construction for KMP

Search Pattern							
a	a	b	a	a	a	b	b

	0	1	2	3	4	5	6	7
a	1	2	2	4	5	6	2	4
b	0	0	3	0	0	3	7	8

j	pattern[1..j]							X	next
0								0	0
1	a							1	0
2	a	b						0	2
3	a	b	a					1	0
4	a	b	a	a				2	0
5	a	b	a	a	a			2	3
6	a	b	a	a	a	b		3	2
7	a	b	a	a	a	b	b	0	4



FSA Construction for KMP

Code for FSA construction in KMP algorithm.

FSA Construction for KMP

```
void kmpinit(char p[], int next[]) {
   int j, X = 0, M = strlen(p);
   next[0] = 0;

   for (j = 1; j < M; j++) {
      if (p[X] == p[j]) {
         next[j] = next[X];
         X = X + 1;
      }
      else {
         next[j] = X + 1;
         X = next[X];
      }
   }
}
```

Specialized KMP Implementation

Specialized C program for `aabaaabb` pattern.

Hardwired FSA for `aabaaabb`

```
int kmpsearch(char t[]) {
   int i = 0;
   s0: if (t[i++] != 'a') goto s0;
   s1: if (t[i++] != 'a') goto s0;
   s2: if (t[i++] != 'b') goto s2;
   s3: if (t[i++] != 'a') goto s0;
   s4: if (t[i++] != 'a') goto s0;
   s5: if (t[i++] != 'a') goto s3;
   s6: if (t[i++] != 'b') goto s2;
   s7: if (t[i++] != 'b') goto s4;
   return i - 8;
}
```

`next[]`

Ultimate search program for `aabaaabb` pattern.

- Machine language version of above.

Summary of KMP

KMP summary.

- Build FSA from pattern.
- Run FSA on text.
- O(M + N) worst case string search.
- Good efficiency for patterns and texts with much repetition.
  - binary files
  - graphics formats
- Less useful for text strings.
- On-line algorithm.
  - virus scanning
  - Internet spying

History of KMP

History of KMP.

- Inspired by theorem of Cook that says O(M + N) algorithm should be possible.
- Discovered in 1976 independently by two groups.
- Knuth-Pratt.
- Morris was hacker trying to build an editor.
  - annoying problem that you needed a buffer when performing text search

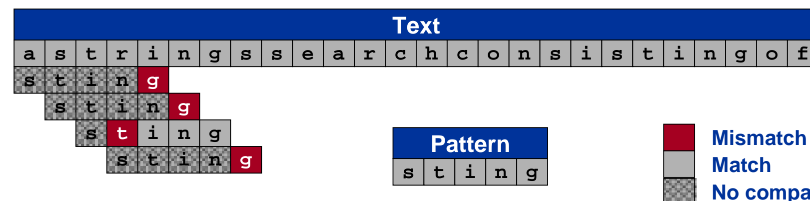
Resolved theoretical and practical problems.

- Surprise when it was discovered.
- In hindsight, seems like right algorithm.

Boyer-Moore

Boyer-Moore algorithm (1974).

- Right-to-left scanning.
  - find offset `i` in text by moving left to right.
  - compare pattern to text by moving right to left.

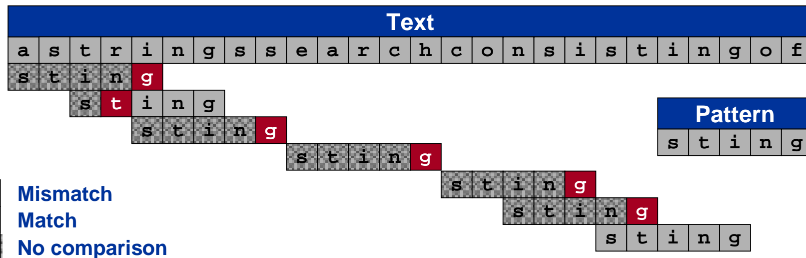


Boyer-Moore

Boyer-Moore algorithm (1974).

- Right-to-left scanning.
- Heuristic 1: advance offset i using "bad character rule."
  - upon mismatch of text character `c`, look up `j = index[c]`
  - increase offset `i` so that `j`th character of pattern lines up with text character `c`

Index	
g	4
i	2
n	3
s	0
t	1
*	-1



Boyer-Moore

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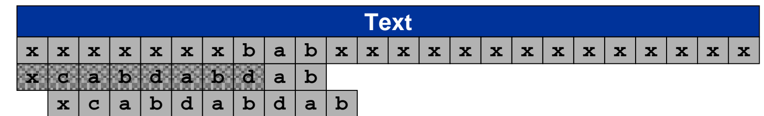
- Right-to-left scanning.
- Heuristic 1: advance offset i using "bad character rule."
  - extremely effective for English text
- Heuristic 2: use KMP-like suffix rule.
  - effective with small alphabets
  - different rules lead to different worst-case behavior



bad character heuristic

Boyer-Moore

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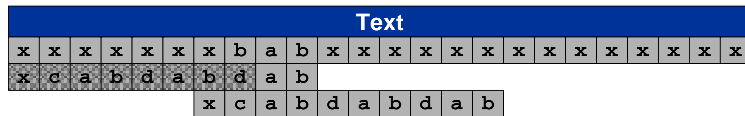

strong good suffix

Boyer-Moore

Boyer-Moore analysis.

- O(N / M) average case if given letter usually doesn't occur in string.
  - English text: 10 character search string, 26 char alphabet
  - time decreases as pattern length increases
  - sublinear in input size!
- O(M + N) worst-case with Galil variant.
  - proof is quite difficult

Karp-Rabin

Idea: use hashing.

- Compute hash function for each text position.
- No explicit hash table!
  - just compare with pattern hash

Example.

- Hash "table" size = 97.

Search Pattern

5	9	2	6	5

59265 % 97 = 95

Search Text

3	1	4	1	5	9	2	6	5	3	5	8	9	7	9	3	2	3	8	4	6

Window	Hash
3 1 4 1 5	31415 % 97 = 84
1 4 1 5 9	14159 % 97 = 94
4 1 5 9 2	41592 % 97 = 76
1 5 9 2 6	15926 % 97 = 18
5 9 2 6 5	59265 % 97 = 95

Karp-Rabin

Idea: use hashing.

- Compute hash function for each text position.

Problems.

- Need full compare on hash match to guard against collisions.
  - 59265 % 97 = 95
  - 59362 % 97 = 95
- Hash function depends on M characters.
  - running time on search miss = MN

Karp-Rabin

Key idea: fast to compute hash function of adjacent substrings.

- Use previous hash to compute next hash.
- O(1) time per hash, except first one.

Example.

- Pre-compute: 10000 % 97 = 9
- Previous hash: 41592 % 97 = 76
- Next hash: 15926 % 97

Observation.

- 15926 ≡ (41592 – (4 * 10000)) * 10 + 6
- 15926 % 97 ≡ (41592 – (4 * 10000)) * 10 + 6
  ≡ (76 – 4 * 9) * 10 + 6
  ≡ 406
  ≡ 18

Karp-Rabin (Sedgewick, p. 290)

```
#define q 3355439                       // table size
#define d 256                           // radix

int rksearch(char p[], char t[]) {
   int i, j, dM = 1, h1 = 0, h2 = 0;
   int M = strlen(p), N = strlen(t);

   for (j = 1; j < M; j++)              // precompute d^M % q
      dM = (d * dM) % q;

   for (j = 0; j < M; j++) {
      h1 = (h1*d + p[j]) % q;           // hash of pattern
      h2 = (h2*d + t[j]) % q;           // hash of text
   }

   for (i = M; i < N; i++) {
      if (h1 == h2) return i - M;       // match found
      h2 = (h2 - a[i-M]*dM) % q;        // remove high order digit
      h2 = (h2*d + a[i])    % q;        // insert low  order digit
   }
   return -1;                           // not found
}
```

Karp-Rabin

Karp-Rabin algorithm.

- Choose table size at RANDOM to be huge prime.
- Expected running time is O(M + N).
- O(MN) worst-case, but this is (unbelievably) unlikely.

Randomized algorithms.

- Monte Carlo: don't check for collisions.
  - algorithm can be wrong but running time guaranteed linear
- Las Vegas: if collision, start over with new random table size.
  - algorithm always correct, but running time is expected linear

Advantages.

- Extends to 2d patterns and other generalizations.