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| PRINCETON UNIVERSITY FALL '18 | COS 597F |
| Homework 1 | |
| Out: *Nov 21* | Due: *Dec 10* |

*You can collaborate with your classmates, but be sure to list your collaborators with your answer. If you get help from a published source (book, paper etc.), cite that.*

*You are encouraged to use your own judgment for the appropriate use of classmates/sources to get the most out of the homework. The homework is intended to be challenging so that you can understand the material better, but not stressful. If you use Piazza for help, I strongly recommend making posts public so that the class can help answer.*

The problems below are designed to develop your comfort with virtual values for continuous distributions.

§1 What is the virtual value function for the uniform distribution on $[0, 1]$? exponential distribution with rate 1?

§2 Let there be $k$ copies of the item (and each bidder wants at most one). What is the revenue-optimal auction?

§3 Provide a counterexample to Bulow-Klemperer (second-price with $n+1$ bidders from $F$ gets more revenue than the optimum for $n$ bidders from $F$) when $F$ is not regular.

§4 Prove that for any regular distribution, setting price equal to the median generates at least half of the optimal achievable revenue (for selling to one bidder). Hint: draw the revenue curve.

§5 Describe the optimal auction when $F_1$ is uniform on $[0, 1]$ and $F_2$ is uniform on $[0, 100]$.

§6 Design an ex-post payment rule that satisfies the payment identity for the (ironed) virtual welfare maximizer, and prove that the resulting mechanism is dominant strategy truthful.

§7 Let Lazy VCG denote the following mechanism: first solicit bids. Then, find the highest bidder. If that bidder's virtual value is non-negative, give them the item. Otherwise, no one wins. Prove that the revenue of Lazy VCG is a 1/2-approximation to the optimum whenever all $F_i$ are regular (but not iid). Hint: try to show that the revenue of VCG plus the revenue of Lazy VCG exceeds the optimum. For one case, count the payments, for the other count the virtual surplus. Prove also that Lazy VCG gets more revenue than VCG.

The problems below are designed to develop your comfort with virtual values for discrete distributions.

§1 Let $X(\cdot)$ be the allocation rule that selects the highest bidder (breaking ties lexicographically). Let the support of each $F_i$ be $\mathbb{N}$. Describe a payment rule that has proper payments and is also DSIC (hint: it should look like a second-price auction with modification).

§2 Using the same Lagrangian multipliers as in lecture, derive the resulting virtual values if the support is discrete, but not necessarily the integers.

§3 Consider discretizing a continuous distribution $F$ by rounding down to the nearest multiple of $F$ and call it $F_\varepsilon$. Let $k$ in the next statement always be an integer. Prove that for all $v$, $\lim_{k\to\infty} \varphi_{F_{v/k}}(v) = \varphi_F(v)$.