1. Is there a faithful functor $F: \mathcal{C} \to \mathcal{D}$ such that there exist distinct arrows $f, g$ in $\mathcal{C}$ with $F(f) = F(g)$? Provide an example, or prove that no example exists.

2. Let $\mathcal{C}$ be a category and $\mathcal{C}_g$ its associated groupoid. Define a faithful functor $\mathcal{C}_g \to \mathcal{C}$. Hence, or otherwise, prove that if $\mathcal{C}$ is equivalent to $\mathcal{C}_g$ then $\mathcal{C}$ is also a groupoid.

3. Let $F: \mathcal{C} \to \mathcal{D}$ be a full and faithful functor.
   (a) Show that $F$ is conservative: For any arrow $f: a \to b$, if $Ff$ is an isomorphism then $f$ is an isomorphism.
   (b) Show that $F$ creates isomorphisms: For any objects $a, b$ in $\mathcal{C}$, if $Fa \cong Fb$ then $a \cong b$.

4. Riehl, Exercise 1.1.iii (p.8)

5. Let $(\mathbb{P}, \leq)$ be a partially ordered set (“poset”).
   (a) Describe a category structure on $\mathbb{P}$ such that there is an arrow between any $a, b \in \mathbb{P}$ iff $a \leq b$.
   (b) Hence, define a category $\textbf{Poset}$ with objects posets and morphisms the order-preserving maps between posets. (Given posets $(\mathbb{P}_1, \prec)$ and $(\mathbb{P}_2, \leq)$ a map $f: \mathbb{P}_1 \to \mathbb{P}_2$ is order preserving iff $a \prec b \Rightarrow f(a) \leq f(b)$).
   (c) Use your previous construction to define a full functor $\textbf{Poset} \to \textbf{Cat}$.
   (d) Prove that $\textbf{Poset}$ is equivalent to the category of (small) categories that have at most one arrow between any two of their objects.

6. Find a set $A$ such that $\textbf{Set}(A, -)$ is naturally isomorphic to the identity functor on $\textbf{Set}$.

7. Let $\mathcal{C}$ be a locally small category. Prove that $f: a \to b$ is an isomorphism if and only if for any $c$ in $\mathcal{C}$ the “precomposition” function

$$\begin{align*}
\mathcal{C}(b, c) & \longrightarrow \mathcal{C}(a, c) \\
g & \mapsto g \circ f
\end{align*}$$

is a bijection.