Random Oracles in a Quantum World

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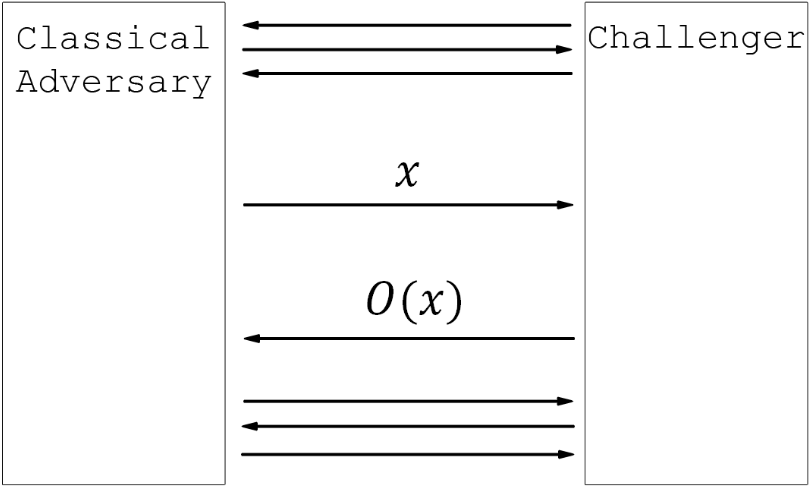
Classical Random Oracle Model Adversaries



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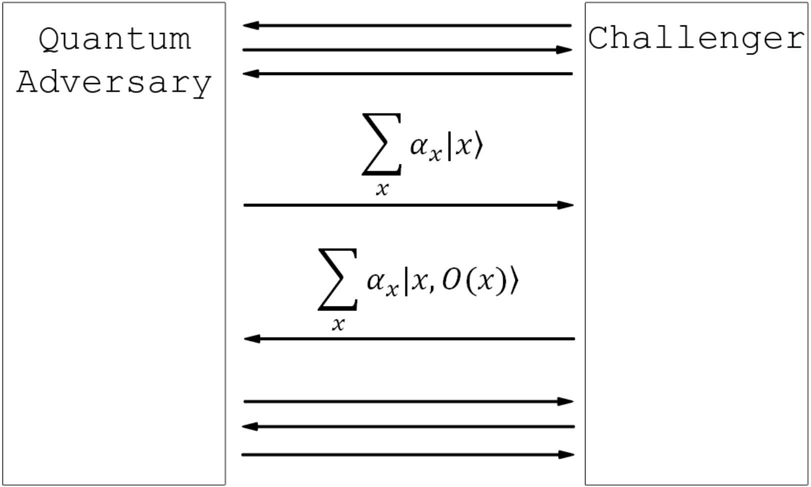

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  Examples:
  - Simulating the random oracle
  - Determining what points the adversary is interested in
  - Programming the random oracle
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- Positive result: Encryption Schemes

Preimage Sampleable Functions

- A preimage sampleable trapdoor function (PSF) $\mathcal{F}$ is a triple of functions (G, f, f^{-1}):
  - $G(1^n)$ outputs $(\mathtt{sk}, \mathtt{pk})$
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- Secure construction from lattices [GPV08]

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Theorem

Suppose $\mathcal{F}$ is a quantum-secure PSF, and that quantum pseudorandom functions exist. Then $\mathcal{S}$ is quantum secure.

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- Prove that security of a certain type of classical reduction (called *history free*) implies security in the quantum setting
- Show that the reduction of [GPV08] is *history free*

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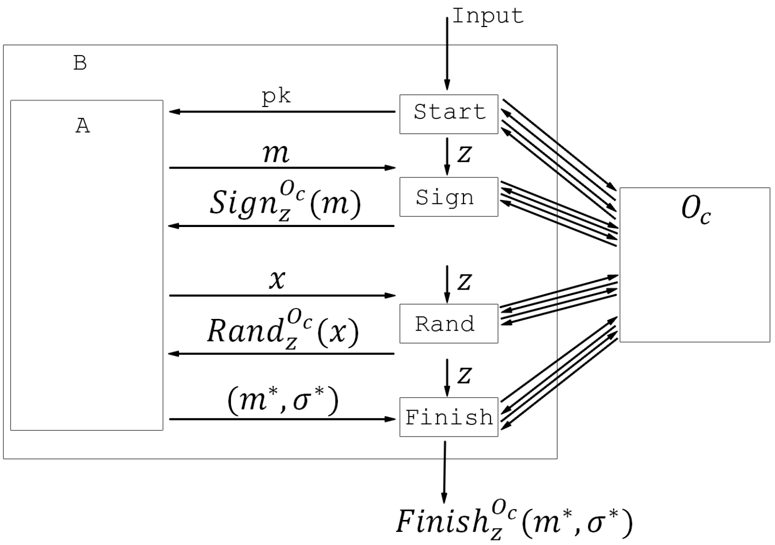
- Random oracle queries answered by $Rand^{O_c}$
    - Truly random
- Signatures answered by $Sign^{O_c}$
    - Consistent with random oracle
    - Distribution identical to actual

(Classical) History Free Reduction

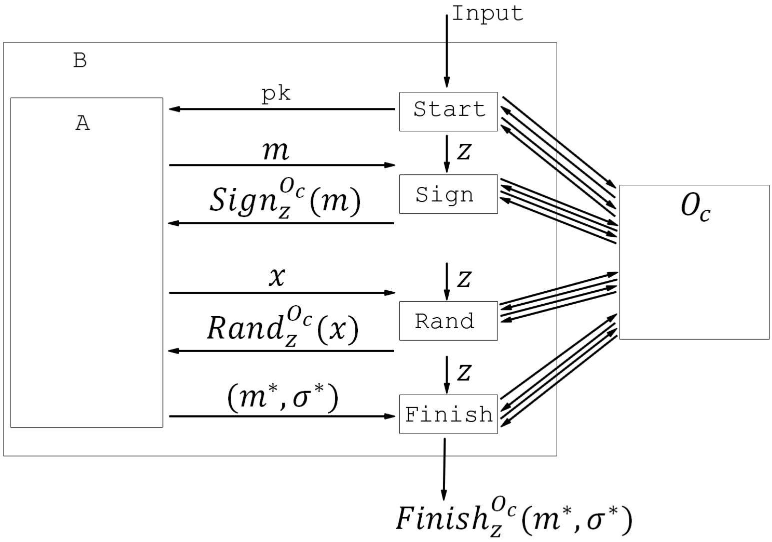


Main Theorem

Theorem

Suppose a random oracle model signature scheme $\mathcal{S}$ has a history-free reduction that transforms any classical adversary A into a classical algorithm B for some hard problem for quantum computers. Suppose further that quantum pseudorandom functions exist. Then $\mathcal{S}$ is secure against quantum adversaries.

Proof


Input
B
A
pk
Start
m
z
Sign
$Sign_z^{O_c}(m)$
O_c
x
z
Rand
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z
(m^*, σ^*)
Finish
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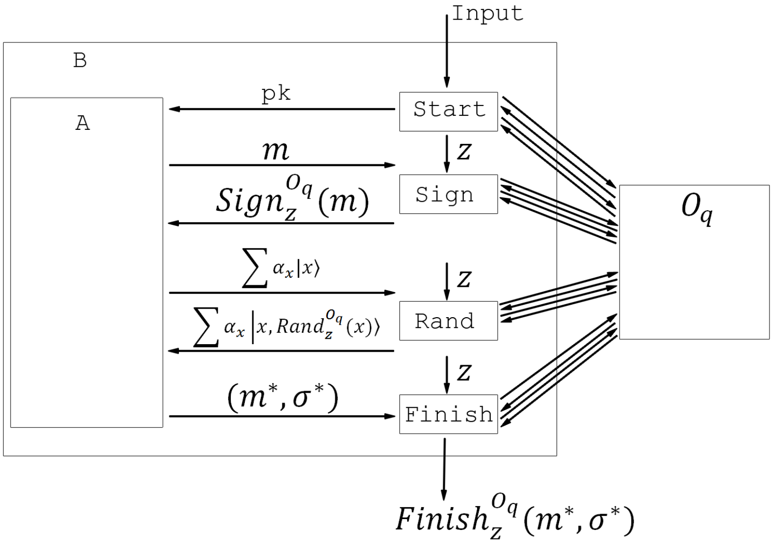
Input
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pk
Start
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z
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Sign
O_q
$\sum \alpha_x |x\rangle$
z
$\sum \alpha_x |x, Rand_z^{O_q}(x)\rangle$
Rand
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Idea: Use a quantum pseudorandom function

Quantum PRF

A quantum pseudorandom function *PRF* is a keyed function that quantum computers cannot tell from a random oracle. Precisely, for all polynomial-time quantum oracle algorithms A,

$$\left|\Pr[A^{PRF_k}() = 1] - \Pr[A^{O_q}() = 1]\right| < \text{negl}$$

Where the left probability is over k and the right is over O_q, both chosen randomly.

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No known provably secure constructions!

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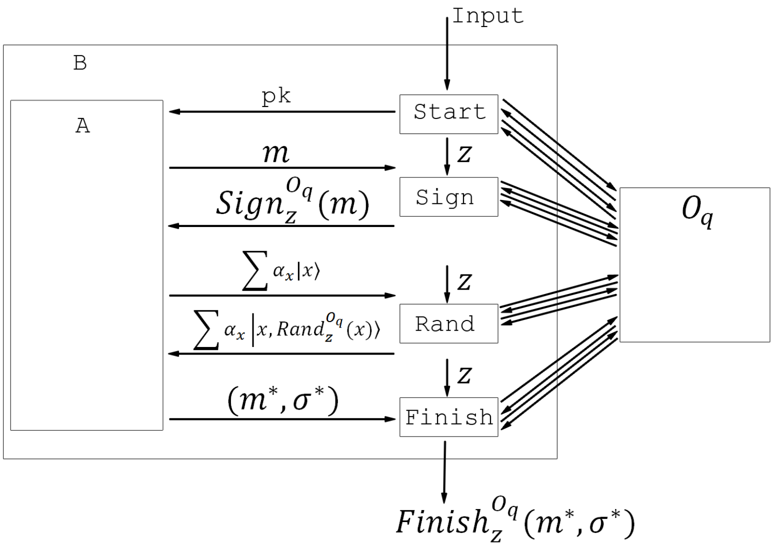
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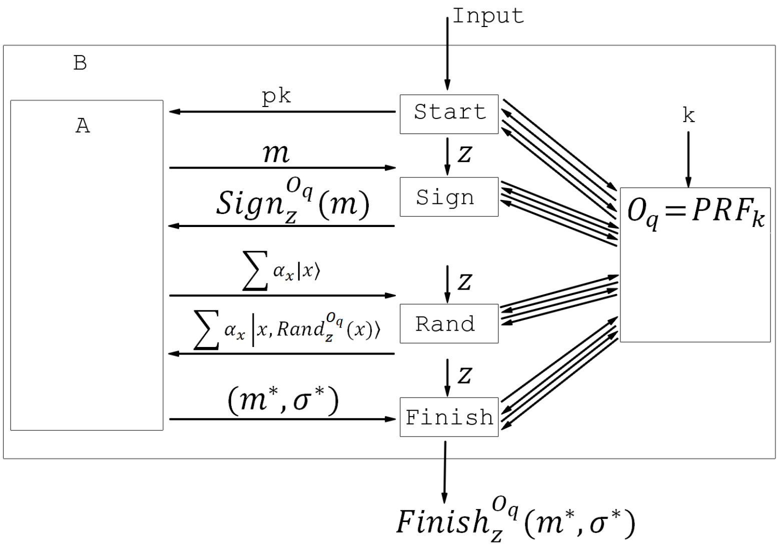
Input
B
A
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Start
m
z
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Sign
k
$O_q = PRF_k$
$\sum \alpha_x |x\rangle$
z
$\sum \alpha_x \left|x, Rand_z^{O_q}(x)\right\rangle$
Rand
z
(m^*, σ^*)
Finish
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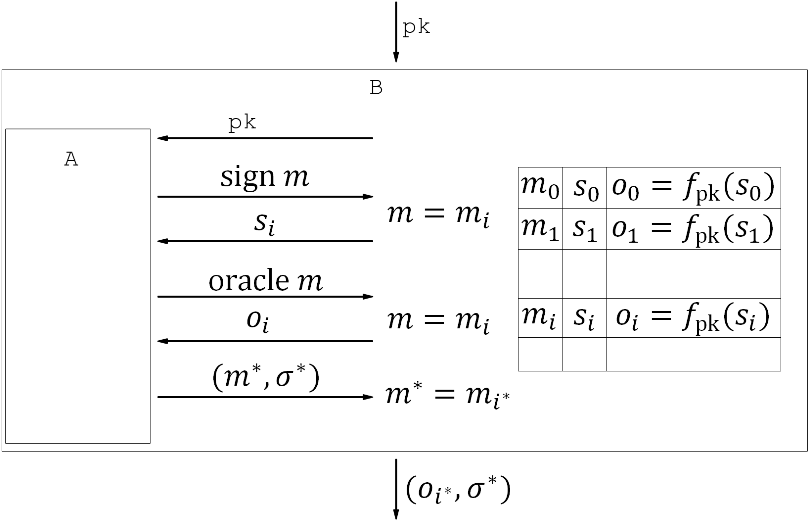
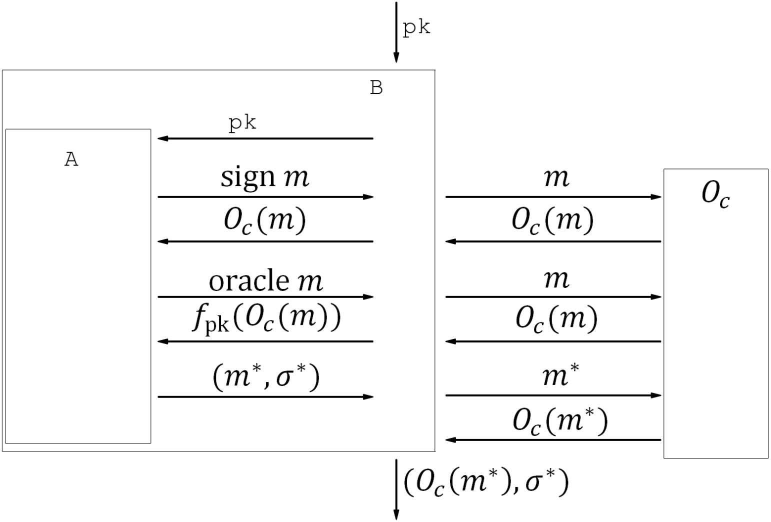
GPV Reduction



Modified GPV Reduction



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- Need to relax definition of history freeness to allow indistinguishable (by quantum adversaries)

Other History-Free Reductions

- Full Domain Hash from claw-free permutations ([Cor00]).
- Katz-Wang Signatures (KW03)

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where f is a trapdoor permutation.

- CCA-security of hybrid encryption scheme:

$$E_{\mathrm{pk}}(m) = f_{\mathrm{pk}}(r) || \left(E_S\right)_{O(r)}(m) \text{ for a random r}$$

where f is a trapdoor permutation and E_S is CCA-secure private key encryption.

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- GPV Signatures are secure

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- Quantum PRFs from one-way functions