A Comparison of Bilateral and Multilateral Exchanges for Peer-Assisted Content Distribution

Christina Aperjis, Michael J. Freedman, Ramesh Johari

Abstract. Peer-assisted content distribution matches user demand for content with available supply at other peers in the network. Inspired by this supply-and-demand interpretation of the nature of content sharing, we employ *price theory* to study peer-assisted content distribution. In this approach, the market-clearing prices are those which exactly align supply and demand, and the system is studied through the characterization of price equilibria. We rigorously analyze the efficiency and robustness gains that are enabled by price-based multilateral exchange. We show that multilateral exchanges satisfy several desirable efficiency and robustness properties that bilateral exchanges do not, *e.g.*, equilibria in bilateral exchange may fail to exist, be inefficient if they do exist, and fail to remain robust to collusive deviations even if they are Pareto efficient. Further, we show that an equilibrium in bilateral exchange corresponds to a multilateral exchange equilibrium if and only if it is robust to deviations by coalitions of users.

1 Introduction

Peer-to-peer systems have been wildly successful as a disruptive technology for content distribution. Varying accounts place peer-to-peer (P2P) traffic as comprising anywhere between 35% and 90% of "all" Internet traffic [1]. Early P2P systems did not provide any incentives for participation, leading to extensive freeloading [2, 3]. The P2P community responded with mechanisms to prevent freeloading by incentivizing sharing on a *bilateral barter* basis, as used by BitTorrent [4] and its variants [5, 6], where peers can achieve better download performance from peers to which they are simultaneously uploading.

While BitTorrent's usage numbers are certainly impressive, it can only perform bilateral barter by matching up well-suited pairs of nodes that have disjoint subsets of a file (or, more generally, files), and it is often hard to find good reciprocation with bilateral barter alone. Furthermore, potential "free-riding" attacks have been observed [7, 8, 9, 5], and altruistic uploading often turns out to be critical for providing continued content availability [10].

Another alternative is to use *market-based multilateral exchange* to match user demand for content to available supply at other peers in the system. This approach uses virtual currency and assigns a budget to each peer, which decreases when downloading and increases when uploading. Monetary incentives in a virtual currency have been previously proposed to incentivize uploading in P2P systems [11, 12, 13, 14]. In this paper, we compare bilateral and multilateral exchanges for peer-assisted content distribution.

We provide a formal comparison of P2P system designs with bilateral barter, such as BitTorrent, and a market-based exchange of content enabled by a price mechanism to match supply and demand. We start in Section §2 with a fundamental abstraction of content exchange in systems with bilateral barter: *exchange ratios*. The exchange ratio from one peer to another gives the download rate received per unit upload rate. Exchange ratios are a useful formal tool because they directly allow us to compare bilateral P2P systems with price-based multilateral P2P systems.

In §3 and §4, we compare bilateral and multilateral P2P systems through the allocations that arise at equilibria. In particular, we show that a multilateral price-based exchange scheme satisfies a number of desirable properties lacking in bilateral exchange, *e.g.*, equilibria in bilateral exchange may fail to exist, be inefficient if they do exist, and fail to remain robust to collusive deviations even if they exist and are efficient. We show that with an additional technical condition, a bilateral equilibrium corresponds to a multilateral equilibrium if and only if it is robust to deviations by coalitions of users.

2 Exchange Ratios in Bilateral Protocols

The BitTorrent protocol and its variants enable exchange on a *bilateral* basis between peers: a peer i uploads to peer j if and only if peer j uploads to peer i in return. While such protocols are traditionally studied solely through the rates that peers obtain, in this section we provide an interpretation of these protocols through *exchange ratios*. As exchange ratios can be interpreted in terms of prices, these ratios will allow us to compare bilateral and multilateral P2P systems in the following section.

Let r_{ij} denote the rate sent from peer i to peer j in an instantiation of a BitTorrent swarm. We define the *exchange ratio* between peer i and peer j as the ratio $\gamma_{ij} = r_{ji}/r_{ij}$; this is the download rate received by i from j, per unit of rate uploaded to j. By definition, $\gamma_{ij} = 1/\gamma_{ji}$. Clearly, a rational peer i would prefer to download from peers with which he has higher exchange ratios.

The exchange ratio has a natural interpretation in terms of prices. An equivalent story emerges if we assume that peers charge each other for content in a common monetary unit, but that all transactions are *settlement-free*, *i.e.*, no money ever changes hands. In this case, if peer i charged peer j a price p_{ij} per unit rate, the exchange of content between peers i and j must satisfy $p_{ij}r_{ij} = p_{ji}r_{ji}$. Note that the preceding condition thus shows the exchange ratio is equivalent to the ratio of bilateral prices: $\gamma_{ij} = p_{ij}/p_{ji}$ (as long as the prices and rates are nonzero). The rates achieved by the BitTorrent and BitTyrant [5] protocols can be naturally modeled through exchange ratios [15].

The preceding discussion highlights the fact that the rates in a bilateral P2P system can be interpreted via exchange ratios. Thus far we have assumed that *transfer rates* are given, and exchange ratios are computed from these rates. In the next section, we turn this relationship around: we explicitly consider an abstraction of bilateral P2P systems where peers react to given exchange ratios, and compare the resulting outcomes to price-based multilateral exchange.

3 Bilateral and Multilateral Equilibria

Motivated by the discussion in the preceding section, this section rigorously analyzes the efficiency properties of price-based bilateral and multilateral mechanisms. Assuming that peers explicitly react to exchange ratios or prices, we compare the schemes through their resulting price equilibria.

In the formal model we consider, a set of peers N shares a set of files F. Peer i has a subset of the files $F_i \subseteq F$, and is interested in downloading files in $T_i \subseteq F - F_i$. We use

Bilateral Peer Optimization:		**Multilateral Peer Optimization:**	
maximize	$V_i(\mathsf{d}_i)$	maximize	$V_i(\mathsf{d}_i)$
subject to	$d_{if} = \sum_j r_{jif}$ for all f	subject to	$d_{if} = \sum_j r_{jif}$ for all f
	$r_{ijf} = 0$ if $f \notin F_i$		$r_{ijf} = 0$ if $f \notin F_i$
	$\sum_{j,f} r_{ijf} \leq B_i$		$\sum_{j,f} r_{ijf} \leq B_i$
	$\sum_f r_{jif} = \gamma_{ij} \sum_f r_{ijf}$ for all j		$\sum_{j,f} p_j r_{jif} = p_i \sum_{j,f} r_{ijf}$
	$\mathsf{r} \geq 0.$		$\mathsf{r} \geq 0.$

Fig. 1: **Optimization problems for price-based exchange.**

r_{ijf} to denote the rate at which user i uploads file f to user j. We then let $d_{if} = \sum_j r_{jif}$ be the total rate at which user i downloads file f. We use sans serif to denote vectors, *e.g.*, $\mathsf{d}_i = (d_{if}, f \in T_i)$ is the vector of download rates for user i. We measure the desirability of a download vector to peer i by a *utility function* $V_i(\mathsf{d}_i)$ that is nondecreasing in every d_{if} for $f \in T_i$. We ignore any resource constraints within the network; we assume that transfers are only constrained by the upload capacities of peers. The upload capacity of peer i is denoted B_i.

We start by considering peers' behavior in bilateral schemes, given a vector of exchange ratios (γ_{ij}). Peer i solves the bilateral optimization problem given in Figure 1. Note that we allow peers to bilaterally exchange content over multiple files, as the more general design, even though this is not typically supported by swarming systems like BitTorrent. This more general design makes it possible to explicitly identify the relative demand for files and reward peers that share more popular content.

By contrast, in a *multilateral price-based exchange*, the system maintains one price per peer, and peers optimize with respect to these prices. In a slight abuse of notation, we denote the price of a peer i by p_i. Figure 1 also gives the peer optimization problem in multilateral price-based exchange. Note that the first three constraints (giving download rates, ensuring peers only upload files they possess, and meeting the upload capacity constraint) are identical to the bilateral peer optimization. While the bilateral exchange implicitly requires peer i to download only from those peers to whom he uploads, no such constraint is imposed on multilateral exchanges: peer i accrues capital for uploading, and he can spend this capital however he wishes for downloading.

For bilateral (resp., multilateral) exchange, an *equilibrium* is a combination of a rate allocation vector and an exchange ratio vector (resp., price vector) such that all peers have solved their corresponding optimization problems. In this case, the exchange ratios (resp., prices) have exactly aligned supply and demand: for any i, j, f, the transfer rate r_{ijf} is simultaneously an optimal choice for both the uploader i and downloader j.

Definition 1. *The rate allocation* r^* *and the exchange ratios* $(\gamma^*_{ij}, i, j \in N)$ *with* $\gamma^*_{ij} > 0$ *for all* $i, j \in N$ *constitute a* ***bilateral equilibrium*** *if for each peer* i, r^* *solves the Bilateral Peer Optimization problem given exchange ratios* $(\gamma^*_{ij}, j \in N)$.

Definition 2. *The rate allocation* r^* *and the peer prices* $(p^*_i, i \in N)$ *with* $p^*_i > 0$ *for all* $i \in N$ *constitute a* ***multilateral equilibrium*** *if for each peer* j, r^* *solves the Multilateral Peer Optimization problem given prices* $(p^*_i, i \in N)$.

This latter is the traditional notion of competitive equilibrium in economics [16]. A multilateral equilibrium can be shown to exist under general conditions in our setting [14].

Moreover, the corresponding allocation is Pareto efficient, *i.e.*, there is no way to increase the utility of some peer without decreasing the utility of some other peer. A bilateral equilibrium, on the other hand, does not always exist, and, even when it exists, the allocation may not be (Pareto) efficient as the following examples illustrate.

Example 1. Consider a system with n peers and n files, for $n > 2$. Each peer i has file f_i and wants $f_{(i \bmod n)+1}$. With these utilities, no bilateral exchange can satisfy all peers, and a bilateral equilibrium does not exist.

Example 2. Consider a system with peers $\{1,2,3\}$ and files $\{f_1, f_2, f_3\}$. Peer i has file f_i and wants the other two files. The peer's utilities are, $V_1(d_{13}, d_{12}) = \ln(d_{13}) + 9\ln(d_{12})$, $V_2(d_{21}, d_{23}) = \ln(d_{21}) + 9\ln(d_{23})$, and $V_3(d_{32}, d_{31}) = \ln(d_{32}) + 9\ln(d_{31})$, where d_{ij} is the rate at which peer i downloads file f_j. The unique bilateral equilibrium is inefficient, because each peer is allocated a smaller rate of the file it values more.

These observations are intuitive: after all, exchange is far more restricted in a bilateral equilibrium than in a multilateral equilibrium. In the next section, we will focus on determining conditions under which a bilateral equilibrium yields a multilateral equilibrium.

We conclude this section by justifying our choice of pricing per peer instead of pricing per file. In the P2P setting we are considering, the two pricing schemes are equivalent in terms of equilibria, since the resource that is being priced is the upload capacity of a peer. A peer with multiple files will only upload his most "expensive" files at equilibrium. We chose to price per peer for simplicity. In the bilateral setting, pricing per file would require to have an exchange ratio $\gamma_{ij,fg}$ for each pair of files f, g that peers i, j can exchange, while pricing per peer only requires one exchange ratio for each pair of peers.

4 Robustness to Collusive Deviations

In this section we demonstrate that a bilateral equilibrium may not be robust to collusive deviations and show that a bilateral equilibrium corresponds to a multilateral equilibrium if and only if it is robust to deviations by coalitions of users. The following is a key step in establishing the relationship between bilateral and multilateral equilibrium. For clarity, all proofs are in the Appendix.

Proposition 1. *Consider a bilateral equilibrium with exchange ratios γ_{ij} for every pair of peers i, j. If there exist prices p_i for all $i \in N$ such that $\gamma_{ij} = p_i/p_j$ for all $i, j \in N$, then the bilateral equilibrium allocation is also a multilateral equilibrium allocation.*

This proposition is quite revealing: it shows that if exchange ratios are "fair," in the sense that they yield a unique price per peer, then the bilateral equilibrium allocation is also a multilateral equilibrium allocation, and thus is efficient.

We have already seen that a bilateral equilibrium need not exist and need not be Pareto efficient when it exists, whereas multilateral equilibria exist under general conditions and are Pareto efficient; thus, the two concepts are not equivalent. We now show that even an efficient bilateral equilibrium does not necessarily yield one price per peer, and thus is not always equivalent to a multilateral equilibrium.

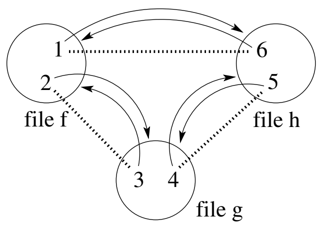

Fig. 2: **Bilateral equilibrium for Ex.3. Peers $\{1,2\}$ have file f, peers $\{3,4\}$ have file g, and peers $\{5,6\}$ have file h. Solid arrows are drawn from a peer to its desired file, *e.g.*, 1 and 4 want h. Heavy dotted lines are between peers bartering at the unique bilateral equilibrium.**

Example 3. There are 6 peers ($\{1,2,3,4,5,6\}$) and 3 files ($\{f,g,h\}$) in the system, with file allocation and demand as shown in Figure 2. The upload capacities of peers are $B_1 = 2$, $B_j = 1, \forall j \neq 1$. At the unique bilateral equilibrium, the following pairs exchange: $\{1,6\}$, $\{2,3\}$ and $\{4,5\}$, and the allocation is Pareto efficient. Thus for these pairs, the equilibrium exchange ratios must be $\gamma_{16} = 1/2$, $\gamma_{32} = 1$ and $\gamma_{54} = 1$.

The optimality conditions in the bilateral equilibrium must ensure that peer 1 does not wish to download from peer 5 instead of peer 6, which implies that we must have $\gamma_{15} \leq \gamma_{16} = 1/2$. Similarly, we must have $\gamma_{53} \leq \gamma_{54} = 1$, and $\gamma_{31} \leq \gamma_{32} = 1$. Note that we thus have $\gamma_{15}\gamma_{53}\gamma_{31} \leq 1/2$. However, this implies there do not exist prices per peer p_i such that $\gamma_{ij} = p_i/p_j$, since such a price vector would imply $\gamma_{15}\gamma_{53}\gamma_{31} = 1$. Thus the bilateral equilibrium cannot be a multilateral equilibrium.

Given the equilibrium exchange ratios of Example 3, peers $\{1,3,5\}$ can benefit by deviating together. By choosing upload rates $r'_{13} = 1/3, r'_{35} = 1/4, r'_{51} = 1/5$, while reducing upload rates to their original trading partners accordingly, each peer in $\{1,3,5\}$ obtains a download rate strictly larger than 1 (the download rate each of these peers gets at the bilateral equilibrium). For example, user 1 obtains a total download rate of $1/5$ (from user 5) plus $5/6$ (from user 6, who in turn gets $r'_{16} = 5/3$), which results in a rate greater than 1.

Inspired by this observation, we show next that if a bilateral equilibrium is robust to deviations by a coalition of peers, and if each peer is only uploading one file, then it corresponds to a multilateral equilibrium. We formalize this result adapting the notion of the *core* [16] to bilateral exchange. An allocation has the core property with respect to given exchange ratios if no coalition of peers can strictly improve the utility of all its members by bartering with peers outside the coalition, subject to the given exchange ratios. Inside the coalition, peers do not need to follow the exchange ratios, and they may allocate rates in any way subject to bandwidth constraints.

Definition 3. *Given exchange ratios $\gamma = (\gamma_{ij}, i, j \in N)$, an allocation* r *is* feasible *for a set of peers S with respect to γ if:*

(i) $r_{ijf} = 0$ *if* $f \notin F_i$;
(ii) $\sum_{j,f} r_{ijf} \leq B_i$ *for all* $i \in S$;
(iii) $\sum_{i\in S}\sum_f r_{jif} = \sum_{i\in S}\gamma_{ij}\sum_f r_{ijf}$, *for all* $j \notin S$.

The first condition ensures that all peers only upload files they have. The second condition ensures that peers in S do not exceed their upload constraints. The third ensures that exchanges between the coalition S and each peer outside S take place at the given exchange ratios.

Definition 4. *Given fixed exchange ratios γ, a coalition S* blocks *an allocation* r^* *with respect to γ if there exists a feasible allocation* r *for S with respect to γ such that* $V_i(\sum_j r_{jif}, f \in T_i) > V_i(\sum_j r^*_{jif}, f \in T_i)$ *for all $i \in S$.*

Definition 5. *The allocation* r *has* the core property *with respect to exchange ratios γ if it can not be blocked by any coalition of peers.*

We note that the usual definition of the core in microeconomics [16] does not allow exchange with agents outside the coalition. Our definition of the core is distinct and more appropriate to model collusion in a bilateral exchange setting, as it depends on the exchange ratios.

We first show that the core property is satisfied by any multilateral equilibrium. This is a standard result from microeconomic theory [16]. However, our result is more general, since our core definition allows a coalition to exchange with peers outside the coalition, and thus there are more feasible allocations which may potentially block the multilateral equilibrium allocation.

Proposition 2. *Any multilateral equilibrium allocation has the core property with respect to the equilibrium exchange ratios $\gamma_{ij} = p_i/p_j$.*

We conclude that a bilateral equilibrium that does not have the core property can not correspond to a multilateral equilibrium. We next show that, when each peer is uploading one file, a bilateral equilibrium with the core property is a multilateral equilibrium. The insight is similar to Example 3: it can be shown that if no price vector exists such that $\gamma_{ij} = p_i/p_j$, then there must exist users i_1, i_2, ..., i_k such that $\prod_{i=1}^{k} \gamma_{i,(i \bmod k)+1} < 1$. In that case, there is a coalition of k peers that can block the allocation.

Proposition 3. *Suppose $|F_i| = 1$ for all $i \in N$. If a bilateral equilibrium allocation* r^* *with exchange ratios γ such that $\sum_j r^*_{jif} > 0$ for all $i \in N, f \in T_i$ has the core property, then it is also a multilateral equilibrium allocation.*

A corollary of Proposition 3 is that if a bilateral equilibrium has the core property, then its allocation is Pareto efficient. Proposition 3 requires that $|F_i| = 1$ for all $i \in N$. The result holds more generally if each peer is uploading a unique file at the bilateral equilibrium (peers may have more files). Whether it holds for the general case where peers upload multiple files at the bilateral equilibrium remains an open problem.

5 Conclusions

This paper analyzes the efficiency and robustness gains that are enabled by price-based multilateral exchange. We identify the condition that a bilateral equilibrium needs to satisfy in order to correspond to a multilateral equilibrium. These results help clarify the tradeoffs inherent in choosing between bilateral and multilateral exchanges: simplicity in the former, and efficiency and robustness gains in the latter.

Our novel theoretical results provide insight into the gap between bilateral and multilateral exchange. Even though the two exchanges are compared theoretically in terms of equilibria in this paper, it is possible to design a system that practically realizes the benefits of multilateral exchange. In particular, since it is hard to know the equilibrium prices in advance, peers can update their prices according to supply and demand. A system for currency-backed content exchange is presented in [15].

Bibliography

[1] Bangeman, E.: P2P responsible for as much as 90 percent of all 'Net traffic. ArsTechnica (Sep 3 2007)

[2] Adar, E., Huberman, B.: Free riding on Gnutella. First Monday **5**(10) (2000)

[3] Hughes, D., Coulson, G., Walkerdine, J.: Free riding on Gnutella revisited: The bell tolls? IEEE Distributed Systems Online **6**(6) (2005)

[4] Cohen, B.: Incentives build robustness in BitTorrent. In: Proc. Workshop on Economics of Peer-to-Peer Systems. (2003)

[5] Piatek, M., Isdal, T., Anderson, T., Krishnamurthy, A., Venkataramani, A.: Do incentives build robustness in BitTorrent? In: Proc. Symposium on Networked Systems Design and Implementation. (2007)

[6] Wu, F., Zhang, L.: Proportional response dynamics leads to market equilibrium. In: Proc. Symposium on Theory of Computing. (2007)

[7] Jun, S., Ahamad, M.: Incentives in BitTorrent induce free riding. In: Proc. Workshop on the Economics of Information Security. (2005)

[8] Locher, T., Moor, P., Schmid, S., Wattenhofer, R.: Free riding in BitTorrent is cheap. In: Proc. HotNets. (2006)

[9] Sirivianos, M., Park, J.H., Chen, R., Yang, X.: Free-riding in BitTorrent networks with the large view exploit. In: Proc. International Workshop on Peer-to-Peer Systems. (2007)

[10] Guo, L., Chen, S., Xiao, Z., Tan, E., Ding, X., Zhang, X.: Measurements, analysis, and modeling of BitTorrent-like systems. In: Proc. Internet Measurement Conference. (2005)

[11] Gupta, M., Judge, P., Ammar, M.: A reputation system for peer-to-peer networks. In: Proc. Workshop on Network and Operating System Support for Digital Audio and Video. (2003)

[12] Vishnumurthy, V., Chandrakumar, S., Sirer, E.G.: KARMA: A secure economic framework for P2P resource sharing. In: Proc. Workshop on the Economics of Information Security. (2003)

[13] Sirivianos, M., Park, J.H., Yang, X., Jarecki, S.: Dandelion: Cooperative content distribution with robust incentives. In: Proc. USENIX Technical Conference. (2007)

[14] Aperjis, C., Johari, R.: A peer-to-peer system as an exchange economy. In: Proc. GameNets. (2006)

[15] Aperjis, C., Freedman, M.J., Johari, R.: The role of prices in peer-assisted content distribution. Technical Report TR-814-08, Princeton University, Computer Science (2008)

[16] Mas-Colell, A., Whinston, M.D., Green, J.R.: Microeconomic Theory. Oxford University Press (1995)

A Proofs

Proof of Proposition 1: Substituting $\gamma_{ij} = p_i/p_j$ in the fourth set of constraints of the Bilateral Peer Optimization problem, we get the constraints $p_j \sum_f r_{jif} = p_i \sum_f r_{ijf}$ for all j. The Multilateral Peer Optimization problem has a larger feasible region; however, the two optimization problem have the same optimal value. In particular, if r is feasible for the Multilateral Optimization problem of peer i, then we can construct $\bar{\mathsf{r}}$ such that (i) $\bar{\mathsf{r}}$ is feasible for the Bilateral Optimization problem of peer i, and (ii) $\bar{\mathsf{r}}$ gives the same utility as r to user i. For example, we can achieve this by setting $\bar{r}_{jif} = r_{jif}$ for all j, f and then choosing $\bar{r}_{ijf}$ for each j such that $\sum_f \bar{r}_{ijf} = (p_j/p_i)\sum_f \bar{r}_{jif}$. This shows that an optimal solution for the Bilateral Peer Optimization problem is also optimal for the Multilateral Peer Optimization problem, and concludes the proof. ∎

Proof of Proposition 2: Consider a competitive equilibrium and suppose that there exists a coalition S that blocks it and let r be the corresponding rate allocation. Then by Definition 4 and the Multilateral Optimization problem, $\sum_{j,f} p_j \cdot r_{jif} > p_i \cdot B_i, \forall i \in S$. Summing over all $i \in S$, $\sum_{i\in S}\sum_{j\in S} p_j \cdot \sum_f r_{jif} + \sum_{i\in S}\sum_{j\notin S} p_j \cdot \sum_f r_{jif} > \sum_{i\in S} p_i \cdot B_i$. Since $\gamma_{ij} = p_i/p_j$, condition (iii) of Definition 3 becomes $p_j \cdot \sum_{i\in S}\sum_f r_{jif} = \sum_{i\in S} p_i \cdot \sum_f r_{ijf}$ for all $i \in S$, $j \notin S$. Combining the two conditions and rearranging, $\sum_{i\in S} p_i \sum_{j,f} r_{ijf} > \sum_{i\in S} p_i \cdot B_i$, which contradicts condition (ii) of Definition 3. ∎

Proof of Proposition 3: We first show that the exchange ratios in a bilateral equilibrium only depend on the files being exchanged, not on the peers' identities. Suppose that $f \in F_{i_1}$, $f \in F_{i_2}$, $g \in F_{j_1}$ and $g \in F_{j_2}$ and at the bilateral equilibrium, peers i_1 and j_1 exchange f and g, and i_2 and j_2 exchange f and g. Then,

$$\gamma_{i_1,j_1} \geq \gamma_{i_1,j_2} \ ; \ \gamma_{j_1,i_1} \geq \gamma_{j_1,i_2} \ ; \ \gamma_{i_2,j_2} \geq \gamma_{i_2,j_1} \ ; \ \gamma_{j_2,i_2} \geq \gamma_{j_2,i_1},$$

since i_1 exchanges with j_1 at equilibrium, not j_2; j_1 exchanges with i_1, not i_2; etc. Combining these inequalities with the fact $\gamma_{ij} = 1/\gamma_{ji}$, we conclude that $\gamma_{i_1,j_1} = \gamma_{i_1,j_2} = \gamma_{i_2,j_1} = \gamma_{i_2,j_2}$. When there is exchange between files f and g in a bilateral equilibrium we define γ_{fg} to be the unique value of the exchange ratio between files f and g.

We define the *bilateral equilibrium exchange graph* to have a node for every file, and an edge between two files if those files are exchanged at the bilateral equilibrium. Suppose that for every cycle $f_1, f_2, ..., f_k, f_1$ in the bilateral equilibrium exchange graph, we have $\prod_{i=1}^{k} \gamma_{f_i,f_{i+1}} = 1$.[1] Then we get a unique price for each file in the following way. We start from a file (say f_1) whose price we set equal to 1 and then take a minimum spanning tree of the graph. We move along the edges of this tree and set prices for other files, so that the exchange ratios are satisfied. In this way we get prices p_f for each file f. Since the optimization problem of peer i with $f \in F_i$ is not affected by γ_{fg} if $g \notin T_i$, we can derive prices per peer by setting $p_i = p_f$ for $f \in F_i$ which do not change the optimal rate allocations for peers. Thus, by Proposition 1, the bilateral equilibrium allocation is also a multilateral equilibrium allocation.

To complete the proof, it suffices to show that if there is a cycle $f_1, f_2, ..., f_k, f_1$ in the bilateral equilibrium exchange graph with $\prod_{i=1}^{k} \gamma_{f_i,f_{i+1}} < 1$, then there is a coalition that blocks the bilateral equilibrium allocation. Let i be some peer that uploads file f_i, and downloads file f_{i+1} at the bilateral equilibrium. It can be shown that the coalition $S = \{1, ..., k\}$ blocks the bilateral equilibrium allocation by demonstrating that there is a way to increase the rates $r_{i,i-1,f_i}$ for $i = 1, ..., k$ so that the utilities of all peers in S strictly increase. Due to space limitations, here we only sketch the proof for the case that $f_{i-1} \notin T_i$ for all $i \in S$. We provide the complete proof in [15].

If $f_{i-1} \notin T_i$ for all $i \in S$, then at the bilateral equilibrium $r^*_{i,i-1,f_i} = 0$ for all $i \in S$. By sending $r_{i,i-1,f_i}$ to peer $i-1$, peer i reduces the rate he gets from outside S by $\gamma_{f_i,f_{i+1}} \cdot r_{i,i-1,f_i}$. So, the coalition increases i's utility if and only if the rate he receives from S, *i.e.*, $r_{i+1,i,f_{i+1}}$, is greater than $\gamma_{f_i,f_{i+1}} \cdot r_{i,i-1,f_i}$. To show that S blocks r^*, it suffices to find $r_{i+1,i,f_{i+1}} \leq B_{i+1}$, such that $r_{i+1,i,f_{i+1}} > \gamma_{f_i,f_{i+1}} \cdot r_{i,i-1,f_i}$, for all $i \in S$. This is possible because $\prod_i \gamma_{f_i,f_{i+1}} < 1$. In particular, we can choose small $\delta, \varepsilon > 0$, and set: $r_{1,k,f_1} = \delta$, $r_{i+1,i,f_{i+1}} = \gamma_{f_i,f_{i+1}} r_{i,i-1,f_i} + \varepsilon$, for all $i \in S$. ∎

---

[1] We denote by f_{i+1} and f_{i-1} the files after and before file f_i with respect to the cycle $f_1, f_2, ..., f_k, f_1$.