Reprinted from THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA, Vol. 47, No. 2, (Part 2), 665–666, February 1970

Input Generators for Digital Sound Synthesis

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A fast method is described for generating a periodic discrete-time signal with harmonics of equal amplitude and a fundamental frequency which is not necessarily an integral fraction of the sampling frequency (as with ordinary pulse generators). Such a signal can be used as input to digital filters for the synthesis of speech and music.

IN USING DIGITAL FILTERS TO GENERATE SOUND, IT IS IMPORTANT to have suitably general periodic input sources. This means that (1) they must be able to contain all possible harmonics (since an absent harmonic is necessarily absent in the output); (2) they should have a reasonably simple spectrum, so as to produce a reasonably simple relation between the system transfer function and the spectral envelope of the output; (3) it should be possible to vary the fundamental frequency of the input from sample to sample without affecting the above properties and without extensive recalculation of parameters.

The input sources commonly used are actually not very satisfactory by these criteria. Thus, pulse generators[1] satisfy (1) and are excellent for (2), since the relation concerned is simply identity; but since the pulses must occur at integral multiples of the sampling interval, the only frequencies obtainable are the set $2\pi/n$ radians, where $n=2, 3, \cdots$. Another well known method[2] is to have one cycle of a suitable input stored in a table of size S and to sample this table (mod S) with an increment $\omega S/2\pi$, where ω is the desired fundamental frequency in radians. In this case, all possible frequencies are obtainable (with some noise due to truncation of the increment, in an amount inversely proportional to S). However, in order to avoid foldover error while still obtaining all possible harmonics, whenever the frequency is changed, the whole table has to be recalculated to add higher harmonics or delete ones which have become too high. Hence, with this method, sample-to-sample frequency variation is still impractical.

These difficulties can be overcome by the following procedure: Starting from the identity

$$1+2(\cos x+\cos 2x+\cdots+\cos Nx)=\frac{\sin[(2N+1)(x/2)]}{\sin(x/2)}, \quad (1)$$

we let $x=k\omega$, where ω is the desired fundamental frequency in radians and k is the sample number, eliminate the zero-frequency component, and scale to a peak amplitude of 1, thus:

$$\frac{1}{N}(\cos k\omega+\cos 2k\omega+\cdots+\cos Nk\omega)$$
$$=\frac{1}{2N}\left(\frac{\sin[(2N+1)(k\omega/2)]}{\sin(k\omega/2)}-1\right). \quad (2)$$

The successive values of this for $k=0, 1, 2, \cdots$ can then be found by sampling a sine table (mod S) with increment

$$\frac{(\omega/2)S}{2\pi}$$

for the denominator, and multiplying (mod S) the table location by $2N+1$ to locate the numerator value in the same table. When the denominator is 0, the desired result is simply 1.

To obtain all possible harmonics, even with a varying frequency, it is only necessary to maintain the relation

$$N=\text{INTEGER PART OF } (\pi/\omega); \quad (3)$$

but an additional benefit is that N can be set to any other (smaller) value as desired.

Acknowledgments: This work was supported by a National Science Foundation grant and by the United States Army Research Office, Durham, N. C., under contract.

[1] See, for example, L. R. Rabiner, "Digital Formant Synthesizer for Speech-Synthesis Studies," J. Acoust. Soc. Amer. 43, 822–828 (1968).
[2] M. V. Mathews, "An Acoustic Compiler for Music and Psychological Stimuli," Bell System Tech. J. 40, 677–694 (1961).