Chapter 1
Soliton-Guided Quantum Information Processing

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Abstract We describe applications of solitons and soliton collisions to the transport, transfer, and beam-splitting of qubits carried by optical photons. The transport and transfer realize the "flying qubits" necessary for quantum information processing, and the beam-splitting leads, in theory, to an implementation of quantum computing using linear optics. These proposed applications are embedded in a uniform optical fiber and require no special device fabrication, no cooling, and no vacuum.

The pioneering papers of Feynman [1] and Deutsch [2] in the 1980s sparked the rapid development of the field of quantum information processing. The theoretical and experimental progress has been remarkable, with the development, for example, of quantum error correction and a fast algorithm for factoring, and the exploration of a wide variety of physical implementations. In the latter category, the optical photon as the carrier of a qubit has played an important role in the experimental demonstration of quantum cryptography and other important applications to communications and information processing. At the same time there has been tremendous progress in our understanding of classical nonlinear waves, and, in particular, solitons in optical fibers. In this chapter we will explore what role solitons and soliton collisions might play in the development of quantum information processing with optical photons. For a more detailed account, the reader is referred to [3, 4, 5], from which the material in this chapter was drawn.

1.1 Photon trapping

A pulse traveling down a fiber forms a soliton when the dispersion, which tends to widen the pulse, is counterbalanced by the nonlinear Kerr effect, whereby the electric field changes the index of refraction of the material. Such a soliton is called a *temporal* soliton, as opposed to a spatial soliton, where a beam is confined spatially when diffraction is counterbalanced by a nonlinear effect in the material. In this chapter we will restrict our attention to temporal solitons. Hasegawa and Tappert predicted that stable optical solitons will form in a fiber in 1973 [6], and they were observed experimentally in 1980 by Mollenauer, Stolen, and Gordon [7]. Since then, because of important potential applications to communications, there has been intense activity in both the theoretical and experimental aspects of solitons in optical fibers.

The induced waveguide   What is important to us here is the fact the soliton creates a local distortion of the index of refraction that travels with it down the fiber. This moving distortion can act as a waveguide that can trap and shepherd another, much weaker, light pulse that can differ from the soliton in both frequency and polarization. The strong soliton pulse is called the *pump*, denoted by P, and the weak, shepherded, pulse is called the *probe*, denoted by u. We will follow the model of such a pump/probe system that was laid out by Manassah [8]. It consists of two coupled equations: the first is the standard, integrable cubic nonlinear Schrödinger equation (3-NLS) that describes the formation of the pump; the second, which describes the propagation of the probe, is, in fact, precisely the *linear* Schrödinger wave equation with a potential determined by the pump.

   The solution for the pump is the well known soliton solution, a complex wave with a carrier and sech-shaped envelope. The relative phase of two of these solitons on collision determines the nature of the collision. In particular, when the relative phase is π, the collision is repulsive, and the induced waveguide will look like a smoothly bent waveguide. We will be using collisions of this type throughout.

   The solution for the probe is, as we might expect, an eigenvalue problem, which we solve by separation of variables, using as ansatz the complex wave

$$u(z,t) = u(t)e^{-iEz} , \qquad (1.1)$$

where z is distance along the fiber, and t is time in the frame moving with the pump soliton, which we refer to as *local time*. In the z direction it is simply a phasor of constant intensity. In the t direction, which we can think of in the *z-t* plane as the lateral direction in the induced waveguide, the probe is more interesting. The reduced equation with independent variable t is the associated Legendre equation, with solutions $u_{\ell m}$ of degree ℓ and order m that are non-singular, physically acceptable, and zero at infinity for integers $\ell \geq m > 0$. Letting $\xi = \tanh(k_R t)$, where k_R is a parameter that determines the energy of the soliton, each $u_{\ell m}$ is the product of $(1-\xi^2)^{m/2}$ and a polynomial in ξ of degree $(\ell - m)$ and parity $(-)^{\ell - m}$, with $(\ell - m)$ zeros in the interval $-1 \leq \xi \leq +1$ [9, 10]. As functions of t the solutions of the re-

duced equation take the form $\operatorname{sech}^m(k_R t)$ times a polynomial in $\tanh(k_R t)$ of degree $(\ell - m)$. The degree ℓ of the wave functions supported in the induced waveguide is determined solely by the ratio of corresponding parameters in the pump and probe equations, and is therefore fixed for any given physical fiber implementation.

Assume, then, that the given fiber implementation is such that ℓ and m are integers, and denote the probe solution of degree ℓ and order m by $|\ell m\rangle$. Then there are exactly ℓ eigenfunctions supported by the induced waveguide, corresponding to $m = 1, \ldots, \ell$, with corresponding energy eigenvalues $E_1, \ldots, E_\ell$. When the superposition of more than one of these co-propagate (the reduced equation is linear) the difference in these energy levels causes beating in the z-direction (see Eq. 1.1), as discussed in [8].

The quantum limit   Up to now we have described the formation of an electromagnetic probe wave trapped in the waveguide induced by a soliton, where this probe is weak compared to the soliton. If we let the probe get weaker and weaker we reach the point at which the probe can no longer behave like a wave, but must behave like a particle—a photon.

We next must consider the critical question of whether it is possible to detect a probe photon in the presence of the (much larger) pump. There are two ways in which we can separate the probe and pump to make this detection feasible: First, they can be orthogonally polarized in a polarization-maintaining fiber. Second, they can be separated in wavelength. As discussed in more detail in [3], it is reasonable to expect the detection of single probe photons in the collisions described here to be possible at a wavelength of 1550 nm within about one or two kilometers of fiber. This experiment would be the next step in pursuing a physical demonstration of the ideas discussed in this chapter.

1.2 Photon transfer

distance along fiber
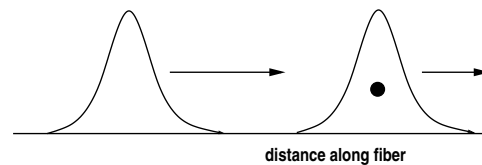
Fig. 1.1 Sketch illustrating the conditions for photon transfer when a fast soliton overtakes a slower one.

We look next at the simple situation where a faster soliton which is not carrying a photon overtakes a slower soliton that is, as sketched in fig. 1.1. What happens, with appropriate choice of parameters, is that the photon will be transferred to the faster soliton, as shown by the numerical simulation illustrated in fig. 1.2. The repulsive
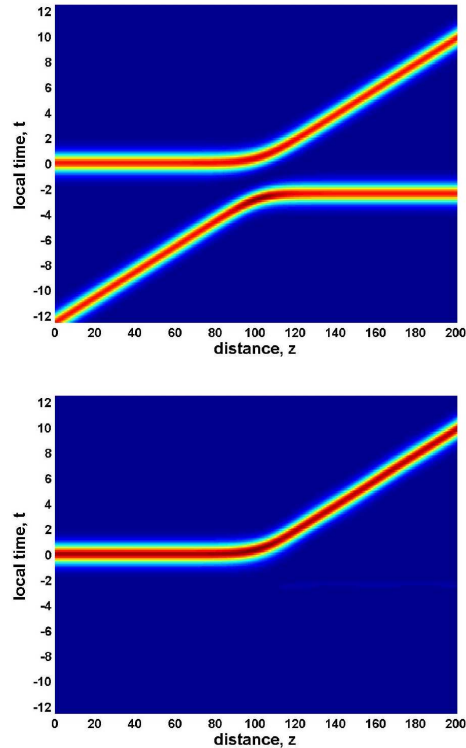
Fig. 1.2 (Color online) An example of photon transfer. *Top*: The pump solitons in a repulsive collision. *Bottom*: The probe when launched in the state $|11\rangle$. The photon is transferred to the overtaking soliton. (See [3] for details.)

collision of the two pump solitons is shown at the top. At the bottom we see the deflection of the probe wave (now a photon) to the faster soliton. The probe in this case is in the single-peaked ground state $|11\rangle$. This setup and the ones described in the following sections correspond exactly to what is known for classical waves as a *directional coupler*, and such couplers induced by spatial solitons have been studied since the 1990s [11, 12, 13].

In some sense the experiment we have just described is analogous to a photon bouncing off a mirror: the photon is simply diverted. If the photon carries a qubit of quantum information in a photon/no-photon representation, it is a flying qubit in the sense described by DiVincenzo [14] in his well known paper outlining the requirements for a physical implementation of quantum computing and communication. The present fiber scheme would then provide a means for routing flying qubits with solitons.

1.3 Beam splitters

If we change the system parameters we can arrange an experiment that corresponds, not to an ordinary mirror, but to a half-silvered mirror—a beam-splitter. For this we use a collision between pump solitons of order 2 instead of 1, and a greater relative velocity, as shown in fig. 1.3.
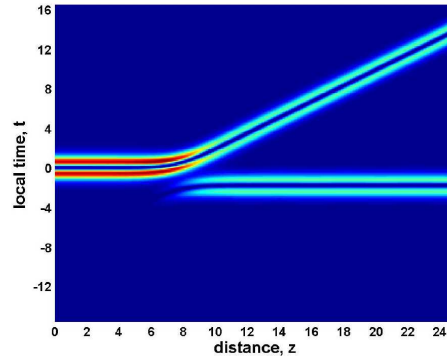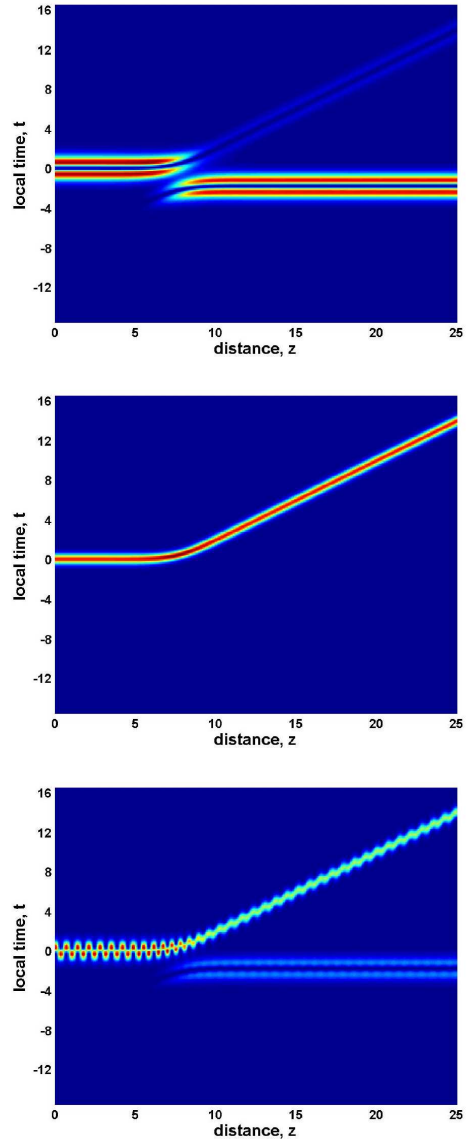

Fig. 1.3 (Color online) The probe when launched in the state $|21\rangle$. In this case the soliton collision takes place at a greater relative velocity and the system acts as an ordinary non-polarizing beam-splitter. (See [3] for details.)

This is analogous to a non-polarizing beam-splitter. The photon is deflected or transmitted with certain probabilities, in the case shown, both 1/2 for a 50/50 beam-splitter. For a system that functions as a polarizing beam-splitter, we can use a probe in a state that is the superposition of states $|22\rangle$ and $|21\rangle$. It turns out that with the proper choice of parameters, the photon in state $|21\rangle$ is not deflected, while the photon in state $|22\rangle$ is transferred. When the superposition of the two photon states is used as the input probe, the modes are separated, just as an ordinary polarizing filter will separate the horizontal and vertical components of a light wave of mixed polarization. An example of a polarizing beam-splitter is shown in fig. 1.4.

Note that in this soliton-induced beam-splitter, the modes of the probe play the role of polarization axes, and these axes should not be confused with the polarization modes of the fiber medium itself. It thus might be more proper to call this system a "mode-separating" beam-splitter.

1.4 Manipulating photon phase

There is a missing piece if we want to accomplish general quantum computing, as we shall see in the next section—we must be able to shift the phase of a single pho-

Fig. 1.4 (Color online) Illustrating a soliton-guided polarizing (mode-separating) beam-splitter. *Top*: The probe when launched in the excited state $|21\rangle$. The photon in this case stays in large part with its original captor soliton. *Center*: The probe when launched in the state $|22\rangle$. The photon is transferred to the faster soliton, as in the $|11\rangle$ case shown in Fig. 1.2. *Bottom*: The probe when an equal linear combination of ground and excited states, $|22\rangle + |21\rangle$, is launched. The system is analogous to a polarizing beam-splitter. (See [3] for details.)
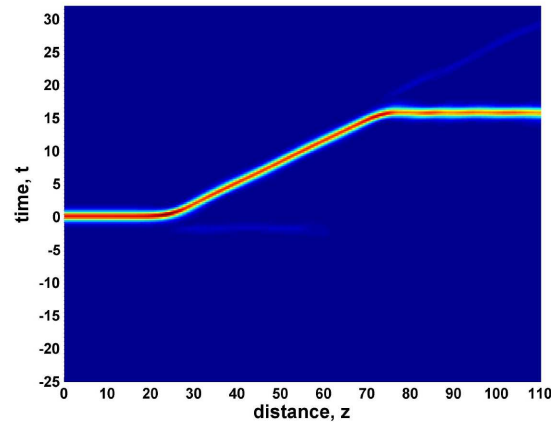
Fig. 1.5 (Color online) The probe signal in a phase shifter. A photon is being carried by soliton A; The faster soliton C overtakes soliton A, picks up the photon, and ferries it to soliton B, where it is deposited. In this example from [4] the overall phase shift achieved is π. (See [4] for details.)

ton. But the same ideas used in the previous sections can be used for this purpose. A phase shifter can work as follows: Two solitons, A and B, are launched at the same velocity in the z direction, first B, then A. Initially, soliton A carries a photon. Soliton C, a third soliton, is then launched at a greater velocity. When C overtakes A, the photon is captured by C; and C carries the photon with it until it overtakes B, at which point the photon is transferred from C to B. The net effect is that soliton C ferries the photon from A to B. The photon accumulates extra phase during the time it travels at an altered velocity, and the amount of the phase shift can be controlled by adjusting that time. Figure 1.5 shows the probe in an example. The reader is referred to [4] for details about how the solitons and probe are designed to accomplish this phase shifting.

1.5 General quantum computing

In 2001 Knill, Laflamme, and G. Milburn [15] published a surprising and important paper: they showed that general quantum computing can be implemented using only components we have described—beam-splitters, phase shifters—plus single-photon sources and photo-detectors. The latter plays a crucial role in providing the necessary nonlinear aspect to the system: feedback from photo-detectors. The reader is referred to [15] for the details of this clever scheme, and to [16] and its references for recent improvements.

We have thus described a way in which soliton-guided photons can be used to implement general quantum computing, at least in theory. We should quickly point

out, however, that the scheme proposed by Knill et al. carries with it an overhead that may be, although polynomial in the problem size as required by the theory, prohibitively large. This is balanced, however, by the relative simplicity of the physical components, most of which are either off-the-shelf or close to it. In addition, the use of soliton-guided flying qubits provides a natural and uniform way to implement the required routing and switching.

1.6 Using dark solitons
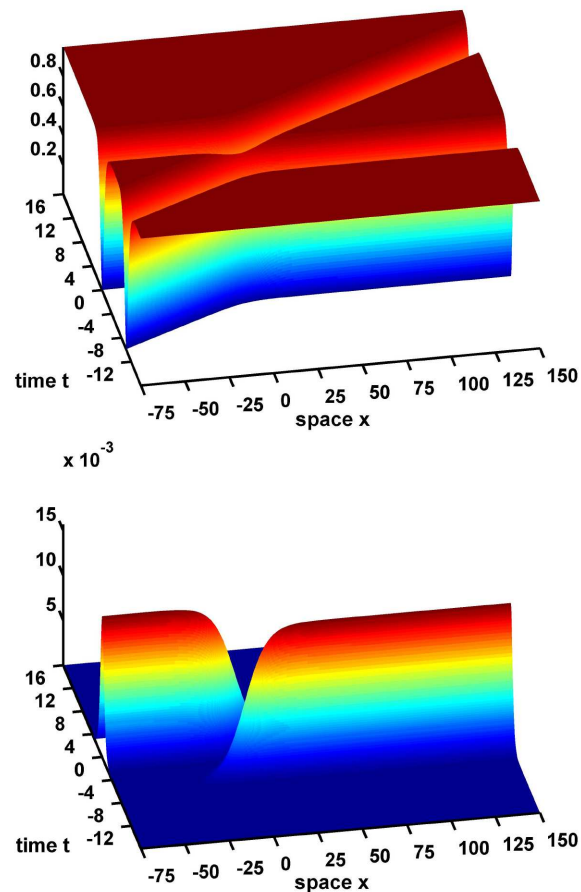
Fig. 1.6 (Color online) *Top*: The pump signal, a dark soliton collision. *Bottom*: The corresponding probe signal illustrating the case when the probe is not affected, the so-called "zero-crosstalk" case [11, 17]. (See [5] for details.)
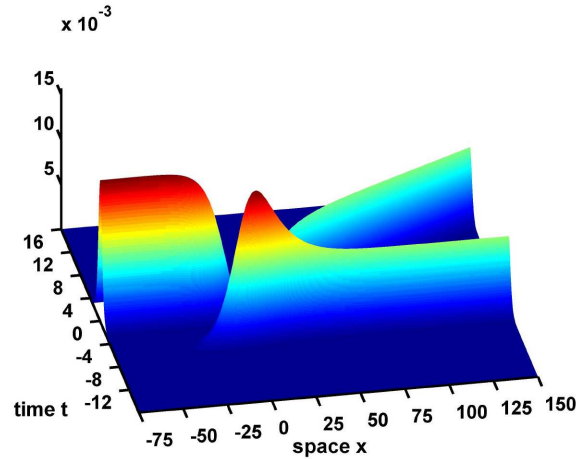
Fig. 1.7 (Color online) The probe signal in the same induced waveguide with the probe parameters adjusted so that probe is split equally. (See [5] for details.)

Dark solitons occur in the normal-dispersion regime of a fiber [18], and occur as dips in a uniform background, in contrast with the bright solitons we have so far been considering. They offer some real advantages over bright soliton collisions in controlling light waves: First, dark solitons are known to be more stable in the presence of noise and are generally more robust than bright solitons [19, 18]. Second, the probe, which is of much lower intensity, peaks at the dip in the intensity of its host soliton, thus increasing the signal-to-noise ratio and making it easier, in principle, to detect. Third, the characteristics of the dark soliton beam splitter do not depend on the relative phase or relative speed of the colliding solitons, whereas bright solitons need to have their phases and speeds carefully controlled to produce a given result. The improvement in signal-to-noise ratio for detecting single photons may prove to be especially important in any practical implementation.

Figure 1.6 shows collisions of two dark solitons, and also illustrates the case when the probe photon is unaffected by the collision, but simply remains with its captor. The waveguides induced by dark solitons can be used to control photon probes in the same way that bright solitons can, in contrast with the zero-crosstalk case reported in [17], provided that the group velocity dispersion and nonlinear coupling parameter for the fiber are chosen appropriately. These degrees of freedom are readily available if we use different wavelengths and polarizations for the pump and probe. For the probes corresponding to the degree-1 associated Legendre modal functions, the dark soliton junctions behave in a way that is very closely analogous to a beam-splitter made of crossed optical polarizers, with a single parameter playing the role of angle between polarizing filters. For the probes corresponding to the degree-2 associated Legendre functions, the junction can act as a mode-separating beam-splitter. Figure 1.7 illustrates such a dark-soliton-guided beam-splitter.

1.7 Conclusion and open problems

We have seen how, in theory (and that is a big qualification), solitons in optical fibers can be used to provide a kind of "substrate" for manipulating qubits. Transport, transfer (and therefore routing), and even general quantum computing, using the scheme of Knill et al. [15], all fit naturally in this picture. At the least, this way of implementing flying qubits may prove of practical use in many quantum communication and cryptographic systems.

Open questions remain concerning the practicality of physical implementation: are fibers, photon sources, and photon detectors available that have the required physical characteristics? Perhaps the most logical next step, and a project of interest in itself, would be the experimental verification of photon capture and transport in an optical fiber, by both bright and dark solitons.

Also of interest are the questions, both theoretical and experimental, of the susceptibility of trapped photons to decoherence, as compared with that of ordinary photons in fibers—a problem that, to the author's knowledge, not been studied. It would also be very interesting if soliton-guided photons could be used to realize quantum repeaters.

1.8 Acknowledgments

References

1. R. P. Feynman. Simulating physics with computers. *Int. J. Theor. Phys.*, 21(6/7):467–488, 1982.
2. D. Deutsch. Quantum theory, the Church-Turing principle and the universal quantum computer. *Proc. Royal Soc. London*, A400:97–117, 1985.
3. K. Steiglitz and D. Rand. Photon trapping and transfer with solitons. *Phys. Rev. A*, 79:021802(R), February 2009.
4. K. Steiglitz. Soliton-guided phase shifter and beam splitter. *Phys. Rev. A*, 81:033835, March 2010.
5. K. Steiglitz. Making beam splitters with dark soliton collisions. *Phys. Rev. A*, 82:043831, October 2010.
6. A. Hasegawa and F. Tappert. Transmission of stationary nonlinear optical physics in dispersive dielectric fibers I: Anomalous dispersion. *Appl. Phys. Lett.*, 23(3):142–144, 1973.
7. L. F. Mollenauer, R. H. Stolen, and J. P. Gordon. Experimental observation of picosecond pulse narrowing and solitons in optical fibers. *Phys. Rev. Lett.*, 45, 1980.
8. J. T. Manassah. Ultrafast solitary waves sustained through induced phase modulation by a copropagating pump. *Opt. Lett.*, 15(12):670–672, June 1990.

9. A. Messiah. *Quantum Mechanics*. North-Holland, Amsterdam, first edition, 1961.
10. L. I. Schiff. *Quantum Mechanics*. McGraw-Hill, New York, third edition, 1968.
11. N. Akhmediev and A. Ankiewicz. Spatial soliton X-junctions and couplers. *Optics Communications*, 100:186–192, 1993.
12. S. Lan, E. DelRe, Z. Chen, M.-F. Shih, and M. Segev. Directional coupler with soliton-induced waveguides. *Opt. Lett.*, 24:475–477, 1999.
13. A. Guo, M. Henry, G. J. Salamo, M. Segev, and G. L. Wood. Fixing multiple waveguides induced by photorefractive solitons: directional couplers and beam splitters. *Optics Letters*, 26:1274–1276, 2001.
14. D. P. DiVincenzo. The physical implementation of quantum computation. *Fort. der Phys.*, 48:771–783, October 2000. http://arxiv.org/pdf/quant-ph/0002077v3.pdf.
15. E. Knill, R. Laflamme, and G. Milburn. A scheme for efficient quantum computation with linear optics. *Nature*, 409(46):46–52, 2001.
16. D. C. Marinescu and G. M. Marinescu. *Classical and quantum information*. Academic Press, New York, NY, 2012.
17. P. D. Miller. Zero-crosstalk junctions made from dark solitons. *Phys. Rev. E*, 53(4):4137–4142, Apr 1996.
18. G. P. Agrawal. *Nonlinear Fiber Optics*. Academic Press, London, UK, fourth edition, 2006.
19. B. Luther-Davies and Y. Xiaoping. Waveguides and Y junctions formed in bulk media by using dark spatial solitons. *Optics Letters*, 17(7):496–498, April 1992.

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