

# Signal Standardization in Collision-based Soliton Computing

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Signal standardization is a key requirement for robust computation. In this paper, we describe a method of achieving this in computing with vector solitons. The state-restoring nature of the construction provides noise immunity and prevents accumulation of errors.

*Keywords:* solitons, collision-based computing, logical gates

## 1. INTRODUCTION

The development of any new computing implementation forces us to consider what key characteristics govern its successful operation. From relays and vacuum tubes to quantum systems and DNA, many physical realizations for computation have been proposed. But what parameters, for example, account for the ubiquity of the solid-state transistor in today's computing devices?

The requirements for computation include cascadability, fanout, and Boolean completeness. The first, cascadability, requires that the output of one device can serve as input to another. Since any useful computation consists of many stages of logic, this condition is essential. The second, fanout, refers to the ability of a logic gate to drive at least two similar gates. Finally, Boolean completeness makes it possible to perform arbitrary computation.

Many physical realizations have been devised which meet these requirements in an ideal world, in which the system is fully isolated from

the environment. The physical realization of a computer, however, must also operate effectively in the presence of noise, which may cause errors. The success of transistor-based logic depends critically on its noise immunity, which is achieved with *state restoration*, meaning that the signal level is regenerated and standardized at the output of each gate. Any accumulation of signal alterations per logic level would produce an error and ultimately destroy the computation. As a result, we must develop ways to combat such a deterioration process. With logic-level restoration, a range of values can be accepted which represent a binary ‘1’ or ‘0’ at the input, and the output is restored to a standardized ‘1’ or ‘0’ level. These ranges of input signals over which the correct outputs are obtained are known as *noise margins*, which provide tolerance in the amounts of variability on the inputs and a means through which to achieve practical cascadability. The ideas presented here on the physical requirements necessary for a computing device were well understood fifty years ago, and were discussed, for example, by Lo as early as 1961 [17].

Even with state restoration, can we compute effectively with noise-prone logic gates? In 1952, von Neumann suggested improving the reliability of computational schemes by executing each operation multiple times, and using majority voting on the output [34]. These architectures require redundancy, which is accompanied by an increase in the resources required, either in space or time, to perform the computation. A computing scheme such as this, where unreliability is compensated for by redundancy, is called *fault tolerant*. This term implies that we can engineer a system to function in a noisy environment. That is, even though logic devices are used which are vulnerable to noise-induced errors, they can still be used for computation.

The recently proposed computing paradigm of quantum computing has been designed to use logic gates that are susceptible to noise. A fault tolerant scheme was developed to show that robust computation could be performed in such a system. It involves encoding input data using error correcting codes, executing the computation on the encoded data, and performing error correction on the output [26]. Error correction makes data, either classical bits or quantum bits, more resilient against the effects of noise. The data may appear corrupted, but can be recovered through decoding. Like von Neumann’s approach, this scheme adds redundancy through the encoding of the error correcting codes, which increases the number of bits. In this way, even if some of the message is destroyed by noise, there is enough redundancy to recover the data. For a review of fault tolerance in quantum computation, see [21].

## 1.1 Physical vs. logical state restoration

At this point, we should emphasize the difference between *physical* and *logical* state restoration. In *physical* state-restoring systems such as solid-state transistors or vacuum tubes, sufficient noise margins allow for variability at the input. Such systems are inherently cascadable, and no redundancy is required. The output is standardized to the proper logic level, provided that the input lies within the specified margin. In *logical* state-restoring systems, redundancy is used to reconstruct the correct output from error-prone data. Logic level standardization, and thus cascadability, is achieved, for example, by a scheme describing majority voting or error correction. The probability that errors occur with logical state restoration remains small if errors are corrected frequently and the system contains sufficient redundancy. It should be noted that a physical state-restoring system in the framework of quantum computation has been proposed recently [16].

An additional, more subtle issue characteristic of logical state-restoring systems involves the inherent unreliability of the error-correcting process. Assuming the hardware with which both computation and error correction are performed is the same, errors will unavoidably occur in both processes. In this respect, in order for such systems to operate with suitable fidelity, the system must be designed to account for both sources of error.

## 1.2 All-optical soliton computing

As data rates in optical communication systems continue to increase, the demand for all-optical signal processing and computing devices does as well. Examples of such devices include the nonlinear optical loop mirror [11], the temporal soliton dragging gate [27], the spatial soliton deflection gate [7], and the TOAD, an asymmetric loop mirror [28]. These devices avoid the bottleneck associated with optical-electrical conversion. For a recent discussion of the requirements necessary for such all-optical devices, see [8].

In this paper, we describe physical state-restoring computation using collisions of optical solitons. This work is part of a larger subject known as *collision-based* computing, sometimes called dynamical computation (for a review, see [2]). Such constructions include ideal collisions of billiard balls [12], Conway's universal game of Life [6], and multidimensional excitable lattices [1]. Early work on soliton



FIGURE 1  
Possible soliton computing device in which spatial solitons interact in a nonlinear medium.

computation involved soliton-like collisions in cellular automata, which demonstrated the ability to embed computation in automata using particles [14,20,33].

A schematic for a possible soliton computing device is shown in Fig. 1. Input solitons enter on the left and exit on the right, representing the result of an arbitrary computation. The computation occurs through collisions, and the configuration of input beams determines the operation performed. This results in a dynamic computer without spatially fixed gates or wires, which is unlike most present-day conceptions of a computer that involve integrated circuits, in which information travels between logical elements that are fixed spatially through fabrication on a silicon wafer. We can call such a scheme “nonlithographic,” in the sense that there is no architecture imprinted on the medium.

We concentrate on solitons described by the cubic nonlinear Schrödinger equation (NLSE), which takes the common dimensionless form

$$i \frac{\partial u}{\partial t} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0 \quad (1)$$

where  $u(x, t)$  is the soliton envelope. Equation (1), known as the *scalar* NLSE, is an integrable system — that is, it can be solved analytically, and collisions between solitons are elastic. In 1971, Zakharov and Shabat first solved this equation analytically using the inverse scattering method [35]. The scalar soliton solutions of Eq. (1) describe temporal solitons in fiber and spatial solitons in various lossless nonlinear media (for a recent review of spatial solitons and their interactions, see [30]). Because of the integrability of Eq. (1), solitons emerge from collisions with all their original energy. Therefore, no spurious radiation emerges from the

collision site, as is characteristic of nonintegrable systems. For this reason, we will focus on integrable systems in this paper.

In order to study soliton interactions in the context of computation, we identify soliton parameters that can act as state variables, which can then be used to carry and transfer information. For scalar solitons, the only appropriate parameters are the carrier phase and position, because these are the only variables affected by collision. The changes in these parameters, however, do not have an effect on the results of subsequent collisions. From the point of view of universal computation, this is an unfortunate result, since solitons must be able to transfer state information in order to be useful in arbitrary computation [14].

Despite this setback, it was soon discovered that a system similar to the NLSE, the Manakov system [18], possesses very rich collisional properties [22] and is integrable as well. Manakov solitons are a specific instance of two-component vector solitons, and it has been shown that collisions of Manakov solitons are capable of transferring information via changes in a complex-valued polarization state [13].

This paper is organized as follows. In Section 2 we describe the mathematical model of the Manakov soliton system. Section 3 discusses how it is possible to create a bistable configuration of such solitons, followed by Section 4, in which we describe a FANOUT gate and a physical state-restoring NAND gate using the bistable soliton collision cycles of Section 3. In Sections 5 and 6, we conclude with a discussion of experimental progress and prospects for future work.

## 2. THE MANAKOV MODEL

The Manakov system consists of two coupled NLSEs [18]:

$$\begin{aligned} i \frac{\partial q_1}{\partial t} + \frac{\partial^2 q_1}{\partial x^2} + 2\mu(|q_1|^2 + |q_2|^2)q_1 &= 0 \\ i \frac{\partial q_2}{\partial t} + \frac{\partial^2 q_2}{\partial x^2} + 2\mu(|q_1|^2 + |q_2|^2)q_2 &= 0 \end{aligned} \quad (2)$$

where  $q_1(x, t)$  and  $q_2(x, t)$  are two interacting optical components,  $\mu$  is a positive parameter representing the strength of the nonlinearity, and  $x$  and  $t$  are normalized space and time, respectively. The two components can be thought of as components in two polarizations, or, as in the case of a photorefractive crystal, two uncorrelated beams [10].

Manakov first solved Eq. (2) by the method of inverse scattering [18]. The system admits single-soliton, two-component solutions that can be characterized by a complex number  $k \equiv k_R + ik_I$ , where  $k_R$  represents the energy of the soliton and  $k_I$  the velocity, all in normalized units. The additional soliton parameter is the complex-valued polarization state  $\rho \equiv q_1/q_2$ , defined as the ratio between the  $q_1$  and  $q_2$  components.

Radhakrishnan *et al.* [22] derived a general two-soliton solution for Eq. (2) and analyzed its asymptotic behavior to explain the collision properties of Manakov solitons. The results showed that collisions are characterized not only by a phase and position shift (similar to scalar soliton collisions), but also an intensity redistribution among the two component fields  $q_1$  and  $q_2$ .

Figure 2 shows the schematic for a general two-soliton collision, with initial parameters  $\rho_1, k_1$  and  $\rho_L, k_2$ , corresponding to the right-moving and left-moving solitons, respectively. The values of  $k_1$  and  $k_2$  remain constant during collision, but the polarization state changes. Let  $\rho_1$  and  $\rho_L$  denote the respective soliton states before impact, and suppose the collision transforms  $\rho_1$  into  $\rho_R$ , and  $\rho_L$  into  $\rho_2$ . It turns out that the state change undergone by each colliding soliton takes on the very simple form of a linear fractional transformation (also called a bilinear or Möbius transformation). Explicitly, the state of the emerging left-moving soliton is given by [13]:

$$\rho_2 = \frac{[(1-g)/\rho_1^* + \rho_L]\rho_L + g\rho_1/\rho_1^*}{g\rho_L + (1-g)\rho_1 + 1/\rho_1^*}, \quad (3)$$

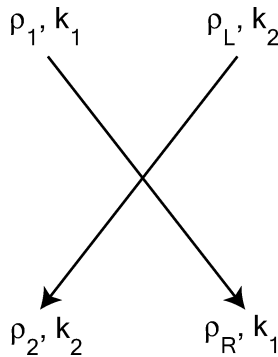


FIGURE 2  
Schematic of a general two-soliton collision. Each soliton is characterized by a complex-valued polarization state  $\rho$  and complex parameter  $k$ .

where

$$g \equiv \frac{k_1 + k_1^*}{k_2 + k_1^*}.$$

The state of the right-moving soliton is obtained similarly, such that

$$\rho_R = \frac{[(1-h^*)/\rho_L^* + \rho_L]\rho_1 + h^* \rho_L / \rho_L^*}{h^* \rho_1 + (1-h^*) \rho_L + 1/\rho_L^*}, \quad (4)$$

where

$$h \equiv \frac{k_2 + k_2^*}{k_1 + k_2^*}.$$

We assume here, without loss of generality, that  $k_{1R}, k_{2R} > 0$ .

These state transformations were first used by Jakubowski *et al.* [13] to describe logical operations such as NOT. Later, it was established in [31] that arbitrary computation was possible through time gating of Manakov (1+1)-dimensional spatial solitons.

The computational architecture of [31] incorporates several notable characteristics. First, it is reversible, in that the system of Eq. (2) is based on physical laws independent of the direction of time. This property does not limit the system's computational power, but would require storage of all outputs for the computation to be run in reverse [5].

The second feature is the nonlithographic nature of the physical realization, and is characteristic of many collision-based computational paradigms. In [31], information is processed through interactions of soliton beams in a nonlinear medium. As described earlier in connection with Fig. 1, this presents an alternative to the conventional, lithographically defined silicon integrated circuit, in which logic gates are defined in space.

From a practical standpoint, successful soliton computation in [31] requires ideal interactions and error-free propagation. In this sense, it is analogous to the construction of Fredkin and Toffoli, in which ideal, elastic collisions of billiard balls were used to achieve universal and reversible computation [12]. In reality, noise will cause variability in soliton propagation and collision, and fault tolerance based on logical state restoration would need to be implemented in order to improve system performance. In a sense, this is an analog rather than a digital computer.

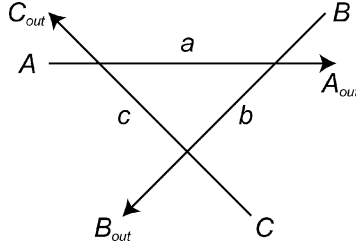


FIGURE 3  
Schematic of a three soliton collision cycle.

The type of soliton computation we highlight in this paper is based on more recent work, which uses bistable configurations of Manakov solitons [23,32]. This scheme takes advantage of the dynamical nature of computation in a nonlithographic medium, with the added benefit of physical state-restoring logic. We show that such a system demonstrates inherent state restoration, resulting in signal standardization.

### 3. A BISTABLE CONFIGURATION OF SOLITONS

A collision cycle of three soliton beams is shown schematically in Fig. 3, with input states  $A$ ,  $B$ , and  $C$ . The intermediate states are denoted by  $a$ ,  $b$ , and  $c$ , with outputs  $A_{out}$ ,  $B_{out}$ , and  $C_{out}$ . Note that we refer, equivalently, to the beams as well as their polarization states, as  $A$ ,  $B$ ,  $C$ , etc. Each collision, interpreted as a line crossing in the schematic of Fig. 3, is governed by the state transformation of Eqs. (3) and (4). Supposing we start with beam  $C$  initially off, so that  $A = a$ , the cycle can be described as follows: Beam  $a$  first hits  $B$ , transforming it to state  $b$ . If beam  $C$  is then turned on, it will hit  $b$ , change to  $c$ , and subsequently collide with  $A$ , closing the cycle. Beam  $a$  is then changed, changing  $b$ , etc., and the cycle of state changes propagates clockwise. Given fixed input beams, such a cycle was shown numerically to converge to exactly one or two foci [32]. Cases with two foci demonstrate bistability in the steady-state values of the *polarization state*.

One example of bistability is shown in Fig. 4. This plot was generated by fixing the input beams and choosing random points  $a$  uniformly distributed over a given range of the complex plane. The cycle described above was then carried out until convergence in the complex numbers  $a$ ,  $b$ , and  $c$  was obtained to within  $10^{-12}$  in norm. The foci, labeled  $f_0$  and  $f_1$ ,



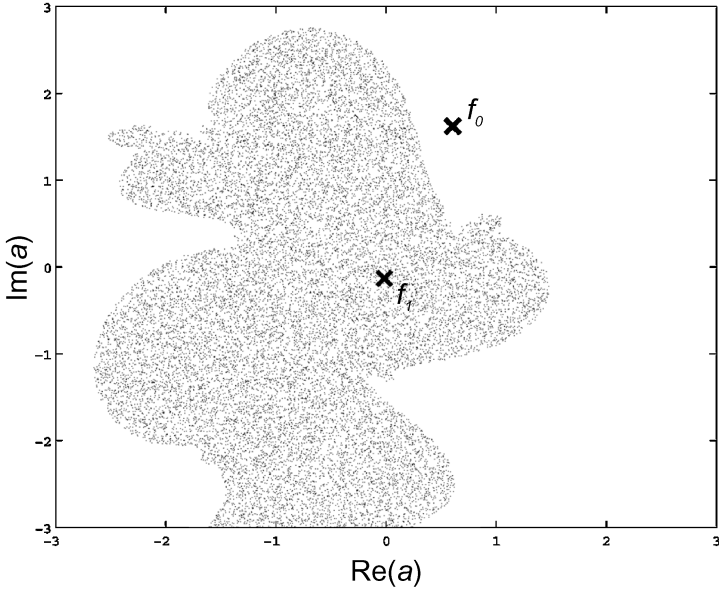


FIGURE 4

Example of bistable soliton collision cycle. The states of the input beams are  $A = 0.7 - 0.3i$ ,  $B = -1.1 - 0.5i$ ,  $C = 0.2 + 0.8i$ , and  $k = 4 \pm i$ .

are the two steady-state values of  $a$ , and correspond to the value of that beam in binary state 0 and 1, respectively. The basins of attraction illustrate those initial values (of state  $a$ ) that converge to each basin. The boundary between the basins of attraction is a kind of 2-D threshold, analogous to the switching in ordinary transistor-based logic. Beams  $A$ ,  $B$ ,  $C$ , and the values  $k_1$  and  $k_2$  remain constant, while  $a$ ,  $b$ ,  $c$ ,  $A_{out}$ ,  $B_{out}$ , and  $C_{out}$  can each have two stable steady-state values, depending on the binary state of the cycle. If  $A$ ,  $B$ , or  $C$  is changed, the basins and foci will change, and we can lose bistability altogether, resulting in only one steady-state focus. In fact, we will use this last observation of monostability to control a bistable cycle.

### 3.1 Controlling a bistable cycle

In order to use these bistable collision cycles for data storage and logic, we need to develop a method by which we can individually address these devices. In other words, given a bistable configuration of Manakov solitons with certain constant inputs (e.g., Fig. 4), we must be able to switch between binary states of the cycle reliably.

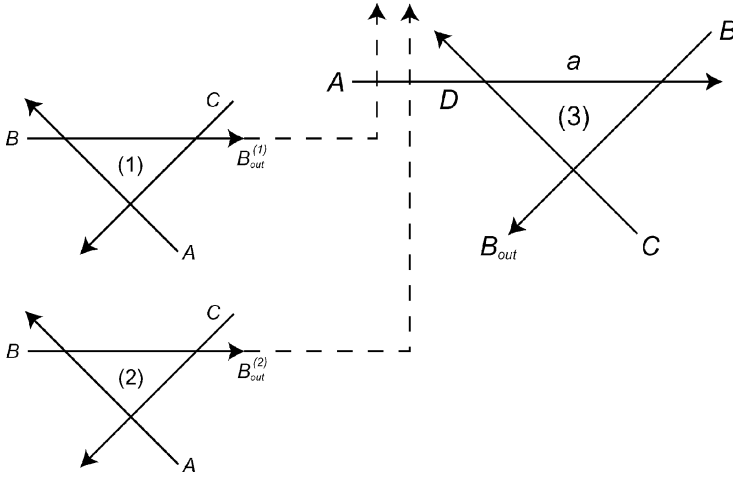


FIGURE 5  
Schematic of NAND gate using bistable collision cycles.

We accomplish this by temporarily disrupting the bistability of the cycle. For example, colliding a control beam, or beams, with  $A$  (as shown by the dashed lines in Fig. 5) changes the input state  $A$  to  $D$ . Through careful design of the control beams, we can ensure that  $A$  changes in such a way that the cycle (cycle (3) in Fig. 5), which demonstrated bistability without the control beams, becomes monostable, yielding only one possible steady-state value for the intermediate and output solitons of cycle (3). Subsequently, when the control beams are turned off,  $A$  equals  $D^1$  and cycle (3) recovers its bistable configuration, but now the initial state of the cycle is known. This initial condition will lie in one of the two basins of attraction, causing the cycle to settle to the focus corresponding to that basin. In this fashion, we control the output state of a bistable soliton collision cycle, where the value of the monostable focus is controlled by changing the state of the control beam.

#### 4. NAND AND FANOUT GATES

The schematic of a NAND gate is shown in Fig. 5. It consists of three cycles: cycles (1) and (2) are the inputs to cycle (3), which represents the

<sup>1</sup> We assume here that there is sufficient separation between collisions to ensure that this equality is true.

actual gate. All three cycles have identical bistable configurations, with input solitons  $A = -0.2 + 0.2i$ ,  $B = 0.9 + 1.5i$ ,  $C = -0.5 - 1.5i$  and  $k = 4 \pm i$ . The output of any cycle is  $B_{out}$ , and an input is described by a collision with  $A$ . Using the method described in Section 3.1, cycles (1) and (2) can be set in either binary state 0 or 1. When the inputs from cycles (1) and (2) are active, cycle (3) will become monostable and, depending on the values of the inputs, there are four possible monostable foci for cycle (3). Turning off the inputs will place cycle (3) in the state corresponding to the NAND operation. By using identical bistable collision cycles, we ensure that the output is standardized and can serve as input for the next level of logic.

The bistable configuration of all three cycles, along with the values of the monostable foci which correspond to the four inputs, are shown in Fig. 6. Only when the inputs are both in state 1 will the cycle be put into state 0. A variability on the inputs will change the position of the monostable foci slightly. We can see from Fig. 6 that this change will not affect the position of the output state, unless the change is greater than a specified amount. Quantifying the noise margins of this system remains a topic for future work.

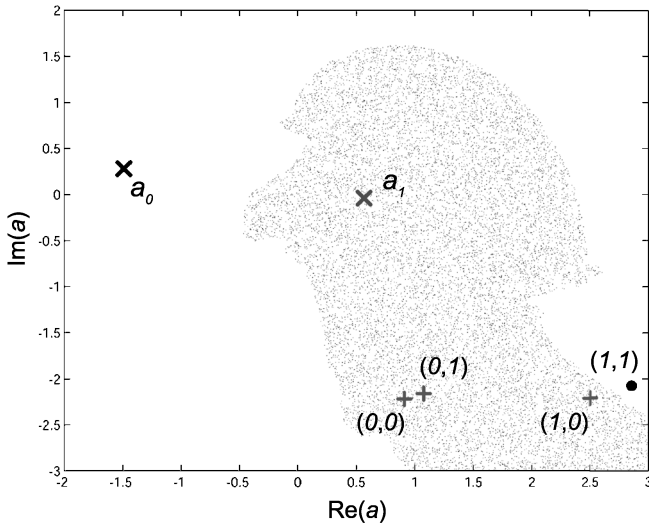


FIGURE 6

Map of beam  $a$  in the complex plane showing NAND gate operation. The two foci,  $a_0$  and  $a_1$ , are shown with their corresponding basins of attraction. The + signs are monostable foci which indicate inputs where the cycle reaches state 1, the  $\bullet$  is the monostable focus acquired with a (1, 1) input.

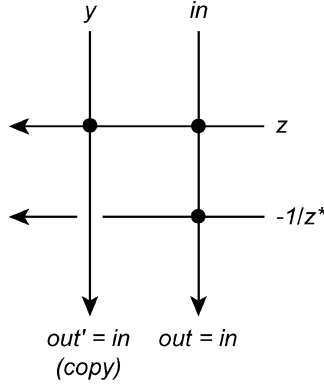


FIGURE 7  
Schematic of FANOUT gate, where each • indicates a collision.

Figure 7 shows the schematic of a FANOUT gate, where solitons  $y$  and  $z$  are chosen in such a way that a copy of soliton  $in$  is created at the output, as indicated by  $out'$ . Explicitly, we define the transformations from Eqs. (3) and (4) as  $\rho_2 \equiv L(\rho_1, \rho_L)$  and  $\rho_R \equiv R(\rho_1, \rho_L)$ , respectively. The value of  $out'$  is then  $R(y, L(in, z))$ . The original input soliton is recovered using the inverse property of Manakov collisions, as described in [13]. When viewed as an operator, each polarization state  $\rho$  has an inverse defined as  $-1/\rho^*$ . As such, an arbitrary soliton  $\rho_1$  which collides with another soliton  $\rho_2$ , followed by a collision with its inverse  $-1/\rho_2^*$ , restores the original state  $\rho_1$ . Thus the original input soliton  $in$  is restored by a collision with the inverse of  $z$ ,  $-1/z^*$ .

As a useful example, we design a FANOUT gate for the case of input soliton  $in = B_{out}$ , where  $B_{out}$  is taken from the output of a NAND gate. The bistable configuration of the NAND gate provides for two possible outputs,  $B_{out0}$  and  $B_{out1}$ , corresponding to binary states 0 and 1, respectively. The FANOUT design stipulates that  $B_{out0}$  map to  $B_{out0}$  and  $B_{out1}$  map to  $B_{out1}$ , which can be expressed by the following two simultaneous complex equations in two complex unknowns:

$$\begin{aligned} R(y, L(B_{out0}, z)) &= B_{out0}, \\ R(y, L(B_{out1}, z)) &= B_{out1}. \end{aligned} \quad (5)$$

Solving Eqs. (5) numerically yields  $y = 0.6240 - 0.4043i$  and  $z = -1.1286 + 0.7313i$ . This example thus demonstrates that the output from a NAND gate can be used to drive two similar NAND gates.

## 5. DISCUSSION

### 5.1 Experimental progress

There exist several candidates for the physical realization of Manakov solitons, including photorefractive crystals [3,4,9,10,25], semiconductor waveguides [15], quadratic media [29], and optical fiber [19].

In [4], Anastassiou *et al.* demonstrated energy exchanging collisions of two Manakov-like solitons. This experiment was performed using a photorefractive crystal, which, in contrast to the Kerr nonlinearity of the Manakov system, has a saturable nonlinearity. However, these systems are similar in the limit of very low intensities [24]. Later, in [3], this work was extended to the case of two collisions, where it was shown that information could be passed from one soliton to another using an intermediate collision.

### 5.2 Reversibility

How is it that the mathematical model of soliton propagation and collision (the Manakov system) is completely reversible, and yet basic logical elements, such as a NAND gate, are not? For example, suppose we operate the NAND gate in Fig. 6 with inputs (0, 1). The NAND gate output will converge to  $a_1$ , which represents binary state 1. However, we could also have arrived at  $a_1$  from the points (1, 0) or (0, 0). The reason we cannot reverse this computation is that the evolution of the state history is erased because of the assumed ambient noise. If we were able to keep the state  $a_1$  to infinite precision, then in fact the system would be reversible.

## 6. CONCLUSION

In this paper, we have shown that arbitrary computation is possible using collisions of Manakov solitons in a nonlithographic, nonlinear optical medium. A FANOUT and a NAND gate were described to demonstrate fanout, cascability, and Boolean completeness, necessary requirements for universal computation. Furthermore, we demonstrated physical state restoration. Signals are standardized to binary logic levels, and, as in solid-state transistor logic, the system can operate reliably in the presence of noise.

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