1. Introduction

Can machine learning agents learn high-level, human-like mathematical reasoning?

- Problem
  1. Automated theorem proving (ATP) manipulates theorems using a low-level representation, making it difficult to benefit from the high-level abstraction common to humans.
  2. Interactive theorem proving (ITP) is close to human mathematical reasoning. But it is labor-intensive, requiring humans to interact with a software system (proof assistant).

- Proposed solution
  1. ITP’s high-level formalism + ATP’s proof automation
  2. Develop machine learning agents to imitate humans for interacting with a proof assistant.

- Contributions
  2. ASTactic: a deep learning model for this task that can prove theorems not provable by existing methods.

2. Interactive Theorem Proving

- Humans interact with proof assistants
  1. The user sees the goal and enters a tactic representing a high-level manipulation, such as induction.
  2. The proof assistant executes the tactic, decomposing the goal into multiple sub-goals.
  3. The process starts with the original theorem as the initial goal, and ends when there is no goal left.

- We train machine learning agents to replace humans in this task.

3. CoqGym: Data and Environment

- A tool for interacting with the Coq proof assistant [1]
- Large
  1. 71K human-written proofs
  2. Over an order of magnitude larger than existing datasets
  3. Sufficient for data-hungry machine learning models
- Diverse:
  1. From 123 Coq projects
  2. Cover a broad spectrum of domains (math, software, etc.)
- Structured data
  1. Proof represented by proof trees
  2. Abstract syntax trees of the expressions in the proof

4. ASTactic: Deep Tactic Generation

Given the current proof goal, an encoder-decoder architecture generates a tactic in the form of abstract syntax tree (AST).

- Term encoder
  - Input terms (multiple assumptions and one conclusion) are parsed into ASTs and embedded by a TreeLSTM network [2].
- Tactic decoder
  - To generate a tactic AST conditioned on the embeddings, the decoder sequentially grows a partial tree by selecting production rules and terminal tokens.
- Search for complete proofs
  1. At each step, ASTactic outputs 20 tactics via beam search.
  2. Treating them as possible actions, we search for a proof via depth-first search.

5. Experiments

- Task: Proving the 13,137 testing theorems in CoqGym. Each proof within 300 tactics AND a wall time of 10 minutes.
- Baselines
  1. Coq’s built-in automated tactics: trivial, auto, intuition, easy
- Results
  1. Our system proves 12.2% of the theorems, outperforming Coq’s built-in tactics (4.9%).
  2. Proves 30.0% of the theorems when combined with hammer, a large improvement of 5.2% over using hammer alone.