Modules and Abstract Data Types

COS 326

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Last Time: Modules

- Before the break, we introduced you to ML's module system:
  - *signatures*: interfaces that mention the names of abstract types and the names/types of operations over them
  - *structures*: implementations of abstract data types (they give concrete definitions to abstract types and code to implement the abstract operations)
  - *functors*: functions from modules to modules (they provide a way to allow us to parameterize our modules)

Functor Kitten
(official mascot of the
OCaml module system)

An Example

```
module type UNSIGNED_BIGNUM =
sig
  type ubignum
  val fromInt : int -> ubignum
  val toInt : ubignum -> int
  val plus : ubignum -> ubignum -> ubignum
  val minus : ubignum -> ubignum -> ubignum
  val times : ubignum -> ubignum -> ubignum
  …
end
```

An Implementation

```
module My_UBignum_1000 : UNSIGNED_BIGNUM =
struct
  let base = 1000

  type ubignum = int list

  let toInt(b:ubignum):int = …

  let plus(b1:ubignum)(b2:ubignum):ubignum = …

  let minus(b1:ubignum)(b2:ubignum):ubignum = …

  let times(b1:ubignum)(b2:ubignum):ubignum = …
  …
end
```

A Simpler (but slower) Implementation

```
module BIGNUM_UNARY : UNSIGNED_BIGNUM =
struct
  type ubignum = Zero | Succ of ubignum

  let rec utoInt (u:ubignum) : int =
    match u with
    | Zero -> 0
    | Succ u’ -> 1 + (toInt u’)

  let rec plus (u1:ubignum) (u2:ubignum) : ubignum =
    match u1 with
    | Zero -> u2
    | Succ u1’ -> uplus u1’ (Succ u2)


  …
end
```

The Abstraction Barrier

Rule of thumb: try to use the language mechanisms (e.g., modules, interfaces, etc.) to *enforce* the abstraction barrier.

- reveal as little information about *how* something is implemented as you can.
- provides maximum flexibility for change moving forward.
- pays off down the line

However, like all design rules, we must be able to recognize when the barrier is causing more trouble than it's worth and abandon it.

- may want to reveal more information for debugging purposes
  - eg: conversion to string so you can print things out

Another Example: Queues or Fifo’s

```
module type QUEUE =
  sig
    type ‘a queue
    val empty : unit -> ‘a queue
    val enqueue : ‘a -> ‘a queue -> ‘a queue
    val is_empty : ‘a queue -> bool
    exception EmptyQueue
    val dequeue : ‘a queue -> ‘a queue
    val front : ‘a queue -> ‘a
  end
```

Another Example: Queues or Fifo’s

```
module type QUEUE =
  sig
    type ‘a queue
    val empty : unit -> ‘a queue
    val enqueue : ‘a -> ‘a queue -> ‘a queue
    val is_empty : ‘a queue -> bool
    exception EmptyQueue
    val dequeue : ‘a queue ->      queue
    val front : ‘a queue -> ‘a
  end
```

These queues are re-usable for different element types.

Here's an exception that client code might want to catch

One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type ‘a queue = ‘a list
    let empty() = []
    let enqueue(x:’a)(q:’a queue) : ‘a queue = q @ [x]
    let is_empty(q:’a queue) =
     match q with
     | [] -> true
     | _::_ -> false

    ...



end
```

One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type ‘a queue = ‘a list
    let empty() = []
    let enqueue(x:’a)(q:’a queue) : ‘a queue = q @ [x]
    let is_empty(q:’a queue) = ...

    exception EmptyQueue
    let deq(q:’a queue) : (‘a * ‘a queue) =
      match q with
      | [] -> raise EmptyQueue
      | h::t -> (h,t)
    let dequeue(q:’a queue) : ‘a queue = snd (deq q)
    let front(q:’a queue) : ‘a = fst (deq q)
end
```

One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type ‘a queue = ‘a list
    let empty() = []
    let enqueue(x:’a)(q:’a queue) : ‘a
    let is_empty(q:’a queue) = ...

    exception EmptyQueue
    let deq(q:’a queue) : (‘a * ‘a queue) =
      match q with
      | [] -> raise EmptyQueue
      | h::t -> (h,t)
    let dequeue(q:’a queue) : ‘a que
    let front(q:’a queue) : ‘a = fst (d
end
```

Notice deq is a helper function that doesn’t show up in the signature.

You can't use it outside the module.

One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type ‘a queue = ‘a list
    let empty() = []
    let enqueue(x:’a)(q:’a queue) : ‘a queue = q @ [x]
    let is_empty(q:’a queue) = ...

    exception EmptyQueue
    let deq(q:’a queue) : (‘a *
      match q with
      | [] -> raise EmptyQueue
      | h::t -> (h,t)
    let dequeue(q:’a queue) : ‘a queue = snd (deq q)
    let front(q:’a queue) : ‘a = fst (deq q)
end
```

Notice enqueue takes time proportional to the length of the queue

Dequeue runs in constant time.

An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type ‘a queue = {front:’a list; rear:’a list}







    ...







end
```

In Pictures

```
let q0 = empty;;          {front=[];rear=[]}

let q1 = enqueue 3 q0;;   {front=[];rear=[3]}

let q2 = enqueue 4 q1 ;;  {front=[];rear=[4;3]}

let q3 = enqueue 5 q2 ;;  {front=[];rear=[5;4;3]}

let q4 = dequeue q3 ;;    {front=[4;5];rear=[]}

let q5 = dequeue q4 ;;    {front=[5];rear=[]}

let q6 = enqueue 6 q5 ;;  {front=[5];rear=[6]}

let q7 = enqueue 7 q6 ;;  {front=[5];rear=[7;6]}
```

An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type ‘a queue = {front:’a list; rear:’a list}

    let empty() = {front=[]; rear=[]}

    let enqueue x q = {front=q.front; rear=x::q.rear}

    let is_empty q =
      match q.front, q.rear with
      | [], [] -> true
      | _, _ -> false

    ...
end
```

An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type ‘a queue = {front:’a list; rear:’a list}
    ...
    exception EmptyQueue

    let deq (q:’a queue) : ‘a * ‘a queue =
      match q.front with
      | h::t -> (h, {front=t; rear=q.rear})
      | [] -> match List.rev q.rear with
              | h::t -> (h, {front=t; rear=[]})
              | [] -> raise EmptyQueue

    let dequeue (q:’a queue) : ‘a queue = snd(deq q)
    let front (q:’a queue) : ‘a = fst(deq q)
  end
```

How would we design an abstraction?

- Write some test cases:
  - what operations might you want?
  - what *abstract* types might you want?
- From this, we can derive a signature
  - list the types
  - list the operations with their types
  - don’t forget to provide enough operations that you can debug!
- Then we can build an implementation
  - when prototyping, build the simplest thing you can.
  - later, we can swap in a more efficient implementation.
  - (assuming we respect the abstraction barrier.)

Common Interfaces

- The stack and queue interfaces are quite similar:

```
module type STACK =
  sig
    type ‘a stack
    val empty : unit -> ‘a stack
    val push  : int -> ‘a stack -> ‘a stack
    val is_empty : ‘a stack -> bool
    exception EmptyStack
    val pop
    val top
  end
```

```
module type QUEUE =
  sig
    type ‘a queue
    val empty : unit -> ‘a queue
    val enqueue : ‘a -> ‘a queue -> ‘a queue
    val is_empty : ‘a queue -> bool
    exception EmptyQueue
    val dequeue : ‘a queue -> ‘a queue
    val front : ‘a queue -> ‘a
  end
```

It’s a good idea to factor out patterns

```
module type CONTAINER =
  sig
    type ‘a t
    val empty : unit -> ‘a t
    val insert : ‘a -> ‘a t -> ‘a t
    val is_empty : ‘a t -> bool
    exception Empty
    val remove : ‘a t -> ‘a t
    val first : ‘a t -> ‘a
  end
```

Slap the same interface on both the Queue module and the Stack module -- interfaces are reuseable in ML!

This lets us write an algorithm, like a tree traversal, using a generic container interface.

To get depth-first traversal, use the stack; to get breadth-first traversal, use the queue; to get prioritized traversal, use a priority queue; etc.

FUNCTORS

Matrices

- Suppose I ask you to write a generic package for matrices.
  - e.g., matrix addition, matrix multiplication
- The package should be parameterized by the element type.
  - We may want to use ints or floats or complex numbers or binary values or … for the elements.
- What we'll see:
  - RING: a signature to describe the type (and necessary operations) for matix elements
  - MATRIX: a signature to describe the available operations on matrices
  - DenseMatrix: a functor that will generate a MATRIX with a specific RING as an element type

Ring Signature

```
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
```

Some Rings

```
module IntRing =
  struct
    type t = int
    let zero = 0
    let one = 1
    let add x y= x + y
    let mul x y = x * y
  end
```

```
module FloatRing =
  struct
    type t = float
    let zero = 0.0
    let one = 1.0
    let add = (+.)
    let mul = ( *. )
  end
```

```
module BoolRing =
  struct
    type t = bool
    let zero = false
    let one = true
    let add x y= x || y
    let mul x y = x && y
  end
```

Matrix Signature

```
module type MATRIX =
  sig
    type elt
    type matrix
    val matrix_of_list : elt list list -> matrix
    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
   end
```

The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with elt = R.t) =
struct

  ...







end
```

The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with elt = R.t) =
struct

   ...

end
```

Argument R must be a RING

Result must be a MATRIX

Specify Result.elt = R.t

The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with elt = R.t) =
struct

  ...





end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

Use DenseMatrix like it is a function from modules to modules

The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX ~~with elt = R.t~~) =
struct

   ...

end
```

redacted

```
module type MATRIX =
  sig
    type elt
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
   end
```

abstract = unknown!

non-existant

```
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX ~~with elt = R.t~~) =
struct
```

redacted

If the "with" clause is redacted then IntMatrix.elt is abstract -- we could never build a matrix because we could never generate an elt

```
module type MATRIX =
  sig
    type elt
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
  end
```

abstract = unknown!

non-existant

```
end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with elt = R.t) =
struct

  ...

end
```

sharing constraint

```
module type MATRIX =
  sig
    type elt = int
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
  end
```

known to be int when R.t = int like when R = IntRing

list of list of ints

```
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with elt = R.t) =
struct



end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

sharing constraint

The "with" clause makes IntMatrix.elt equal to int -- we can build a matrix from any int list list

```
module type MATRIX =
  sig
    type elt = int
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
  end
```

known to be int when R.t = int like when R = IntRing

list of list of ints

Matrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with elt = R.t) =
struct
  type elt = R.t
  type matrix = (elt list) list
  let matrix_of_list rows = rows
  let add m1 m2 =
    List.map (fun (r1,r2) ->
                List.map (fun (e1,e2) -> R.add e1 e2))
                    (List.combine r1 r2))
       (List.combine m1 m2)
  let mul m1 m2 = (* good exercise *)
end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

Satisfies the sharing constraint

Another Functor Example

```
module type BASE =
sig
  val base : int
end

module UbignumGenerator(Base:BASE) : UNSIGNED_BIGNUM =
struct
  type ubignum = int list
  let toInt(b:ubignum):int =
    List.fold_left (fun a c -> c*Base.base + a) 0 b   …
end

module Ubignum_10 =
  UbignumGenerator(struct let base = 10 end) ;;

module Ubignum_2 =
  UbignumGenerator(struct let base = 2 end) ;;
```

Subtyping

- A module matches any interface as long as it provides *at least* the definitions (of the right type) specified in the interface.

- But as we saw earlier, the module can have more stuff.
  - e.g., the deq function in the Queue modules

- Basic principle of subtyping for modules:
  - wherever you are expecting a module with signature S, you can use a module with signature S', as long as all of the stuff in S appears in S'.
  - That is, S' is a bigger interface.

Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

RING is a sub-type of GROUP.

Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

There are *more* modules matching the GROUP interface than the RING one.

Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

Any module expecting a GROUP can be passed a RING.

Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    include GROUP
    val one  : t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

The **include** primitive is like cutting-and-pasting the signature's content here.

Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    include GROUP
    val one  : t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

That *ensures* we will be a sub-type of the included signature.

A Bigger Example

```
module type SET =
  sig
    type elt
    type set
    val empty : set
    val is_empty : set -> bool
    val insert : elt -> set -> set
    val singleton : elt -> set
    val union : set -> set -> set
    val intersect : set -> set -> set
    val remove : elt -> set -> set
    val member : elt -> set -> bool
    val choose : set -> (elt * set) option
    val fold : (elt -> 'a -> 'a) -> 'a -> set -> 'a
  end
```

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
             :  (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
            :  (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

ListSet is a parameterized module – given a module argument for Elt, it generates a new module.

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
               :   (SET with elt   Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

This is a very simple, anonymous signature (it just specifies there's some type t) for the argument to ListSet

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
              :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

This is the signature of the resulting module – we have a set plus the knowledge that the Set's elt type is equal to Elt.t

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
               :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

These are two SET modules that I created with the ListSet functor.

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
             :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set =
...
end


module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

In this case, I'm passing in an anonymous module for Elt that defines t to be int.

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
            :  (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

We know that
IntListSet.elt = int.

Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
             :  (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = [
  let is_empty (s:set
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:el
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

```
module type SET =
  sig
    type elt = int
    type set
    val empty : set
    val is_empty : set -> bool
    val insert : elt -> set -> set
    ...
  end
```

equal to int
so we can actually
build a set using
insertions!

Let’s Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
               : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  ...
end
```

Let’s Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
               :(SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set = ???
end
```

Let’s Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
               : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set =
      s1 @ s2
  ...
end
```

Ugh. Wastes space if s1 and s2 have duplicates. (Also, makes remove harder…)

Let’s Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
             : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set =
      List.fold_right insert s1 s2
  ...
end
```

Gets rid of the duplicates. Now remove can stop once it finds the element.

Let’s Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
               : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set =
      List.fold_right insert s1 s2
  ...
end
```

Gets rid of the duplicates. Now remove can stop once it finds the element.

Let's Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
              : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set =
      List.fold_right insert s1 s2
  ...
end
```

But List.mem and List.fold_right take time proportional to the length of the list. So union is quadratic.

Gets rid of the duplicates. Now remove can stop once it finds the element.

Let’s Write the Rest of the Functor

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  let union (s1:set) (s2:set) : set =
      List.fold_right insert s1 s2
  ...
end
```

If we knew that s1 and s2 were *sorted* we could use the merge from mergesort to compute the sorted union in linear time.

A Sorted List Set Functor

```
module type COMPARATOR = sig
  type t
  val compare : t -> t -> Order.order
end


module SortedListSet (Elt : COMPARATOR)
                   : (SET with elt = Elt.t) =
struct ...
  let rec insert (x:elt) (s:set) : set =
      match s with
      | [] -> [x]
      | h::t -> (match Elt.compare x h with
                 | Less -> x::s
                 | Eq -> s
                 | Greater -> h::(insert x t)) ...
end
```

A Sorted List Set Functor

```
module type COMPARATOR = sig
  type t
  val compare : t -> t -> Order.order
end


module SortedListSet (Elt
                    : (SET with e
struct ...
  let rec insert (x:elt) (s:set)
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      | [] -> [x]
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                 | Less -> x::s
                 | Eq -> s
                 | Greater -> h::(insert x t)) ...
end
```

To support the sorting, I'm passing in a comparison operation to go with the element type.

A Sorted List Set Functor

```
module SortedListSet (Elt : COMPARATOR)
                     : (SET with elt = Elt.t) =
struct ...
  let rec union (s1:set) (s2:set) : set =
      match s1, s2 with
      | [], _ -> s2
      | _, [] -> s1
      | h1::t1, h2::t2 ->
         (match Elt.compare h1 h2 with
            | Less -> h1::(union t1 s2)
            | Eq -> h1::(union t1 t2)
            | _ -> h2::(union s1 t2))
  …
end
```

Simpler

```
module SortedListSet (Elt : COMPARATOR)
                    : (SET with elt = Elt.t) =
struct ...
  let rec union (s1:set) (s2:set) : set = ...

  let insert (x:elt) (s:set) : set = union [x] s ;;

end
```

Another Alternative: Bit Vectors

```
module BitVectorSet (Elt : sig type t
                             val index : t -> int
                             val max : int
                           end)
                  : (SET with elt = Elt.t) =
struct
  type set = bool array
  let empty = Array.create Elt.max false
  let member x s = s.(Elt.index x)
  let union s1 s2 =
       Array.init Elt.max
         (fun i -> s1.(i) || s2.(i))
  let intersect s1 s2 =
       Array.init Elt.max
         (fun i -> s1.(i) && s2.(i))
  ...
```

Another Alternative: Binary Search Trees

```
module BSTreeSet(Elt : sig type t
                           val compare : t -> t -> Order.order
                       end) : (SET with elt = Elt.t) =
struct
  type set = Leaf | Node of set * elt * set
  let empty() = Leaf
  let rec insert (x:elt) (s:set) : set =
      match s with
      | Leaf -> Node(Leaf,x,Leaf)
      | Node(left,e,right) ->
         (match Elt.compare x e with
              | Eq -> s
              | Less -> Node(insert x left, e, right)
              | Greater -> Node(left, e, insert x right))
  let rec member (x:elt) (s:set) : bool =
      match s with
      | Leaf -> false
      | Node(left,e,right) ->
        (match Elt.compare x e with
            | Eq -> true
            | Less -> member x left
            | Greater -> member x right)
  ... end
```

Wrap up and Summary

- It is often tempting to break the abstraction barrier.
  - e.g., during development, you want to print out a set, so you just call a convenient function you have lying around for iterating over lists and printing them out.
- But the whole point of the barrier is to support future change in implementation.
  - e.g., moving from unsorted invariant to sorted invariant.
  - or from lists to balanced trees.
- Many languages provide ways to leak information through the abstraction barrier.
  - “good” clients should not take advantage of this.
  - but they always end up doing it.
  - so you end up having to support these leaks when you upgrade, else you’ll break the clients.

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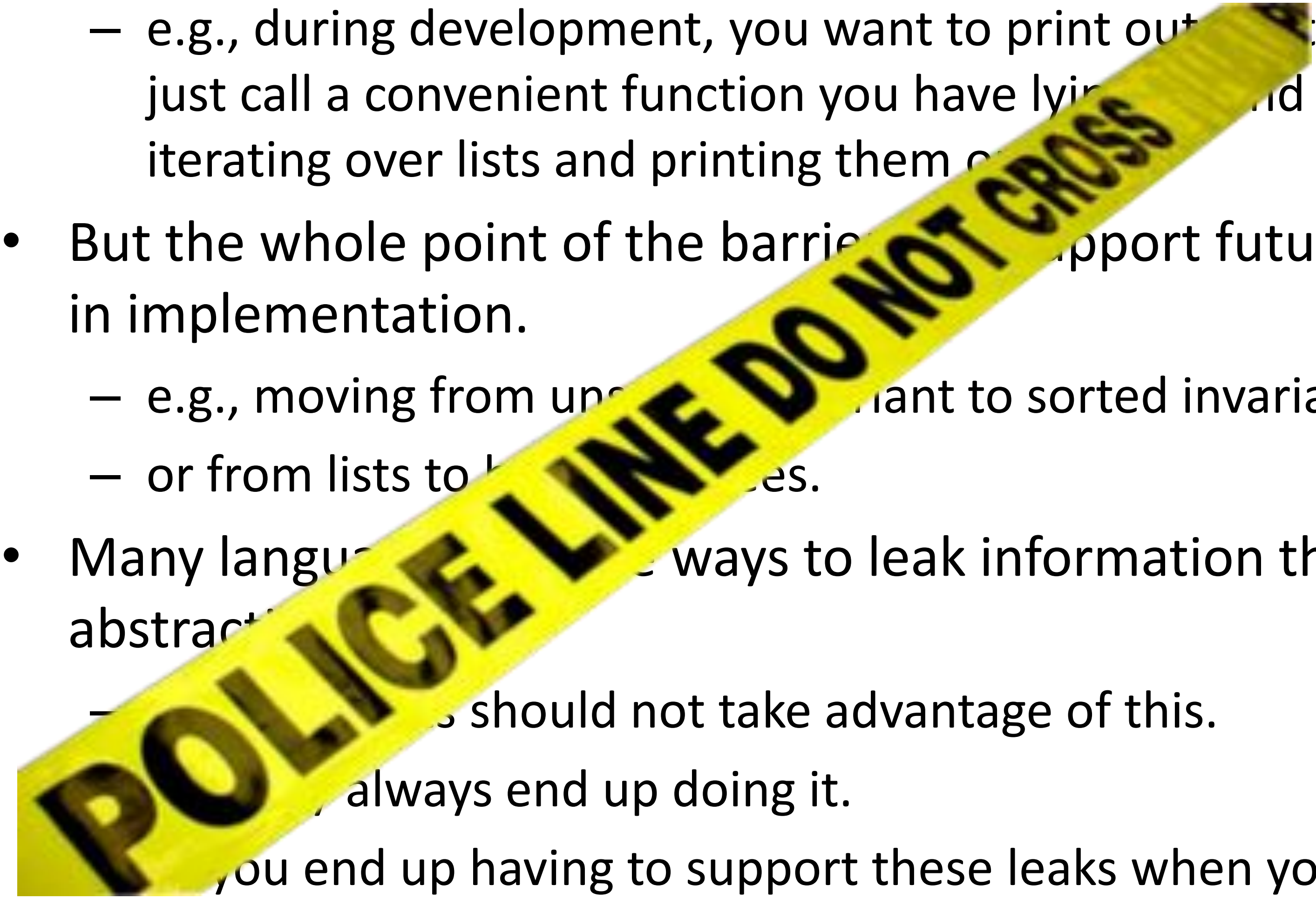

Key Points

- Design in terms of *abstract* types and algorithms.
  - think “sets” not “lists” or “arrays” or “trees”
  - think “document” not “strings”
- Use linguistic mechanisms to insulate clients from implementations of mechanisms.
  - makes it easy to swap in new implementations
  - the *less* you reveal in an interface, the easier it is to replace the implementation
  - on the other hand, you need to reveal enough in the interface to make it useful for clients.
- In Ocaml, we can use the module system
  - provides support for *name-spaces*
  - *hiding information* (types, local value definitions)
  - *code reuse* (via functors, reuseable interfaces, reuseable modules)

END