The text has been assembled with extraordinary care and, for a book of this nature, has remarkably few errors—typographical or otherwise. An on-line errata list included nearly all the errors that this reviewer spotted. Notes at the end of each chapter illuminate the historical background of the ideas and algorithms presented and provide pointers into the more recent literature. Algorithms are presented in a clear ⟵-style pseudocode.

A numbering system whereby a lemma, theorem, and algorithm, introduced (in that order) in Chapter n, are assigned designations "Lemma $n.1$," "Theorem $n.2$," and "Algorithm $n.3$," makes for easy reference. The book's index is excellent and incorporates a name index that points into the end-of-chapter notes. There is also an exhaustive symbol index. The clarity of this navigation system means that the book can function as a work of reference, and one expects it become one of the standard references in the field of computer algebra if this has not happened already.

There are many exercises at the end of each chapter. The ordering of these exercises follows that of the chapter, and more difficult exercises are placed last. Considerable mathematical sophistication is assumed, however. Often enough the very first exercise involves proving some number-theoretic result, say, by mathematical induction. But there is also a wealth of exercises that involve applying the algorithms discussed in the text.

The bibliography provided by Von zur Gathen and Gerhard is extensive—even scholarly, and, in general, their book is informed by a sense of the rich history of mathematics and of algorithms in particular. This reviewer knows of no text on any level that better conveys one's sense that algorithms and the algorithm concept have made a difference for culture.

Review[3] of

The Discrepancy Method —Randomness and Complexity

Author of book: Bernard Chazelle

Cambridge University Press, 460 pages, hardcover, $64.95
ISBN: 0521770939

Author of review: Jin-Yi Cai, University of Wisconsin, Madison

Complexity Theory is the study of quantitative limitations of computation. The central question in Complexity Theory is what can and what cannot be solved by efficient algorithms, and most importantly why certain problems are difficult to solve. For certain problems, provably efficient deterministic algorithms seem to be lacking while efficient randomized algorithms do exist. Sometimes, however, efficient randomized algorithms have been found first and then efficient deterministic algorithms are found afterwards, often by a certain process of derandomization.

One primary focus of current Complexity Theory research is: To what extent is randomization essential in computation? This is a problem the Complexity research community is likely to be occupied with for a long time to come. It is a quest that is both extremely deep technically as well as profound philosophically. The book by Bernard Chazelle, the leading computational geometer of our day, is a masterful exposition of some of the most beautiful aspects of this theory.

The central theme of the book, as the title implies, is Discrepancy Theory, and its applications to the theory of computing, especially to derandomization. ¿From the moment I laid my hand on this book, I had no doubt in my mind that Professor Chazelle has produced a masterpiece that few books published in our field can equal. It is not as encyclopedic as the multi-volume treatise

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by Don Knuth; but my admiration for it is such that, the volumes by Knuth are the only other scholarly books published in computer science that I can think in comparison.

¿From the choice of topics included, to its masterful exposition, I stand in awe of the beauty and elegance of the book, and, above all, the *exquisite taste* of the author. In about 400 pages, excluding bibliography and 3 short (and very elegant) appendices (on Probability, Harmonic Analysis and Convex Geometry, respectively), the book managed to lead us on a magnificent tour of many facets of Discrepancy Theory and applications. These topics include Pseudorandomness, Communication Complexity, rapid mixing Markov Chains, Modular forms, Geometric sampling, VC-dimension Theory, Voronoi diagram, derandomization, linear programming, linear circuit complexity, etc. ¿From the foremost computational geometer of our day, it is also fitting that the book features a tour de force by the author himself, of an optimal deterministic algorithm for convex hulls in all dimensions. The book also presents, as far as I know for the first time in book form, another crowning achievement by the author himself, a deterministic minimum spanning tree algorithm with running time $O(m\alpha(m, n))$, where α is the classical inverse Ackermann function. Both these algorithms are achieved by suitable (and intricate) derandomization based on Discrepancy method. In the minimum spanning tree problem, there is still potentially a gap, for a linear time randomized algorithm is known due to Karger, Klein and Tarjan, which further relies on a linear time verification procedure for MST, an idea goes back to Komlós. The convex hull problem is perhaps the most famous problem in computational geometry, and the study of the minimum spanning tree problem predates the beginning of the field of computer science itself. The various featured topics might be eclectic, but here you will see so many beautiful ideas come together, and come alive, and you can do nothing but be swept away by the sheer elegance.

Given a set system with n points and m subsets, can you color the points to two distinct colors, such that every subset contains roughly an equal number of points with each color? How close can it be? How close can it not be? How good a coloring can one get with an efficient procedure? How about points and subsets with extra regularities such as distributed in Euclidean space? These are the basic questions the Theory of Discrepancy deals with. Often one strives to achieve deterministically what can be shown to hold under uniform or carefully chosen distributions. It is therefore not surprising that the subject has a lot to do with derandomization in computation, and also, certain tools such as Fourier Analysis play an important role here.

The book starts off with a succinct introduction to combinatorial discrepancy theory. You will meet the greedy methods including the method of conditional expectations, and the hyperbolic cosine algorithm. You will meet the entropy method, the Beck-Fiala Theorem, and VC-dimension. You will also be introduced to the Hadamard matrix, eigenvalue bound, and a classical theorem of Roth (the Fields Medalist), and summed up from the point of view of Harmonic Analysis.

The second chapter deals with upper bound techniques in Discrepency Theory. We see Halton-Hammersley Points, Ergodicity of Arithmetic Progressions on a circle, Weyl's Criterion, Quaternions and $SO(3)$ (the special orthogonal group in dimension 3), Spherical Harmonics and the Laplacian, Hecke Operators and the Ramanujan Bound. Also included, is a nice introduction to Modular Group, Modular Forms, Zeta functions and L-functions, with a glimpse of Hasse-Weil, Shimura-Taniyama-Wiles. The author modestly insists that he is "hardly an expert on this", and offers the following: "... apology to Oscar Wilde, he is always ready to give to those who are more experienced than himself the full benefits of his inexperience." I found his little tour of this most sanctified realm of Arithmetic Algebraic Geometry beautiful and refreshing. Of course, with a mere few sections, one can not expect any full proofs. But here, without all the proofs, I found the exposition very appealing intuitively and it gives a very good sense of the big picture. (Of course here I should offer my own apology to Oscar Wilde, squared.)

In chapter 3, the author discusses lower bound techniques. We will see the Method of orthogonal functions. We see the proof of the theorem that states that the mean-square discrepancy for axis-parallel boxes is at least $\Omega(\log n)^{d-1}$ for any set of n points in the unit square of dimension d. We see Haar Wavelets applied. The next section relaxes the restriction to non-axis-parallel boxes. We meet Beck's amplification method, Bessel functions and Fejér Kernel. The chapter finishes off with the finite differencing method.

Chapter 4 deals with Sampling, how to extract a small set of representatives from a large data set. It introduces ϵ-Net and ϵ-Approximation, Sampling in bounded VC-dimension. Along the way it also gives a primer on Hyperbolic Geometry. Some of the sampling techniques are best viewed through the prism of Hyperbolic Geometry. As the author indicates, while it is possible to translate these constructions back to Euclidean geometry, "this would be a mistake, ..., (for) much of the poetry would get lost in the translation". Now, how long ago have you seen a computer science book where the "poetry" of the sublime is of such high priority? Speaking of language, this book uses the word "Method" often (and early! It is in its title.) Such profound insight and depth are codified by the author with this somewhat modest but respectable word "Method", that it just brings a smile to my face whenever I think of the great world of Object Oriented Programming, where words like "Class" and "Method" occupy such an exalted place.

Chapter 5 deals with some Geometric Searching Problems. Chapter 6 deals with complexity lower bounds, especially arithmetic circuits in range searching. Eigenvalue, eigenspace, and entropy are the main tools (the combinatorial matrix rigidity approach of Valiant is not included). There is also a section on geometric databases. Chapter 7 is on Convex Hull and Voronoi Diagrams. Here the optimal deterministic algorithm for convex hull in all dimensions by the author is presented. The presentation follows a simplified form given by a joint work of Brönnimann (a former student of Chazelle), Chazelle, and Matoušek. Chapter 8 deals with linear programming. Also you will see Löwner-John Ellipsoids. Chapter 9 is on Pseudorandomness. We see finite fields and character sums, pairwise independence as a replacement for total independence, universal hash functions, random walks on expanders, pseudorandom bits from quadratic residuosity, and finishes off with a section on polynomials, small Fourier coefficients and low-discrepancy arithmetic progressions. Chapter 10 is on communication complexity. We see the matrix rank bound, and much more. Chapter 11 is on the Minimum Spanning Tree Algorithm mentioned at the beginning. We start with linear searching as low-discrepancy sampling and soft heap, a data structure invented by the author. Then we are onto his glorious deterministic algorithm on Minimum Spanning Tree.

A book of art, a book of love, this book belongs on the shelf of every theoretical computer scientist with a discriminating taste. As I stated earlier, the only other published computer science books I can think of to compare it to are the volumes of Don Knuth. I am confident that we will all agree 25 years from now.