A No-Regret Framework For Deriving Optimal Strategies With Emphasis On Trading In Electronic Markets

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Abstract

We present a no-regret learning framework to analyze behavior of strategic agents within single and multi-player games. We introduce a tool that could be used for any game to calculate optimal strategies for each player. Specifically, we base our tool on two regret minimization algorithms - Multiplicative Weights [5] and EXP3 [6].

We begin by describing each of the two regret minimization algorithms used followed by justification of our system design. Then, for proof of concept, we test our tool on two known games. Then, given that the main intention of this work is to measure the viability of applying a regret minimization framework within a trading environment, we move our discussion to describing our market framework followed by testing on an abstract trading game. That trading game is one that is designed based on certain abstractions, which will be described in detail. We then evaluate the application of our framework for that trading game with future potential of applying to further trading environments.

A significant amount of effort was dedicated towards understanding the market microstructure of an electronic market order book system. We will describe in detail our implementation of a *zero-intelligence* [3] market environment, and the implications and results for using the same. The trading game developed and tested is inherently linked to this market environment and the randomness of the market was instrumental in designing the game used.

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I would further like to thank Dr. Robert Almgren, Adjunct Professor in the Department of Operations Research and Financial Engineering, for his patience in explaining to me the dynamics of the electronic market microstructure and suggesting to me the paper on zero-intelligence, which formed the backbone of our market environment. He did not have any official affiliation with me or my project, so his generosity in this regard, especially during a time when I was lost while designing a market environment without accounting for market impact to execute trades on, will never be forgotten.

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1 Introduction

In this paper, we discuss the implementation of a no-regret learning framework to games. The games could be single player and multi-player, with specific payoffs/rewards. Emphasis is placed on the background of two no-regret algorithms - Multiplicative Weights and EXP3, as well as the design of the tool that incorporates these learning algorithms within a game theoretic setting. We describe in detail our architectural decisions behind the tool. The evaluation of our tool is done on two known games to calculate optimal strategies of two players playing the game - Rock-Paper-Scissors and Prisoner's Dilemma. We show that our framework yields the expected cyclical behavior of strategies on the first game, and the correct Nash Equilibrium for the second [1]. This step is essential to ensure the correctness of our framework to any known game.

With the intention of applying no-regret learning within the context of trading, we then discuss our implementation of a market model, based on electronic limit order books. We use the zero-intelligence market model primarily to account for the problem of measuring market impact of traders' actions within the market. After ascertaining the correctness of our market model, we then construct an abstract trading game based on a few assumptions, and try to construct a game out of a general type of market manipulation algorithms known as spoofing [4]. In short, spoofing is a means of manipulating markets by executing a fake order to generate market behavior which a trader correctly foresees and benefits from. With certain abstractions of the market place, we created a game and analyzed the behavior of traders under a situation when there are two active spoofers.

We were inspired to create this project from scratch because of our shared interests in game theory and understanding human behavior in dynamic environments.

The markets are perhaps the most dynamic environment there is and we wanted to build a tool that could help traders strategize their activities, to bring in new rationality to their behaviors, especially when known events and market forces are seen again.

2 Background

In this section we will discuss concepts and work related to this project. We will first describe what we mean by *no-regret learning* and the *regret-minimization problem*. Then, we will give an overview of Multiplicative Weights and EXP3 algorithms and explain the reasons for selecting these algorithms within our framework. We will then discuss the zero-intelligence market model, and why it is desirable for the purposes of our project. Finally, we give an overview of our architectural considerations while designing the tool.

2.1 The Regret-Minimization Problem

An attractive feature of *no-regret dynamics* in multi-player games is their rapid convergence to an approximate equilibrium [5]. Let us begin by looking at the *regret-minimization problem*, which studies a single decision-maker playing a game against an adversary. Then we will connect the single-player setting to multi-player games.

The following is the setup for a set of actions A. The number of actions $n \geq 2$. [5]

- At time $t = 1, 2, ..., T$:
  - A decision maker picks a mixed strategy p^t - that is, a probability distribution over its actions A.
  - An adversary picks a cost vector $c^t : A- > [0, 1]^1$.
  - An action a^t is chosen according to the distribution p^t, and the decision maker incurs cost $c^t(a^t)$. The decision maker learns the entire cost vector c^t, not just the realized cost $c^t(a^t)$.

Definition 2.1.1 (Regret) The time-averaged *regret* of the action sequence $a^1, ..., a^T$ with respect to the action a is $\frac{1}{T}[\sum_{t=1}^{T} c^t(a^t) - \sum_{i=1}^{T} c^t(a)]$. [5]

Definition 2.1.2 (No-Regret Algorithm) Let α be an online decision-making problem.

1. An adversary for α is a function that takes as input the time t, the mixed strategies $p^1, ..., p^t$ produced by α on the first t times, and the realized actions $a^1, ..., a^{t-1}$ of the time $(t-1)$ times, and produces as output a cost vector $c^t : [0,1]- > A$.

2. An online decision-making algorithm has *no regret* if for every adversary for it, the expected regret with respect to every action $a \in A$ is $o(1)$ as $T- > \infty$. [5]

Result 2.1.1 (Regret Lower Bound) With n actions, no algorithm has expected regret vanishing faster than $\Theta(\sqrt{(\ln n)/T}$. [5]

2.2 The Multiplicative Weights Algorithm

The design of Multiplicative Weights algorithm has the following design principles: [5]

1. Past performance of actions should guide which action is chosen now.

2. Many instantiations of the above idea yield no-regret algorithms. For optimal regret bounds however, it is important to aggressively punish bad actions - when a previously good action turns sour, the probability with which it is played should decrease at an exponential rate.

The formal description of the algorithm is as follows: [5]

1. Initialize $w^1(a) = 1$ for every $a \in A$.

2. For $t = 1, 2, ..., T$:

   (a) Play an action according to the distribution $p^t = w^t/\tau^t$, where $\tau^t = \sum_{a \in A} w^t(a)$ is the sum of the weights.

   (b) Given the cost vector c^t, decrease weights using the formula $w^{t+1}(a) = w^t(a) * (1 - \epsilon)^{(c^t(a))}$ for every action $a \in A$.

   ϵ is the learning rate, which will be between 0 and $\frac{1}{2}$.

The fundamental reason for using Multiplicative Weights as our no-regret learning algorithm is that the (per-time-step) regret is at most $2\sqrt{(\ln n)/T}$ [5], which is the regret lower bound result from **Result 2.1.1**. The learning rate parameter ϵ is chosen as $\epsilon = \sqrt{(\ln n)/T}$, which achieves the regret lower bound on the algorithm.

Remark 2.2.1 (When T is Unknown) $\epsilon = \sqrt{(\ln n)/\hat{T}}$, where $\hat{T}$ is the smallest power of 2 greater than or equal to t.

Now we will extend Multiplicative Weights from single-player to multi-player settings. In each time step $t = 1, 2, ..., T$ of no-regret learning in a multi-player setting (*no-regret dynamics*):

1. Each player i simultaneously and independently chooses a mixed strategy p_i^t using a no-regret algorithm.

2. Each player i receives a cost vector c_i^t, where $c_i^t(s_i)$ is the expected cost of strategy s_i when the other players play their chosen mixed strategies.

2.3 The EXP3 Algorithm

The EXP3 Algorithm works within the *MultiArmed Bandit* (MAB) problem framework. In this model, there is a set of N actions from which the player has to choose in every time step $t \in T$. After that action is chosen, the player can see the loss of that action, but not of that of the other possible actions. An example of the MAB problem includes choosing a route to take to work, where the driver does not know the losses of routes outside of the route that they end up taking. The EXP3 Algorithm is one of the regret minimization algorithms within the category of MAB problems.

The general idea of the EXP3 algorithm is the following: [6]

- At time t, the player chooses actions according to the distribution p_t - i.e., choose action i with probability $p_t(i)$.
- The player receives a profit for action i of $g_t(i)$. (In EXP3, the objective is to maximize profits instead of minimizing losses.)
- The player updates the estimate for the sum of profits of action i by $\frac{g_t(i)}{p_t(i)}$.

The expectation of the profit of i is roughly the sum: $E[\text{estimate of } i\text{'s profit}] = \sum_t \frac{g_t(i)}{p_t(i)}$

The EXP3 algorithm is based on the exponential weights regret minimization algorithm. We have a learning parameter $\eta \in [0, 1]$. Each action is originally assigned a weight $w_i = 1$, i.e., $\forall i \in [1, 2, ..., N], w_i = 1$. W_t is defined as the sum of the weights of the actions at time t: $W_t = \sum_{i=1}^{N} w_i(t)$. Then, W_t is used to normalize the probabilities p_i of actions. [6]

At time t, [6]

1. $p_i(t) = (1 - \eta)\frac{w_i(t)}{W_t} + \eta\frac{1}{N}$, i.e., $p_i(t)$ is proportional to the relative weight of i with a small correction to ensure that p_i is never too close to 0.
2. Player chooses an action i_t (a random variable) according to the distribution $p_1(t), ..., p_N(t)$.

3. The player receives a profit $g_i(t) \in [0, 1]$.

4. Define $\hat{g}_j(t) = \{ \; g_j(t)p_j(t), if j = i_t$

   $0, otherwise$

5. Update the weights of the actions: $w_j(t+1) = w_j(t)e^{\eta \hat{g}_j(t)/N}$

In short, the weights give an estimate of how good the actions are. Actions get chosen with probability relative to their weights, and the weights are updated in an exponential fashion.

Remark 2.2.1 (Regret Bound of the EXP3 Algorithm) EXP3 reaches a regret bound of $O\sqrt{T}$ by combining the exploration and exploitation stages of the MAB algorithm. [6]

2.4 The Zero-Intelligence Market Model [3]

A significant amount of time was devoted to finding the right market model to test our no-regret learning framework on within the context of trading activities. The most significant issue faced was the question of how to account for market impact of executed strategies. This quest to understand a market model which would allow us to operate under conditions of predictable market impact of our trades, or at least one that would allow us to not assume our cost of market activities, led to adapting the *zero-intelligence model*, also known as the *Santa-Fe Model*. The model also does a good job of predicting the average spread, and the price diffusion rate. [3]

The main components of the *zero-intelligence* market model will be described in the sections below.

2.4.1 Continuous Double Auction [3]

The *zero-intelligence* market assumes a continuous double auction, the most widely used method of price formation in modern electronic markets. The orders are categorized into two types: *market orders*, which are requests to buy or sell a desired number of shares immediately at the best available price, i.e., the market quote. *Limit orders* are the other type of orders which specify a limit below (or above) which a sell (or buy) order will not execute. Limit orders may not result in an immediate transaction, in which case are stored in a queue called the *limit order book* (see Figure 1). As each buy order arrives it is transacted against accumulated sell limit orders that have a lower selling price, in priority of price and arrival time, and vice-versa for sell orders. The *best ask,* $a(t)$ is the lowest selling price offered in the book at any point in time. The *best bid,* $b(t)$ is the highest buying price offered in the book at any point in time.

Figure 1: A random process model of the continuous double auction. Stored limit orders are shown stacked along the price axis, with sell orders (supply) stacked above the axis at higher prices and buy prices (demand) stacked below the axis at lower prices. New sell limit orders are visualized as randomly falling down, and new buy orders as randomly falling up. New sell orders can be placed anywhere above the best buying price, and new buy orders anywhere below the best selling price. Limit orders can be removed spontaneously (e.g. because the agent changes their mind or the order expires) or they can be removed by market orders of the opposite type. This can result in changes in the best prices, which in turn alters the boundaries of the order placement process. It is this feedback between order placement and price formation that makes this model interesting, and its predictions non-trivial. [3]

2.4.2 Review of the Model [3]

The model calls market order placing agents as *impatient agents*, and limit order placing agents as *patient agents*. It assumes that they are zero intelligence agents that place and cancel orders randomly, as shown in Figure 1. Impatient agents place market orders of size σ, which arrive at a rate μ *shares per time*. Patient agents plae limit orders of the size σ, which arrive with a constant rate density α *shares per price per time*. Queued limit orders are canceled at a constant rate δ, with dimensions of $1/time$. Prices change in discrete increments called *ticks*, of size *dp*. The rates of buying and selling are considered to be equal, and limit/market order placement and cancellation processes are Poisson processes. All of these processes are independent except for coupling through their boundary conditions: Buy limit orders arrive with a constant density α over the interval $-\infty < p < a(t)$, where p is the logarithm of the price; and sell limit orders arrive with a constant density α over the interval $b(t) < p < \infty$. As a result of the random order arrival processes, *a(t)* and *b(t)* each make random walks, but because of coupling of the buying and selling processes the bid-ask *spread* $s(t) = a(t) - b(t)$ is a stationary random variable.

New orders may alter the best prices *a(t)* and *b(t)*, which in turn changes the boundary conditions for subsequent limit order placement. For example, the arrival of a limit buy order inside the spread will alter the best bid *b(t)*, which then alters the boundary condition for sell limit order placement, and vice-versa. This feedback between order placement and price diffusion makes this model interesting.

2.4.3 Assumptions of the model

The model makes some strong assumptions.

- It assumes that the rates for buying and selling are equal.

- The sizes of limit and market orders are the same.
- Limit order deposition is uniform on semi-infinite intervals.
- Rates of order submissions are unaffected by price changes.

The only participants in this market are zero-intelligence noise traders that randomly supply and demand liquidity. This model is intended to be used as primarily to provide a background market to execute trades on, where the question of market impact is readily answered and without assumptions.

2.5 Architectural Considerations while designing the tool

Our main tool consists of the framework that allows single- and multi-player games to be played and analyzed. The main design choices implemented -

- Automatic input of the game participants, its number, strategies available per participant, and payoff matrix through files.
- Ability to spawn multiple threads per round wherein each player could independently implement their next action.

Our implementation of the zero-intelligence market model makes the following design and architectural considerations and assumptions -

- Prices were assumed to be in absolute units and not logarithmic.
- Order size of market and limit orders (σ) were assumed to be unit sized.
- Tick size of 1 price unit was used.
- 5 units of price from the best market price of both ask and bid were used to place the minimum and maximum range of bids and sells, respectively. For example: if the lowest ask $a(t) = 104$, then bids $(\underline{t})$ could be in the range $98 < b(t) < 104$,

spaced by tick size. Similarly, if the highest bid $b(t) = 102$, then asks t) could be in the range $102 < a(t) < 108$, spaced by tick size.

- Extensive use of multi-threaded programming and multi-programming while executing the zero-intelligence market model.

3 Our No-Regret Framework

In this section, we describe our tool for finding optimal strategies in single- and multi-player games, followed by an in-depth look at our design of the zero-intelligence market model. We also describe the nature of input to our tool.

The development of the tool took place in stages. We started off by creating a manual-input version for deciding payoffs/costs per strategy while playing a game, eventually moving on to file input where information about the payoff/cost matrix, number of players and number of strategies per player is pre-written. Here, we will only describe our finished tool where multiple players execute their strategies and update actions' weight matrices per time step in independent threads, with input being fed from files.

The framework is the same for both Multiplicative Weights and the EXP3 algorithms, with the underlying changes only being with respect to the learning parameters used (ϵ and η, respectively) and the means with which the players' weights for strategies are changed for the next round. Finally, in order to visualize which strategies are winning out, for each player, at each time step, we normalize weights for each strategy.

3.1 Structure of the Input File

Each game has a corresponding input file that stores the following information:

- Number of strategies per player (N) - at the beginning of the game, this gets read into a variable.
- Number of players ($NumPl$) - at the beginning of the game, this gets read into a variable.

- Cost(Payoff) Matrix describing what the cost(payoff) would be for Multiplicative Weights(EXP3) per player per strategy depending on what the other players' strategies are. It is of dimensions (N^{NumPl} x $NumPl$). This gets stored into a multi-dimensional cost(payoff) array during the initialization of the game.

3.2 Objects Used to Store Information

We employ the following classes to store information about and execute the game -

1. **Environment** Class - the environment that runs the game. Has the following data variables and member function -
   (a) Number of strategies per player (N)
   (b) Number of players ($NumPl$)
   (c) Cost(Payoff) Matrix in a multi-dimensional array
   (d) generateRewards() function - this function will, based on strategies of all players during t^{th} round, return an array of cost(payoff) values per player.
2. **Player** Class - used to store each players' strategies and how they are performing. Has the following data variables and member functions -
   (a) Total time for which the game will be played (T).
   (b) Number of strategies (N).
   (c) Weight Matrix for actions ($w_{N \times T}$)
   (d) Normalized Weight Matrix for actions ($W_{N \times T}$)
   (e) **(Only for EXP3)** A Probability array of size $(1 \times N)$ proportional to the relative weight of each strategy.
   (f) Learning parameter (ϵ and η corresponding to Multiplicative Weights and EXP3, respectively). $\epsilon = \sqrt{\ln N/T}$ and $\eta = 10 * \sqrt{(N * \ln N)/(2 * T)}$. The

choice of ϵ and η are picked based on no-regret bound theorems described in **Result 2.1.1** and **Theorem 4.1** of [6]. The factor 10 is added to η to make the weight plots more prominent, and serves no theoretical purpose.

(g) pickStrategy() function - picks a strategy according to the probability distribution discretized over weights of individual actions. **(For EXP3, directly use the probability array belonging to the player. For Multiplicative Weights, generate the probability distribution over the individual weights within function.)**

(h) changeWeight() function - Change weights of strategies for the $(t+1)^{th}$ step depending on the cost(payoff) incurred during the t^{th} step. **(Note that Multiplicative Weights and EXP3 differ in *exploration* vs. *exploitation* [6] in that the former updates the weights of all strategies not taken per the strategies taken by the other players, whereas the latter only updates weight for the strategy taken in the previous step. Multiplicative Weights is an example of *exploration*; EXP3 is an example of *exploitation*.)** Finally, normalize the weights in this function so as to visualize the effect of playing the game over time on deriving winning/optimal strategies.

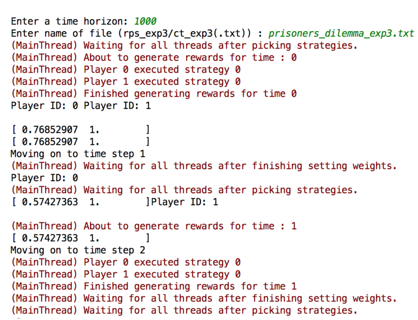
3.3 Framework for playing the game

- Initialize the environment and player objects based on input from the file storing game structure, and time over which to analyze strategies (provided manually).
- At each time step t -
  1. Spawn $NumPl$ number of threads, each for a player to pick their strategy for time step t (call pickStrategy() function for each player).

2. Wait for all threads to join, at which point generate the rewards (call generateRewards() function of environment) for each player based on the actions chosen. It is important for threads to join before changing weights for two reasons -
   (a) Time step for each player remains the same, i.e., all players move from time t to $t+1$ only after they have all finished picking strategies.
   (b) Multiplicative Weights needs to know the information for strategies picked by other players in order to set weights of actions not taken at time step t (*exploration* [6]).
3. Spawn another set of $NumPl$ threads, each for a player to change weights of their actions (call changeWeights() function for each player).
4. Wait for all threads to join, then move to the next time step.

- When T time steps have been played for each player, then finish the game and show the results of players' game theoretic behavior through plotting the normalized plots for their individual weight strategies per time step.

A snapshot of execution of the game is presented in Figure 2 below.


```
Enter a time horizon: 1000
Enter name of file (rps_exp3/ct_exp3(.txt)) : prisoners_dilemma_exp3.txt
(MainThread) Waiting for all threads after picking strategies.
(MainThread) About to generate rewards for time : 0
(MainThread) Player 0 executed strategy 0
(MainThread) Player 1 executed strategy 0
(MainThread) Finished generating rewards for time 0
Player ID: 0 Player ID: 1

[ 0.76852907  1.        ]
[ 0.76852907  1.        ]
Moving on to time step 1
(MainThread) Waiting for all threads after finishing setting weights.
Player ID: 0
(MainThread) Waiting for all threads after picking strategies.
[ 0.57427363  1.        ]Player ID: 1

(MainThread) About to generate rewards for time : 1
[ 0.57427363  1.        ]
Moving on to time step 2
(MainThread) Player 0 executed strategy 0
(MainThread) Player 1 executed strategy 0
(MainThread) Finished generating rewards for time 1
(MainThread) Waiting for all threads after finishing setting weights.
(MainThread) Waiting for all threads after picking strategies.
```

Figure 2: Behavior of our tool as players pick and execute strategies from time step 0 to time step 1. Note the messages in red show multi-threaded programming in action; The vectors shown are weight matrices of Player0 and Player1 (there is a time lag to standard output due to different thread executions simultaneously hence the misplacement of messages with respect to the weight vectors on the main method).

4 Our implementation of the Zero-Intelligence Market

We employ the following classes to interface directly with the market -

1. **Limit Order Book** Class - the limit order book of the market. Consists of the following data members and member functions -

    (a) Limit Buys - Will store the limit buy order queue. Limit buys are 3-dimensional vectors of (price, time, order id). They will be arranged in decreasing order of price and increasing order of time, i.e., the highest bid offer at the earliest time will be put in front of the queue.

    (b) Limit Sells - Will store the limit sell order queue. Limit sells are 3-dimensional vectors of (price, time, order id). They will be arranged in increasing order of price and increasing order of time, i.e., the lowest sell offer at the earliest time will be put in front of the queue.

    (c) Limit Buy Count - to keep count of the number of outstanding limit buy offers that have not yet expired.

    (d) Limit Sell Count - to keep count of the number of outstanding limit sell offers that have not yet expired.

    (e) addLimitOrder() - a function to add a new limit order to the limit order book.

    (f) delLimitBuy() - a function that matches a market sell order with the limit buy atop the limit buys queue and deletes the limit buy count by 1.

    (g) delLimitSell() - a function that matches a market buy order with the limit sell atop the limit sells queue and deletes the limit sell count by 1.

    (h) getHighestBid() - returns the highest and earliest bid at any point in time.

(i) getLowestAsk() - returns the lowest and earliest ask at any point in time.

(j) showLOB() - show the contents of the limit order book.

2. **Limit Order** Class - the limit order generated by each market participant. Consists of the following data members and member functions -

   (a) id - to store the order identity (distinguish between orders).

   (b) buyOrSell - indicate whether it is a buy or sell order by an indicator variable passed to each Limit Order class object.

   (c) price - the price for bid or ask.

   (d) timeBegan - when the order originated.

   (e) cancellationTime - for how long will the order stay.

   (f) orderSize - the size of the order (constant σ used).

   (g) endTime - $timeBegan + cancellationTime$. Indicates when the limit order expires.

   (h) lob - the Limit Order Book object where limit orders are being submitted and stored.

   (i) showCharacteristics() - a function to show the characteristics of the limit order.

   (j) poke() - a function that spawns a background thread per limit order that keeps poking the limit order book 'lob' to check whether or not the limit order has expired. If it has, it removes the corresponding order from the limit order book.

3. **Trader** Class - to generate limit orders based on trading strategies. This is a basic class in which the trader will, based on the limit order book information, implement a trading strategy and generate a limit order on the buy or sell side. Its data members and member functions include -

(a) lob - the Limit Order Book object where limit orders are being submitted and stored.

(b) generateLimitOrder() - function to generate a limt order based on specific trading strategies informed by the limit order book.

Note: The Trader class handles simple trading strategies and interfaces directly with the market; however due to being unable to account for calculating proper cost(reward) functions within the zero-intelligence framework, it is being kept open for future extension as well as use within our larger no-regret framework.

4.1 The Zero-Intelligence Market

Definition 4.1.1 A Poisson process is $F(t) = 1 - e^{-\lambda t}$, where t is the elapsed time and $F(t)$ is the probability that the poisson process will happen t time units into the future. λ is the rate parameter of the Poisson process.

Definition 4.1.2 If a process follows the Poisson process, then the time for the next event is : $t = -\ln(1 - F(t))/\lambda$, $F(t) \in [0.0, 1.0)$, λ is the rate parameter.

Limit buy orders, limit sell orders, market buy orders and market sell orders each have their own Poisson process for generating orders, which means they have their own rate parameter λ. Similarly, the order cancellation rate for each limit order is a separate Poisson process with its own rate parameter λ.

4.1.1 Initializing the Market

First, select values for lowest ask and highest bid, and the rate parameters for Poisson processes. Initially fill in the limit order book before taking in market orders. In order to do this, for a certain period of time, say 20 seconds, we do the following in a loop:

1. Create multiple processes to find the next time that a limit buy and limit sell is generated, using their respective λ's and using **Definition 4.1.2**.

2. Wait for both processes to join.

3. Pick the earliest between the two.

4. Now, since limit orders are poisson processes over (price x time), once the time is picked, it is time to pick a price. We select a range of prices to be picked uniformly over a discrete distribution described in **Section 2.5**.

5. Add the picked process to the limit order book.

At this point the limit order book is filled (Figure 3).

4.1.2 Creating the rest of the market processes

Once the market has been initialized, it is time to generate the rest of the zero-intelligence market activity. Do the following in an infinite loop:

1. Create multiple processes to find the next time that a limit buy, limit sell, market buy and market sell is generated, using their respective λ's and using **Definition 4.1.2**.

2. Wait for all 4 processes to join.

3. Pick the earliest among the 4, and then spawn another thread to create the next market or limit order based on the picked process to be generated next. That thread has to join in before the next iteration of the loop continues.

   (a) If it is a market sell that was picked, then simply delete the first entry within the limit buy queue (Figure 4).

   (b) If it is a market buy that was picked, then simply delete the first entry within the limit sell queue (Figure 5).

(c) If it is a limit sell that was picked, then if matched with a limit buy, delete the first entries of both limit buy and limit sell queues (Figure 6). Vice-versa for limit buy.

(d) If a limit sell or limit buy wasn't matched, then add to the appropriate position within the corresponding queue in the limit order book (Figure 7). In this case, we also spawn a thread to begin the poke() process of the limit order (sell or buy). This process will run in the background until either the limit order gets matched with a market or limit order of the opposite direction, or reaches its expiration time (the order cancellation rate is used to calculate the cancellation time for a limit order). **Note:** The price picking process for limit orders is similar to the one described for limit orders during the initialization phase in **Section 4.1.1**.

We also have a separate thread running in the background that pokes prospective traders (each with their own trading strategies) for their orders. We have provided the framework for traders to execute their trading strategies within this zero-intelligence market once questions about calculating the right rewards(payoffs) per trading strategy are answered. For the time-being, we only describe a fully-functioning zero-intelligence market.

The figures following (3-7) show various functions during execution of the zero-intelligence market.

```
At the end of steady state. Highest Bid = 102.000000, Lowest Ask = 104.000000, Spread = 2.000000
Limit Buy Orders.....
[[  1.02000000e+02   5.80170155e-02   5.00000000e+00]
 [  1.02000000e+02   3.61821413e-01   3.40000000e+01]
 [  1.02000000e+02   6.52333975e-01   5.60000000e+01]
 ...,
 [  9.70000000e+01   1.95589666e+01   1.84400000e+03]
 [  9.70000000e+01   1.96977923e+01   1.86200000e+03]
 [  9.70000000e+01   2.02436769e+01   1.87900000e+03]]
Limit Sell Orders.....
[[  1.04000000e+02   1.82158232e-01   1.20000000e+01]
 [  1.04000000e+02   2.48025894e-01   1.90000000e+01]
 [  1.04000000e+02   2.55534649e-01   2.60000000e+01]
 ...,
 [  1.09000000e+02   1.98963382e+01   1.88300000e+03]
 [  1.09000000e+02   1.99773488e+01   1.88700000e+03]
 [  1.09000000e+02   2.00495431e+01   1.89000000e+03]]
Total buy orders = 937, Total sell orders = 960
```

Figure 3: Initializing the zero-intelligence market with limit orders.

```
Previous highest bid = 102.000000, Previous lowest ask = 104.000000, Previous spread = 2.000000
Current highest bid = 102.000000, Current lowest ask = 104.000000, Current spread = 2.000000
```

Figure 4: Market sell order generated and it simply removes the first element in limit buy queue. The highest bid, lowest ask, and spread values are then updated.

```
Previous highest bid = 102.000000, Previous lowest ask = 104.000000, Previous spread = 2.000000
Current highest bid = 102.000000, Current lowest ask = 104.000000, Current spread = 2.000000
```

Figure 5: Market buy order generated and it simply removes the first element in limit sell queue. The highest bid, lowest ask, and spread values are then updated.

```
Waiting....Iteration 31
Previous highest bid = 102.000000, Previous lowest ask = 104.000000, Previous spread = 2.000000
Limit sell price = 102.000000
Current highest bid = 102.000000, Current lowest ask = 102.000000, Current spread = 0.000000
Finished waiting...Iteration 31
Matched limit buy with limit sell...
Deleting Highest bid = 102.000000, Deleting Lowest sell = 102.000000
New Highest bid = 102.000000, New Lowest sell = 104.000000
```

Figure 6: Showing Matching of limit buy with limit sell, and updating of the highest bid, lowest ask values.

```
Iteration 43, Market Order Buy Time = 0.153750, Market Order Sell Time = 0.045400, Limit Order Buy
Previous highest bid = 102.000000, Previous lowest ask = 104.000000, Previous spread = 2.000000
Limit sell price = 107.000000
Current highest bid = 102.000000, Current lowest ask = 104.000000, Current spread = 2.000000
Waiting....Iteration 43
Finished waiting...Iteration 43
No trading took place...
```

Figure 7: A limit order was generated but it was pushed down in the limit order book queue and no trading took place.

5 Results

5.1 Rock-Paper-Scissors

Rock-Paper-Scissors is a popular hand-game that is played between two players, each having three strategies - Rock, Paper and Scissors. The structure of the game is -

- Paper beats Rock
- Scissors beats Paper
- Rock beats Scissors

Accordingly, we set our game and payoff matrix and run our two no-regret learning algorithms to find optimal strategies in both players.
In our simulation, we set -

- Strategy0 to be Rock
- Strategy1 to be Paper
- Strategy2 to be Scissors

5.1.1 Multiplicative Weights

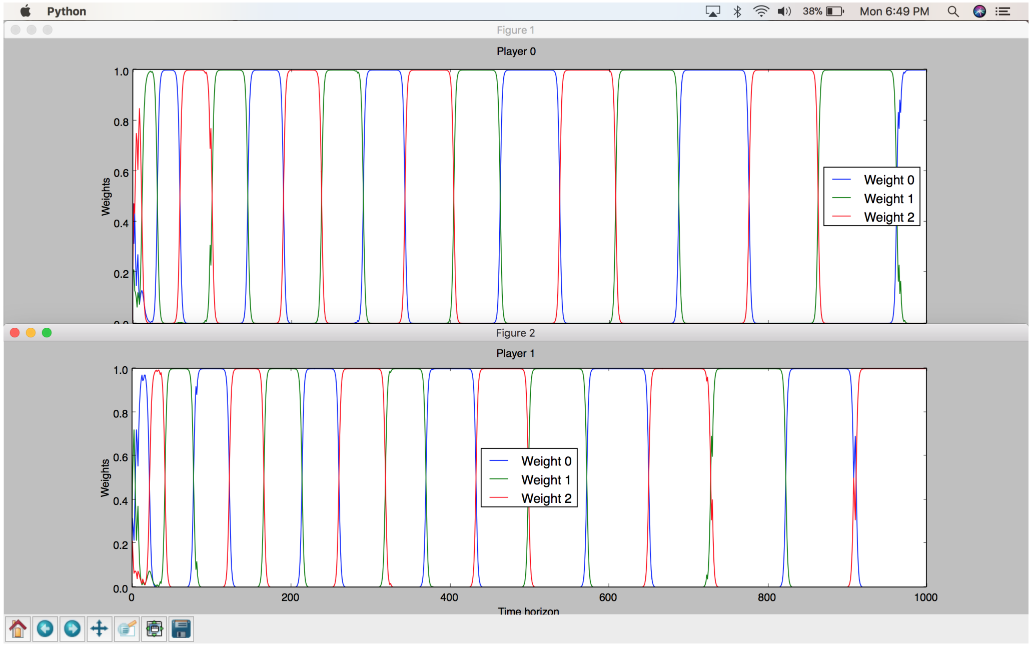


Figure 8: Multiplicative Weights Algorithm results on Rock-Paper-Scissors game. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1, **Weight 2** corresponds to the weight of Strategy2.
We note that both players exhibit a cyclical behavior in their strategies picked over time. As one player is strong on Scissors, the other player begins to realize that the winning strategy is to pick Rock. As the latter's Rock strategy begins to win more often, the first player begins to realize that Paper could beat the opposing player's Rock. Scissors then beats Paper, and this cycle repeats itself until the end of the simulation, leading to an optimal game strategy for Rock-Paper-Scissors.

5.1.2 EXP3

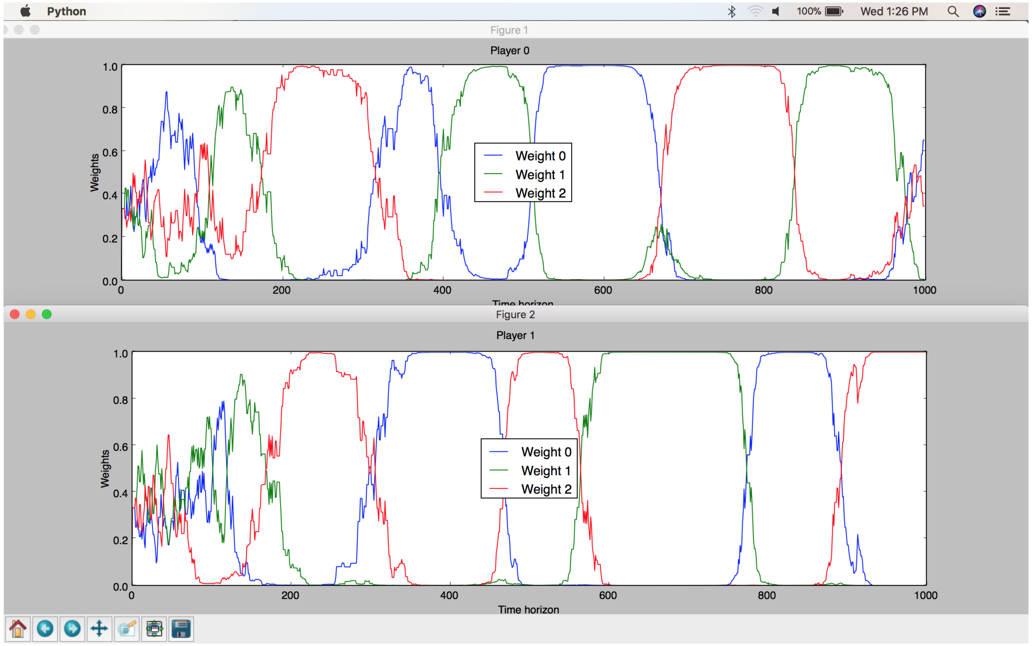


Figure 9: EXP3 Algorithm results on Rock-Paper-Scissors game. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1, **Weight 2** corresponds to the weight of Strategy2.
We note that both players exhibit a cyclical behavior in their strategies picked over time. As one player is strong on Scissors, the other player begins to realize that the winning strategy is to pick Rock. As the latter's Rock strategy begins to win more often, the first player begins to realize that Paper could beat the opposing player's Rock. Scissors then beats Paper, and this cycle repeats itself until the end of the simulation, leading to an optimal game strategy for Rock-Paper-Scissors.
Note that the EXP3 graphs are a little less smooth compared to Multiplicative Weights' for the same game, but the cyclical behavior remains the same.

5.2 Prisoner's Dilemma

Prisoner's Dilemma is a famous game analyzed in game theory. It consists of two prisoners (players) held without sufficient evidence on a crime, each with two strategies (cooperate or do not cooperate with the police). The cooperation or lack thereof of one prisoner is with regards to informing on the other prisoner of a more serious crime. The structure of the game is as follows [1]-

- If both prisoners cooperate with the authorities and inform on the other, then they each get a jail sentence of 2 years.

- If prisoner A cooperates and prisoner B does not cooperate, then prisoner A will be set free and prison B will get a jail sentence of 3 years. And vice-versa.

- If neither prisoners cooperate, then they each get a jail sentence of 1 year on lesser charges.

The context of the game is that the police do not have sufficient evidence to indict either prisoner on a more serious offense. Also the prisoners have no means of communicating with each other, and it is further implied that their actions will have no consequences outside of the prison.

If the prisoners were purely rational and acted in self-interest, then they would each cooperate with the police and betray the other. However, if both acted in self-interest, then the outcome for both would be worse than if they had cooperated with each other (or not cooperated with the police). Analysis by game theory shows that the optimal strategy for both player is to not cooperate with the police, even though pursuing their own self-interest says otherwise.

Our no-regret framework for prisoner's dilemma produces the same results as expected from its game theory analysis.

In our simulation, we set -

- Strategy0 - Cooperate with the police

- Strategy1 - Not Cooperate with the police

5.2.1 Multiplicative Weights

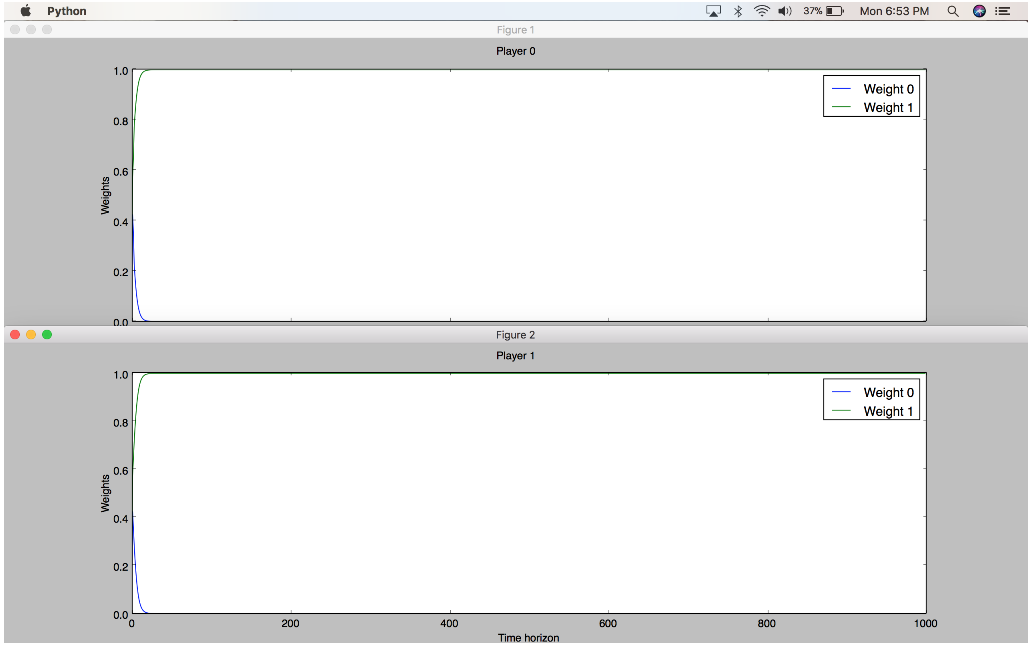


Figure 10: Multiplicative Weights Algorithm results on Prisoner's Dilemma game. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1. As shown above, both prisoners decide to pursue Strategy1 (co-operate with each other, not cooperate with the police, and to not pursue individual self-interest).

5.2.2 EXP3

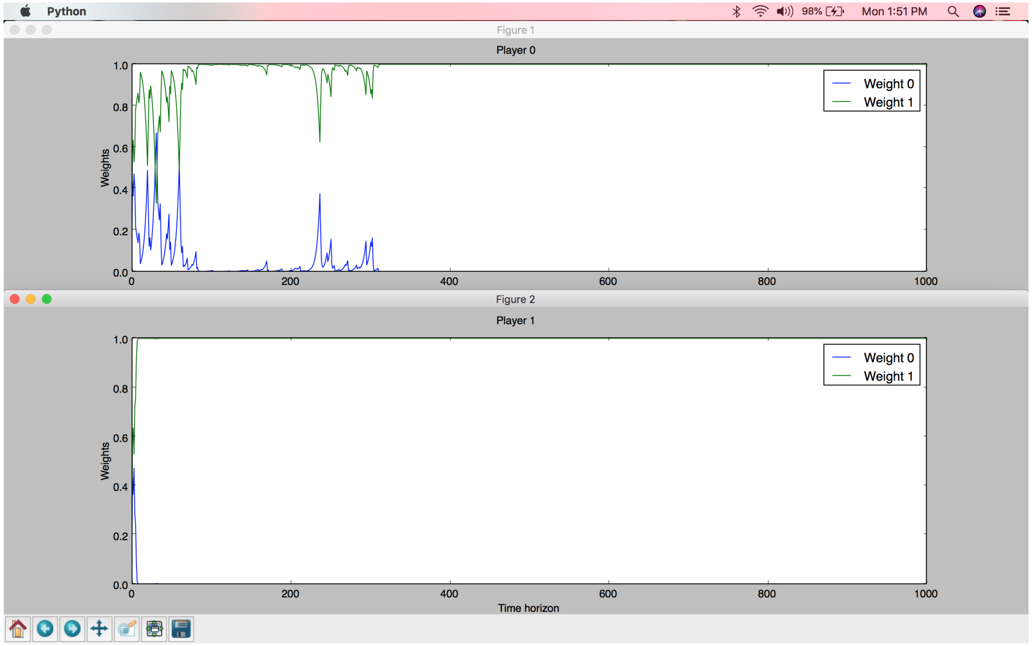


Figure 11: EXP3 Algorithm results on Prisoner's Dilemma game. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1. As shown above, both prisoners decide to pursue Strategy1 (cooperate with each other, not cooperate with the police, and to not pursue individual self-interest).

5.3 Abstract Two Spoofer game

Here, we define an abstract game of two spoofers trying to manipulate the markets to show proof of concept of our no-regret framework. Spoofers are market manipulators who try to increase the selling price of a security by creating a fake buy side order of large volume, in order to entice market participants to increase their buying pressure. The spoofer hopes to take advantage of this increase in demand to sell the security at a higher price than the one it is currently trading on. Lastly, the spoofer cancels the fake buy order so as to not be on the hook to buy when they had no intention of buying.

Due to the aforementioned issues with deciding proper reward structure per trading strategies within our zero-intelligence market, including being unable to settle on how to represent real-world market dynamics, we decided to create an abstract game in which two spoofers try to spoof the same security at the same time. We set a very simple payoff structure and set spoofers' actions depending on how they see the spoof is going.

- Spoofer sees that the spoof is going awry (the market is not behaving as they expected), so instead of being on the hook for buying when they didn't intend to, decides to cancel their spoof after a certain time. This is **Strategy0** in our simulation.

- Spoofer sees that the spoof is going well (the market is behaving as they expected), so they decide to let their spoof go through. This is **Strategy1** in our simulation.

- Spoofer sees a mixed reaction in the markets, maybe due to the other spoofer's large order which is making the market suspicious. They decide to settle on a strategy to cancel a portion of their spoof order while having had to buy some of their fake orders since the market behaved unexpectedly. This is **Strategy2**.

We use our no-regret framework as proof of concept that shows that it is in the best-interest of both spoofers to either execute **Strategy1**, or enter into a compromise between them (one or both of them executes **Strategy2**).

5.3.1 Multiplicative Weights

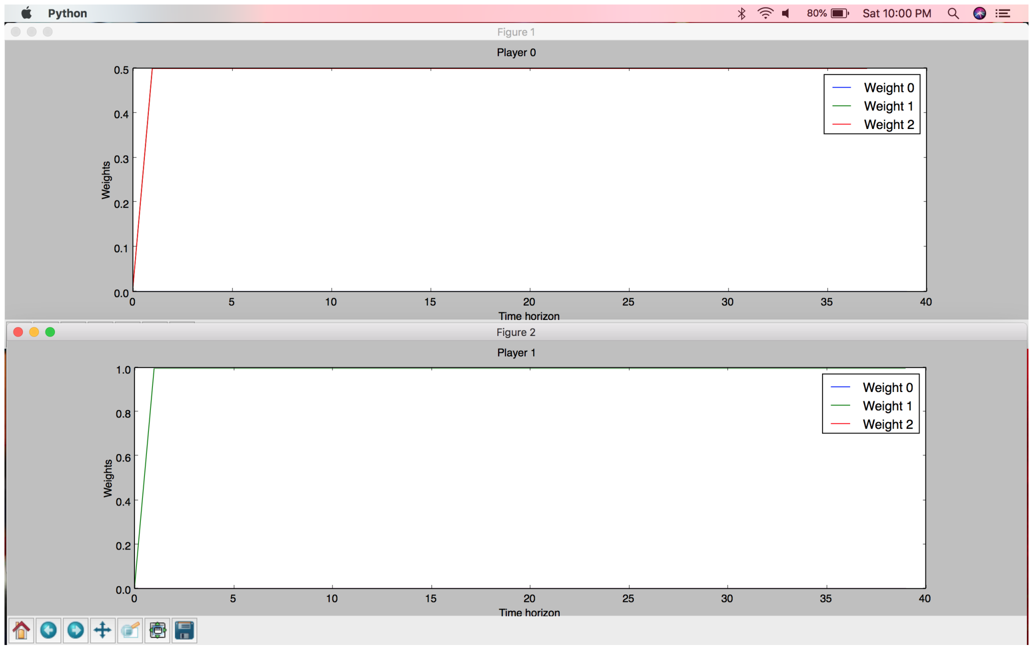


Figure 12: Multiplicative Weights Algorithm results on Two Spoofer game - result I. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1, **Weight 2** corresponds to the weight of Strategy2. Spoofer0 compromises while Spoofer1 goes for the win.

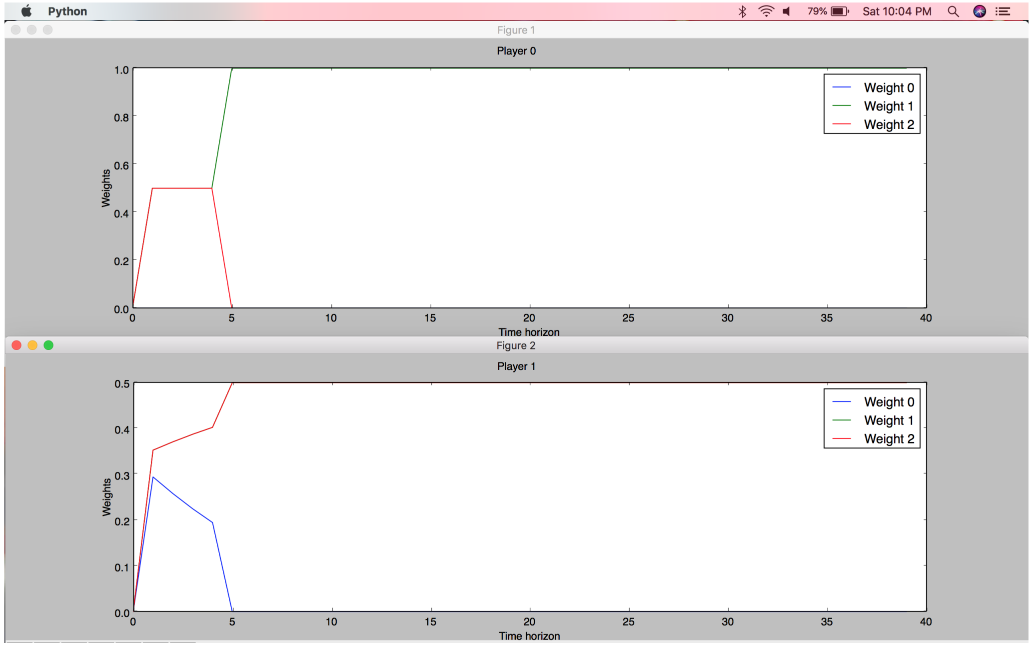


Figure 13: Multiplicative Weights Algorithm results on Two Spoofer game - result II. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1, **Weight 2** corresponds to the weight of Strategy2. Spoofer1 compromises while Spoofer0 goes for the win.

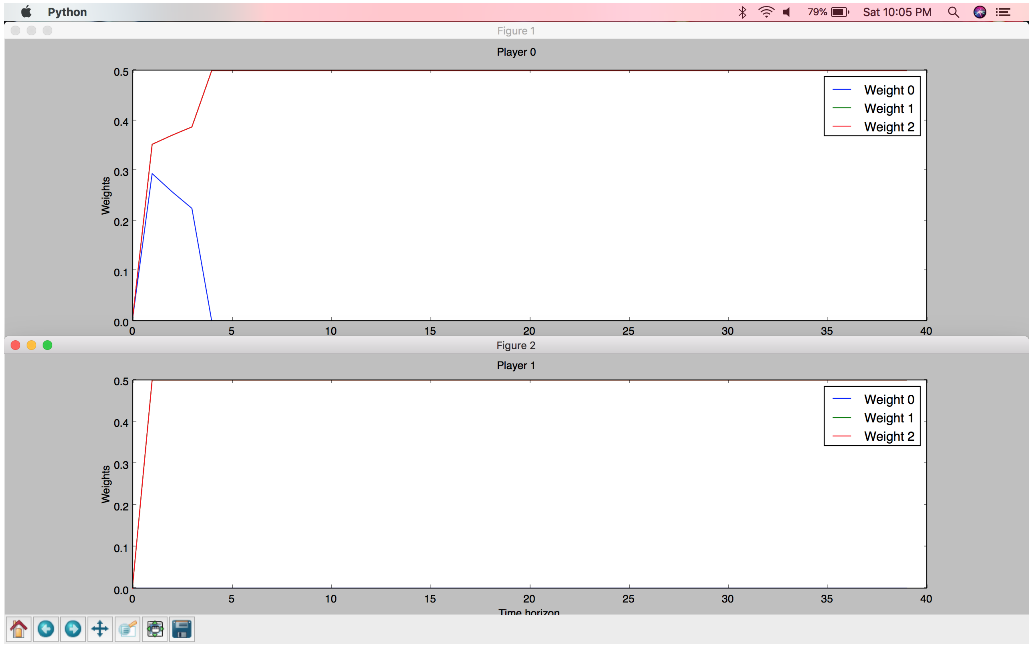


Figure 14: Multiplicative Weights Algorithm results on Two Spoofer game - result III. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1, **Weight 2** corresponds to the weight of Strategy2. Both spoofers decide to compromise here.

5.3.2 EXP3

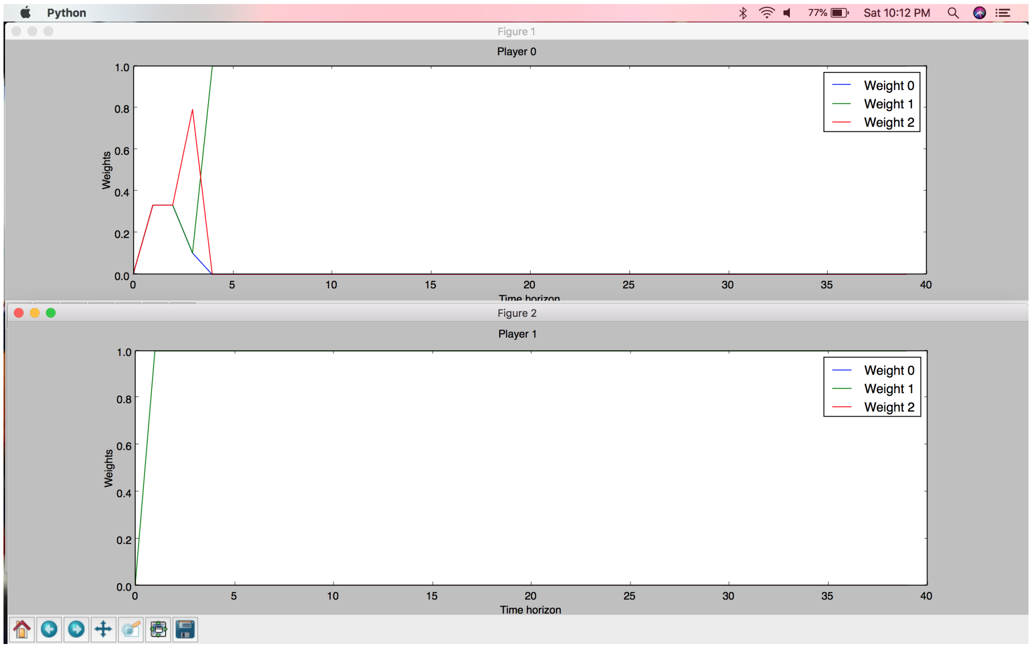


Figure 15: EXP3 Algorithm results on Two Spoofer game - result I. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1, **Weight 2** corresponds to the weight of Strategy2. Both spoofers decide to go for the win.

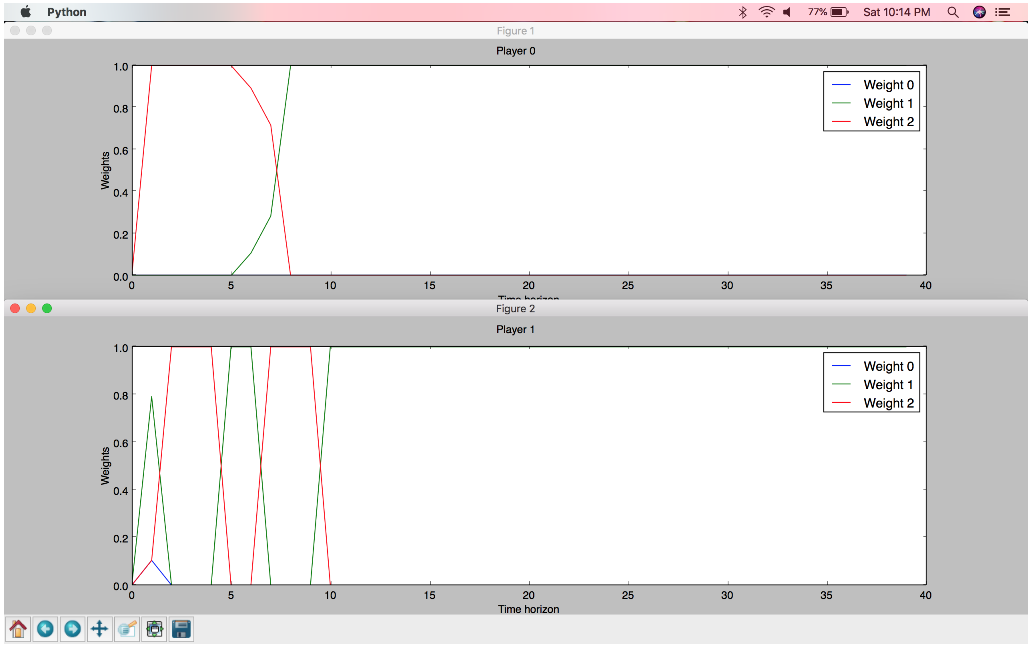


Figure 16: EXP3 Algorithm results on Two Spoofer game - result II. **Weight 0** corresponds to the weight of Strategy0, **Weight 1** corresponds to the weight of Strategy1, **Weight 2** corresponds to the weight of Strategy2. Both spoofers decide to go for the win.
Interesting to note the dynamics in this simulation between compromise and no compsomise, before finally setting on no compromise.

6 Conclusion

We have built a tool in which multiple players can derive optimal strategies while playing against one another. The tool can handle any kind of game. Using this tool, players would know that their strategy at each step is the best strategy and one that would lead them to the most optimal regret. We implemented Multiplicative Weights and EXP3 within our framework, based on the difference between *exploration* and *exploitation* [6], which sometimes could yield differing optimal strategies depending on what is most valuable to the player playing the game. We have further built a *zero-intelligence* market model in which trading actions could be simulated while measuring their impact in a way that is consistent with macroscopic market properties such as movement of the bid-ask spread, price diffusion (volatility) and dynamics of demand and supply [3]. With proper assumptions and heuristics, this market could be used to simulate trading activities that are scalable with real-world trading and risk measurement systems, but that work would involve further knowledge of stochasticity within market behavior and extensive financial engineering expertise. With the right heuristics in place, and under practical assumptions, real-world trading strategies could be simulated for market impact, whose rewards could then feed into the no-regret framework, which is the crux of this project. Nevertheless, we have reverse-engineered a simple spoofing mechanism under two-player setting and shown that the behavior of participants under conditions of no-regret are consistent with what we would expect.

7 Future Work

One possible extension of the no-regret framework is to combine it with any reward generating simulation/game, the latter portion being abstracted away and feeding directly into the former instead of using manually defined files pertaining to each game as input. This project has a lot of future potential within a practical financial engineering context. The underlying no-regret learning framework is correct and could be applied with other algorithms such as EXP4 [6]. The most challenging aspect then would be to use the created market model in such a way that trading strategies are measured appropriately and in a way that is scalable in practical trading and risk management systems. An interdisciplinary effort among financial engineers, economists and computer scientists could provide the right set of answers that we couldn't quite address during the duration of this project, particularly as it relates to what kind of heuristics to use to generate trading strategies, what is the right proportion of zero-intelligence to intelligent agents within the markets, how to scalably measure trades within limited trader interactions to the larger worldwide financial markets, how to account for the interactions among traders and zero-intelligence agents, etc. All of these were questions we dealt with for the longest period of time, but ultimately were unsuccessful in deriving answers that we felt made sense; hence we stopped trading directly within this market. Another important practical extension of this project would be the ability to directly interact between the market and the no-regret framework. Setting the right reward/payoff function for measuring trading strategies is another important point of future consideration. In the real world, utility functions are used to measure the viability and success of a trading strategy. Integrating that within this market framework in such a way that is consistent with the strongest concepts of financial engineering was a source of trouble for us, and one that we hope will be addressed. The market does however, handle simple strategies such as monitoring relative bids-to-asks ratio at the spread, and some heuristics like *Volume-Weighted*

Mid Price [2] and *Mean Volume-Weighted Mid Price* could be easily applied to generate a trading strategy. However, the issue regarding interactions with the random poisson processes of market and limit order generations, leading to unreliable calculations of reward functions, was hard to ignore. Future researchers could adequately find answers to these questions, and take this project to its full potential through complete integration with the underlying no-regret learning framework, so long as trading strategies among traders are handled within the context of games. Finally, the intention of this project is to be used as a portfolio and risk management tool, in which depending on market conditions, the right signal is generated corresponding to an already simulated strategy or set of strategies for the most optimal returns.

References

[1] Prisoner's dilemma. https://en.wikipedia.org/wiki/Prisoner%27s_dilemma.

[2] To spoof, or not to spoof, that is the question. http://www.automatedtrader.net/articles/strategies/155465/to-spoof--or-not-to-spoof--that-is-the-question, 2016.

[3] Ilija I. Zovko J. Doyne Farmer, Paolo Patelli. The predictive power of zero intelligence in financial markets. In *Proceedings of the National Academy of the National Academy of Sciences of the United States of America*. PNAS, 2005.

[4] Matthew Leising. Spoofing. https://www.bloomberg.com/quicktake/spoofing, 2017.

[5] Tim Roughgarden. No-regret dynamics. http://theory.stanford.edu/~tim/f13/l/l17.pdf.

[6] Shai Vardi Yishay Mansour. Multiarmed bandit in the adversarial model. https://www.tau.ac.il/~mansour/advanced-agt+ml/scribe4-MAB.pdf.