Transforming MIMO BPSK Maximum Likelihood Detection into QUBO Form

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1. Introduction

One of the most important challenges in the design of next-generation wireless communication systems is to meet users' ever-increasing demand for capacity and throughput. Large Multiple Input-Multiple Output (MIMO) systems with spatial multiplexing are one of the most promising ways to satisfy this demand. We propose work on new decoding methods utilizing Adiabatic Quantum Computation (AQC) for faster detection of transmitted symbols. We believe the AQC technique may be able to reduce computational complexity and guarantee high throughput at the same time in the above, critically-important MIMO performance regimes. To use AQC, the Maximum Likelihood (ML) problem has to be transformed into the Quadratic Unconstrained Binary Optimization (QUBO) form. In this document, we demonstrate how to transform 2×2 BPSK and 3×3 BPSK ML detection problems into the QUBO form.

2. 2x2 BPSK

For $N \times N$ MIMO communications, the Maximum Likelihood (ML) detection solves

$$\arg\min_{\hat{x}} \|\mathbf{H}\hat{\mathbf{x}} - \mathbf{y}\|^2 \tag{1}$$

where $H \in R^{N \times N}$ is complex MIMO channel, $\hat{x} \in R^N$ is transmitter's symbol candidate, and $y \in R^N$ is received symbol.

In 2×2 BPSK MIMO,

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{I,11} & h_{I,12} \\ h_{I,21} & h_{I,22} \end{bmatrix} + j \begin{bmatrix} h_{Q,11} & h_{Q,12} \\ h_{Q,21} & h_{Q,22} \end{bmatrix}, \tag{2}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_{I,1} \\ y_{I,2} \end{bmatrix} + j \begin{bmatrix} y_{Q,1} \\ y_{Q,2} \end{bmatrix}, \tag{3}$$

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}, \ where\ \hat{x}_1, \hat{x}_2 \in \{-1, 1\}. \tag{4}$$

The equation (1)'s norm can be expressed as

$$\|\mathbf{H}\hat{\mathbf{x}} - \mathbf{y}\|^2 = \left\| \begin{matrix} h_{11}\hat{x_1} + h_{1_2}\hat{x_2} - y_1 \\ h_{21}\hat{x_1} + h_{2_2}\hat{x_2} - y_2 \end{matrix} \right\|^2$$

which is,

$$\begin{aligned}
&= \left\| \begin{matrix} (h_{I,11} + jh_{Q,11})\hat{x_1} + (h_{I,12} + jh_{Q,12})\hat{x_2} - (y_{I,1} + jy_{Q,1}) \\ (h_{I,21} + jh_{Q,21})\hat{x_1} + (h_{I,22} + jh_{Q,22})\hat{x_2} - (y_{I,2} + jy_{Q,2}) \end{matrix} \right\|^2 \\
&= \left\| \begin{matrix} (h_{I,11}\hat{x_1} + h_{I,12}\hat{x_2} - y_{I,1}) + j(h_{Q,11}\hat{x_1} + h_{Q,12}\hat{x_2} - y_{Q,1}) \\ (h_{I,21}\hat{x_1} + h_{I,22}\hat{x_2} - y_{I,2}) + j(h_{Q,21}\hat{x_1} + h_{Q,22}\hat{x_2} - y_{Q,2}) \end{matrix} \right\|^2 \\
&= \{(h_{I,11}\hat{x_1} + h_{I,12}\hat{x_2} - y_{I,1})\}^2 + \{(h_{Q,11}\hat{x_1} + h_{Q,12}\hat{x_2} - y_{Q,1})\}^2 \\
&+ \{(h_{I,21}\hat{x_1} + h_{I,22}\hat{x_2} - y_{I,2})\}^2 + \{(h_{Q,21}\hat{x_1} + h_{Q,22}\hat{x_2} - y_{Q,2})\}^2. \qquad (5)
\end{aligned}$$

The QUBO form is

$$\min_{q_i, q_j} \sum_{i \le j = 1}^{N} Q_{ij} q_i q_j \qquad (6)$$

where Q $\in R^{N \times N}$ is an upper triangular matrix, and $q \in \{0, 1\}$ is binary so $q = q^2$. In our problem, q denotes qubit and N the number of qubits. Note that in BPSK, $\hat{x}$ can be mapped onto $2q - 1$. Using this relationship, we can express the equation (5) as

$$\begin{aligned}
\|\mathbf{H}\hat{\mathbf{x}} - \mathbf{y}\|^2 = &\{(h_{I,11}(2q_1 - 1) + h_{I,12}(2q_2 - 1) - y_{I,1})\}^2 \\
&+ \{(h_{Q,11}(2q_1 - 1) + h_{Q,12}(2q_2 - 1) - y_{Q,1})\}^2 \\
&+ \{(h_{I,21}(2q_1 - 1) + h_{I,22}(2q_2 - 1) - y_{I,2})\}^2 \\
&+ \{(h_{Q,21}(2q_1 - 1) + h_{Q,22}(2q_2 - 1) - y_{Q,2})\}^2. \qquad (7)
\end{aligned}$$

Expand the equation (7), and then

$$
\begin{aligned}
\|\mathbf{H}\hat{\mathbf{x}}-\mathbf{y}\|^2 = \{&2h_{I,11}^2+2(2h_{I,11})(-h_{I,11}-h_{I,12}-y_{I,1}) \\
&+2h_{Q,11}^2+2(2h_{Q,11})(-h_{Q,11}-h_{Q,12}-y_{Q,1}) \\
&+2h_{I,21}^2+2(2h_{I,21})(-h_{I,21}-h_{I,22}-y_{I,2}) \\
&+2h_{Q,21}^2+2(2h_{Q,21})(-h_{Q,21}-h_{Q,22}-y_{Q,2})\}q_1 \\
&+\{2h_{I,12}^2+2(2h_{I,12})(-h_{I,11}-h_{I,12}-y_{I,1}) \\
&+2h_{Q,12}^2+2(2h_{Q,12})(-h_{Q,11}-h_{Q,12}-y_{Q,1}) \\
&+2h_{I,22}^2+2(2h_{I,22})(-h_{I,21}-h_{I,22}-y_{I,2}) \\
&+2h_{Q,22}^2+2(2h_{Q,22})(-h_{Q,21}-h_{Q,22}-y_{Q,2})\}q_2 \\
&+\{2(2h_{I,11})(2h_{I,12})+2(h_{Q,11})(h_{Q,12}) \\
&+2(2h_{I,21})(2h_{I,22})+2(h_{Q,21})(h_{Q,22})\}q_1q_2 \\
&+\{(-h_{I,11}-h_{I,12}-y_{I,1})^2+(-h_{Q,11}-h_{Q,12}-y_{Q,1})^2 \\
&+(-h_{I,21}-h_{I,22}-y_{I,2})^2+(-h_{Q,21}-h_{Q,22}-y_{Q,2})^2\}.
\end{aligned} \tag{8}
$$

The equation (8) is the objective function of our minimization problem, so one can ignore the last constant term. Then, the minimization of this objective function is equivalent to the equation (6).

Transforming the problem into the QUBO form :

$$
\min_{q_i,q_j}\sum_{i\le j=1}^{2} Q_{ij}q_iq_j, \text{where } Q=\begin{bmatrix} Q_{11} & Q_{12} \\ 0 & Q_{22}\end{bmatrix}
$$

$$
\begin{aligned}
Q_{11} = 2&h_{I,11}^2+2(2h_{I,11})(-h_{I,11}-h_{I,12}-y_{I,1}) \\
&+2h_{Q,11}^2+2(2h_{Q,11})(-h_{Q,11}-h_{Q,12}-y_{Q,1}) \\
&+2h_{I,21}^2+2(2h_{I,21})(-h_{I,21}-h_{I,22}-y_{I,2}) \\
&+2h_{Q,21}^2+2(2h_{Q,21})(-h_{Q,21}-h_{Q,22}-y_{Q,2})
\end{aligned}
$$

$$
\begin{aligned}
Q_{22} = 2&h_{I,12}^2+2(2h_{I,12})(-h_{I,11}-h_{I,12}-y_{I,1}) \\
&+2h_{Q,12}^2+2(2h_{Q,12})(-h_{Q,11}-h_{Q,12}-y_{Q,1}) \\
&+2h_{I,22}^2+2(2h_{I,22})(-h_{I,21}-h_{I,22}-y_{I,2}) \\
&+2h_{Q,22}^2+2(2h_{Q,22})(-h_{Q,21}-h_{Q,22}-y_{Q,2})
\end{aligned}
$$

$$
\begin{aligned}
Q_{12} = &2(2h_{I,11})(2h_{I,12})+2(h_{Q,11})(h_{Q,12}) \\
&+2(2h_{I,21})(2h_{I,22})+2(h_{Q,21})(h_{Q,22})
\end{aligned}
$$

3. 3x3 BPSK

The same process is applied to 3×3 BPSK. In this way, the ML detection for $N \times N$ BPSK can be generalized.

In 3×3 BPSK,

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \begin{bmatrix} h_{I,11} & h_{I,12} & h_{I,13} \\ h_{I,21} & h_{I,22} & h_{I,23} \\ h_{I,31} & h_{I,32} & h_{I,33} \end{bmatrix} + j \begin{bmatrix} h_{Q,11} & h_{Q,12} & h_{Q,13} \\ h_{Q,21} & h_{Q,22} & h_{Q,23} \\ h_{Q,31} & h_{Q,32} & h_{Q,33} \end{bmatrix},$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} y_{I,1} \\ y_{I,2} \\ y_{I,3} \end{bmatrix} + j \begin{bmatrix} y_{Q,1} \\ y_{Q,2} \\ y_{Q,3} \end{bmatrix},$$

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix}, \; where \; \hat{x}_1, \hat{x}_2, \hat{x}_3 \in \{-1, 1\}.$$

The QUBO form for 3×3 BPSK is

$$\min_{q_i, q_j} \sum_{i \le j = 1}^{3} Q_{ij} q_i q_j, \text{where}\, Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ 0 & Q_{22} & Q_{23} \\ 0 & 0 & Q_{33} \end{bmatrix}.$$

The rest steps are exactly same as the 2×2 BPSK problem.

$$\left\| H\hat{x} - y \right\|^2 = \left\| \begin{matrix} h_{11}\hat{x_1} + h_{1_2}\hat{x_2} + h_{13}\hat{x_3} - y_1 \\ h_{21}\hat{x_1} + h_{2_2}\hat{x_2} + h_{23}\hat{x_3} - y_2 \\ h_{31}\hat{x_1} + h_{3_2}\hat{x_2} + h_{33}\hat{x_3} - y_3 \end{matrix} \right\|^2$$

$$= \left\| \begin{matrix} (h_{I,11}\hat{x_1} + h_{I,12}\hat{x_2} + h_{I,13}\hat{x_3} - y_{I,1}) + j(h_{Q,11}\hat{x_1} + h_{Q,12}\hat{x_2} + h_{Q,13}\hat{x_3} - y_{Q,1}) \\ (h_{I,21}\hat{x_1} + h_{I,22}\hat{x_2} + h_{I,23}\hat{x_3} - y_{I,2}) + j(h_{Q,21}\hat{x_1} + h_{Q,22}\hat{x_2} + h_{I,33}\hat{x_3} - y_{Q,2}) \\ (h_{I,31}\hat{x_1} + h_{I,32}\hat{x_2} + h_{I,33}\hat{x_3} - y_{I,3}) + j(h_{Q,31}\hat{x_1} + h_{Q,32}\hat{x_2} + h_{I,33}\hat{x_3} - y_{Q,3}) \end{matrix} \right\|^2$$

In BPSK, $\hat{x} = 2q - 1$, where $q \in \{0, 1\}$.

$$\begin{aligned} \|\mathbf{H}\hat{\mathbf{x}} - \mathbf{y}\|^2 = &\{(h_{I,11}(2q_1 - 1) + h_{I,12}(2q_2 - 1) + h_{I,13}(2q_3 - 1) - y_{I,1})\}^2 \\ &+\{(h_{Q,11}(2q_1 - 1) + h_{Q,12}(2q_2 - 1) + h_{Q,13}(2q_2 - 1) - y_{Q,1})\}^2 \\ &+\{(h_{I,21}(2q_1 - 1) + h_{I,22}(2q_2 - 1) + h_{I,23}(2q_3 - 1) - y_{I,2})\}^2 \\ &+\{(h_{Q,21}(2q_1 - 1) + h_{Q,22}(2q_2 - 1) + h_{Q,23}(2q_3 - 1) - y_{Q,2})\}^2 \\ &+\{(h_{I,31}(2q_1 - 1) + h_{I,32}(2q_2 - 1) + h_{I,33}(2q_3 - 1) - y_{I,3})\}^2 \\ &+\{(h_{Q,31}(2q_1 - 1) + h_{Q,32}(2q_2 - 1) + h_{Q,33}(2q_3 - 1) - y_{Q,3})\}^2 \end{aligned}$$

$$
\begin{aligned}
&= \{2h_{I,11}^2 + 2(2h_{I,11})(-h_{I,11} - h_{I,12} - h_{I,13} - y_{I,1}) \\
&+2h_{Q,11}^2 + 2(2h_{Q,11})(-h_{Q,11} - h_{Q,12} - h_{Q,13} - y_{Q,1}) \\
&+2h_{I,21}^2 + 2(2h_{I,21})(-h_{I,21} - h_{I,22} - h_{I,23} - y_{I,2}) \\
&+2h_{Q,21}^2 + 2(2h_{Q,21})(-h_{Q,21} - h_{Q,22} - h_{Q,23} - y_{Q,2}) \\
&+2h_{I,31}^2 + 2(2h_{I,31})(-h_{I,31} - h_{I,32} - h_{I,33} - y_{I,3}) \\
&+2h_{Q,31}^2 + 2(2h_{Q,31})(-h_{Q,31} - h_{Q,32} - h_{Q,33} - y_{Q,3})\}q_1 \\
&+\{2h_{I,12}^2 + 2(2h_{I,12})(-h_{I,11} - h_{I,12} - h_{I,13} - y_{I,1}) \\
&+2h_{Q,12}^2 + 2(2h_{Q,12})(-h_{Q,11} - h_{Q,12} - h_{Q,13} - y_{Q,1}) \\
&+2h_{I,22}^2 + 2(2h_{I,22})(-h_{I,21} - h_{I,22} - h_{I,23} - y_{I,2}) \\
&+2h_{Q,22}^2 + 2(2h_{Q,22})(-h_{Q,21} - h_{Q,22} - h_{Q,23} - y_{Q,2}) \\
&+2h_{I,32}^2 + 2(2h_{I,32})(-h_{I,31} - h_{I,32} - h_{I,33} - y_{I,3}) \\
&+2h_{Q,32}^2 + 2(2h_{Q,32})(-h_{Q,31} - h_{Q,32} - h_{Q,33} - y_{Q,3})\}q_2 \\
&+\{2h_{I,13}^2 + 2(2h_{I,13})(-h_{I,11} - h_{I,12} - h_{I,13} - y_{I,1}) \\
&+2h_{Q,13}^2 + 2(2h_{Q,13})(-h_{Q,11} - h_{Q,12} - h_{Q,13} - y_{Q,1}) \\
&+2h_{I,23}^2 + 2(2h_{I,23})(-h_{I,21} - h_{I,22} - h_{I,23} - y_{I,2}) \\
&+2h_{Q,23}^2 + 2(2h_{Q,23})(-h_{Q,21} - h_{Q,22} - h_{Q,23} - y_{Q,2}) \\
&+2h_{I,33}^2 + 2(2h_{I,33})(-h_{I,31} - h_{I,32} - h_{I,33} - y_{I,3}) \\
&+2h_{Q,33}^2 + 2(2h_{Q,33})(-h_{Q,31} - h_{Q,32} - h_{Q,33} - y_{Q,3})\}q_3 \\
&+\{2(2h_{I,11})(2h_{I,12}) + 2(h_{Q,11})(h_{Q,12}) \\
&+2(2h_{I,21})(2h_{I,22}) + 2(h_{Q,21})(h_{Q,22}) \\
&+2(2h_{I,31})(2h_{I,32}) + 2(h_{Q,31})(h_{Q,32})\}q_1q_2 \\
&+\{2(2h_{I,11})(2h_{I,13}) + 2(h_{Q,11})(h_{Q,13}) \\
&+2(2h_{I,21})(2h_{I,23}) + 2(h_{Q,21})(h_{Q,23}) \\
&+2(2h_{I,31})(2h_{I,33}) + 2(h_{Q,31})(h_{Q,33})\}q_1q_3 \\
&+\{2(2h_{I,12})(2h_{I,13}) + 2(h_{Q,12})(h_{Q,13}) \\
&+2(2h_{I,22})(2h_{I,23}) + 2(h_{Q,22})(h_{Q,23}) \\
&+2(2h_{I,32})(2h_{I,33}) + 2(h_{Q,32})(h_{Q,33})\}q_2q_3 \\
&+\{(-h_{I,11} - h_{I,12}h_{I,13} - y_{I,1})^2 + (-h_{Q,11} - h_{Q,12} - h_{Q,13} - y_{Q,1})^2 \\
&+(-h_{I,21} - h_{I,22} - h_{I,23} - y_{I,2})^2 + (-h_{Q,21} - h_{Q,22} - h_{Q,23} - y_{Q,2})^2 \\
&+(-h_{I,31} - h_{I,32} - h_{I,33} - y_{I,3})^2 + (-h_{Q,31} - h_{Q,32} - h_{Q,33} - y_{Q,3})^2\}
\end{aligned}
$$

Therefore,

$$\begin{aligned}
Q_{11} = 2h_{I,11}^2 + 2(2h_{I,11})(-h_{I,11} - h_{I,12} - h_{I,13} - y_{I,1}) \\
+2h_{Q,11}^2 + 2(2h_{Q,11})(-h_{Q,11} - h_{Q,12} - h_{Q,13} - y_{Q,1}) \\
+2h_{I,21}^2 + 2(2h_{I,21})(-h_{I,21} - h_{I,22} - h_{I,23} - y_{I,2}) \\
+2h_{Q,21}^2 + 2(2h_{Q,21})(-h_{Q,21} - h_{Q,22} - h_{Q,23} - y_{Q,2}) \\
+2h_{I,31}^2 + 2(2h_{I,31})(-h_{I,31} - h_{I,32} - h_{I,33} - y_{I,3}) \\
+2h_{Q,31}^2 + 2(2h_{Q,31})(-h_{Q,31} - h_{Q,32} - h_{Q,33} - y_{Q,3})
\end{aligned}$$

$$\begin{aligned}
Q_{22} = 2h_{I,12}^2 + 2(2h_{I,12})(-h_{I,11} - h_{I,12} - h_{I,13} - y_{I,1}) \\
+2h_{Q,12}^2 + 2(2h_{Q,12})(-h_{Q,11} - h_{Q,12} - h_{Q,13} - y_{Q,1}) \\
+2h_{I,22}^2 + 2(2h_{I,22})(-h_{I,21} - h_{I,22} - h_{I,23} - y_{I,2}) \\
+2h_{Q,22}^2 + 2(2h_{Q,22})(-h_{Q,21} - h_{Q,22} - h_{Q,23} - y_{Q,2}) \\
+2h_{I,32}^2 + 2(2h_{I,32})(-h_{I,31} - h_{I,32} - h_{I,33} - y_{I,3}) \\
+2h_{Q,32}^2 + 2(2h_{Q,32})(-h_{Q,31} - h_{Q,32} - h_{Q,33} - y_{Q,3})
\end{aligned}$$

$$\begin{aligned}
Q_{33} = 2h_{I,13}^2 + 2(2h_{I,13})(-h_{I,11} - h_{I,12} - h_{I,13} - y_{I,1}) \\
+2h_{Q,13}^2 + 2(2h_{Q,13})(-h_{Q,11} - h_{Q,12} - h_{Q,13} - y_{Q,1}) \\
+2h_{I,23}^2 + 2(2h_{I,23})(-h_{I,21} - h_{I,22} - h_{I,23} - y_{I,2}) \\
+2h_{Q,23}^2 + 2(2h_{Q,23})(-h_{Q,21} - h_{Q,22} - h_{Q,23} - y_{Q,2}) \\
+2h_{I,33}^2 + 2(2h_{I,33})(-h_{I,31} - h_{I,32} - h_{I,33} - y_{I,3}) \\
+2h_{Q,33}^2 + 2(2h_{Q,33})(-h_{Q,31} - h_{Q,32} - h_{Q,33} - y_{Q,3})
\end{aligned}$$

$$\begin{aligned}
Q_{12} = 2(2h_{I,11})(2h_{I,12}) + 2(h_{Q,11})(h_{Q,12}) \\
+2(2h_{I,21})(2h_{I,22}) + 2(h_{Q,21})(h_{Q,22}) \\
+2(2h_{I,31})(2h_{I,32}) + 2(h_{Q,31})(h_{Q,32})
\end{aligned}$$

$$\begin{aligned}
Q_{13} = 2(2h_{I,11})(2h_{I,13}) + 2(h_{Q,11})(h_{Q,13}) \\
+2(2h_{I,21})(2h_{I,23}) + 2(h_{Q,21})(h_{Q,23}) \\
+2(2h_{I,31})(2h_{I,33}) + 2(h_{Q,31})(h_{Q,33})
\end{aligned}$$

$$\begin{aligned}
Q_{23} = 2(2h_{I,12})(2h_{I,13}) + 2(h_{Q,12})(h_{Q,13}) \\
+2(2h_{I,22})(2h_{I,23}) + 2(h_{Q,22})(h_{Q,23}) \\
+2(2h_{I,32})(2h_{I,33}) + 2(h_{Q,32})(h_{Q,33})
\end{aligned}$$