Effect of Speed on Handwriting Shape: A Descriptive Generalization[1]

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Research Report CS-TR-497-95
August 1995

Abstract

In this report, we characterize the natural simplifying transformations of a handwritten curve. We argue that the number and type of singularities in a geometric curve determine the amount of time required for the human handwriting system to produce that curve to a given level of accuracy. Therefore, the behavior of the human handwriting system under increasing time pressure may be characterized by a set of stochastic transformations that replace costly curve segments with less costly segments, or remove segments entirely.

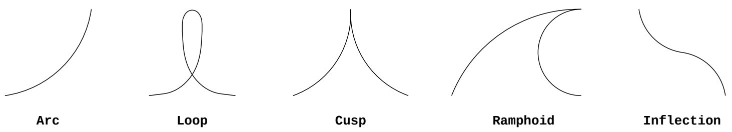
Keywords: handwriting, motor complexity, speed–accuracy tradeoff.

[1]The author is greatful to F.A. Mussa-Ivaldi and E. Bizzi for their advice, and to the participants of IGS'95 for their comments. This research was funded in part by National Science Foundation Young Investigator Award IRI-9258517 to the author and by an equipment grant from CalComp.

1 Introduction

This report attempts to characterize the natural diachronic and synchronic transformations that apply to handwritten curves. We would like to explain why sigma evolved into word-final sigma and then lunate-sigma in the Greek alphabet, and why the Roman `s` further evolved into the Palmer `s`. We would also like to account for the one-sided nature of spontaneous transformations observed in modern American and European handwriting, such as why the Roman `c` and `i` are often handwritten as the cursive `e`, while the Roman `e` rarely (if ever) appears as the cursive `c` or `i`.

We hypothesize that the complexity of a planar geometric curve is determined primarily by the number of curvature reversals and geometric singularities that it contains. In particular, segments of a planar geometric curve are ordered by increasing complexity as follows:[2]



The natural transformations of handwritten curves are exactly those that locally replace a given curve segment with one of lower complexity. We predict that a ramphoid will be replaced by a cusp or a loop, but not the other way round. This suffices to account for the diachronic and synchronic changes mentioned above, along with many other changes.

The most natural explanation of such handwriting change is arguably in terms of the writer's attempt to maximize the communicative bandwidth, by reducing the time required to handwrite a given curve without completely sacrificing its intelligibility. Consequently, our hypothesized transformations are not solely due to the writer's desire to reduce the motor complexity of a planar curve. Rather, they arise from the writer's attempt to minimize motor complexity while maintaining intelligibility.

In order to obtain direct empirical evidence bearing on our hypothesis, we have conducted the following two experiments. Both experiments are designed to measure the complexity of the geometric singularities inherent the Roman alphabet while accurately recreating the task constraints of natural handwriting. The results of both experiments strongly confirm our complexity hypothesis.

[2] An *arc* is a smooth curve with constant curvature. A *loop* is a smooth curve with a single geometric node (ie., intersection point). A *cusp* is a piecewise smooth curve containing one singular point belonging to two smooth curves with identical sign of curvature. A *ramphoid* is a piecewise smooth curve containing one singular point belonging to two smooth curves with opposite sign of curvature. An *inflection* is a smooth curve whose curvature changes sign exactly once (ie., the curvature has exactly one zero-crossing). Strictly speaking, an inflection point is not a geometric singularity.

2 Experimental Design

2.1 Subjects

Our subjects consisted of four male, able-bodied undergraduate students in the age range 20–22. Two were right-handed and two were left-handed. Their participation was voluntary and without monetary compensation.

2.2 Apparatus

The subjects sat at a desk and wrote on a piece of lined paper that was secured to the top of an opaque CalComp 3300 Drawing Board II digitizer tablet. The digitizer tablet was connected to a dedicated Compaq 386/33 personal computer with 64mb RAM and 1GB disk, whose sole function was to write the digitizer output to disk for future analysis.

2.3 Procedure

The first experiment is designed to measure complexity directly, as the minimal time required to perform a specified task to a specified accuracy. Subjects are instructed to reproduce a set of planar curves with maximum accuracy, under no explicit time constraint. (It is important, however, to realize that there is always an implicit time constraint arising from the impatience of our subjects.) Each curve contains a sequence of geometric singularities. Accuracy is explained to the subject as the smooth reproduction of smooth curves in the path, and the accurate reproduction and localization of geometric singularities in the path. Subjects are instructed to practice each path under the guidance of the experimenter until they are unable to further increase the accuracy of their handwriting. After a brief resting period, the subject is then asked to reproduce the path under experimental conditions. We compare the time required to do this for each of the different segments. Our hypothesis predicts that paths containing more complex segments will require more time to complete to the same accuracy.

The second experiment is designed to measure complexity indirectly, by studying the path transformations that occur when a fixed task is performed with decreasing amounts of time. Subjects are instructed to reproduce the same set of geometric curves under increasingly stringent time requirements (fast, faster, fastest). We characterize the transformations that occur during such high-speed writing. These transformations are natural transformations of the handwritten path that reduce the complexity of the path. Our hypothesis predicts that inflections are replaced by ramphoids, ramphoids by cusps, cusps by loops, and loops by arcs at greater writing speeds.

A third experiment, suggested to us by F.A. Mussa-Ivaldi (personal communication), was designed to measure the complexity of handwritten paths, inde-

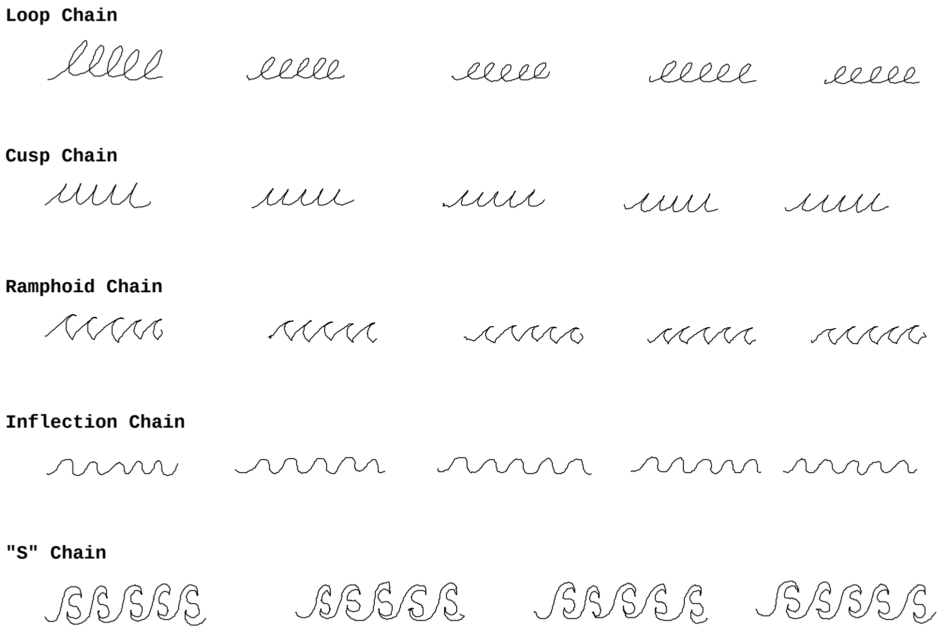


Figure 1: Five sets of planar curves used to test our complexity hypothesis.

pendent of the Roman alphabet and an externally-defined accuracy requirement. In this case, the target path is not specified. Rather, subjects are instructed to "doodle" within a given region without lifting the tip of their pen from the writing surface. Our hypothesis predicts that less complex geometric singularities should appear with greater frequency in these doodles. We were not able to run this experiment for all subjects, and so our preliminary results will not be discussed here.

2.4 Discussion

In all cases, our experimental tasks were carefully designed to recreate the essential motor features of handwriting, so that the gross motor behavior of writers would be indistinguishable from their natural motor behavior during actual handwriting. Writers are instructed to grasp the writing instrument with the same hand and in the same manner as when writing notes to themselves. We have also attempted to naturally reproduce the wrist saccade: the fact that the vast majority of writers rest their forearm and wrist on the writing surface while producing a word, and invariably advance their wrist at word boundaries.

The wrist saccade is not a biomechanical necessity. It is all the more interesting because it is at odds with elementary school handwriting instruction, which encourages writers to maintain a constant rightward wrist velocity [1]. The wrist saccade appears to arise primarily from the writer's conceptualization of the handwriting task as organized into a hierarchy of linguistic subtasks: the utterance, the word, and the character. The wrist saccade marks the completion of one complex motor task and the initiation of the next task. Accordingly, we organized our patterns into a hierarchy (three lines of four sets of five connected segments, with a space in between each set) and explained the tasks to our subjects in these exact terms. Subjects were additionally instructed to think of the cusp chain as a series of cusps (`i`'s) rather than as a series of arcs (`u`'s). The net effect is that our subjects reliably performed a wrist saccade at the completion of each set of five connected curve segments (ie., before beginning the next set of five segments).

3 Results

3.1 Temporal Requirements by Curve Segment

In the first experiment, subjects were instructed to produce the five curves at maximum accuracy. We measured the average velocity throughout each handwriting session. We observed that the writers required increasing amounts of time to perform the loop chain, cusp chain, ramphoid chain, and S chain. Our results are exemplified by figure 2 below. In no case did we observe a meaningful difference between the left- and right-handed writers.

The one apparent exception related to the inflection chain, which was performed with higher average velocity than the allegedly less complex ramphoid chain. A closer look at the handwriting produced by the subjects revealed that no writer was able to create a smooth inflection point. Rather, the inflection chains appeared as as ramphoid chains with lessened angular change at the geometric singularity. This subtlety may not be obvious in the figure below, due to the low resolution of the word processing software. (The output of the digitizer tablet at 1000 pts/inch was converted to PostScript via xfig at 80 pts/inch.)

3.2 Transformations under Time Pressure

In the second experiment, subjects were instructed to produce the five geometric curves under increasing time pressure. In each case, we observed that their handwriting degraded exactly in accordance with our complexity hypothesis: inflections became ramphoids, ramphoids became cusps, cusps became loops, and loops became arcs. This behavior is exemplified by the handwriting displayed in figure 3 below, which contains all the world's `s`'s. Figure 4 plots the frequency of geometric singularities as a function of average writing velocity. There was no meaningful difference between the left- and right-handed writers.

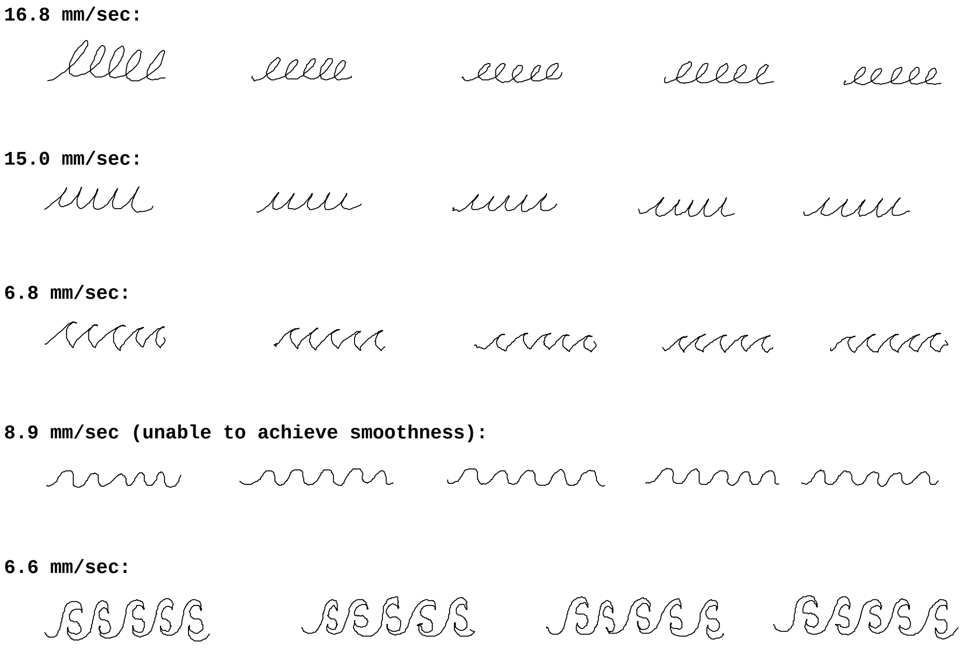


Figure 2: The time required to produce a geometric path with maximum accuracy increases according to the number and type of geometric singularities in the path.

The appendix contains three more trials of this experiment.

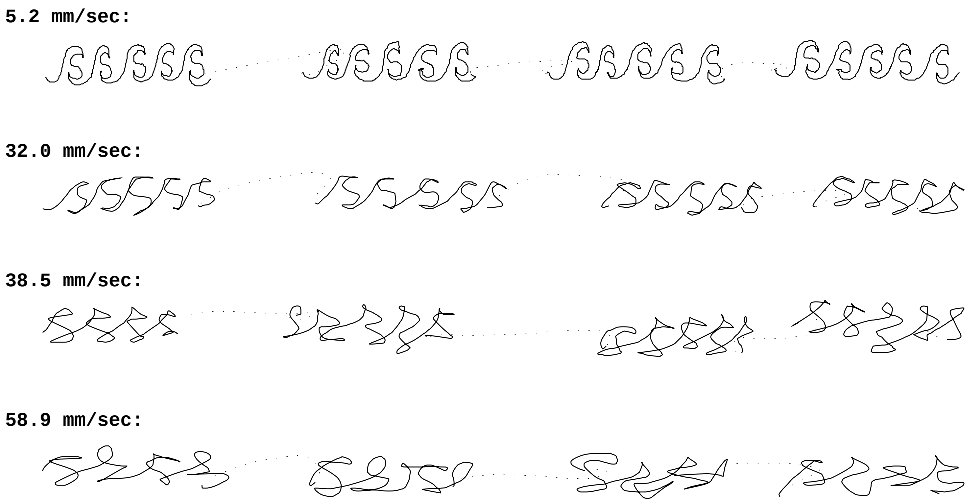


Figure 3: The number of geometric singularities in a geometric path changes under increasing time pressure. Inflections are replaced with ramphoids, ramphoids are replaced with cusps, cusps are replaced with loops, and loops are replaced with arcs. All the world's `s`'s may be found in this writer's handwriting.

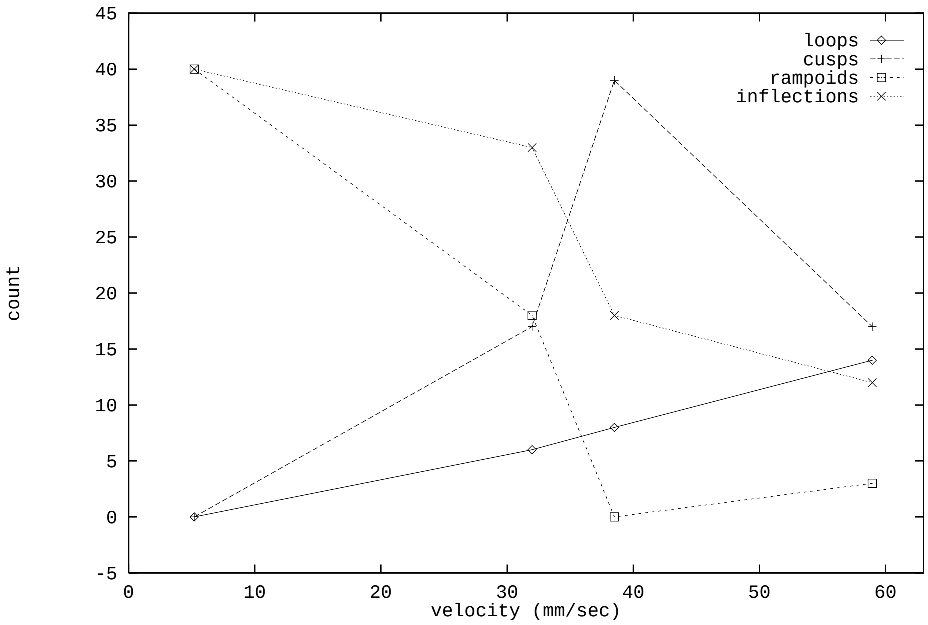


Figure 4: The frequency of geometric singularities in a handwritten planar curve is largely a function of the average pen tip velocity.

4 Conclusion

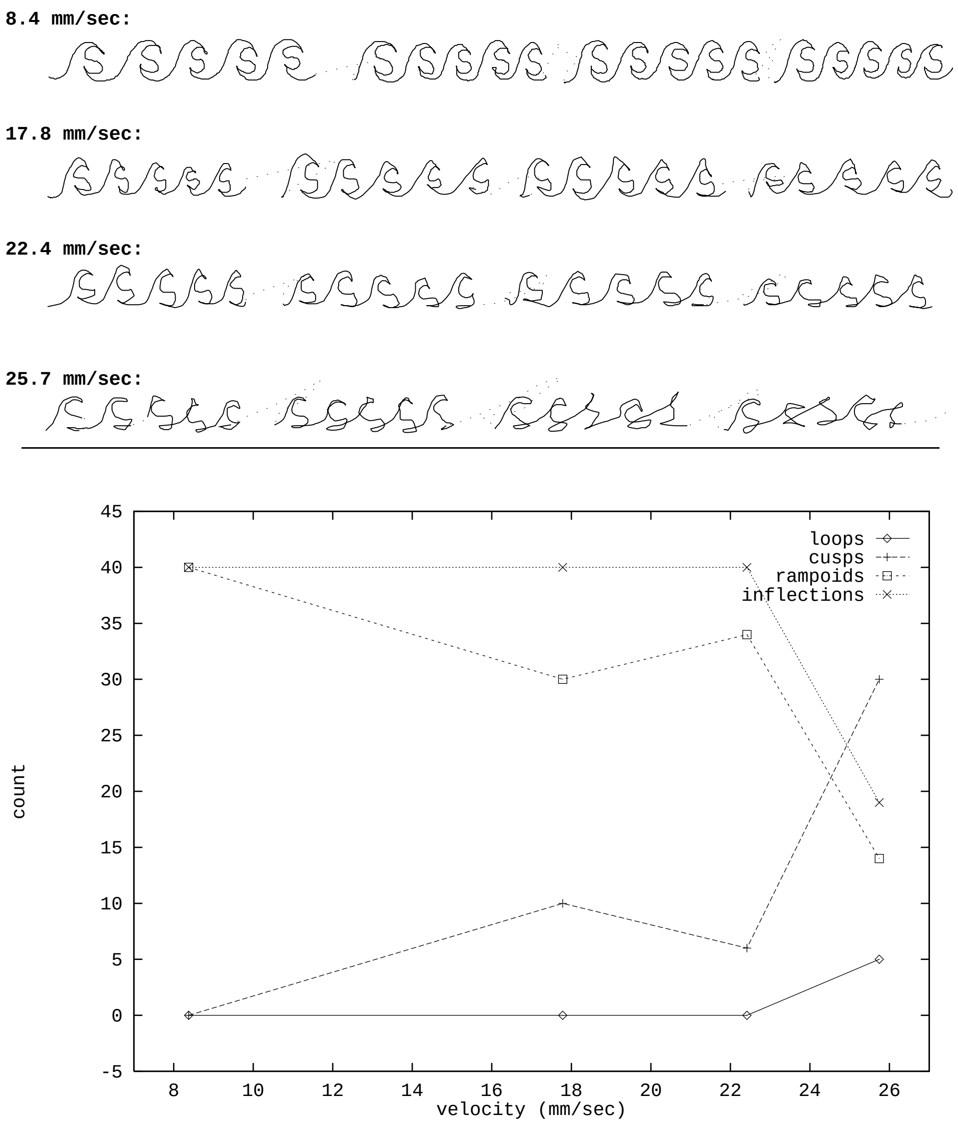
Our two experiments confirmed our initial hypothesis that the complexity of a geometric curve is primarily a function of the number and type of geometric singularities in the curve. In ongoing work, we are attempting to model the time required to reproduce a planar curve to a given accuracy as a linear combination of the number of geometric singularities of the various types. We are also evaluating an alternative hypothesis, that the complexity of a planar curve is simply a function of the curve's total arc length and total angular change. Both approaches require more a ambitious experimental regime, using more accurate instrumentation.

References

[1] Slote, C. T. *Improve Your Handwriting.* McGraw Hill, New York, 1956.

A More Trials

This appendix contains three more trials of the second experiment, for three different subjects.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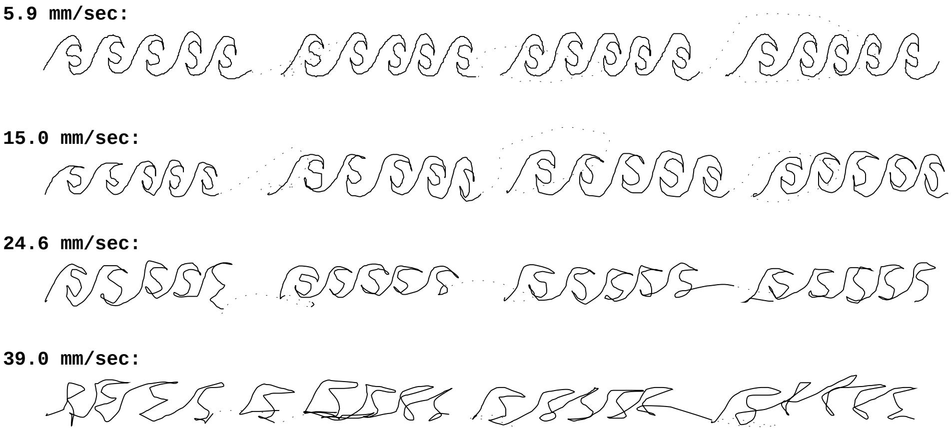
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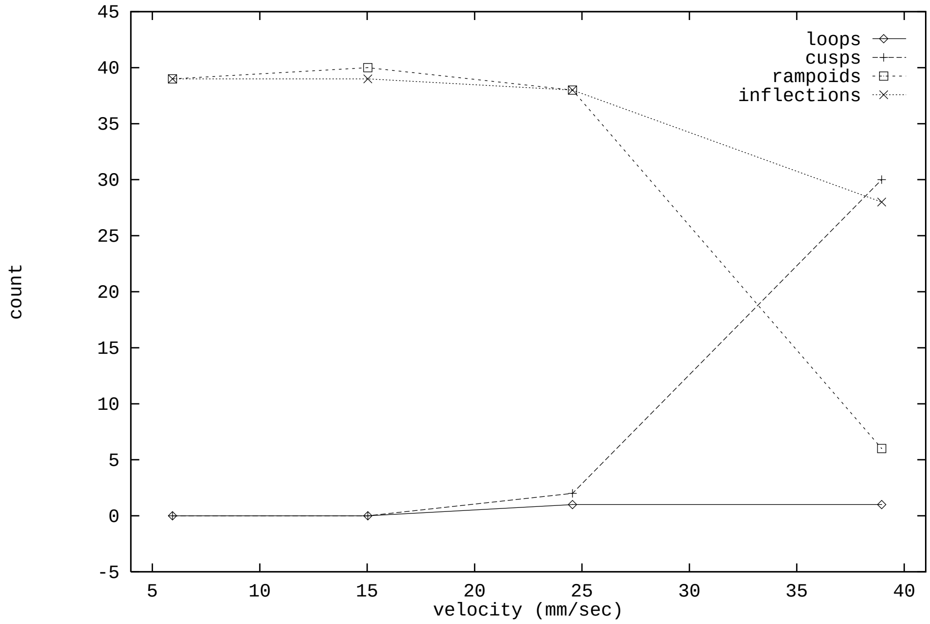
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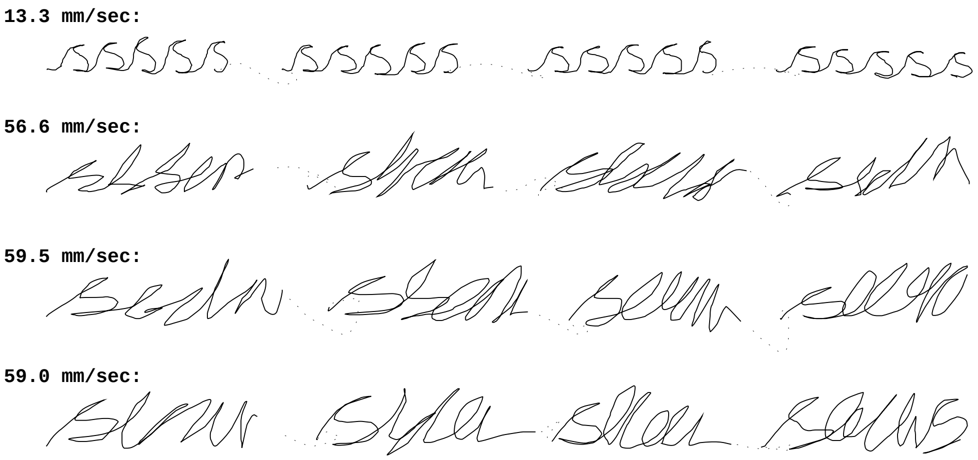
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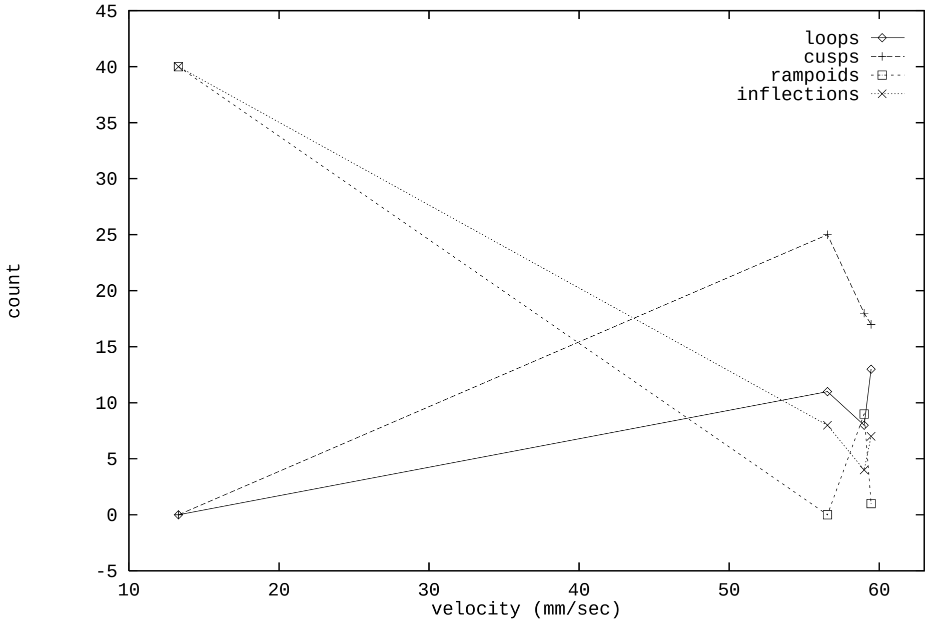
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