A Catalog of Higher Pairs and their Configuration Spaces

Leo Joskowicz
IBM T.J. Watson Research Center
P.O. Box 704
Yorktown Heights, NY 10598

Elisha Sacks
Computer Science Department
Princeton University
Princeton, NJ 08544

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Abstract

We illustrate the HIPAIR kinematic analysis program for higher pairs on twenty examples from Artobolevsky's encyclopedia of mechanisms. For each example, we show the pair, present its configuration space, explain its structure and behavior, and list its reference number in the encyclopedia. All the figures are direct program output. HIPAIR is the first comprehensive kinematic analysis program for higher pairs. It covers pairs of 2.5D parts with two degrees of freedom, including ones with intermittent, simultaneous, and degenerate contacts. This class contains 90% of 2.5D pairs and 80% of all higher pairs according to our survey of 2500 mechanisms. It analyzes pairs of average shape complexity in five seconds and pairs with several thousand features in one minute. HIPAIR facilitates the design and analysis of multipart mechanisms that contain higher pairs. It helps designers detect part interference, establish part clearances, identify functional surfaces, and find jamming and lock-up configurations.

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Introduction

This paper illustrates our research in automating the design and analysis of mechanisms. We address the analysis of kinematic pairs, which are the building blocks of mechanisms. Kinematic pairs are pairs of rigid parts that exhibit useful behavior, such as motion generation, motion transformation, indexing, and locking. Parts that interact via point or curve contacts are called higher pairs, for example gears, cams, ratchets, and escapements. The contacts can be permanent or intermittent. Parts that interact via permanent surface contacts are called lower pairs, for example pin and screw joints. Kinematic analysis determines the possible behaviors of a pair consistent with the shapes and degrees of freedom (allowable motions) of its parts.

Commercial mechanism analysis software fully automates the kinematic analysis of lower pairs. The analysis is straightforward because there are only six types of surface contact, each of which constrains the touching parts in a simple, known manner. Automating the kinematic analysis of higher pairs is much harder because there are infinitely many types of contact, each of which constrains the touching parts in a different, generally complex way. Previous research addresses only pairs with a single, permanent contact and without interference from other contacts. Yet, pairs with multiple and intermittent contacts play an important role in mechanism design.

We have developed a comprehensive kinematic analysis program, called HIPAIR, for 2.5D higher pairs with two degrees of freedom. Each part consists of one or more slices parallel to a reference plane. Each slice is formed by extruding a closed boundary parallel to the reference plane. Each part has one degree of freedom: translation or rotation relative to a fixed spatial axis (Figure 1). HIPAIR covers 90% of 2.5D pairs and 80% of all higher pairs according to our survey of 2500 mechanisms. We have tested it on over 100 parametric variations of 25 pairs, including non-standard gears, escapements, ratchets, latches, and indexing pairs. It produces correct results for every pair to within a user-defined tolerance. It takes five seconds to analyze pairs of average shape complexity and one minute for pairs with several thousand contacts. HIPAIR facilitates the design and analysis of multipart mechanisms that contain higher pairs. It helps designers detect part interference, establish part clearances, identify functional surfaces, and find jamming and lock-up configurations.

We perform kinematic analysis within the configuration space representation, which provides a concise, yet complete description of rigid body interaction. The configuration space of a kinematic pair is the space of configurations (positions and orientations) of its parts. It decomposes into blocked space where the parts overlap and into free space where they do not overlap. Only configurations in free space are physically realizable. The boundary between free and blocked space contains the configurations where the parts touch.

HIPAIR computes configuration spaces for higher pairs. It generates the contact constraints for every pair of feature contacts, intersects them, and partitions the resulting space into regions. Each region is defined by equality and inequality constraints that represent

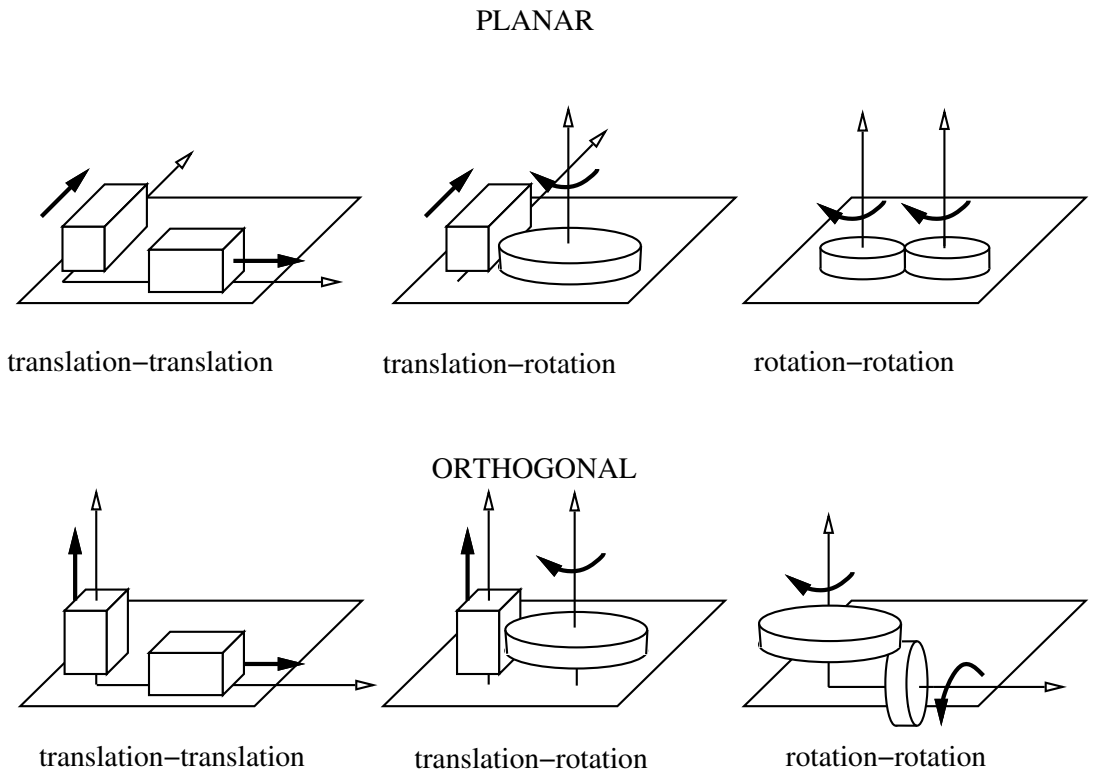


Figure 1: The motion types covered by the HIPAIR program.

possible feature contacts. Contact changes occur when the configuration shifts between regions.

We illustrate the power of HIPAIR and the richness of higher pairs with a catalog of twenty examples from Artobolevsky's encyclopedia of mechanisms [1]. For each example, we show the pair, present its configuration space, explain its structure and behavior, and list its reference number in the encyclopedia. All the figures are direct HIPAIR output and were produced in under a minute. We draw the pairs in a planar format. We project the parts onto their reference planes, except for one of the orthogonal gears in Example 8. We show only the boundaries of most parts, but in a few pairs we fill in the boundary of one part for clarity. A small circle marks the center of rotation of each rotating part. We present the configuration spaces in a uniform format. The axes are labeled with the degrees of freedom and their ranges. The labels x and y indicate translation along the horizontal and vertical axes; the labels θ and ω indicate rotation in the xy plane (around the z axis). The blocked space, representing configurations where parts overlap, is shaded. A small black circle marks the configuration in which the parts are displayed. We describe the configuration space of the first example in detail for those unfamiliar with this representation.

The HIPAIR program is part of a kinematic analysis and simulation program for multipart mechanisms. The program generates the configuration space of a mechanism comprised of lower and higher pairs. It decomposes the mechanism into subassemblies of lower pairs (linkages) and of higher pairs, computes subassembly configuration spaces, and composes the results. It computes motion paths by simulating driving motions and external forces. We describe HIPAIR [4], kinematic analysis of mechanisms [2], and kinematic simulation [3] in the references listed below.

References

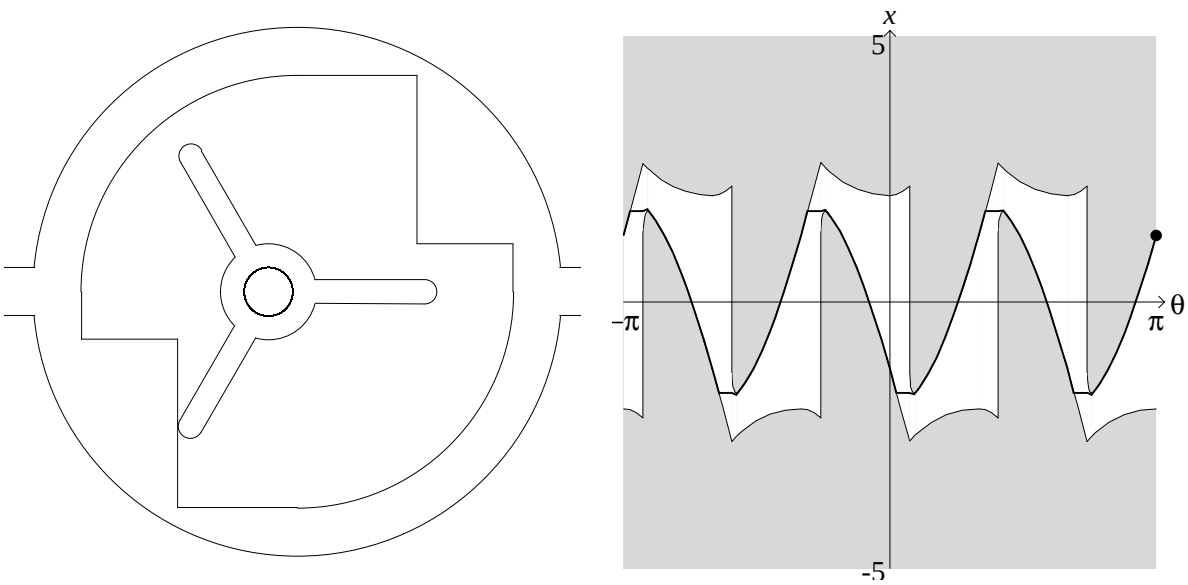
[1] Artobolevsky, I. *Mechanisms in Modern Engineering Design*, volume 1–4. (MIR Publishers, Moscow, 1979). English translation.

[2] Joskowicz, L. and Sacks, E. Computational kinematics. *Artificial Intelligence* **51** (1991) 381–416.

[3] Sacks, E. and Joskowicz, L. Automated modeling and kinematic simulation of mechanisms. *Computer-Aided Design* **25** (1993) 106–118.

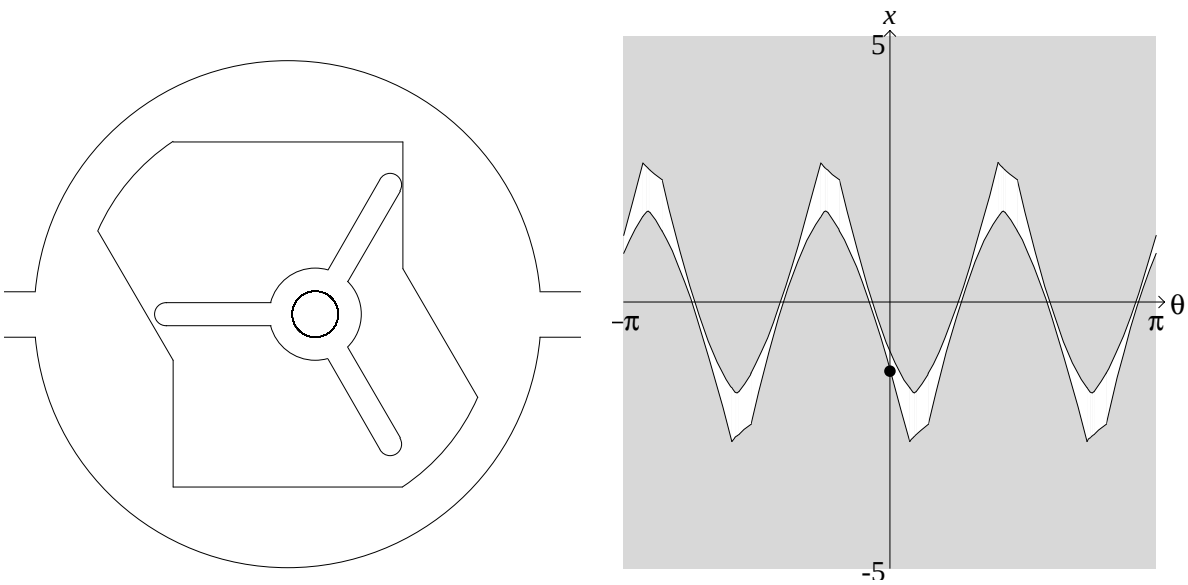
[4] Sacks, E. and Joskowicz, L. Comprehensive kinematic analysis of higher pairs with two degrees of freedom. Technical Report CS-TR-421-93, Princeton University, 1993. submitted to *ASME Journal of Mechanical Design.*

1 Three finger cam



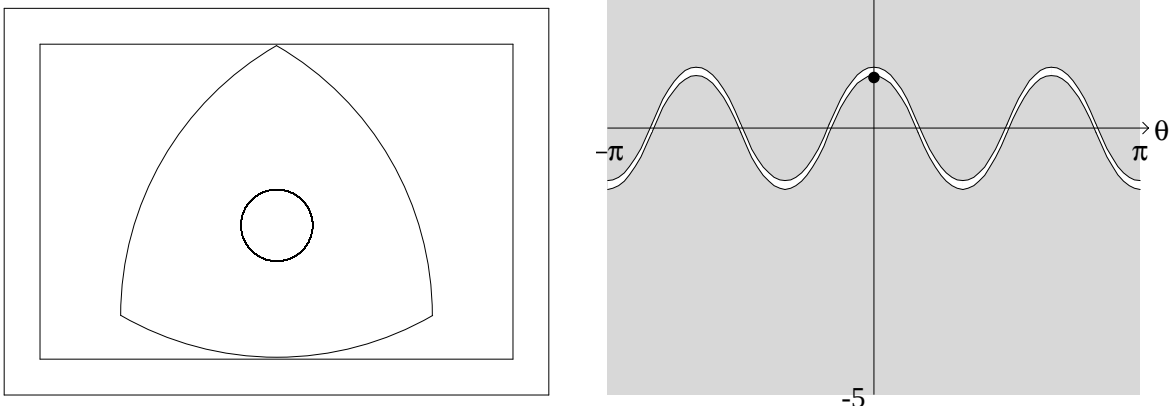
The pair consists of a rotating three-finger cam that drives a horizontally translating follower. A full clockwise rotation of the cam drives the follower left and right three times with short dwell periods. The cam fingers alternately push the upper vertical segment of the follower right and the lower vertical segment left. Rotating the cam counterclockwise causes it to block when one of its fingers hits a horizontal segment of the follower. In the configuration space, the orientation of the cam is θ and the displacement of the follower is x. The pair follows the thick line from right to left as the cam rotates clockwise, starting at configuration $(\pi, 1.25)$. The horizontal segments represent the dwell periods when the cam does not touch the follower. The rising and falling segments represent the periods when the cam pushes the follower right and left. A cam finger breaks contact with one side of the follower at the start of each dwell period. The next finger makes contact with the other side of the cam at the end of the dwell period. (2985)

2 Bidirectional three finger cam



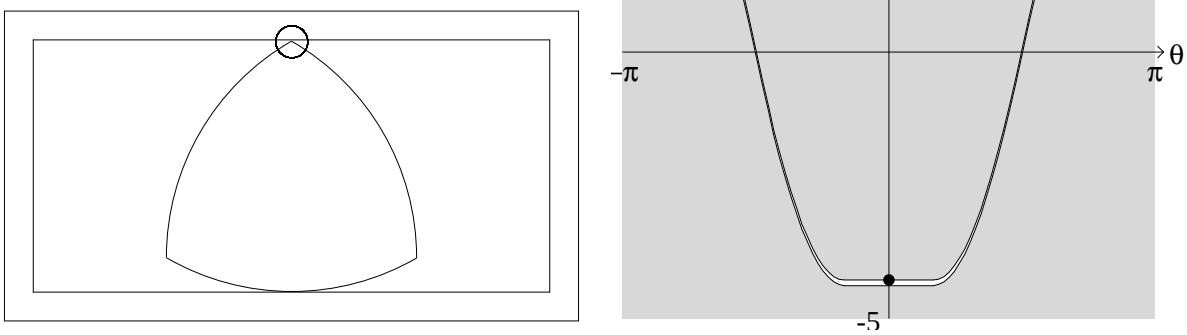
The pair consists of a rotating three-finger cam that drives a horizontally translating follower. A full clockwise rotation of the cam drives the follower left and right three times with short dwell periods. The cam fingers alternately push the upper vertical segment of the follower right and the lower vertical segment left. A full counterclockwise rotation of the cam drives the follower in the same way, but the cam fingers now push the lower slanted segment right and the upper ones left. In the configuration space, the orientation of the cam is θ and the displacement of the follower is x. (adapted from 2985)

3 Three arc cam



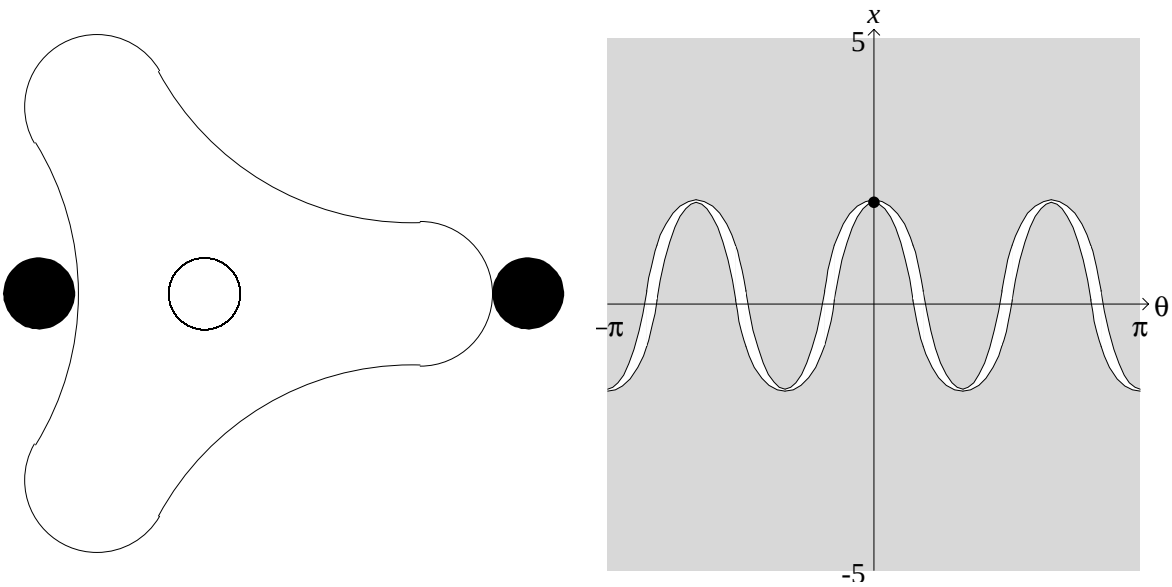
The pair consists of a rotating cam that drives a vertically translating follower. The cam contour consists of three arc segments whose endpoints lie on a circle described from the cam's center of rotation. The follower consists of a frame whose parallel horizontal faces touch the cam. The distance between the upper and lower contact points is constant, so the cam always tightly fits inside the follower without blocking as it rotates. A full rotation of the cam drives the follower up and down three times at evenly spaced intervals. In the configuration space, the orientation of the cam is θ and the displacement of the follower is y. The channel width shows the amount of backlash. (2984)

4 Offset three arc cam



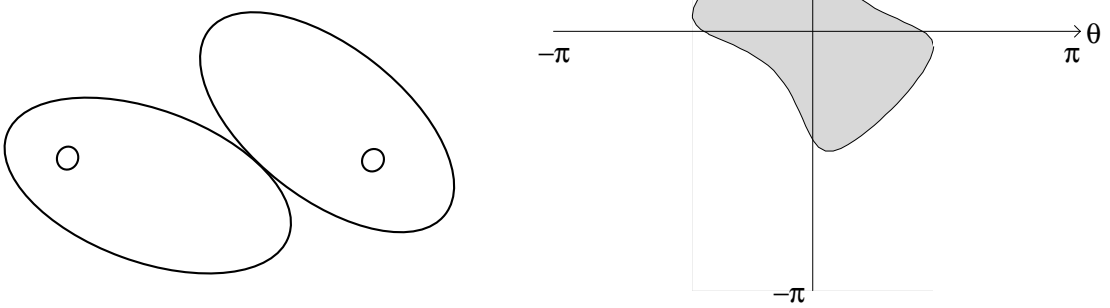
The pair consists of a rotating cam that drives a vertically translating follower. The cam contour consists of three arc segments whose endpoints lie on a circle. The center of rotation coincides with the common endpoint of two arcs. The follower consists of a frame whose parallel horizontal faces touch the cam. A full rotation of the cam drives the follower up and down with dwell periods in between. The dwell periods occur at configurations where the common endpoint at the center of rotation touches the upper and lower horizontal faces of the frame. The contact point between the third arc segment and the follower lies on a circle of constant radius. In the configuration space, the orientation of the cam is θ and the displacement of the follower is y. The channel width shows the amount of backlash. (2994)

5 Six arc cam



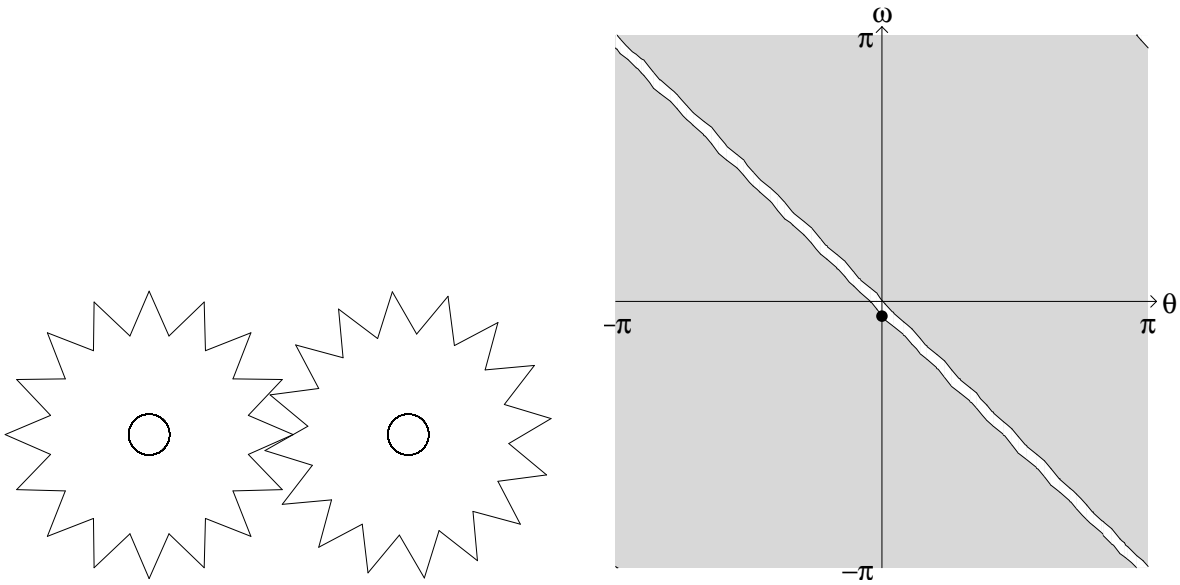
The pair consists of a rotating cam that drives a horizontally translating follower. The cam contour consist of three circular arc lobes connected by three three tangent circular arcs. The follower consist of two circular pins mounted on a frame (not shown). A full rotation of the cam drives the follower left and right three times at evenly spaced intervals. The distance between the left and right contacts is constant, so the cam always tightly fits between the the two pins without blocking as it rotates. In the configuration space, the orientation of the cam is θ and the displacement of the follower is x. The channel width shows the amount of backlash. (2983)

6 Elliptical cam



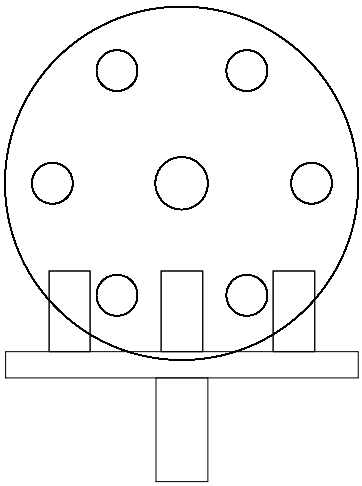
The pair consists of identical rotating ellipses whose centers of rotation coincide with their left focal points. The transmission ratio varies with the orientations of the ellipses and reverses sign at the lowest, highest, leftmost, and rightmost contact configurations. In the configuration space, the orientation of the left ellipse is θ and the orientation of the right ellipse is ω. (2289)

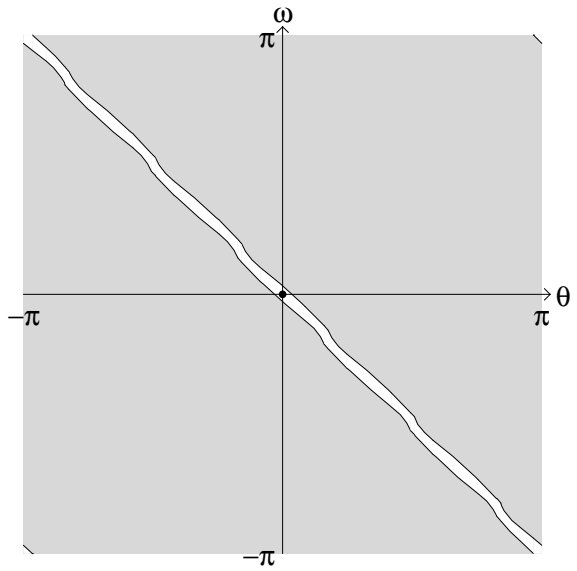
7 Gears



The pair consists of identical rotating toothed gears. Each gear has 16 teeth with triangular profiles. When either gear rotates in either direction, the other gear rotates in the opposite direction by the same angle. Two pairs of teeth are engaged at all times. The transmission ratio is almost linear. In the configuration space, the orientation of the left gear is θ and the orientation of the right gear is ω. The slanted channels represent the gear engagements; the rough edges reflect chatter and slight variations in the motion transmission ratio. The channel width shows the amount of backlash. (2301)

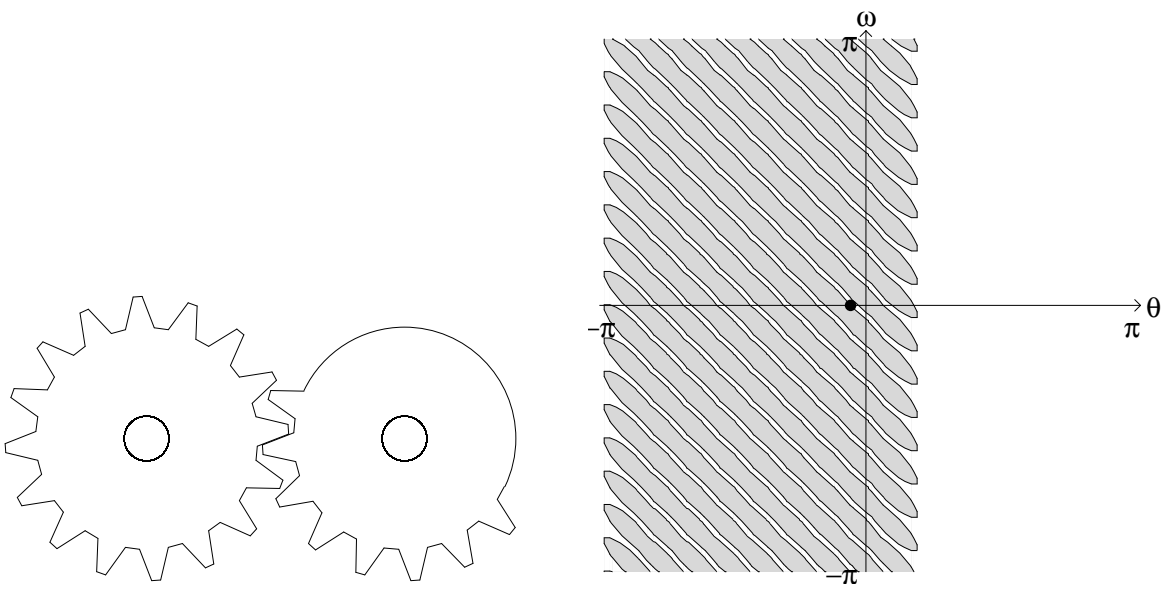
8 Orthogonal gears





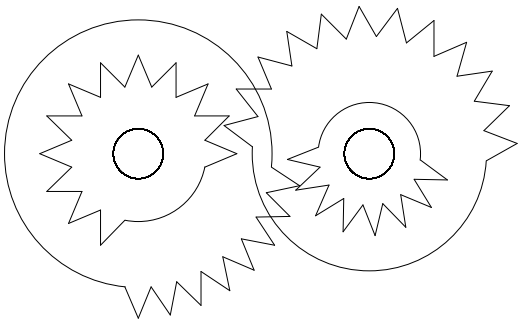
The pair consists of identical toothed gears rotating around orthogonal axes. The top gear rotates around an axis normal to the page. The bottom gear rotates around a vertical axis parallel to the page. Each gear consists of six cylindrical teeth mounted on a cylindrical basis at evenly spaced orientations. When either gear rotates in either direction, the other gear rotates in the opposite direction by the same angle. Two pairs of teeth are engaged at all times. The transmission ratio is almost linear. In the configuration space, the orientation of the top gear is θ and the bottom gear is ω. The slanted channels represent the gear engagements; the rough edges reflect chatter and slight variations in the motion transmission ratio. The channel width shows the amount of backlash. (adapted from 2388)

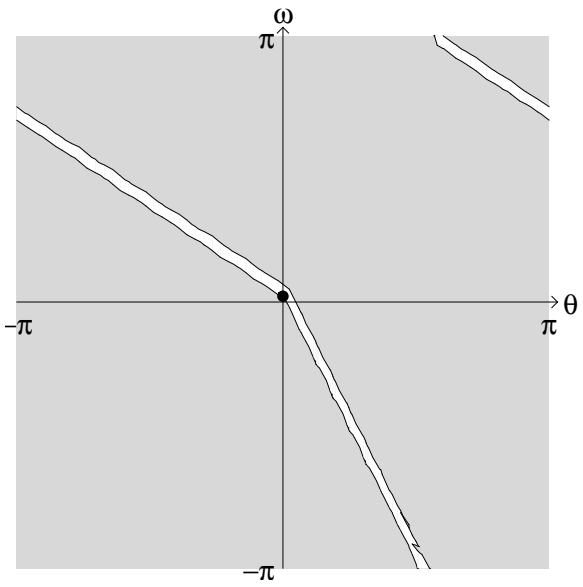
9 Sector gears



The pair consists of a full gear and a sector gear of equal radii. The sector gear has half as many teeth as the full gear. Both have teeth with trapezoidal profiles. When the gears are engaged and one gear rotates in either direction, the other gear rotates in the opposite direction by the same angle. Two pairs of teeth are engaged at all times. The transmission ratio is almost linear with slight backlash and chatter. When the gears are disengaged, their rotations are independent. Continuously rotating the sector gear makes the full gear rotate with a dwell period after each half turn. In the configuration space, the orientations of the sector gear and the full gear are θ and ω. (adapted from 2314)

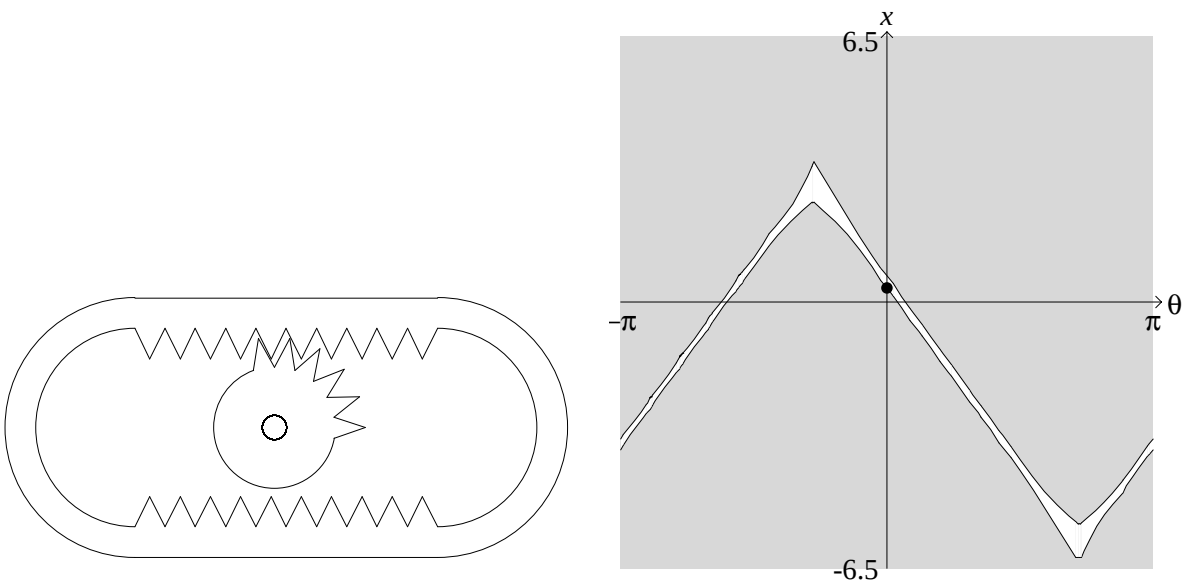
10 Double sector gears





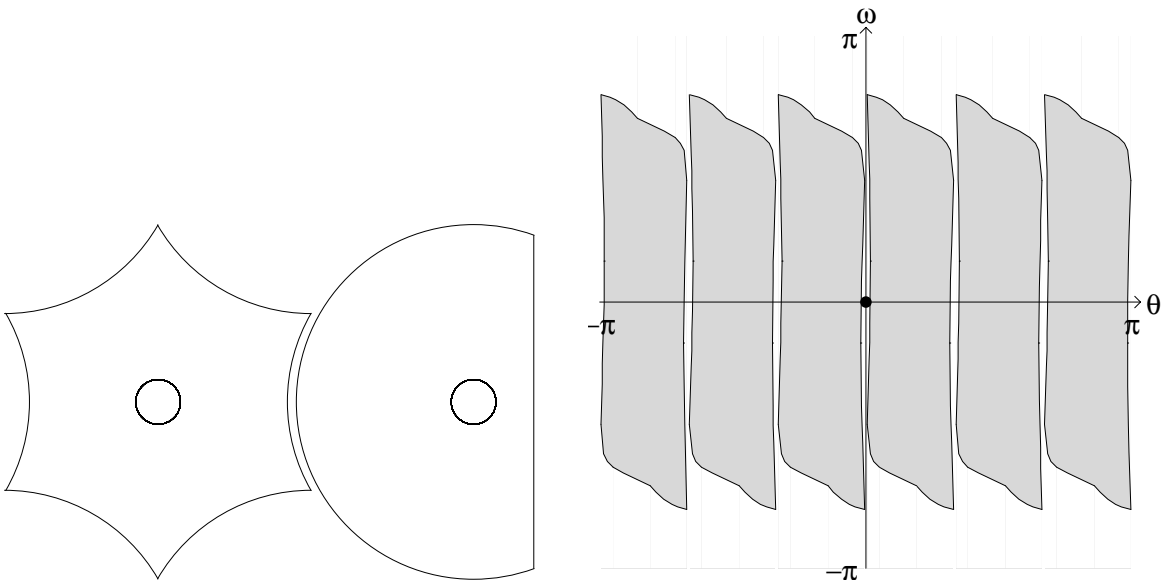
The pair consists of two rotating parts. Each parts consists of two concentric gears one on top of the other. In the left part, the lower gear has angular extent $2\pi/3$ and radius 1, while the upper gear has angular extent $\pi/2$ and radius 2. In the right part, the lower gear has angular extent $\pi/2$ and radius 2, while the upper gear has angular extent π and radius 1. Rotating either part once, causes the other part to rotate once in the opposite direction. The transmission ratio is 1/2 while the lower gears mesh and 2 while the upper gears mesh. The transmission ratios are almost linear, with slight backlash and chatter. In the configuration space, the orientations of the left and right parts are θ and ω. (2314)

11 Double rack gear



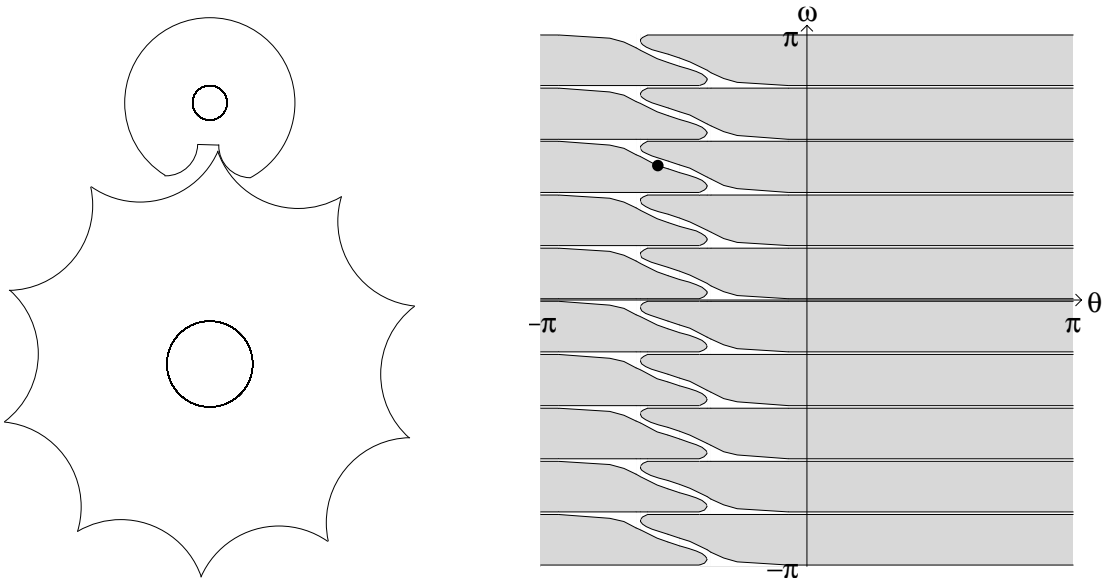
The pair consists of a rotating sector gear and a horizontally translating follower. The sector gear has teeth on less that half of its circumference. The follower has geared racks at the top and the bottom of its inner perimeter. The sector gear alternately engages the upper and lower racks, pushing the follower left and right. Continuously rotating the sector gear makes the follower move left and right without dwell. The maximum displacement of the follower is equal to the perimeter of the sector gear. In the configuration space, the orientation of the sector gear is θ and the displacement of the follower is x. (2321)

12 interlock



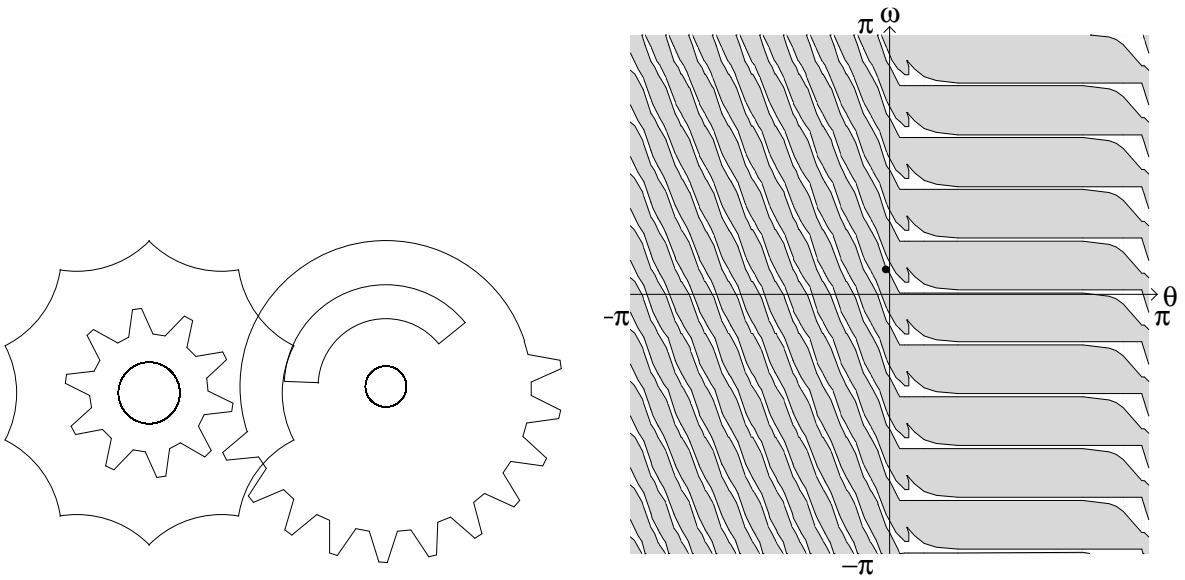
The pair consists of a rotating star-shaped part (left) and a rotating locking part (right). The star consists of six arc segments whose centers coincide with the center of rotation of the locking part. The boundary of the locking part consists of an arc segment centered at the center of rotation and of a line segment. The locking part locks the star at six orientations. The star is disengaged by turning the locking part so that the line segment faces it vertically. In the configuration space, the orientations of the star and of the locking part are θ and ω. The narrow vertical channels represent locking configurations. (261)

13 Intermittent gear



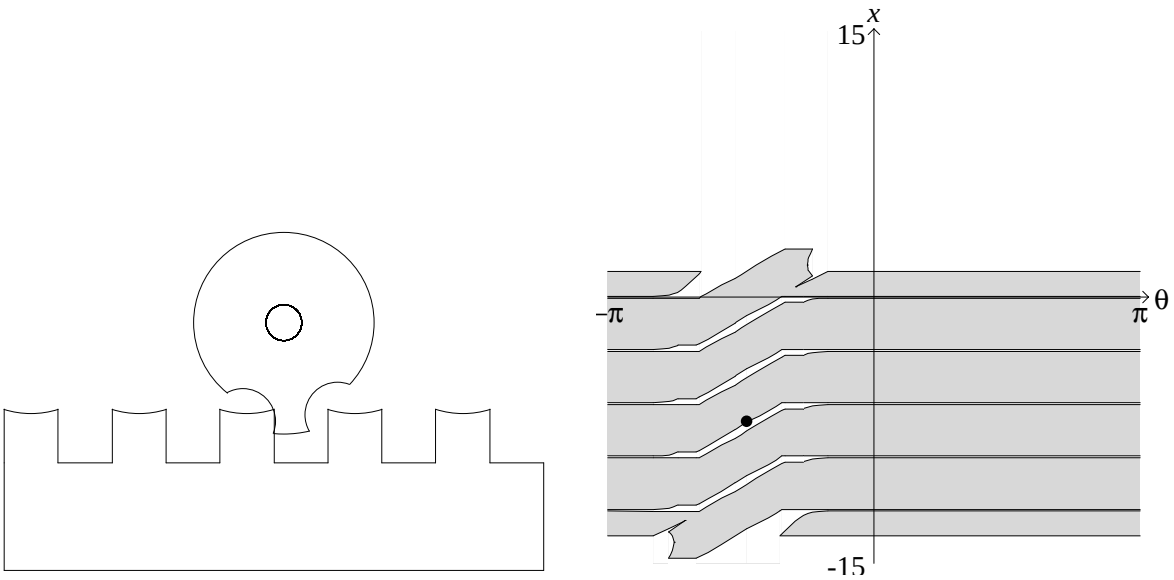
The pair consists of a rotating driver (top) and of a rotating star-shaped part (bottom). The driver consists of a disk with a recess, which allows the arc segment cusps to pass underneath. The star consists of ten arc segments whose centers coincide with the center of rotation of the driver. A spring (not shown) applies a clockwise torque to its hub. A full counterclockwise rotation of the driver advances the star clockwise by 1/10th of a turn and locks it for the rest of the rotation. In the configuration space, the orientations of the driver and of the star are θ and ω. (262)

14 Dwell gear



The pair consists of a rotating driver (right) and a rotating driven part (left). The driver is a concentric arc segment mounted on a gear sector. The driven part is a star-shaped disk mounted on a gear wheel. As the driver rotates clockwise, the gear sector engages and turns the gear wheel. When the sector disengages, the concentric arc segment engages one of the ten concave surfaces of the star-shaped disk, thereby locking it in a dwell position. A full rotation of the driver advances the driven part by 1/10th of a rotation. In the configuration space, the orientations of the driver and of the driven part are θ and ω. The horizontal channels in the configuration space represent the dwell period. The slanted channels represent the gear engagements; the rough edges reflect chatter and slight variations in the motion transmission ratio. The channel width shows the amount of backlash. (2377)

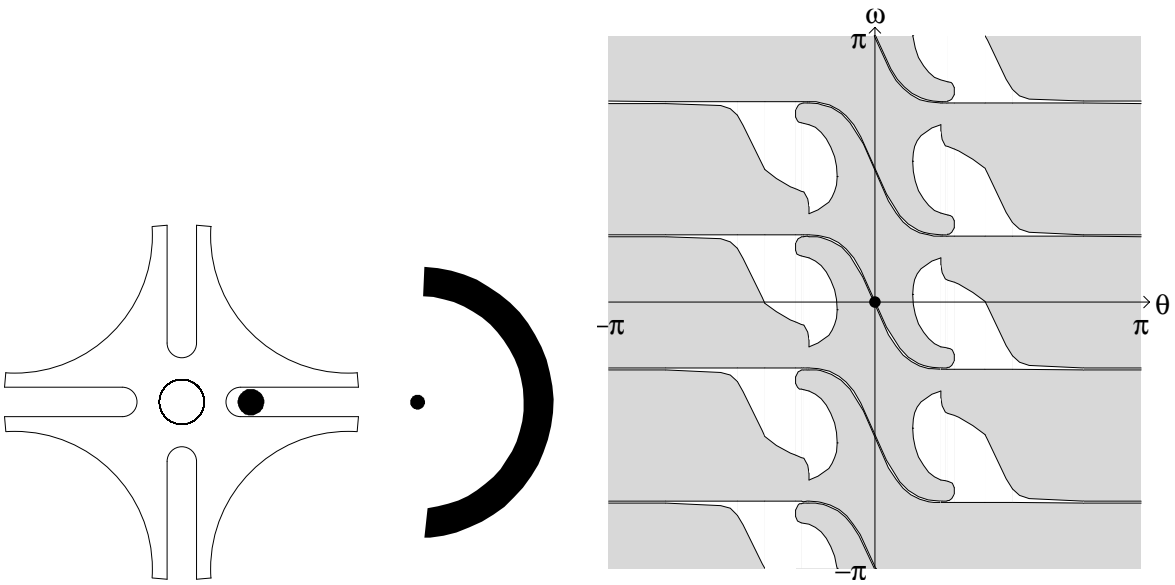
15 Interlock drive



The pair consists of a rotating single-tooth driver and a horizontally translating toothed rack. Each rotation, the driver tooth first engages one tooth of the rack and pushes it then the rounded part of the driver engages the concave top of the tooth and locks the rack. Continuously rotating the driver makes the rack translate with dwell periods. In the configuration space, the orientation of the driver is θ and the displacement of the rack is x.

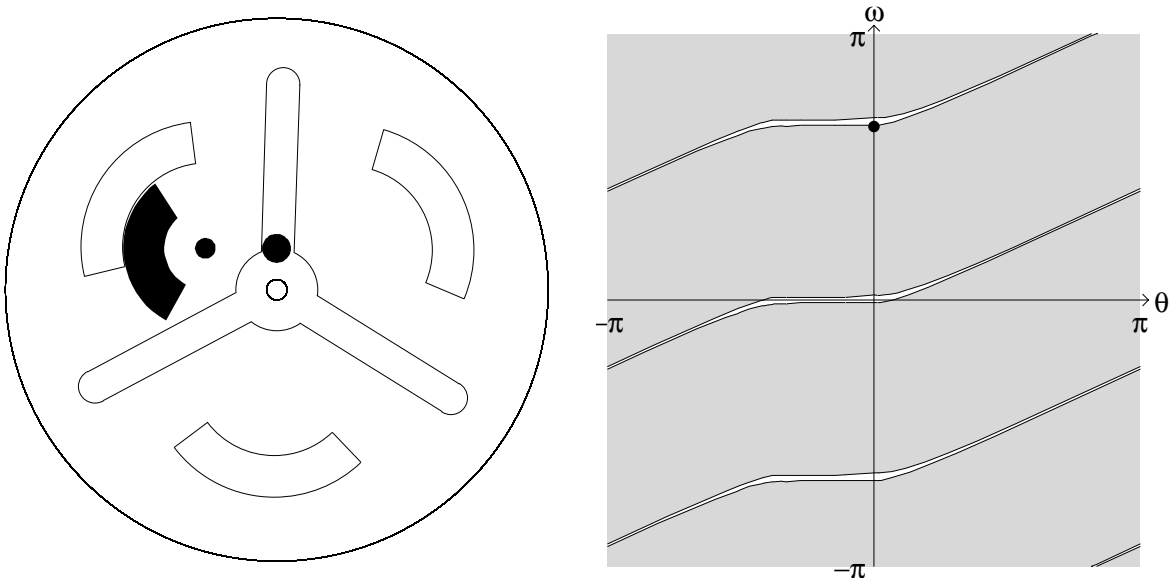
(2354)

16 Geneva wheel



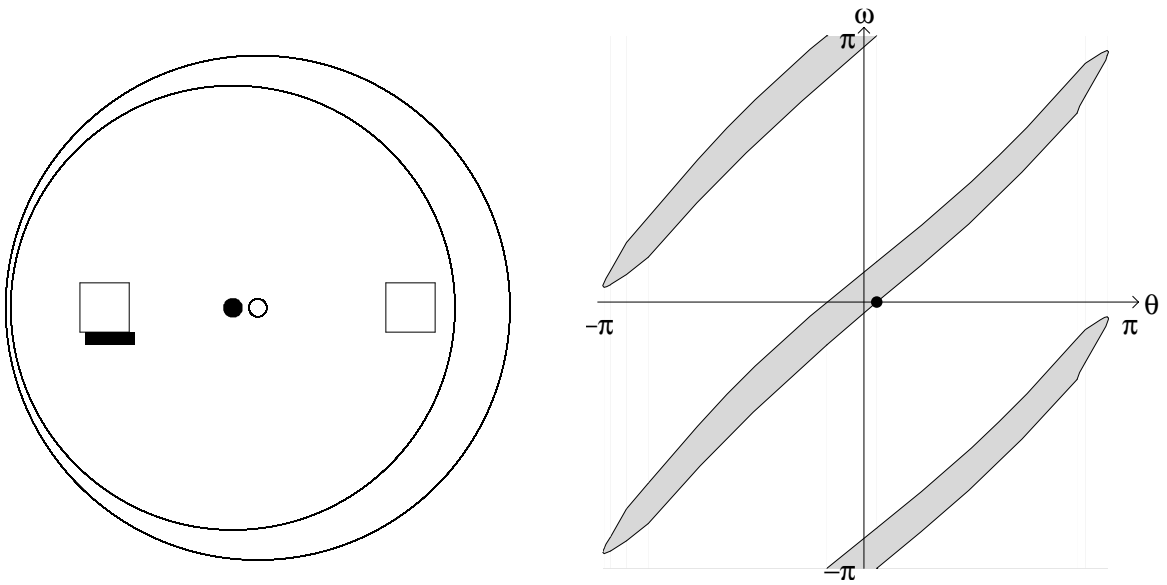
The pair consists of a rotating driver (solid black profiles) and a rotating geneva wheel. The driver consists of a driving pin and a concentric locking arc segment. The geneva wheel consists of a cylindrical basis with four slots on which four locking arc segments are symmetrically mounted. As the driver rotates, the pin engages the geneva wheel slots and rotates it nonuniformly in the same direction. The locking pieces of the driver engage those of the wheel during the idle periods, thereby preventing the wheel from rotating. A full rotation of the driver causes a nonuniform, intermittent rotation of the wheel with three dwell periods and three drive periods. In the configuration space, the orientations of the driver and of the geneva wheel are θ and ω. (2626)

17 Inverse geneva wheel



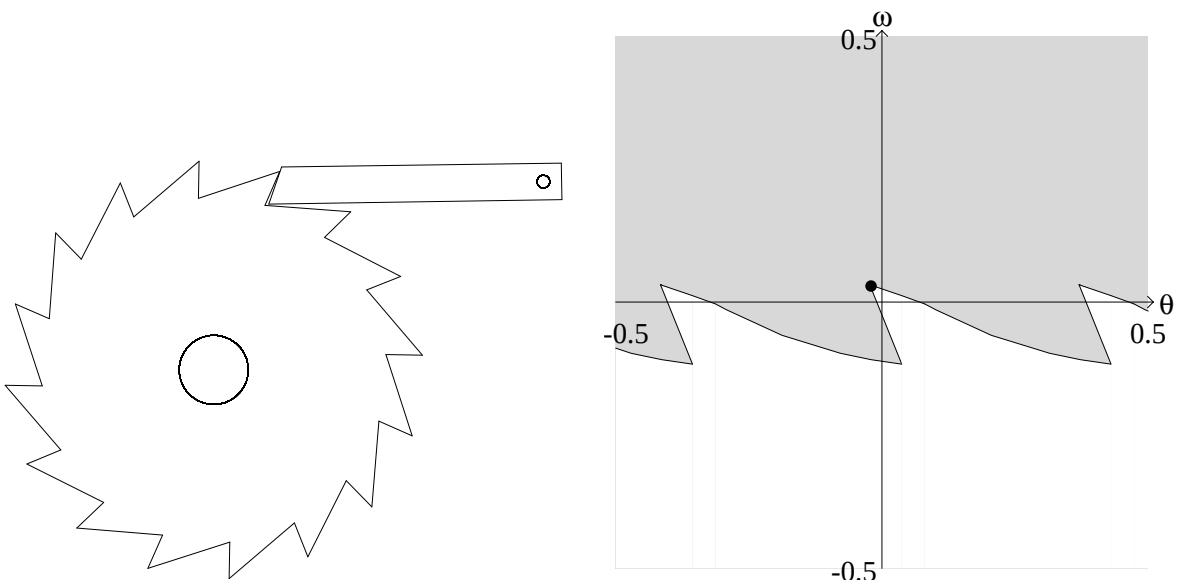
The pair consists of a rotating driver (solid black profiles) and a rotating geneva wheel. The drive consists of a driving pin and a concentric locking arc segment. The geneva wheel consists of a cylindrical basis with three slots on which three locking arc segments are symmetrically mounted. As the driver rotates, the pin engages the geneva wheel slots and rotates it nonuniformly in the same direction. The locking pieces of the driver engage those of the wheel during the idle periods, thereby preventing the wheel from rotating. A full rotation of the driver causes a nonuniform, intermittent rotation of the wheel with three dwell periods and three drive periods. In the configuration space, the orientations of the driver and of the geneva wheel are θ and ω. The open "bubbles" correspond to configurations where the driver does not fully lock the rotation of the wheel. (2626)

18 Intermittent cam



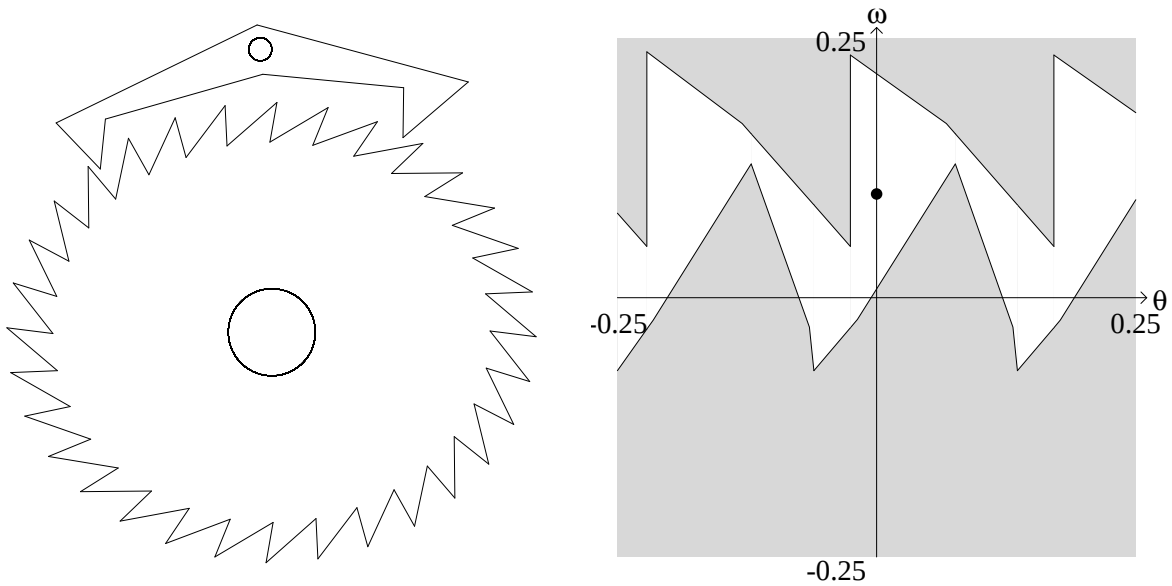
The pair consists of a rotating driving cam (solid black profile) and a rotating follower. The cam consists of a rectangular pin mounted on a basis. The follower consists two square pins symmetrically mounted on a basis. The centers of rotation of the cam and of the follower are offset. As the cam rotates, its rectangular pin pushes the follower's square pin for half a revolution. The pins then slide out of engagement because of the eccentricity of the axes of rotation. The follower is stationary during the second half of the cam's rotation. The rectangular pin of the cam then engages the second square pin of the follower for another half revolution. In the configuration space, the orientations of the cam and of the follower are θ and ω. The free space consists of two strips, which represent the interactions between the rectangular pin of the cam and the square pins of the follower. The pair is bidirectional. (3098)

19 Ratchet



The pair consists of a rotating ratchet (left) and a rotating pawl (right). The ratchet consists of 16 triangular teeth. The pawl is a quadrangle whose tip matches the shape of the ratchet teeth. Rotating the ratchet counterclockwise makes the pawl rise then fall under its own weight, following the profile of the teeth. The pawl blocks clockwise rotation of the ratchet after at most 1/16th of a rotation. In the configuration space, the orientations of the ratchet and of the pawl are θ and ω. The figure shows the portion of configuration space generated by two consecutive ratchet teeth. (2660)

20 Escapement



The pair consists of a rotating wheel and a rotating anchor. Attached to the anchor is a pendulum (not shown) that oscillates with a constant period. The wheel rotates counterclockwise due to a torque acting on its hub. As the anchor oscillates, the pallets alternately engage teeth on the left and right sides of the wheel. The wheel advances one notch each oscillation. It rotates at a constant velocity determined by the period of the anchor. In the configuration space, the orientations of the wheel and of the anchor are θ and ω. The figure shows the portion of the configuration space generated by two consecutive teeth of the wheel. The upper and lower boundaries correspond to contacts of the right and left anchor arms with the teeth. The interleaved boundaries prevent the wheel from rotating freely.