COS 594: Advanced Algorithms and Complexity **Spring 1999**

Assignment #1

Due: Wednesday, February 17 *Sanjeev Arora*

The first two questions directly test your understanding of lecture material. The other three require creativity.

Suggested reading (for lectures 1,2,3): Spielman's Lecture Notes 1, 4, 5.

Problems (from lectures 1, 2, 3):

1. Show that $\Sigma_2^p = \mathrm{NP}^{\mathrm{SAT}}$.
2. Modify the proof of the Karp-Lipton-Sipser theorem to show that if coNP $\subseteq$ P/*poly* then PH $= \Sigma_3^p$.
3. Show that PSPACE $\neq$ TIME($2^{O(n)}$).
4. Show that for every $k > 0$ there is a language in $\Sigma_4^p \cap \Pi_4^p$ that does not have circuits of size $O(n^k)$. (A Hint: Try to first prove that there is a language that has circuits of size n^{2k+1} but no circuits of size n^k.) Conclude using the Karp-Lipton-Sipser theorem that in fact such a language exists in $\Sigma_2^p \cap \Pi_2^p$.
5. Show that if P $\neq$ NP, then there are languages $A, B \in$ NP such that neither of the two is polynomial-time reducible to the other.