COS 226 Lecture 10: Radix trees and tries

Symbol Table, Dictionary

- records with keys
- INSERT
- SEARCH

Balanced trees, randomized trees

- use O(lgN) comparisons

Hashing

- uses O(1) probes
- but time proportional to key length

Is key length required?

- (no)

Are comparisons necessary?

- (no)

Best possible: examine lgN BITS

Digital search trees (DSTs)

Easy way to balance tree

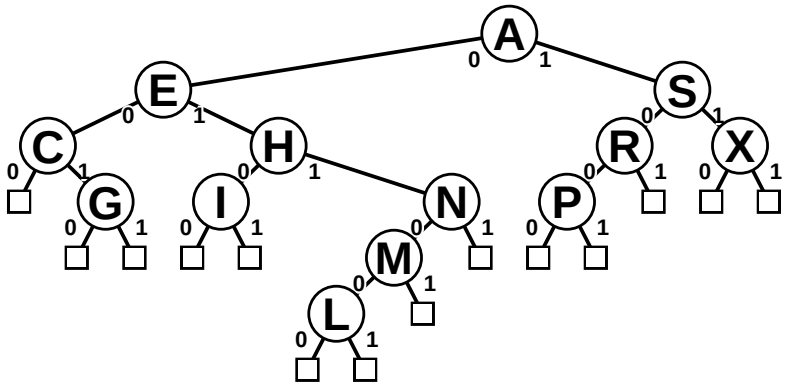
Use bits of key to direct search

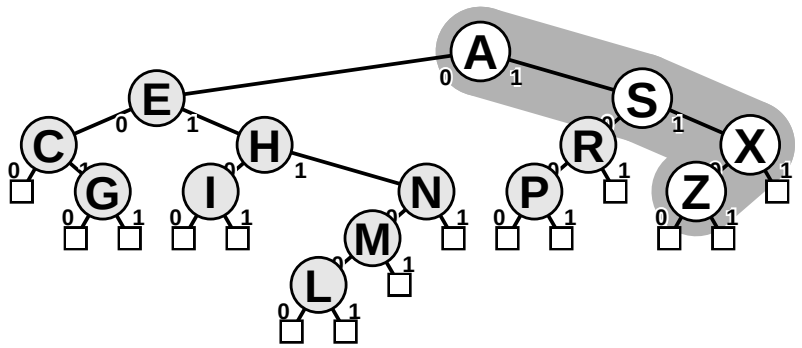
Otherwise identical to BST

```
#define bit(A, B) digit(A, B)
Item searchR(link h, Key v, int w)
  { Key t = key(h->item);
    if (h == z) return NULLitem;
    if eq(v, t) return h->item;
    if (bit(v, w) == 0)
          return searchR(h->l, v, w+1);
    else  return searchR(h->r, v, w+1);
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
link insertR(link h, Item item, int w)
  { Key v = key(item), t = key(h->item);
    if (h == z) return NEW(item, z, z, 1);
    if (digit(v, w) == 0)
          h->l = insertR(h->l, item, w+1);
    else  h->r = insertR(h->r, item, w+1);
    return h;
  }
void STinsert(Item item)
  { head = insertR(head, item, 0); }
```

Digital tree insertion

A	00001
S	10011
E	00101
R	10010
C	00011
H	01000
I	01001
N	01110
G	00111
X	11000
M	01101
P	10000
L	01100





Trees NOT ordered

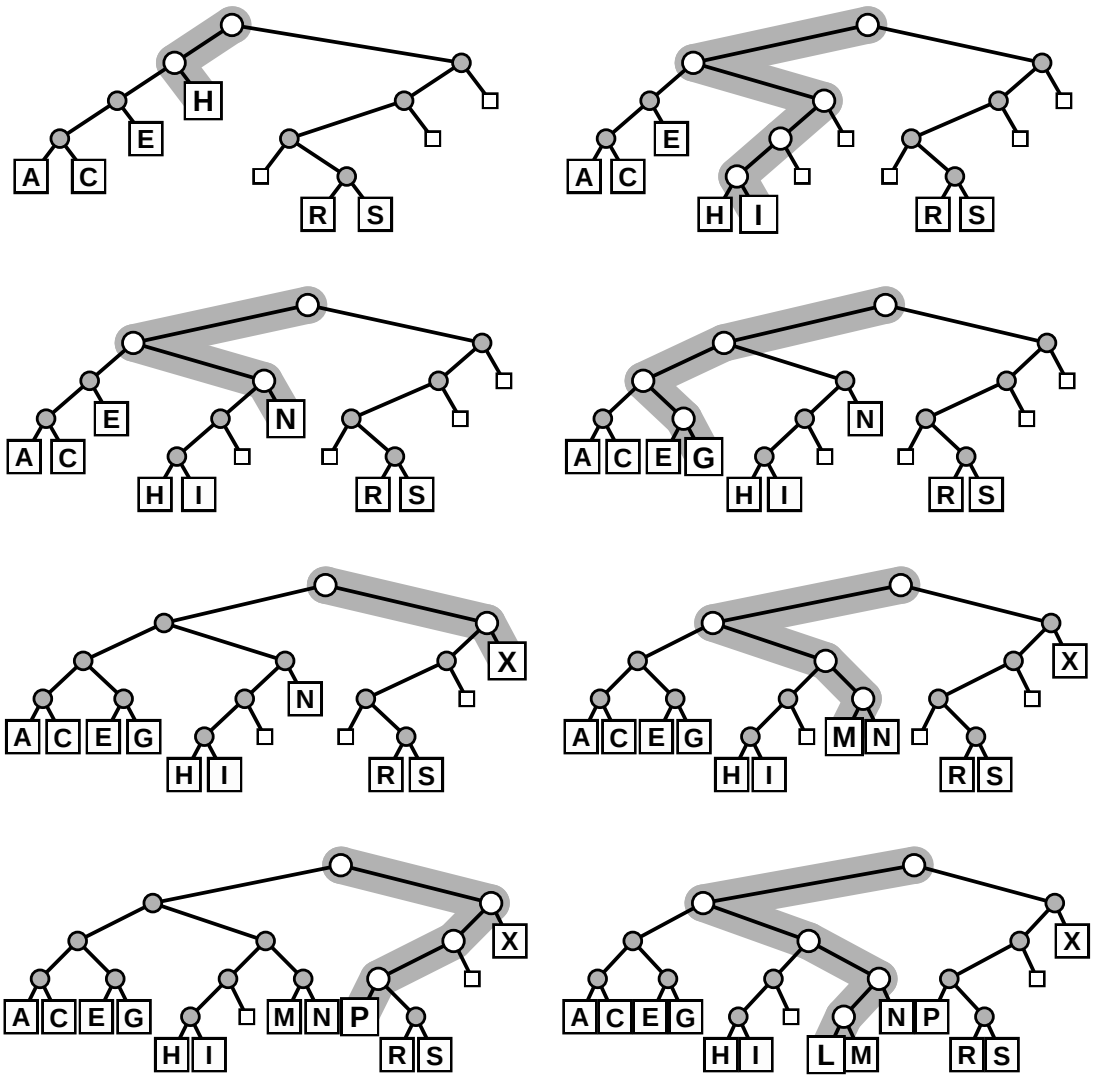
can’t support SORT and SELECT

Tries

Branch according to bits in keys

No keys in internal nodes

Records/keys in external nodes



Structure depends on keys, not insertion order

Examine lgN BITS to distinguish key

independent of key length!

Problems:

44% space waste from 1-way branching

multiple node types

Trie search implementation

Branch according to bits, as in DST

Three outcomes:

- null link
- key in external node matches search key
- key in external node differs from search key

```
#define external(A) ((h->l == z) && (h->r == z))
Item searchR(link h, Key v, int w)
  { Key t = key(h->item);
    if (h == z) return NULLitem;
    if (external(h))
      return eq(v, t) ? h->item : NULLitem;
    if (digit(v, w) == 0)
          return searchR(h->l, v, w+1);
    else  return searchR(h->r, v, w+1);
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
```

Trie insertion implementation

Two possible search miss outcomes

- null link: replace with link to new node
- ext. node: recursive split to distinguish new

```
void STinit()
  { head = (z = NEW(NULLitem, 0, 0, 0)); }
link split(link p, link q, int w)
  { link t = NEW(NULLitem, z, z, 2);
    switch(bit(p->item, w)*2+bit(q->item, w))
    { case 0: t->l = split(p, q, w+1); break;
      case 1: t->l = p; t->r = q; break;
      case 2: t->r = p; t->l = q; break;
      case 3: t->r = split(p, q, w+1); break; }
    return t;
  }
link insertR(link h, Item item, int w)
  { Key v = key(item), t = key(h->item);
    if (h == z) return NEW(item, z, z, 1);
    if (external(h))
      { return split(NEW(item, z, z, 1), h, w); }
    if (digit(v, w) == 0)
          h->l = insertR(h->l, item, w+1);
    else  h->r = insertR(h->r, item, w+1);
    return h;
  }
void STinsert(Item item)
  { head = insertR(head, item, 0); }
```

Patricia tries

Digression: cute nomenclature

Practical
Algorithm
To
Retrieve
Information
Coded
In
Alphanumeric

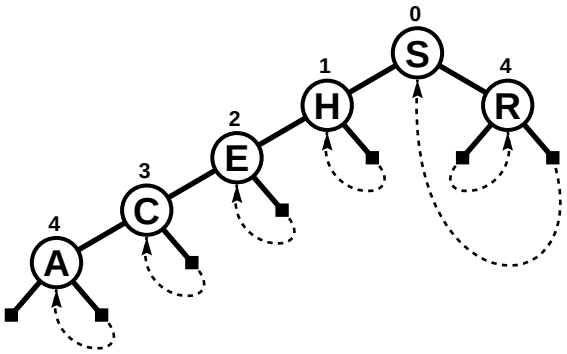
information reTRIEval
(but pronounced "try")

Patricia:
* collapse one-way branches in tries
* "thread" tree to eliminate multiple node types
Quintessential search algorithm

Patricia trie search and insertion

SEARCH

branch according to specified bit

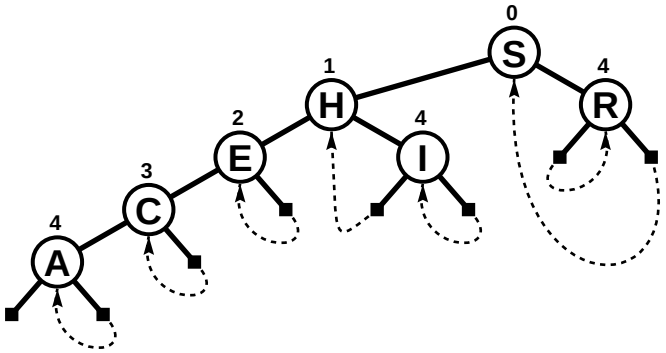


INSERTION

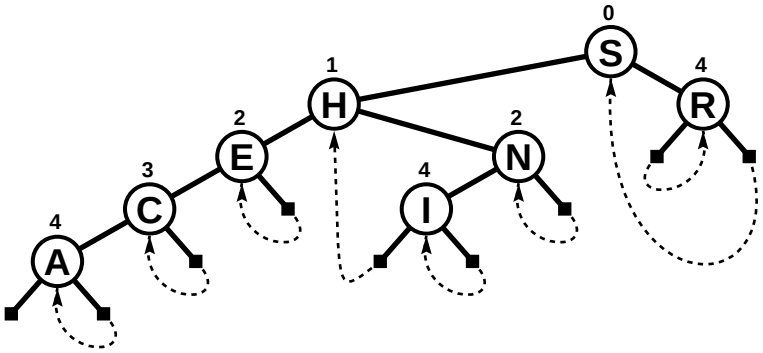
search, then find leftmost bit that

distinguishes new key from key found

Easy (external) ca



Harder (internal



R-way digital tree

Generalize to R links per node

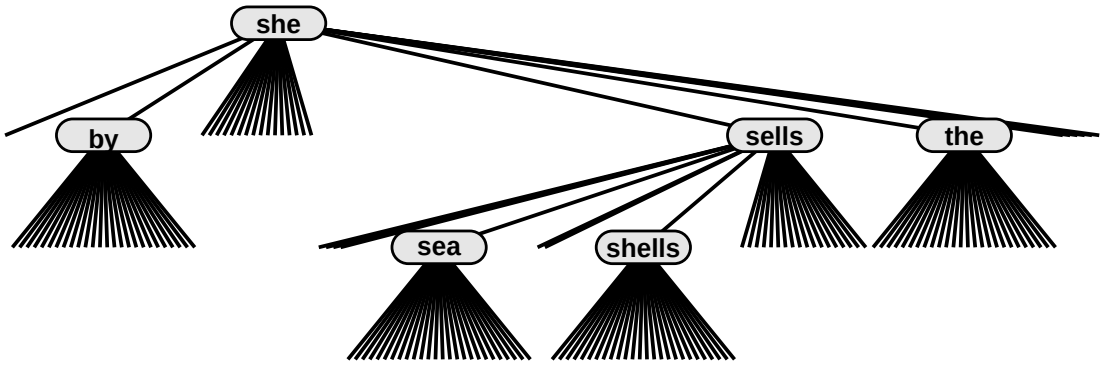
R-way branching gives fast search

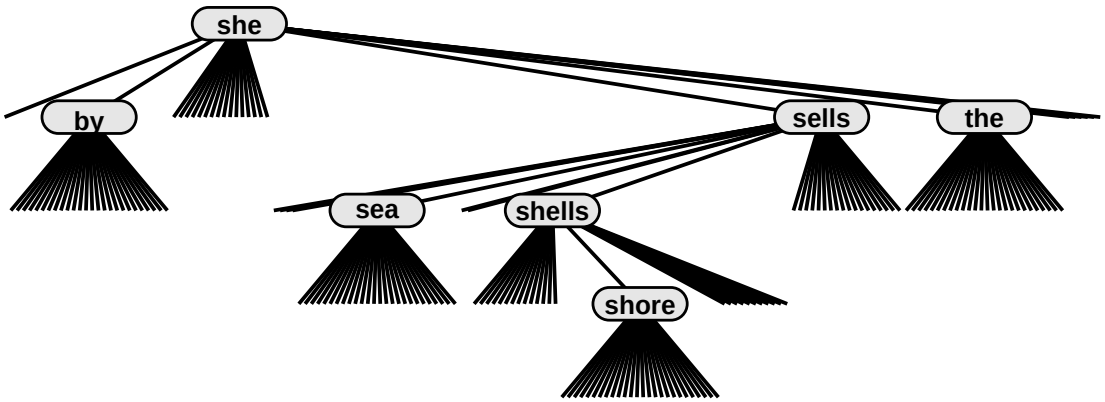
```
struct STnode
   { Item item; link next[26]; };
link NEW(Item);

Item searchR(link h, Key v, int w)
   { int i = digit(v, w);
      if (h == z) return NULLitem;
      if eq(v, key(h->item)) return h->item;
      return searchR(h->next[i], v, w+1);
   }
Item STsearch(Key v)
   { return searchR(head, v, 0); }
```

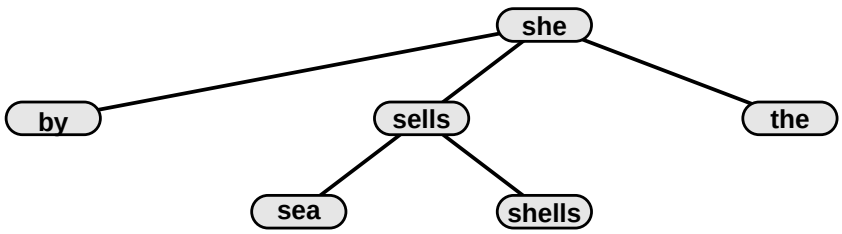
Simple, fast

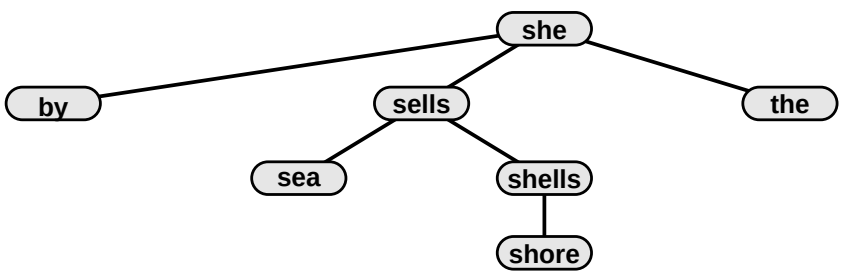
(but uses a lot of space)

R-way digital tree example









R-way digital tree analysis

N keys

N internal nodes
R links per node

Space: N*R
Time: lgN/lgR comparisons

Ex: R=26, N=20000
    500,000 links
    tree height 3-4

Ex: R=16, N=1M
    16M links
    tree height 5

Plus:
    one node type, easy implementation
Minus:
    full comparison at each node
    can't support SORT, SELECT

R-way trie

Generalize trie to R-way branching

Nodes contain characters

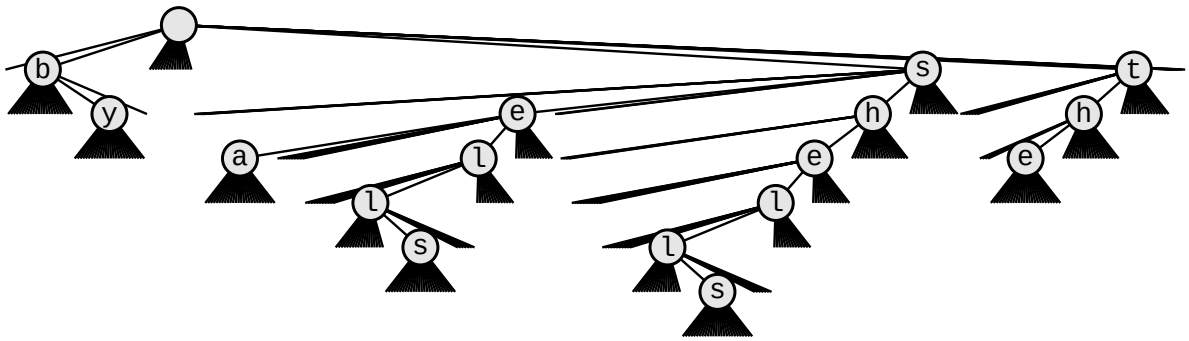
R-way branching on next character

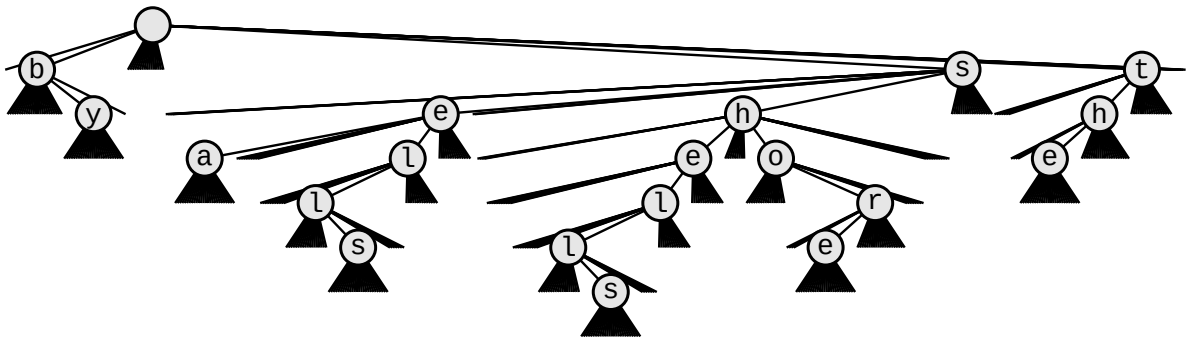
```
typedef struct STnode* link;
struct STnode
   { Item item; link next[R]; };

Item searchR(link h, Key v, int w)
   { int i = digit(v, w);
      if (h == z) return NULLitem;
      if (internal(h))
         return searchR(h->next[i], v, w+1);
      if eq(v, key(h->item)) return h->item;
      return NULLitem;
   }
Item STsearch(Key v)
   { return searchR(head, v, 0); }
```

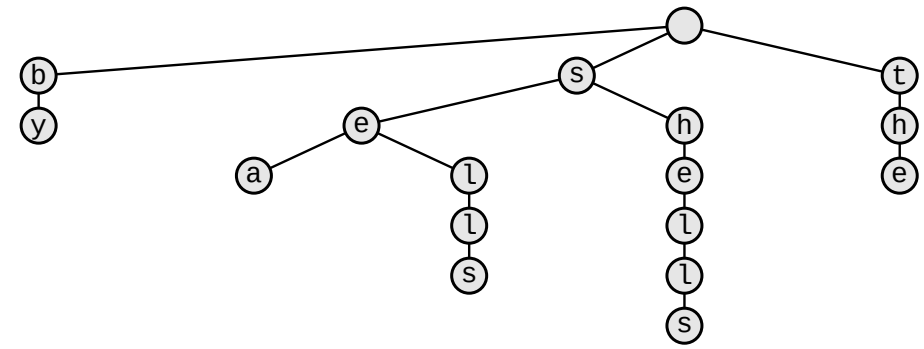
Return match on prefix, or make keys prefix-free

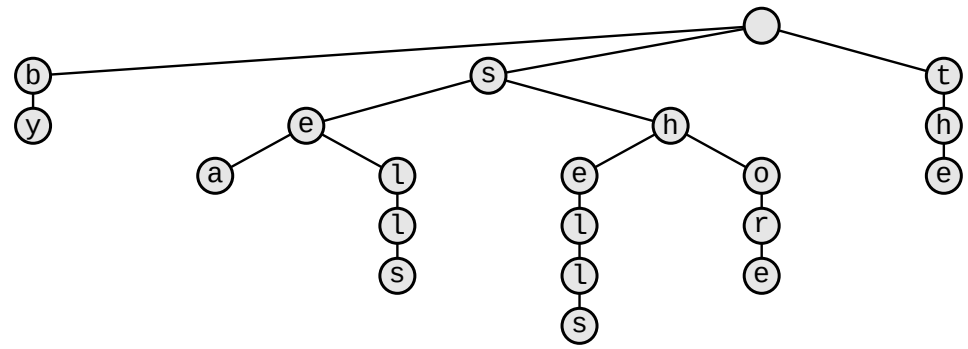
- end-of-key character
- suffix trie

R-way trie example









R-way trie existence table

No data in trie

Keys encoded in trie structure

Easy to implement

May use excessive space

```
Item searchR(link h, Key v, int w)
  { int i = digit(v, w);
    if (h == z) return NULLitem;
    if (i == NULLdigit) return v;
    return searchR(h->next[i], v, w+1);
  }
Item STsearch(Key v)
  { return searchR(head, v, 0); }
```

R-way trie implementation

Generalize binary trie implementation

```
link split(link p, link q, int w)
   { link t = NEW(NULLitem);
      int pd = digit(p->item, w),
           qd = digit(q->item, w);
      if (pd == qd)
             { t->next[pd] = split(p, q, w+1); }
      else  { t->next[pd] = p; t->next[qd] = q; }
      return t;
   }
link insertR(link h, Item item, int w)
   { Key v = key(item);
      int i = digit(v, w);
      if (h == z) return NEW(item);
      if (!internal(h))
         { return split(NEW(item), h, w); }
      h->next[i] = insertR(h->next[i], v, w+1);
      return h;
   }
void STinsert(Item item)
   { head = insertR(head, item, 0); }
```

R-way trie analysis

Assume

one-way links collapsed

link to remainder of key in external nodes

N keys, total of C characters in keys

approx. N internal nodes

R links per node

Space: N*R + C

Time: lgN/lgR CHARACTER comparisons

Ex: M=26, N=20000

5R0,000 links

tree height 3-4

Ex: R=16, N=1M

16M links

tree height 5

Faster than hashing

particularly for unsuccessful search

Problems

* eliminating 1-way branches
* multiple node types
* too much space for null links

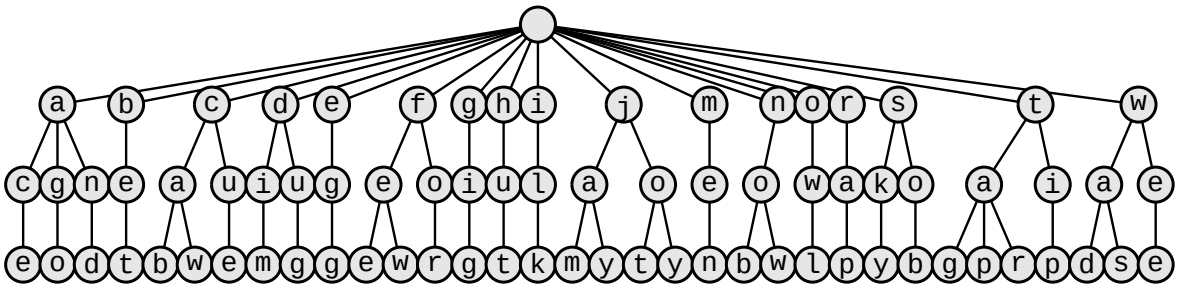
Correspondence with sorting algs

BSTs correspond to Quicksort recursive structure

Roles of TIME and SPACE interchanged

- TIME:
  - path in tree
  - size of subfile in sort
- SPACE:
  - stack size in sort
  - branching factor in tree

Tries correspond to MSD radix sort



3-way tries correspond to 3-way Quicksort

Three-way radix tries

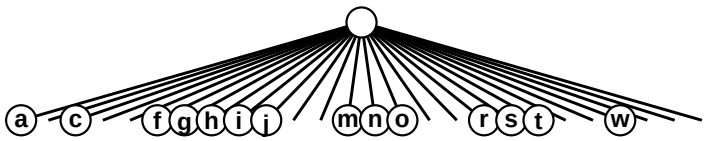
Nodes contain characters

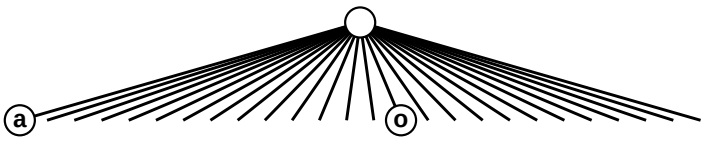
three-way branching on next character

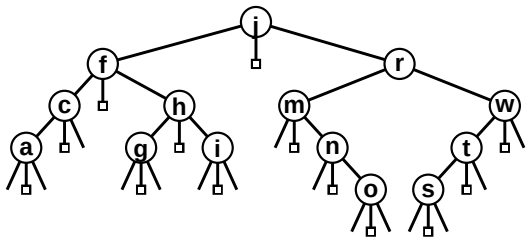
- left: key character less
- middle: key character equal
- right: key character greater

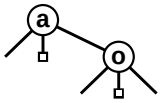
Equivalent to replacing trie node with

- BST on character









Space savings: can use existence trie

Adds factor of 2lnM to search cost

- constant no. of BYTES for unsucc. search

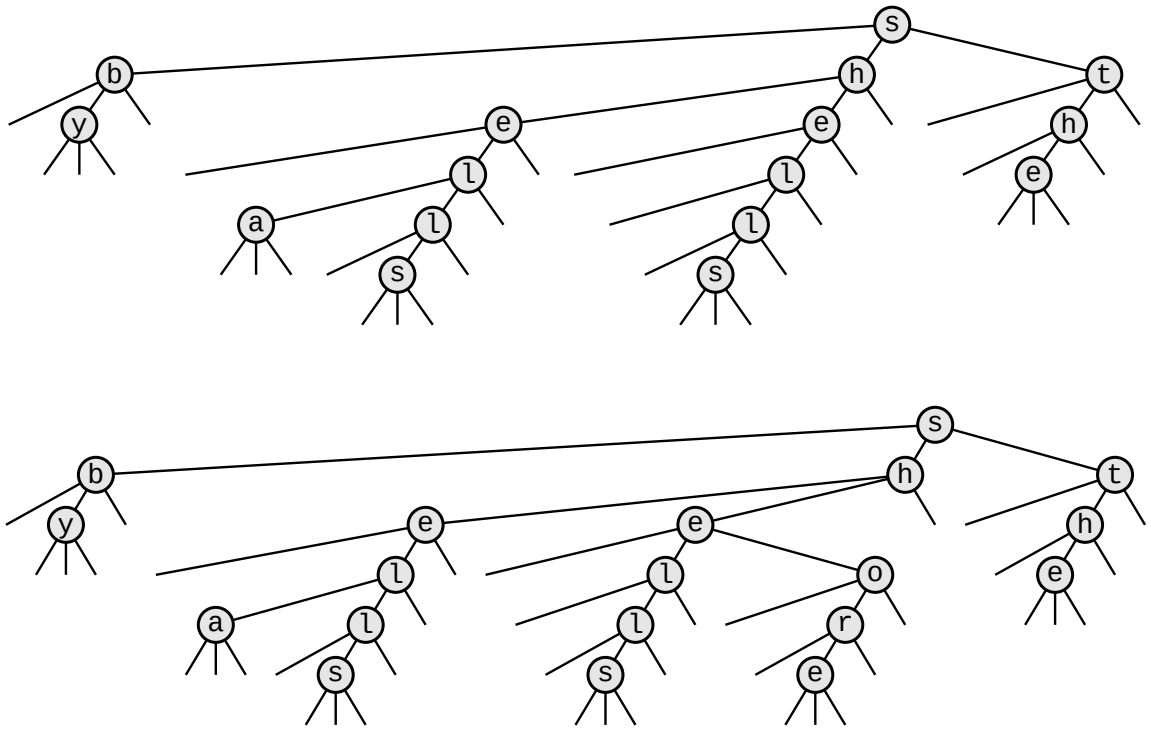
Easy recursive sort

Most important advantage:

- adapts well to strange keys

Faster than hashing

Three-way radix search trie example




```
Item searchR(link h, Key v, int w)
   { int i = digit(v, w);
      if (h == z) return NULLitem;
      if (i == NULLdigit) return v;
      if (i < h->d) return searchR(h->l, v, w);
      if (i == h->d) return searchR(h->m, v, w+1);
      if (i > h->d) return searchR(h->r, v, w);
   }
Item STsearch(char *v)
   { return searchR(head, v, 0); }
```

Three-way trie insertion implementation

```
typedef struct STnode* link;
struct STnode
   { Item item; int d; link l, m, r; };
void STinit() { head = z; }
link NEW(int d)
   { link x = malloc(sizeof *x);
      x->d = d; x->l = z; x->m = z; x->r = z;
      return x;
   }
link insertR(link h, Item item, int w)
   { Key v = key(item);
      int i = digit(v, w);
      if (h == z) h = NEW(i);
      if (i == NULLdigit) return h;
      if (i < h->d) h->l = insertR(h->l, v, w);
      if (i == h->d) h->m = insertR(h->m, v, w+1);
      if (i > h->d) h->r = insertR(h->r, v, w);
      return h;
   }
void STinsert(Key key)
   { head = insertR(head, key, 0); }
```

Empirical studies

random 32-bit keys

	BUILD				SEARCH			
.	bst	dst	trie	pat	bst	dst	trie	pat
. 5000	4	5	7	7	3	2	3	2
. 12500	18	15	20	18	8	7	9	7
. 25000	40	36	44	41	20	17	20	17
. 50000	81	80	99	90	43	41	47	36
.100000	176	167	269	242	103	85	101	92
.200000	411	360	544	448	228	179	211	182

words in Moby Dick

	BUILD				SEARCH			
.	bst	hash	rst	fast	bst	hash	rst	fast
. 5000	5	4	3	3	3	2	2	1
. 12500	11	8	9	7	9	5	5	3
. 25000	23	15	17	13	19	12	10	7
. 50000	50	29	31	25	43	25	21	15

library call numbers

	BUILD				SEARCH			
.	bst	hash	rst	fast	bst	hash	rst	fast
. 5000	19	16	21	20	10	8	6	4
. 12500	48	48	54	97	29	27	15	14
. 25000	118	99	188	156	67	59	36	30
. 50000	230	191	333	255	137	113	70	65