COS 226 Lecture 5: Priority Queues

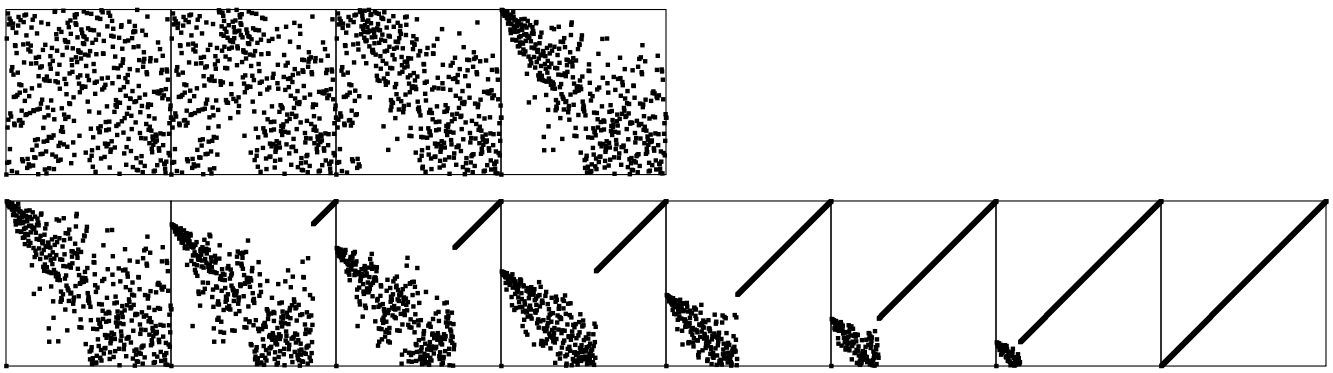
Abstract data types
- client
- interface
- implementation

Priority queue ADT
- insert
- remove the largest

Brute-force implementations

HEAPS

Heapsort



BINOMIAL QUEUES

Abstract data types

Separate INTERFACE and IMPLEMENTATION

- easier maintainence of large programs
- build layers of abstraction
- reuse software
- elementary example: pushdown stack

INTERFACE:

- description of data type, basic operations

CLIENT:

- program using operations defined in interface

IMPLEMENTATION:

- actual code implementing operations

Client can't know details of implementation

- (many implementations to choose from)

Implementation can't know details of client needs

- (many clients use the same implementation)

Modern programming languages support ADTs

- C++, Modula-3, Oberon, Java (C)

Caveats

- perspective needed
- performance matters

Priority queue ADT

Records with keys (priorities)

Two basic operations

* insert
* delete the largest

generic operations common to many ADTs

* create
* test if empty
* destroy (often ignored if not harmful)

INTERFACE for basic operations

```
.      void PQinit();
.      void PQinsert(Item);
.      Item PQdelmax();
.       int PQempty();
```

Should also specify

constraints and error conditions

Example applications

* simulation
* numerical computation
* compression algorithms
* graph searching algorithms

Sample client program

Find the M SMALLEST of N items

Typical values: M=100, N=1000000

```
PQinit();
for (k = 0; k < M; k++)
     PQinsert(nextItem());
for (k = M; k < N; k++)
   {
     PQinsert(nextItem());
     t = PQdelmax();
   }
for (k = 0; k < M; k++)
     a[k] = PQdelmax(());
```

Cost? Depends on PQ implementation

Reasonable assumption for any implementation:

space proportional M

Time bounds for standard implementations:

brute-force: N M

best: N lg M

Typical brute-force implementation

Use an unordered array

```
static Item *pq;
static int N;

PQinsert(Item v)
   { pq[N++] = v; }
Item PQdelmax()
   {
      int j, max = 0;
      for (j = 1; j < N; j++)
         if (less(pq[max], pq[j])) max = j;
      exch(pq[max], pq[N]);
      return pq[--N];
   }
void PQinit(int maxN)
   { pq = malloc(maxN*sizeof(Item)); N = 0; }
int PQempty()
   { return N == 0; }
```

Should also test and report errors

Other options for brute-force implementations

* ordered array
* unordered singly-linked list
* ordered singly-linked list

Client/Interface/Implementation

Separate compilation
(Freudian slip: "complication")

INTERFACE
* define data types
* declare functions
* in C, use ".h" file (no executable code)

CLIENT:
* include ".h" file
* call functions

IMPLEMENTATION:
* include ".h" file
* give code for functions

Client and implementation can be compiled at different times, then function calls LINKED to their implementations

Details: Sedgewick, Chapter 4; COS 217

Modular programming
separate compilation abstraction
essential in implementing big systems
useful in putting good algs to work

Priority queue ADT (continued)

Other useful operations

* construct a PQ from N items
* return the value of the largest
* delete a specified item
* change an item's priority
* merge together two PQs

Interface more complicated

need HANDLES for records

need HANDLES for priority queues

where's the data?

(client, implementation, or both?)

First-class ADT solution

```
.  typedef struct pq* PQ;
.  typedef struct PQnode* PQlink;
.       PQ PQinit();
.      int PQempty(PQ);
.   PQlink PQinsert(PQ, Item);
.     Item PQdelmax(PQ);
.     void PQchange(PQ, PQlink, Item);
.     void PQdelete(PQ, PQlink);
.       PQ PQjoin(PQ, PQ);
```

PQ and PQlink are pointers to structures

to be specified in the implementation

More info: section 4.8 in Sedgewick; lecture 7

PQ implementations cost summary

Worst-case per-operation time cost
as a function of PQ size

	insert	delete max	delete	find max	change key	join
ordered						
array	**N**	**1**	**N**	**1**	**N**	**N**
list	**N**	**1**	**1**	**1**	**N**	**N**
unordered						
array	**1**	**N**	**1**	**N**	**1**	**N**
list	**1**	**N**	**1**	**N**	**1**	**1**
heap	**lg N**	**lg N**	**lg N**	**1**	**lg N**	**N**
binomial queue	**lg N**	**lg N**	**lg N**	**lg N**	**lg N**	**lg N**
best in theory	**1**	**lg N**	**lg N**	**1**	**1**	**1**

Algorithm design success story:

* nearly optimal worst-case cost
* simple (but ingenious!) algorithms

Costs lower in practice

average-case analysis

amortized analysis

Heap-ordered complete binary trees

COMPLETE BINARY TREE:

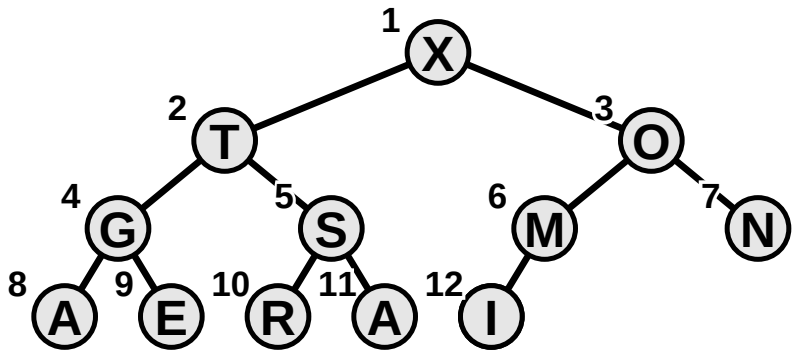
leaves on two levels, on left at bottom level

HEAP-ORDERED:

parent larger than both children

therefore, largest at root

can define for any tree, not just complete



Array representation:

root in a[1]

children of root in a[2] and a[3]

...

children of i in a[2i] and a[2i+1]

parent of i in a[i/2]

No explicit links needed for tree

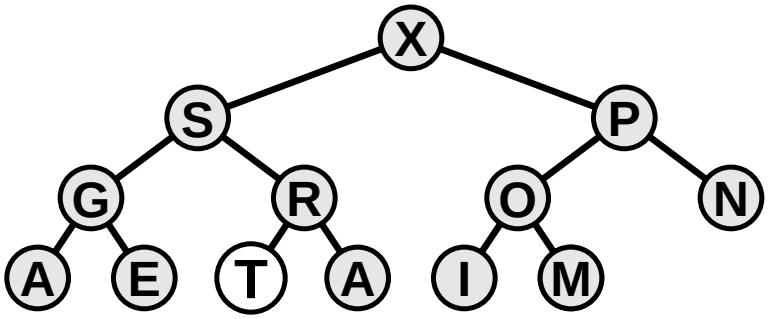
.	0	1	2	3	4	5	6	7	8	9	10	11	12
.		X	T	O	G	S	M	N	A	E	R	A	I

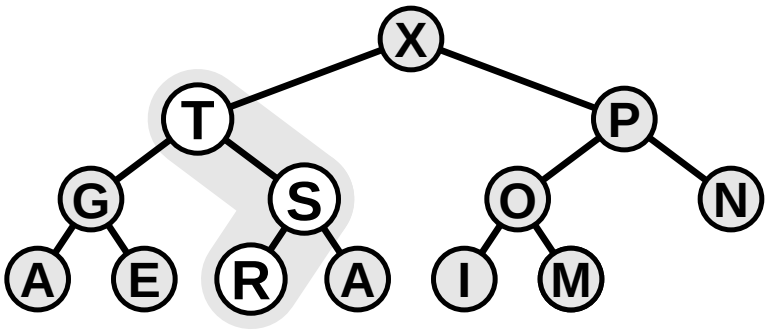
Promotion (bubbling up in a heap)

Change key in node at the bottom of the heap

To restore heap condition:

exchange with parent if necessary





"How to Succeed" model (Peter Principle)

```
fixUp(Item a[], int k)
  {
    while (k > 1 && less(a[k/2], a[k]))
      { exch(a[k], a[k/2]); k = k/2; }
  }
```

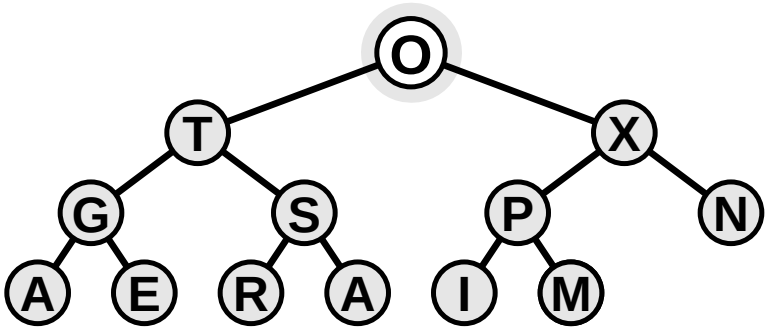
easy to make code twice as fast ala insertion

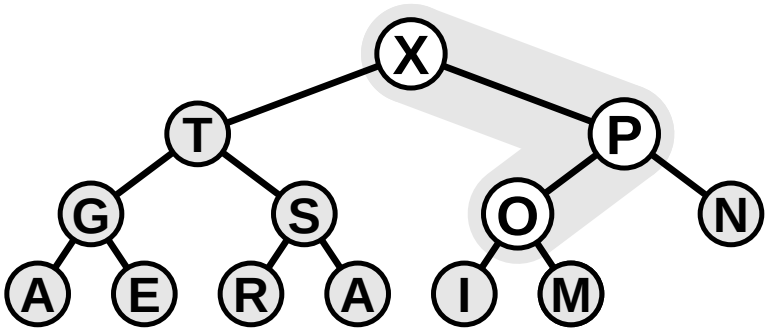
Demotion (sifting down in a heap)

Change key in node at the top of the heap

To restore heap condition:

exchange with larger child if necessary





"Boss's child" model (Royal succession?)

```
fixDown(Item a[], int k, int N)
   { int j;
      while (2*k <= N)
         { j = 2*k;
            if (j < N && less(a[j], a[j+1])) j++;
            if (!less(a[k], a[j])) break;
            exch(a[k], a[j]); k = j;
         }
   }
```

PQ implementation with heaps

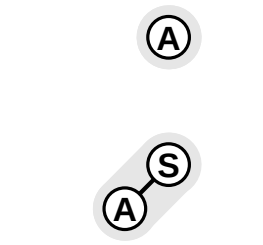
PQinsert: add node at bottom

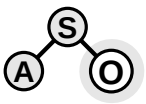
PQdelmax: exch root with node at bottom

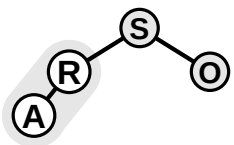
```
static Item pq[maxPQsize+1];
static int N;

void PQinit(int maxN)
   { pq = malloc(maxN*sizeof(Item)); N = 0; }
int PQempty()
   { return N == 0; }
void PQinsert(Item v)
   { pq[++N] = v; fixUp(pq, N); }
Item PQdelmax()
   {
      exch(pq[1], pq[N]);
      fixDown(pq, 1, N-1);
      return pq[N--];
   }
```

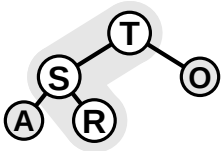
No more than lg N steps (GUARANTEED)

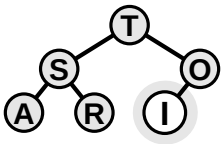
Constructing a heap (top-down)

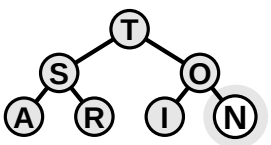


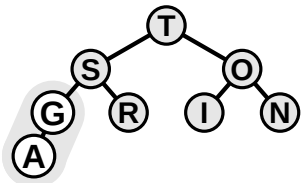


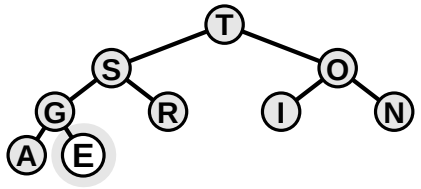


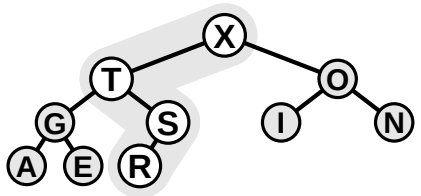


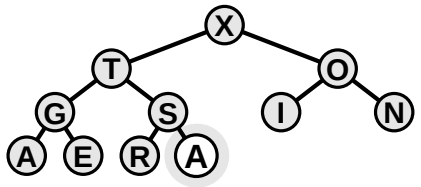


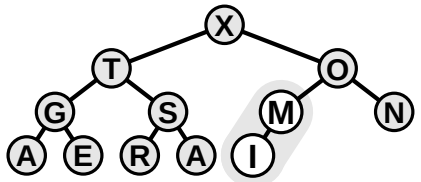


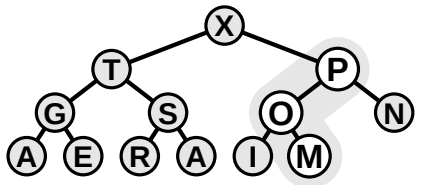


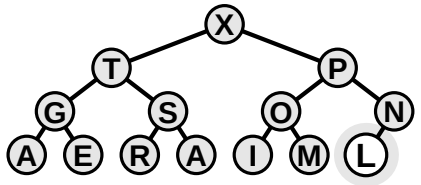


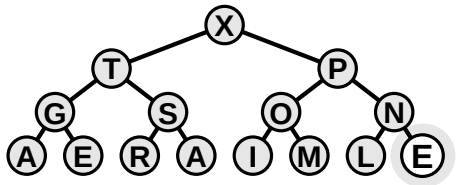




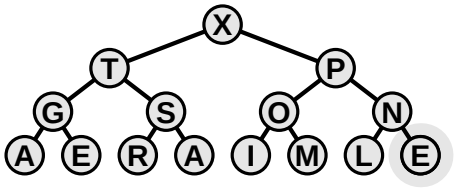


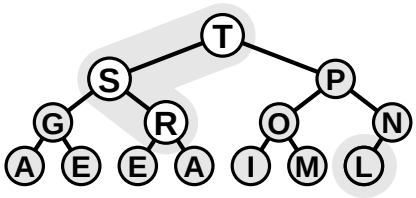


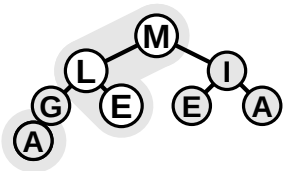


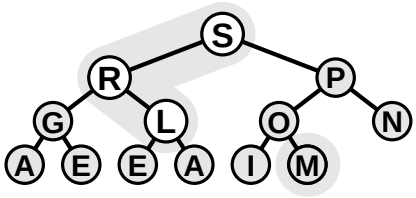


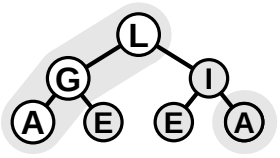
Sorting down a heap

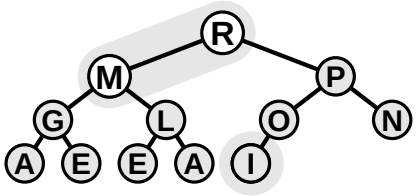


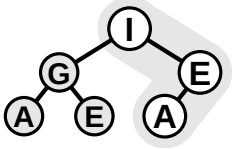


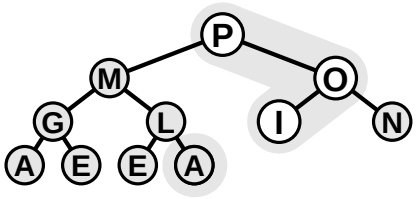


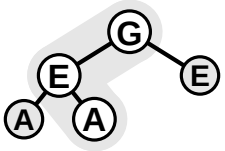


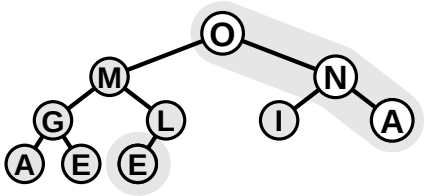


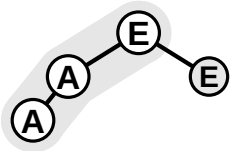


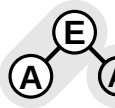


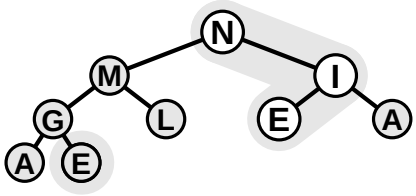


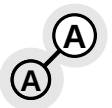










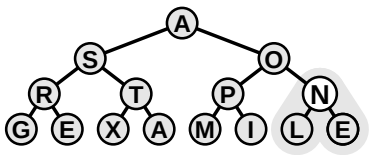


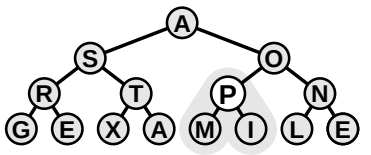
Heapsort

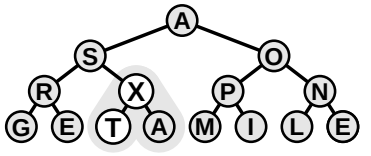
Abandon ADT concept to save space

Faster to construct heap from the top down

```
#define pq(A) a[l-1+A]
void heapsort(Item a[], int l, int r)
   { int k, N = r-l+1;
      for (k = N/2; k >= 1; k--)
         fixDown(&pq(0), k, N);
      while (N > 1)
         {
            exch(pq(1), pq(N));
            fixDown(&pq(0), 1, --N)
         }
   }
```

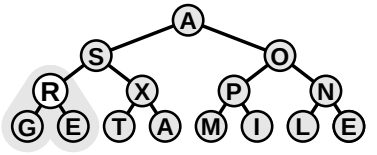


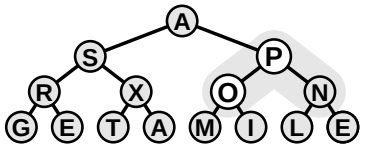


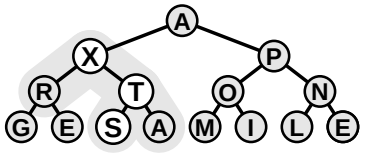


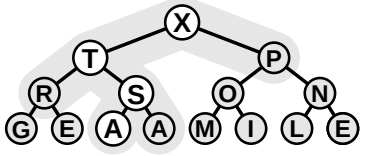
Widely used sorting method

* inplace (no extra space)
* guaranteed NlgN time









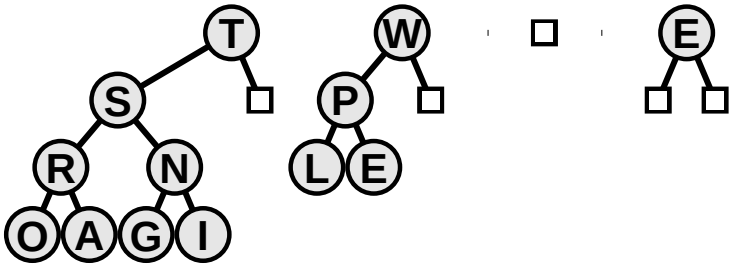
Binomial queues

Support ALL PQ operations in lgN steps
(Heaps have slow merge)

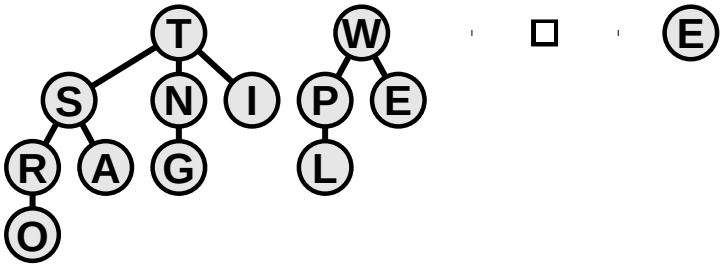
Def: In a LEFT HEAP-ORDERED tree, each node is larger than all nodes in left subtree

Def: A POWER-OF-2 TREE is a binary tree with
* left subtree of root complete
* right subtree empty
(therefore, 2^n nodes)

Def: A BINOMIAL QUEUE of size N is a forest of left heap-ordered power-of-2 trees one for each 1 bit in binary rep. of N



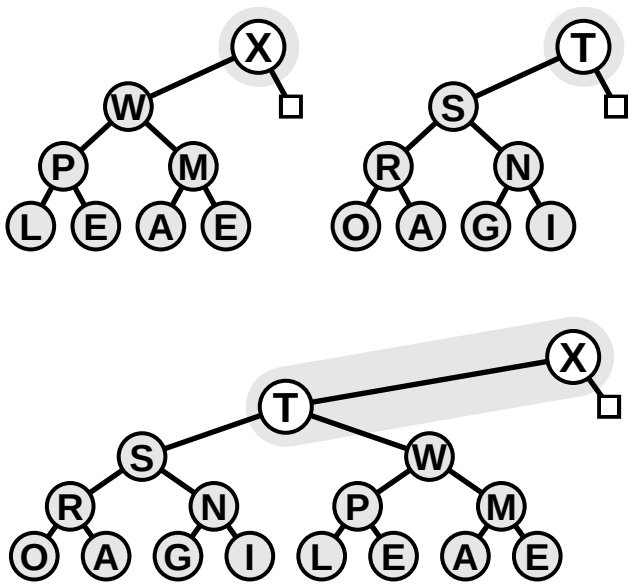
Corresponds to heap-ordered forest:

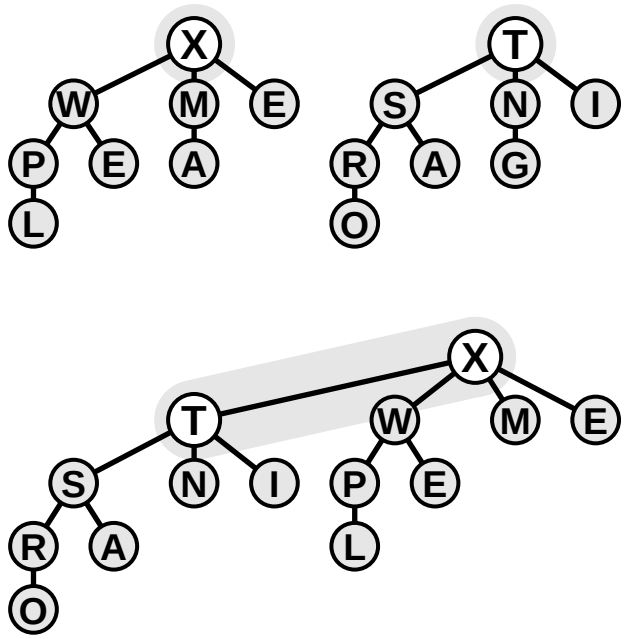


Joining power-of-2 heaps

Constant-time operation

* larger of two roots at top
* left subtree to right subtree of other root
* result is left-heap-ordered if inputs are





Joining power-of-2 heaps (code)

Representation

* two pointers per node
* need HANDLE (pointer to node)

```
struct PQnode
  { Item key; PQlink l, r; };
struct pq { PQlink *bq; };
```

Join operation

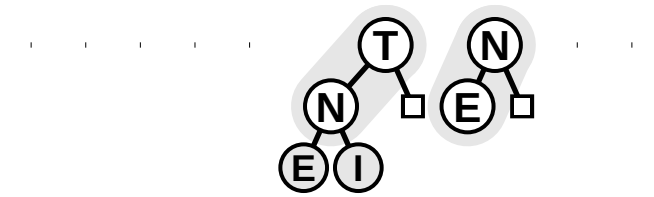
* takes two BQ nodes as input
* returns one BQ nodes as output

```
PQlink pair(PQlink p, PQlink q)
  { PQlink t;
    if (less(p->key, q->key))
      { p->r = q->l; q->l = p; return q; }
    else
      { q->r = p->l; p->l = q; return p; }
  }
```

Joining binomial queues

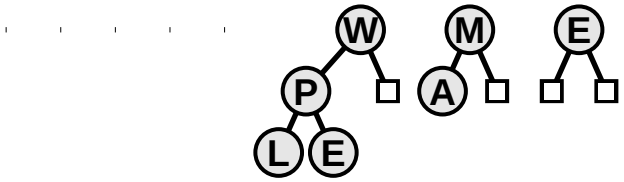
Corresponds to adding binary numbers

* 1 bits correspond to power-of-2 heaps
* 1+1=10 corresponds to carry

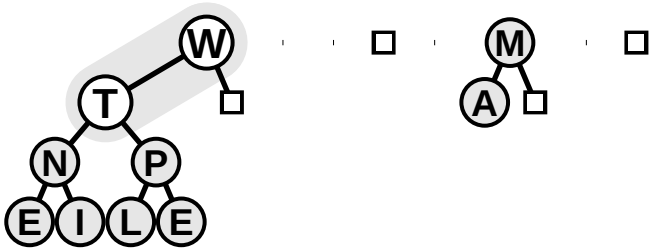


011

111



1010



* lgN time (guaranteed)
* all PQ operations easy to implement with join

Joining binomial queues (code)

c	b	a	a	c
0	0	0	a	0
0	0	1	a	0
0	1	0	b	0
0	1	1	0	a+b
1	0	0	c	0
1	0	1	0	a+c
1	1	0	0	b+c
1	1	1	a	b+c

```
#define test(C, B, A) 4*(C) + 2*(B) + 1*(A)
void PQjoin(PQlink *a, PQlink *b)
{ int i; PQlink c = z;
   for (i = 0; i < maxBQsize; i++)
      switch(test(c != z, b[i] != z, a[i] != z))
      {
      case 2: a[i] = b[i]; break;
      case 3: c = pair(a[i], b[i]);
                a[i] = z; break;
      case 4: a[i] = c; c = z; break;
      case 5: c = pair(c, a[i]);
                a[i] = z; break;
      case 6:
      case 7: c = pair(c, b[i]); break;
      }
}
```

Binomial queues summary

BQ of size N is array of power-of-two heaps
one for each bit in binary rep. of N

Joining two BQs is like adding binary numbers
* insert is like incrementing
* delete, delmax are like decrementing
* heap-like promotion, demotion works
for "change priority"

All operations guaranteed lgN

Actually faster, in "amortized" sense

Example: PQinsert N items, then one more
* N even, just insert item
* N = ...01, just two steps
* N = ..011, just three steps
...

total cost LINEAR
N/2 + 2(N/4) + 3(N/8) + 4(N/16) + ...

Basis for modifications getting most operations
down to constant worst-case time
(some of which are practical)

Good candidate for "system" PQ routine