COS320: Compiling Techniques

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January 29, 2026

Logistics

- Last HW is due on Dean’s date. You will implement:
  - The worklist algorithm for dataflow analysis
  - Constant propagation
  - Alias analysis & dead code elimination
  - Register allocation

Loop transformations

Loops

- Almost all execution time is inside loops
- Many optimizations are centered around transforming loops
    - Loop invariant code motion: hoist expressions out of loops to avoid re-computation
    - Strength reduction: replace a costly operation inside a loop with a cheaper one
    - Loop unrolling: avoid branching by excecuting several iterations of a loop
    - Lots more: parallelization, tiling, vectorization, ...

What is a loop?

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  - Typically no explicit loop syntax at the IR level
  - Not sensitive to syntax of source language (loops can be created with `while`, `for`, `goto`, ...)

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  - Too general – makes it difficult to apply transformations
- Desiderata:
  - Want to *at least* capture loops that would result from structured programming (programs built with `while`, `if`, and sequencing (no `goto`!))
  - Many loop optimizations require inserting code *immediately before* the loop enters, so loop definition should make that easy

What is a loop?

- A loop of a control flow graph is a set of nodes S such that with a distinguished *header* node h such that
  1. S is strongly connected
     - There is a directed path from h to every node in S
     - There is a directed path from any to in S to h
  2. There is no edge from any node *outside* of S to any node *inside* of S, except for h
     - Implies h dominates all nodes in S: every path from entry to a node in S **must** go through h

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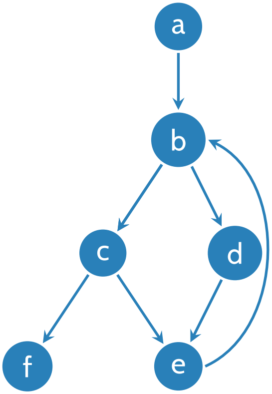
- Observe: a loop has one entry, but may have multiple exits (or none)
  - A *loop entry* is a node with some predecessor outside the loop
  - A *loop exit* is a node with some successor outside the loop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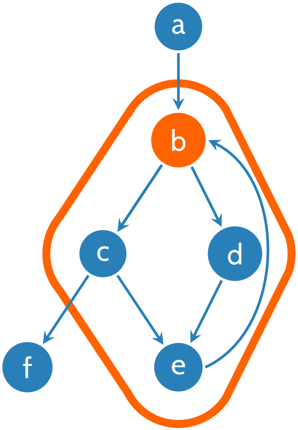
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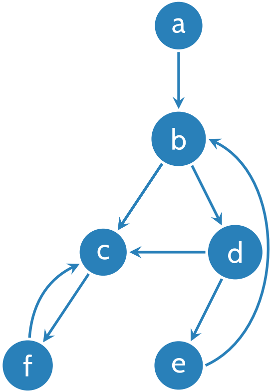
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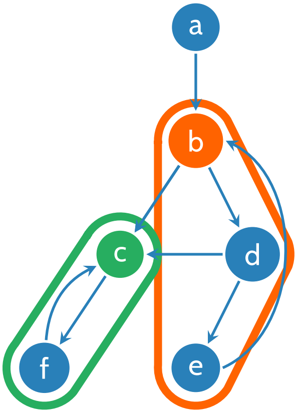
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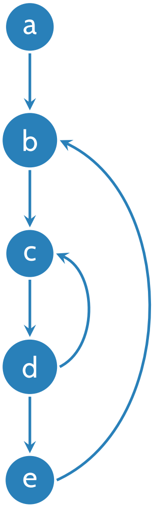
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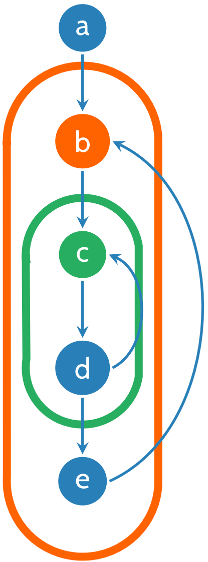
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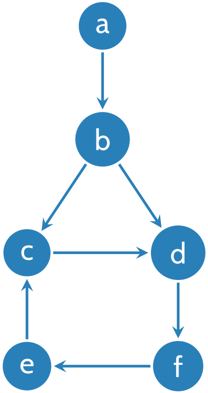
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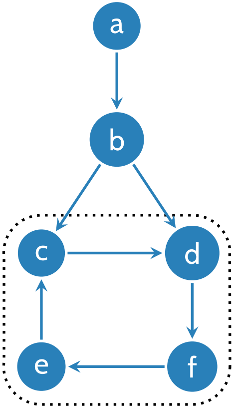
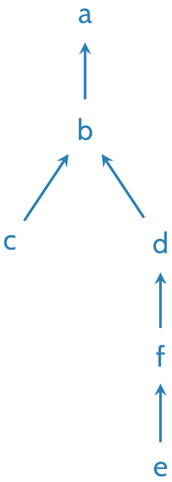


Strongly connected subgraph

Dominator tree



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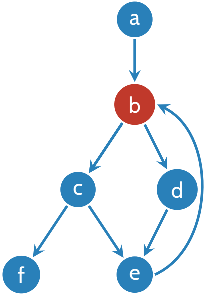

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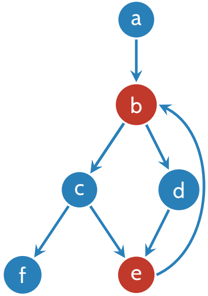

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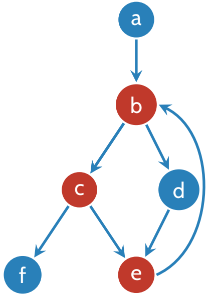

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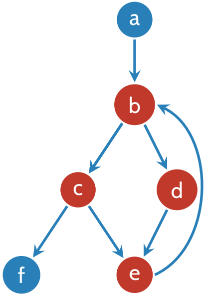

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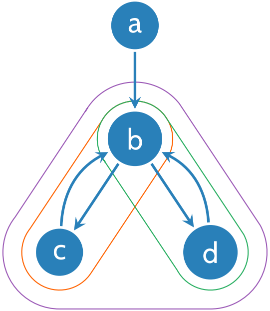
But not every loop is natural:



Nested loops

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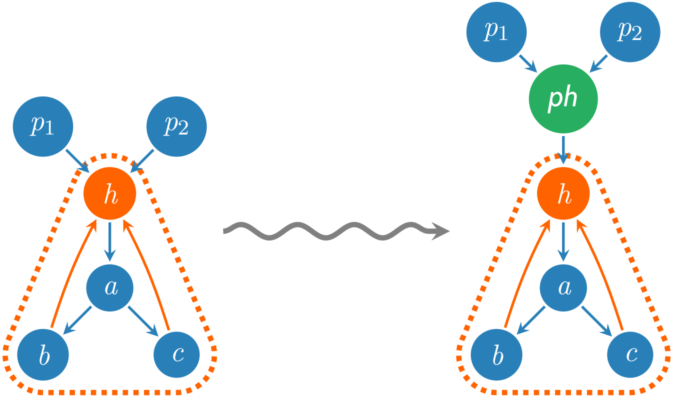
  - Loops obtained by merging natural loops with the same header are either disjoint or nested
  - Loops can be organized into a forest
- We typically apply loop transformations "bottom-up", starting with innermost loops

Loop preheaders

- Some optimizations (e.g., loop-invariant code motion) require inserting statements immediately before a loop executes
- A *loop preheader* is a basic block that is inserted immediately before the loop header, to serve as a place to store these statements



Loop invariant code motion

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  - An operand is *invariant* in a loop L if
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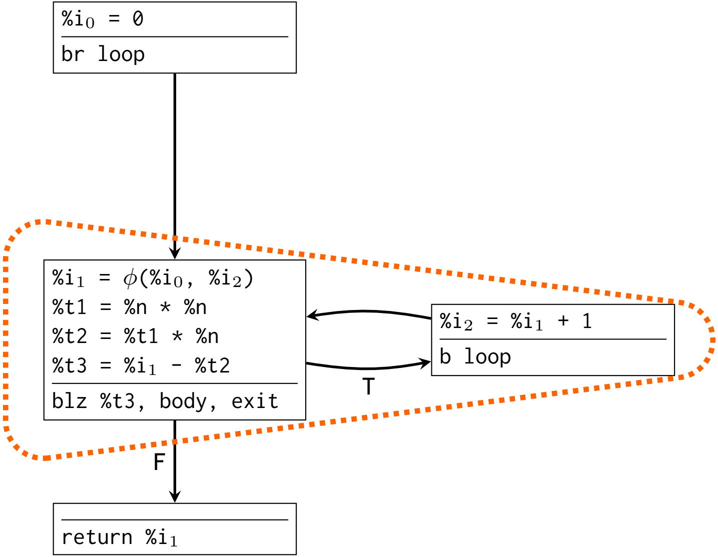
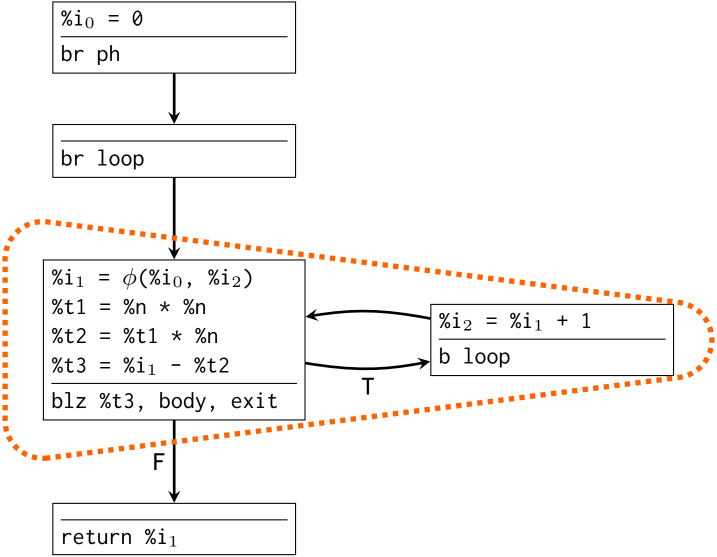
  - For each computation $\%x = opn_1\ op\ opn_2$, if opn_1 and opn_2 are both invariant, move $\%x = opn_1\ op\ opn_2$ to pre-header
  - This moves definition of $\%x$ outside of the loop, so $\%x$ is now invariant


%i0 = 0
br loop
%i1 = φ(%i0, %i2)
%t1 = %n * %n
%t2 = %t1 * %n
%t3 = %i1 - %t2
blz %t3, body, exit
%i2 = %i1 + 1
b loop
T
F
return %i1



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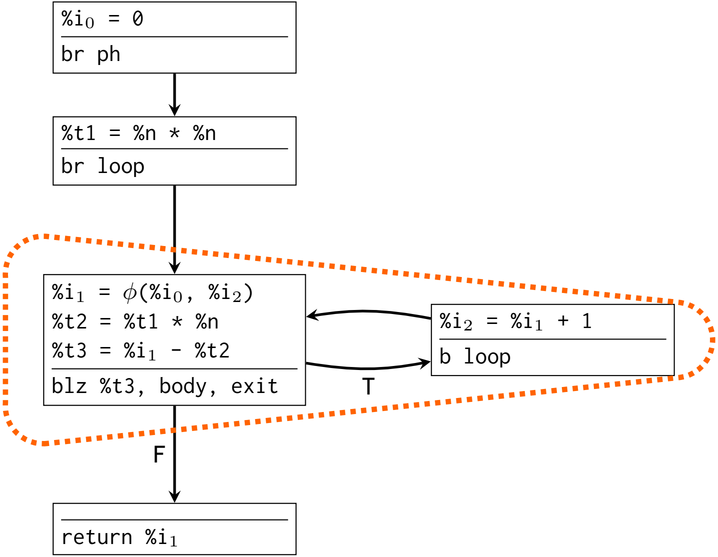
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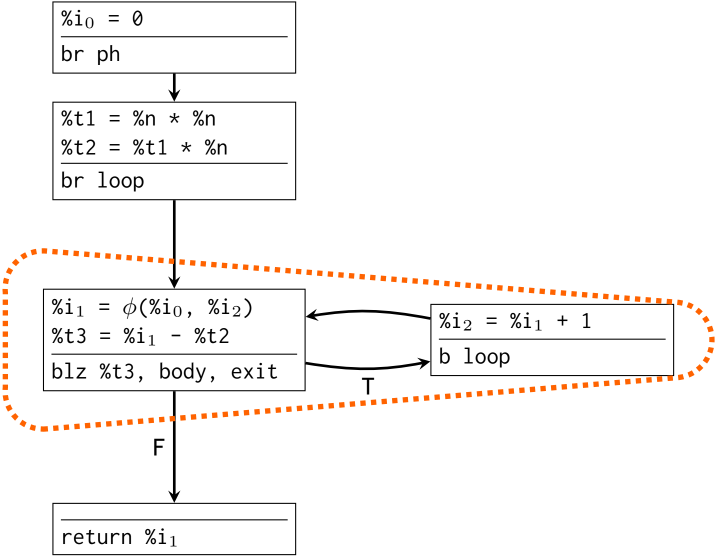
%i_2 = %i_1 + 1
b loop
T
F
return %i_1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%i2 = %i1 + 1
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Induction variables

- An *induction variable* is a variable $\%x$ such that the difference between successive values of $\%x$ in a loop is constant.
  - Common example: the loop counter in a `for` loop

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    for (int i = 0; i < n; i++)
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  - Common example: the loop counter in a `for` loop
    `for (int i = 0; i < n; i++)`
- Useful for several optimizations
  - Strength reduction, loop unrolling, induction variable elimination, parallelization, array bound-check elision

Induction variables, formally

- Use $\%x(k)$ to denote the value of $\%x$ in the kth iteration of a loop. $\%x$ is an induction variable if there is some constant (loop-invariant) $\Delta(\%x)$ such that

$$\%x(k+1) = \%x(k) + \Delta(\%x)$$

  for all k

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- A variable $\%x$ is an *basic induction variable* for a loop L if it is increased / decreased by a fixed loop-invariant quantity in any iteration of the loop.
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  - $\%x(i+1) = \%x(i) + c \Rightarrow \Delta(\%x) = c$
- A variable $\%y$ is an *derived induction variable* for a loop L if it is an affine function of a basic induction variable
  - $\%y(i) = a \cdot \%x(i) + b \Rightarrow \Delta(\%y) = a \cdot c$

Finding induction variables

- Basic induction variable detection:
  - Look for ϕ statements $\%x = \phi(\%x_1, ..., \%x_n)$ in header
    - Each position $\%x_i$ corresponding to a back edge of the loop must be the same uid, say $\%x_k$
  - Find chain of assignments for $\%x_k$ leading back to $\%x$, such that each either adds or subtracts an invariant quantity. Success $\Rightarrow$ $\%x$ is an basic induction var.

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- To detect derived induction variables:
  - Choose a basic induction variable $\%x$
  - Find assignments of the form $\%y =$ *opn*$_1$ *op* *opn*$_2$ where
    - *op* is $+$ or $-$ and *opn*$_1$ and *opn*$_2$ are either $\%x$, derived induction variables of $\%x$, or loop invariant quantities
    - *op* is $*$ and *opn*$_1$ and *opn*$_2$ are as above, and at least one is a loop invariant quantity

Strength reduction

Idea: replace expensive operation with cheaper one (e.g., replace multiplication w/ addition).

```
long trace (long *m, long n) {
  long i;
  long result = 0;
  for (i = 0; i < n; i++) {
    result += *(m + i*n + i);
  }
  return result;
}
```

→

```
long trace (long *m, long n) {
  long i;
  long result = 0;
  long *next = m;
  for (i = 0; i < n; i++) {
    result += *next;
    next += i + 1;
  }
  return result;
}
```

%i$_1$ = ϕ(%i$_0$, %i$_2$)
%result$_1$ = ϕ(%result$_0$, %result$_2$)
%t1 = %i$_1$ - %n
blz %t1, body, exit

%t2 = %i$_1$ * %n
%t3 = %m + %t2
%t4 = %t3 + %i$_1$
%t5 = load %t4
%result$_2$ = %result$_1$ + %t5
%i$_2$ = %i$_1$ + 1
b loop

```
%i₁ = φ(%i₀, %i₂)
%result₁ = φ(%result₀, %result₂)
%t1 = %i₁ - %n
blz %t1, body, exit
```

```
%t2 = %i₁ * %n
%t3 = %m + %t2
%t4 = %t3 + %i₁
%t5 = load %t4
%result₂ = %result₁ + %t5
%i₂ = %i₁ + 1
b loop
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%t1 = %i₁ - %n
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%i₂ = %i₁ + 1
b loop
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i := i + 1

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%t1 = %i₁ - %n
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i := i + 1

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b loop
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$\%i_1 = \phi(\%i_0, \%i_2)$ i := i + 1

$\%result_1 = \phi(\%result_0, \%result_2)$

$\%t1 = \%i_1 - \%n$ t1 := i + n

blz %t1, body, exit

$\%t2 = \%i_1 * \%n$ t2 := n*i

%t3 = %m + %t2

$\%t4 = \%t3 + \%i_1$

%t5 = load %t4

$\%result_2 = \%result_1 + \%t5$

$\%i_2 = \%i_1 + 1$

b loop

%i$_1$ = ϕ(%i$_0$, %i$_2$) i := i + 1

%result$_1$ = ϕ(%result$_0$, %result$_2$)

%t1 = %i$_1$ - %n t1 := i + n

blz %t1, body, exit

%t2 = %i$_1$ * %n t2 := n*i

%t3 = %m + %t2 t3 := n*i + m

%t4 = %t3 + %i$_1$

%t5 = load %t4

%result$_2$ = %result$_1$ + %t5

%i$_2$ = %i$_1$ + 1

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%result_1 = φ(%result_0, %result_2)
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blz %t1, body, exit


%t2 = %i_1 * %n                                 t2 := n*i
%t3 = %m + %t2                                  t3 := n*i + m
%t4 = %t3 + %i_1                                t4 := (n+1)*i + m
%t5 = load %t4
%result_2 = %result_1 + %t5
%i_2 = %i_1 + 1
b loop
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%t2$_0$ = 0
%t3$_0$ = %m
%t4$_0$ = %m

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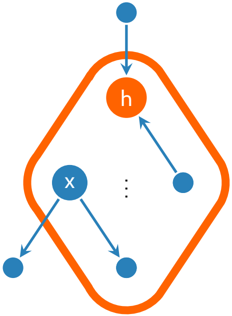
%t3$_1$ = ϕ(%t3$_0$, %t3$_2$)
%t4$_1$ = ϕ(%t4$_0$, %t4$_2$)
%result$_1$ = ϕ(%result$_0$, %result$_2$)
%t1 = %i$_1$ - %n   t1 := i + n
blz %t1, body, exit

%t2$_2$ = %t2$_1$ + %n   t2 := n*i
%t3$_2$ = %t3$_1$ + %n   t3 := n*i + m
%t6 = %t4$_1$ + %n
%t4$_2$ = %t6 + 1   t4 := (n+1)*i + m
%t5 = load %t4$_2$
%result$_2$ = %result$_1$ + %t5
%i$_2$ = %i$_1$ + 1
b loop

Loop unrolling

- Can expose opportunities for using Single Instruction Multiple Data (SIMD) instructions
- Some loops are so small that a significant portion of the running time is due to testing the loop exit condition
  - We can avoid branching by executing several iterations of the loop at once
- Loop unrolling trades (potential) run-time performance with code size.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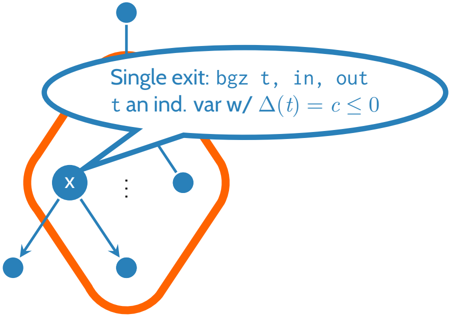
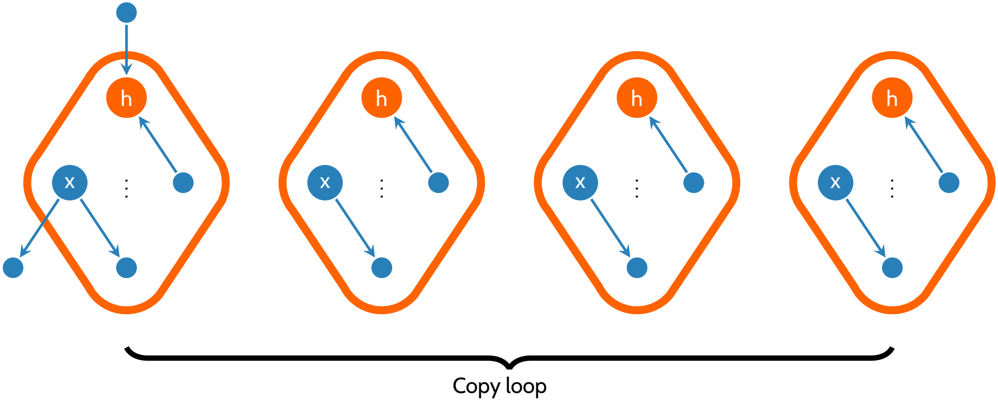
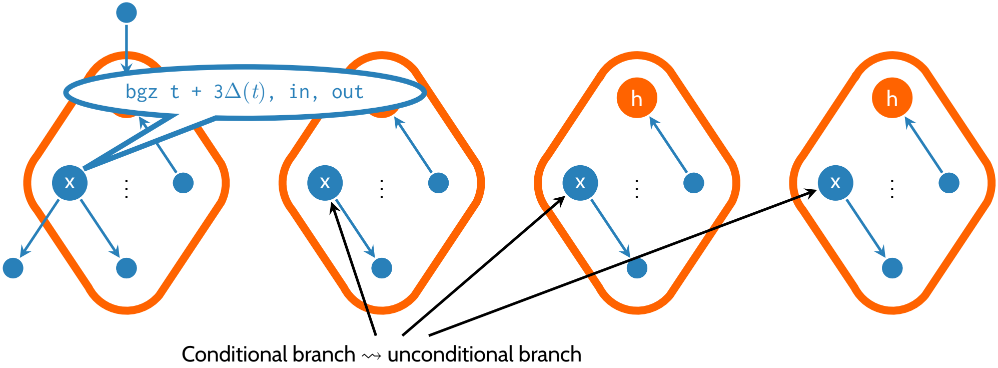
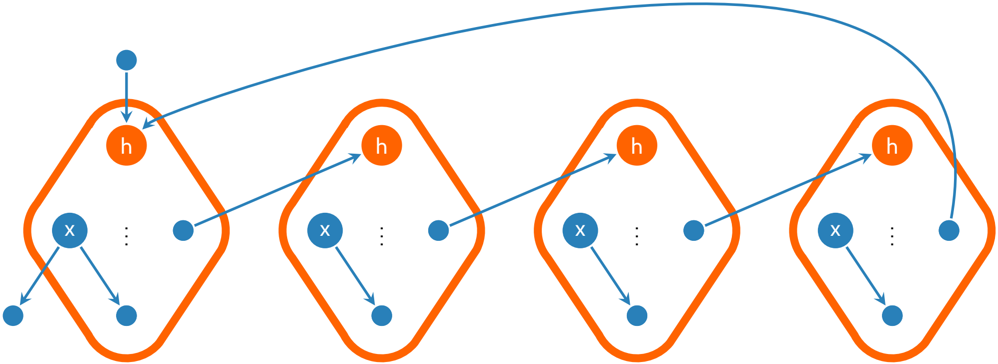
⋮



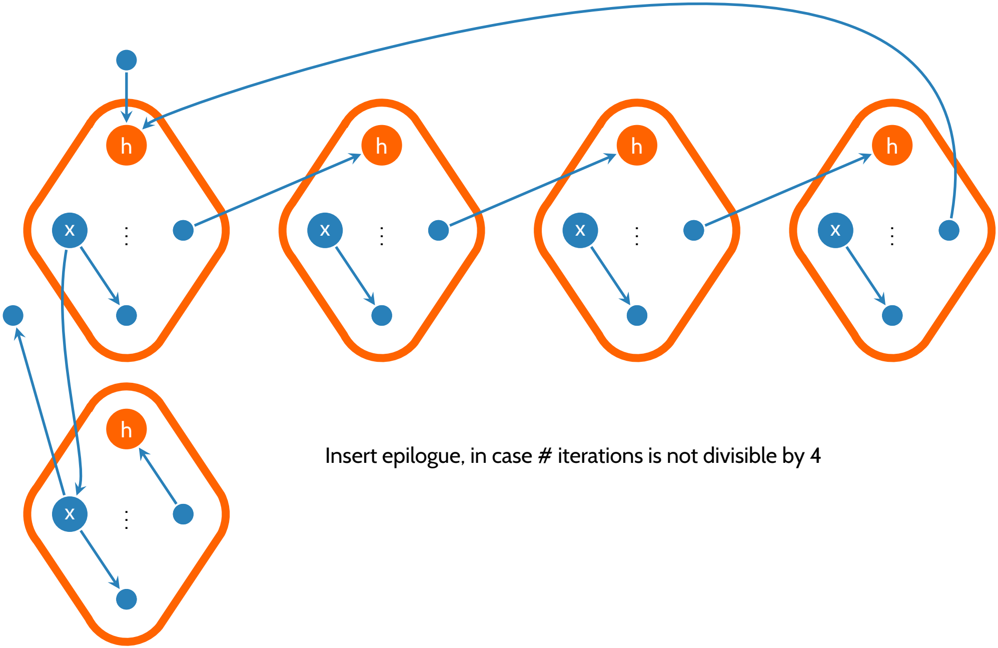
Single exit: bgz t, in, out
t an ind. var w/ $\Delta(t) = c \leq 0$
x


h
x
h
x
h
x
h
x
Copy loop


bgz t + 3Δ(t), in, out
x
x
x
h
x
h
Conditional branch ⇝ unconditional branch



Redirect back-edges to next loop copy



Insert epilogue, in case # iterations is not divisible by 4

Optimization wrap-up

- Optimizer operates as a series of IR-to-IR transformations
- Transformations are typically supported by some analysis that proves the transformation is safe
- Each transformation is simple
- Transformations are mutually beneficial
    - Series of transformations can make drastic changes!