Precept Outline

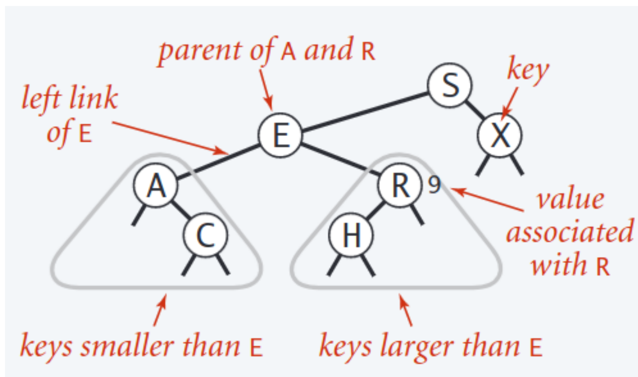
- Review of Lectures 9 and 10:
  - Binary Search Trees
  - Balanced Binary Search Trees
- Midterm Review

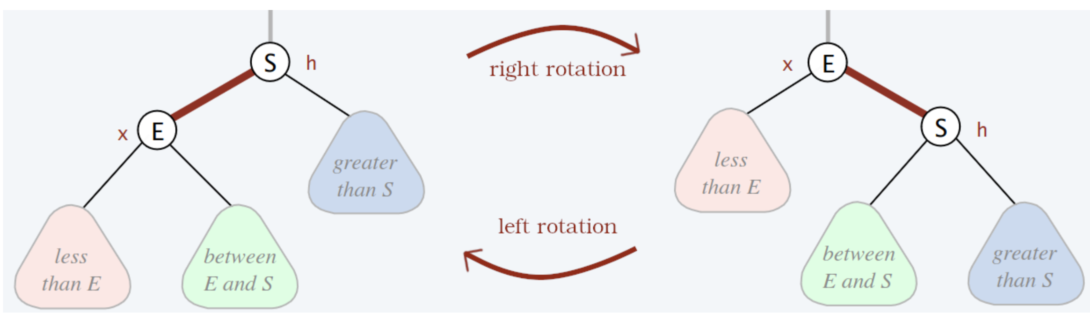
Relevant Book Sections

- Book chapters: 3.1, 3.2 and 3.3

A. Review: Binary Search Trees and Red-Black Trees

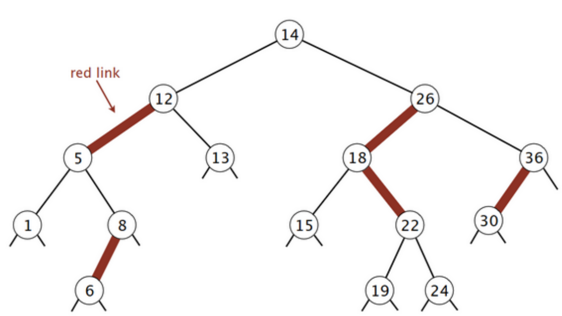
Your preceptor will briefly review key points of this week's lectures. Here are some images reminding you of some of the key definitions from lecture.





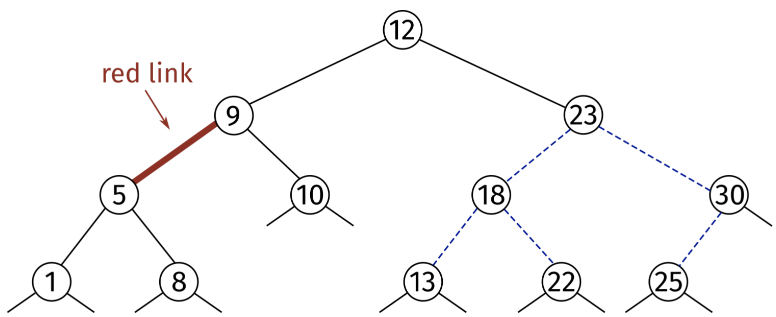
B. Red-Black Trees (Spring'23 Midterm)

The following binary search tree satisfies perfect black balance, but violates color invariants:



Give a sequence of 4 elementary operations (*color flip, rotate left or rotate right*) that restore the color invariants.

Consider the following left-leaning red-black BST with some of the edge colors suppressed:



Which keys in the above tree are red? (Recall that a key is red if the link to its parent is red.)

C. Data Structure Design - Midterm Review

Suppose that there are two teams of players that can play each other in head-to-head matches. Each player has a certain rating, which is an integer value, and two players can play each other if they have the same rating (otherwise it isn't a balanced match). Design a data structure that computes the maximum number of distinct matches that the two teams can play at any point. The data structure should support 2 operations. The first, `addPlayer(rating, team)` adds a new player of rating equal to `rating` and adds it to team `team`, which can be 1 or 2. The second operation, `numberOfMatches()`, returns the maximum number of distinct matches that can be played by elements of team 1 versus elements of team 2 (see the example below for more information).

public class MatchMaker

`MatchMaker()`	*creates two empty teams*
`void addPlayer(int rating, int team)`	*adds player of rating to team*
`int numberOfMatches()`	*returns the maximum number of matches that can be played*

Full credit: The `addPlayer()` method should run in $O(\log n)$ time in the worst case and the `numberOfMatches()` method should run in $\Theta(1)$ time.
Partial credit: The `addPlayer()` method should run in $O(n)$ time in the worst case and the `numberOfMatches()` method should run in $\Theta(1)$ time.

Example

```
MatchMaker mm = new MatchMaker();
mm.addPlayer(100, 1); // Team 1: {100: 1}
mm.addPlayer(200, 1); // Team 1: {100: 1, 200: 1}
mm.addPlayer(100, 2); // Team 2: {100: 1}, Matches = 1
mm.addPlayer(200, 2); // Team 2: {100: 1, 200: 1}, Matches = 2
mm.addPlayer(100, 1); // Team 1: {100: 2, 200: 1}, Matches = 2
StdOut.println(mm.numberOfMatches()); // Output: 2
mm.addPlayer(100, 2); // Team 2: {100: 2, 200: 1}, Matches = 3
StdOut.println(mm.numberOfMatches()); // Output: 3
```

The first call of `numberOfMatches()` outputs 2 since we can match the two 200 rating players together as well as the one 100 rating player from team 2 with one of the 100 rating players from team 1. The second call can match each player to a different player, resulting in 3 total matches.

In the space provided, give a concise English description of your solution to the constructor, the `addPlayer()` *and the* `numberOfMatches()` *methods. You may use any of the algorithms and data structures that we have considered in this course (e.g., lectures, precepts, textbook, assignments) as subroutines. If you modify any of them, be sure to describe the modification. Feel free to use code or pseudocode to improve clarity.*

D. Algorithm Design - Midterm Review

A length-n integer array `a[]` is *single-peaked* if there exists $0 \leq k \leq n-1$ such that the subarray from index up to (and including) k is strictly increasing, and the subarray from index k until $n-1$ is strictly decreasing. The entry `a[k]` is called the *peak*. For example, the array `a[] = {3, 6, 7, 10, 4, 1}` is a single-peaked with peak `10`, but the array `a[] = {3, 6, 7, 10, 4, 5}` is not.

Design an algorithm that receives as input a single-peaked array with n *distinct elements*, and outputs the peak of the array. *Specify the running time of your solution.*

Full credit: The running time of the algorithm must be $O(\log n)$.
Partial credit: The running time of the algorithm must be $O(n)$.

In the space provided, give a concise English description of your algorithm for solving the problem. You may use any of the algorithms that we have considered in this course (e.g., lectures, precepts, textbook, assignments) as subroutines. If you modify such an algorithm, be sure to describe the modification. Feel free to use code or pseudocode to improve clarity.

E. Extra Practice

Part 1: Sorting Compares (Fall'17 Midterm)

Consider an array that contains two successive copies of the integers 1 through n, in ascending order. For example, here is the array when $n = 8$:

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \,|\, 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

Note that the length of the array is $2n$, not n.

How many compares does *selection sort* make to sort the array as a function of n? Use tilde notation to simplify your answer.

How many compares does *insertion sort* make to sort the array as a function of n? Use tilde notation to simplify your answer.

How many compares does *mergesort* make to sort the array as a function of n? You may assume n is a power of 2. Use tilde notation to simplify your answer.

Part 2: Heaps

Consider the following binary heap representation of a maximum-oriented priority queue (with `pq[0]` unused).

pq[]	–	50	40	20	10	30	5
	0	1	2	3	4	5	6

Suppose that you *insert* the key 45 into the binary heap. Which keys would be involved in a *compare*? And which keys would be involved in an *exchange*?

Suppose that you perform a `delMax()` operation in the original binary heap. Which keys would be involved in a *compare*? And which keys would be involved in an *exchange*?

Part 3: Linear or Not?

Which of the following are $O(n)$?

( ) The number of compares needed to apply a `delMin()` operation in a minimum-oriented binary heap with n elements.

( ) The number of compares used by the best non-stable compare-based sorting algorithm that sorts n integers.

( ) The number of times a resizing array resizes in order to perform n operations in a row.

( ) The number of times `hello` is printed by the following code:

```
for (int i = 1; i <= n; i *= 2)
    for (int j = 1; j <= i; j++)
            StdOut.println("hello");
```