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1.3 STACKS AND QUEUES I

- *APIs*
- *array implementations*
- *resizable arrays*
- *amortized analysis*
- *generics*

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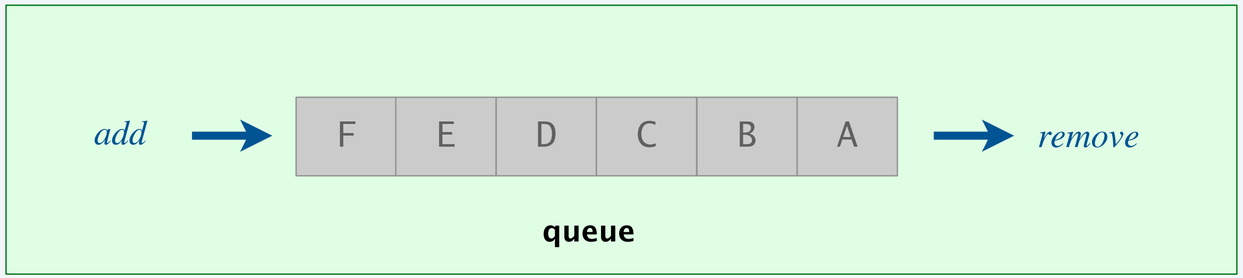
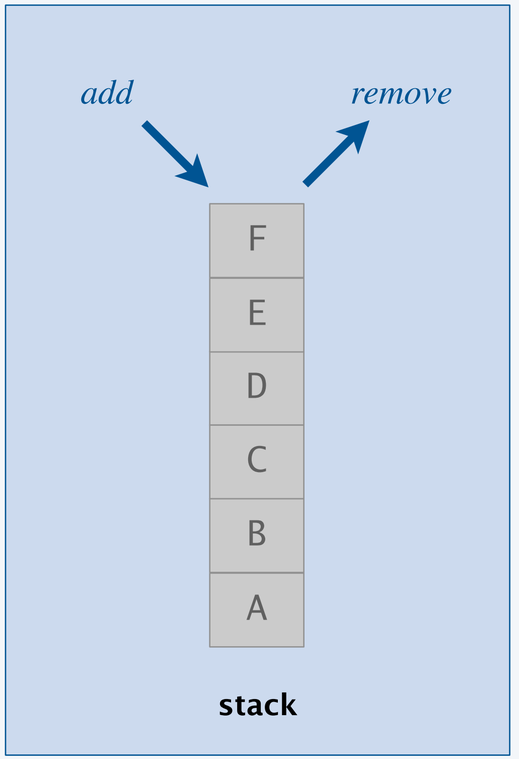
Stacks and queues

Fundamental data types.

- Value: collection of objects.
- Operations: add, remove, iterate, size, test if empty.
- Intent is clear when we add.
- Which item do we remove?

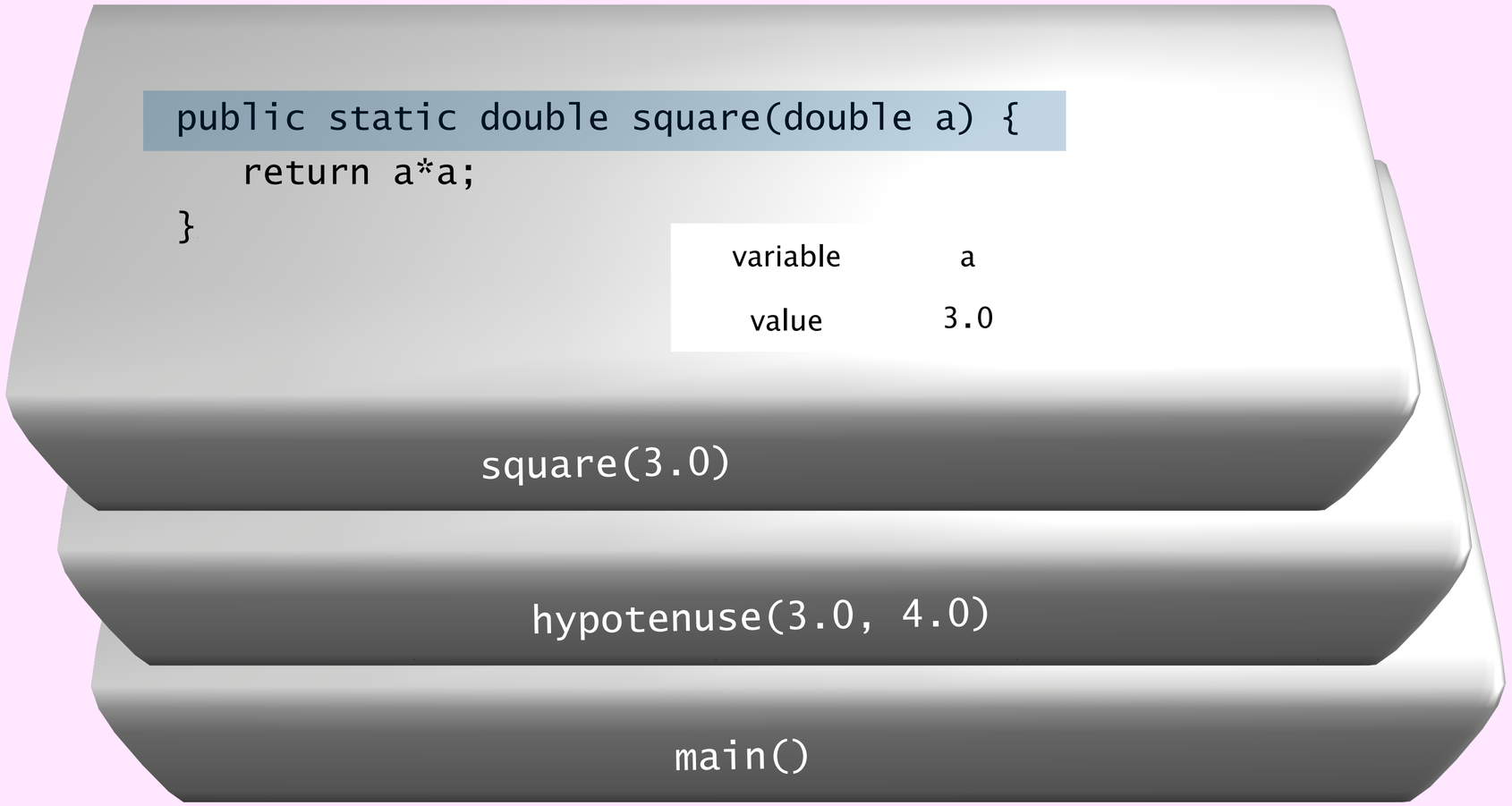




Stack. Remove the item most recently added. ⟵ *LIFO = "last in first out"*

Queue. Remove the item least recently added. ⟵ *FIFO = "first in first out"*

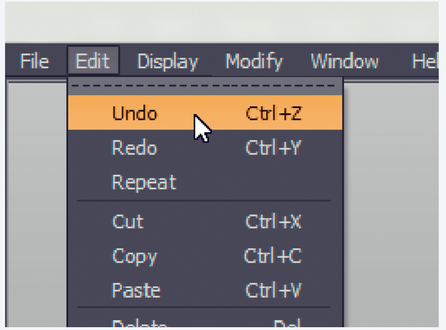
Function-call stack demo

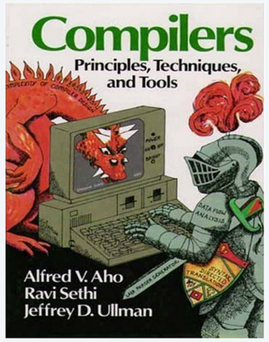


function-call stack

Stack applications

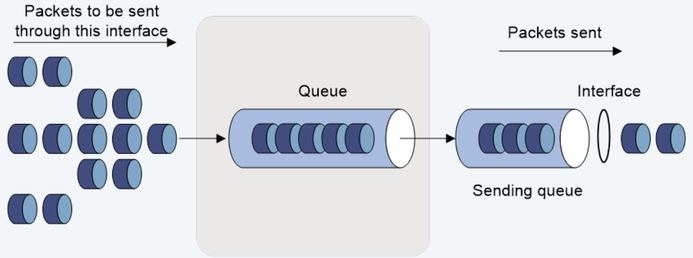
- Rendering text and graphics: PostScript, PDF, …
- Web browser history: back button.
- Function calls: Java virtual machine, Linux kernel, ...
- Undo: text editors, photo editors, games, …
- Compilers: evaluating expressions, parsing syntax, balanced parentheses, …
- …





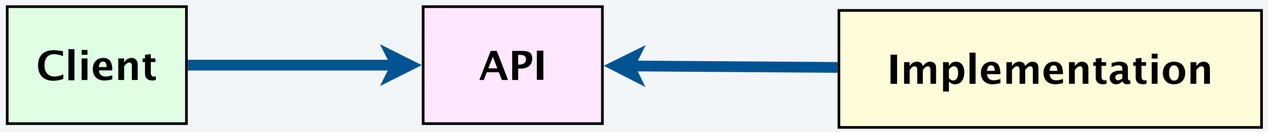
Queue applications

- Media playlists: jukebox, Spotify, Netflix, Peloton, …
- Requests on a shared resource: printer, CPU, GPU, …
- Asynchronous data transfer: file I/O, pipes, sockets, …
- Data buffers: sound card, streaming video, input devices, …
- Simulations of the real world: customer service, traffic analysis, baggage claim, …
- …



Data type design: API, client, and implementation

Separate client and implementation via API.



API: operations that characterize the behavior of a data type.

Client: code that uses a data type through its API.

Implementation: code that implements the API operations.

Benefits.

- Design: develop and maintain reusable code.
- Performance: substitute faster implementations.

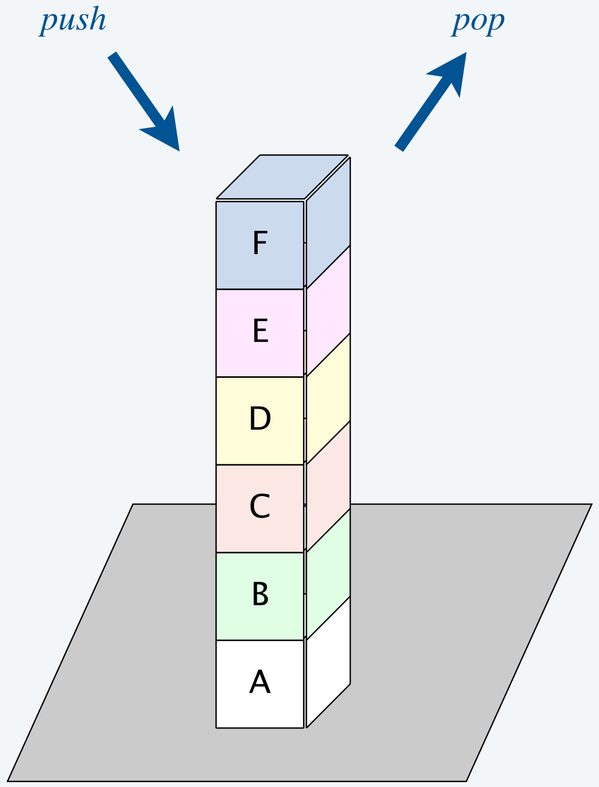
Ex. Stack, queue, priority queue, symbol table, set, union-find, ...

Stack API

Stack data type. Our textbook data type for stacks. ← *available with* `javac-algs4` *and* `java-algs4` *commands*

"generic type parameter"

`public class Stack<Item>`	description
`Stack()`	*create an empty stack*
`void push(Item item)`	*add a new item to the stack*
`Item pop()`	*remove and return the item most recently added*
`boolean isEmpty()`	*is the stack empty?*



Performance goals. Every operation takes $\Theta(1)$ time; stack with n items uses $\Theta(n)$ memory.

Queue API

Queue data type. Our textbook data type for queues.

available with `javac-algs4` *and* `java-algs4` *commands*

`public class Queue<Item>`	description
`Queue()`	*create an empty queue*
`void enqueue(Item item)`	*add a new item to the queue*
`Item dequeue()`	*remove and return the item least recently added*
`boolean isEmpty()`	*is the queue empty?*

Performance goals. Every operation takes $\Theta(1)$ time; queue with n items uses $\Theta(n)$ memory.

Warmup client

Goal. Read strings from standard input and print in reverse order.

access library in `algs4.jar` *(typically omitted)*

```
import edu.princeton.cs.algs4.Stack;
import edu.princeton.cs.algs4.StdIn;
import edu.princeton.cs.algs4.StdOut;

public class Reverse {
   public static void main(String[] args) {
      Stack<String> stack = new Stack<String>();

      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         stack.push(s);
      }

      while (!stack.isEmpty()) {
         String s = stack.pop();
         StdOut.print(s + " ");
      }
      StdOut.println();

   }
}
```

"type argument" (can be any reference type)

declare and create stack

read strings from standard input and push onto stack

pop all strings and print to standard output

```
~/cos226/stacks> javac-algs4 Reverse.java

~/cos226/stacks> java-algs4 Reverse
I have a dream today
<Ctrl-D>
today dream a have I
```

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Stack API (warmup)

Warmup API. Stack of strings data type, with fixed maximum capacity.

```
public class FixedCapacityStackOfStrings
```

	`FixedCapacityStackOfStrings(int capacity)`	*create an empty stack*
`void`	`push(String item)`	*add a new string to stack*
`String`	`pop()`	*remove and return the string most recently added*
`boolean`	`isEmpty()`	*is the stack empty?*

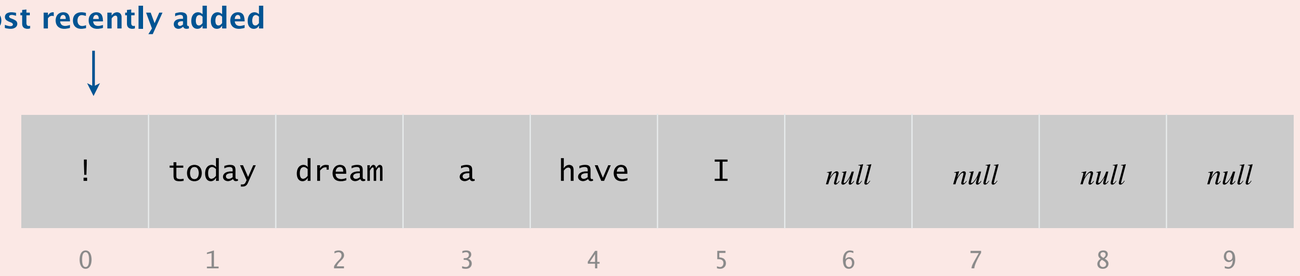
artificial limit (stay tuned)

Stacks and queues I: poll 1

How to implement efficiently a fixed-capacity stack with an array?

A.

B.

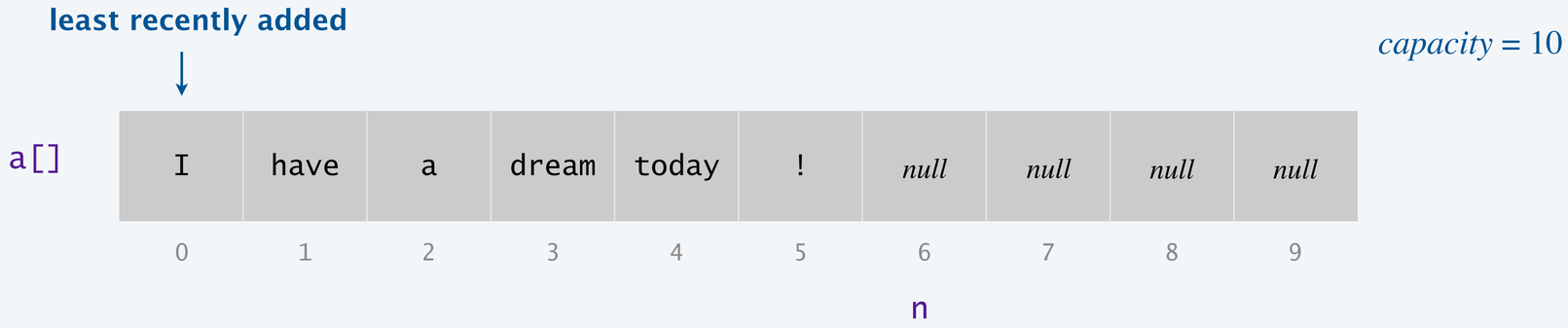


C. *Both A and B.*

D. *Neither A nor B.*

Fixed-capacity stack: array implementation

- Use array `a[]` to store `n` items on stack.
- Push: add new item at `a[n]`.
- Pop: remove item from `a[n-1]`.



Defect. Stack overflows when `n` exceeds `capacity`. [stay tuned]

Fixed-capacity stack: array implementation

```
public class FixedCapacityStackOfStrings {
   private String[] a;
   private int n = 0;

   public FixedCapacityStackOfStrings(int capacity) {
      a = new String[capacity];
   }

   public boolean isEmpty() {
      return n == 0;
   }

   public void push(String item) {
      a[n++] = item;
   }

   public String pop() {
      return a[--n];
   }

}
```

post-increment operator: use as index into array; then increment n

pre-decrement operator: decrement n*; then use as index into array*

Stack considerations

Underflow. Throw exception if `pop()` called when stack is empty.

Overflow. Use "resizable array" to avoid overflow. [next section]

Null items. For simplicity, we allow `null` items to be added.

Loitering. Holding an object reference when it is no longer needed.

I	have	a	dream	today	!	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

n

```
public String pop() {
   return a[--n];
}
```

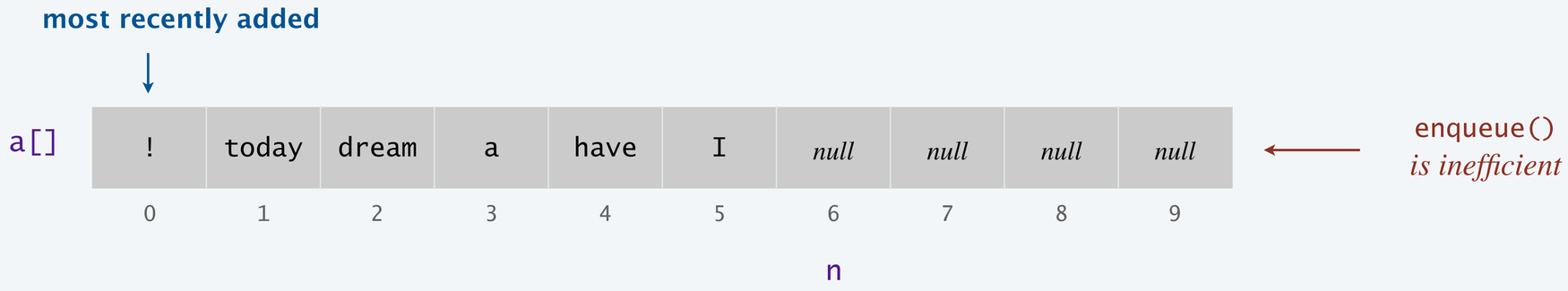
loitering

```
public String pop() {
   String item = a[n-1];
   a[n-1] = null;
   n--;
   return item;
}
```

no loitering

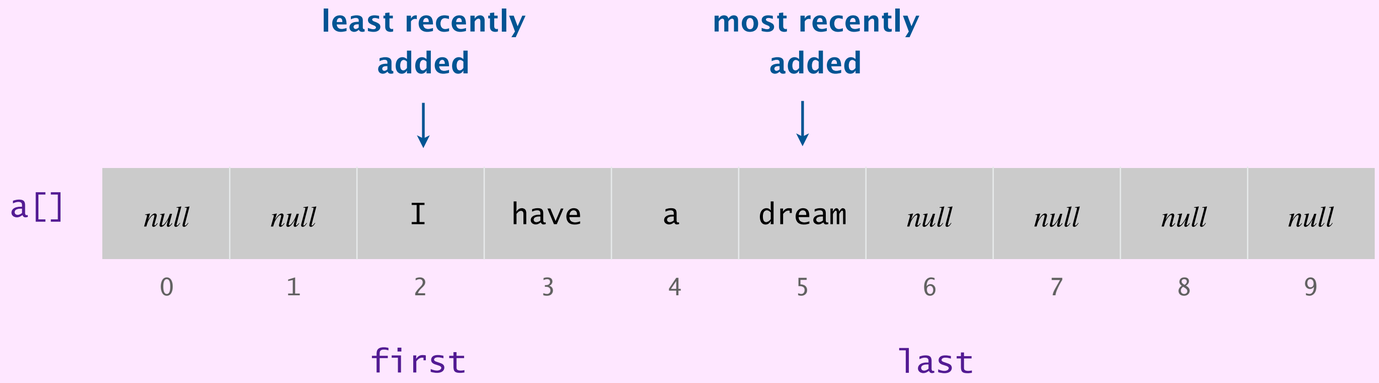
Fixed-capacity queue: array implementation

Goal. Implement a queue using a fixed-capacity array so that all operations take $\Theta(1)$ time.



Fixed-capacity queue: array implementation demo

Goal. Implement a queue using a fixed-capacity array so that all operations take $\Theta(1)$ time.



Fixed-capacity queue: array implementation

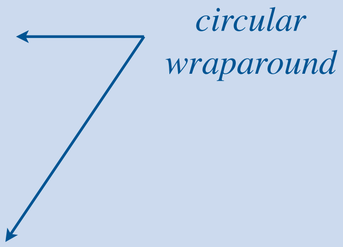
```
public class FixedCapacityQueueOfStrings {
   private String[] a;
   private int first = 0;
   private int last = 0;

   public FixedCapacityQueueOfStrings(int capacity) {
      a = new String[capacity];
   }

   public void enqueue(String item) {
      a[last] = item;
      last++;
      if (last == a.length) last = 0;
   }

   public String dequeue() {
      first++;
      if (first == a.length) first = 0;
      return a[first];
   }

}
```



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1.3 Stacks and Queues I

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Stacks and queues I: poll 2

How to grow and shrink the array length?

A. Increase by 1 before each *push*;

decrease by 1 after each *pop*.

B. Increase by $2\times$ in *push* when array becomes full;

decrease by $2\times$ in *pop* when array becomes 50% full.

C. *Either A or B.*

D. *Neither A nor B.*

Stack: resizable-array implementation

Problem. Requiring client to provide maximum capacity does not implement API!

Q. How to grow and shrink the array automatically?

referred to as a {resizable, dynamic, extendable} array

Naive approach.

- Push: increase length of array `a[]` by 1.
- Pop: decrease length of array `a[]` by 1.

Too expensive.

- Need to copy all items to a new array, for each push/pop.
- Array accesses to add item k : $1 + 2(k - 1)$ — *to copy $k-1$ elements from old array to new array (ignoring cost to create new array)*
- Array accesses to add first n items: $n + (2 + 4 + 6 + \ldots + 2(n-1)) \sim n^2$.

add items to array

array resizing to lengths $2, 3, 4, \ldots, n$

$\Theta(n^2)$ infeasible for large n

Challenge. Ensure that array resizing happens infrequently.

Stack: resizable-array implementation

Q. How to grow the array?

"geometric expansion"

A. If array is full, create a new array of twice the length, and copy items.

```
public class ResizableArrayStackOfStrings {
   private String[] a;
   private int n = 0;

   public ResizableArrayStackOfStrings() {
      a = new String[1];
   }

   public void push(String item) {
      if (n == a.length) resize(2 * a.length);
      a[n++] = item;
   }

   private void resize(int capacity) {
      String[] copy = new String[capacity];
      for (int i = 0; i < n; i++)
         copy[i] = a[i];
      a = copy;
   }

}
```

if the array if full, double its length

helper method (to resize the array)

Stack: resizable-array implementation

Q. How to grow the array?

"geometric expansion"

A. If array is full, create a new array of twice the length, and copy items.

Cost is reasonable.

- Still need to copy all items to a new array but, now, that happens infrequently.
- Array accesses to add first $n = 2^i$ items: $n + (2 + 4 + 8 + 16 + \ldots + n) \sim 3n$.

add items to array

array resizing to lengths $2, 4, 8, 16, \ldots, n$

$\Theta(n)$ *feasible for large n*

Q. Can I use a growth factor other than $\alpha = 2$?

A. Yes. Classic time-space tradeoff.

language	data type	α
Java	`ArrayList`	1.5
C++	`vector`	1.5
Python	`list`	1.125
⋮	⋮	⋮

Stack: resizable-array implementation

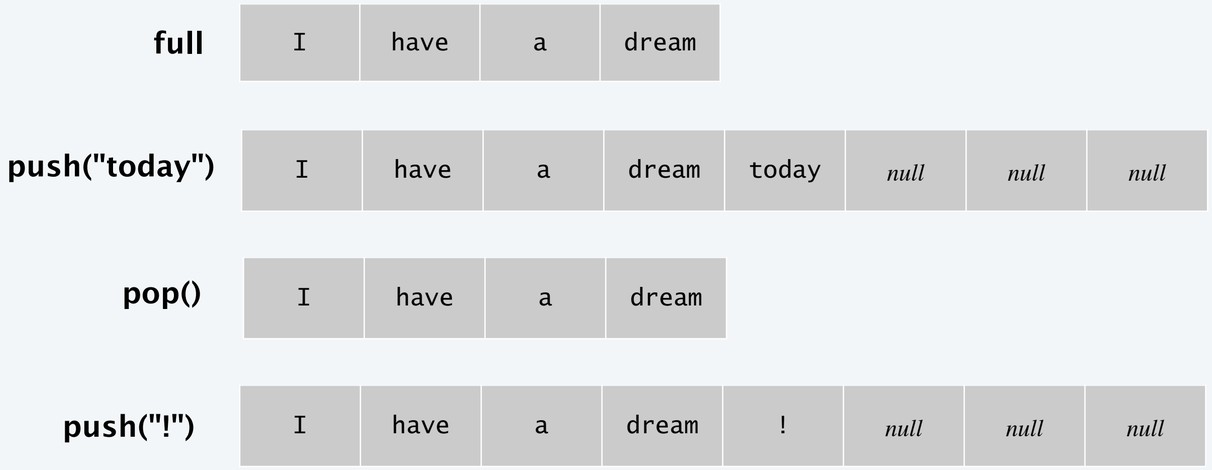
Q. How to shrink the array?

First try.

- Push: double length of array `a[]` when array is full.
- Pop: halve length of array `a[]` when array is one-half full.

Too expensive for some sequences of operations.

- Push $n = 2^i$ items to make array full; then, alternate n push and pop operations.
- Each alternating operation triggers an array resizing and takes $\Theta(n)$ time.



Stack: resizable-array implementation

Q. How to shrink the array?

Efficient solution.

- Push: double length of array `a[]` when array is full.
- Pop: halve length of array `a[]` when array is one-quarter full.

```
public String pop() {
   String item = a[--n];
   a[n] = null;
   if (n > 0 && n == a.length/4)
      resize(a.length/2);
   return item;
}
```

if the array is one-quarter full, halve its length

so, on average, each of the m operation takes $\Theta(1)$ time

Proposition. Starting from an empty stack, any sequence of m push/pop operations takes $\Theta(m)$ time.

Intuition. After array resizes to length n, at least $\Theta(n)$ push/pop operations before next array resizing.

Stack resizable-array: memory usage

Proposition. A `ResizableArrayStackOfStrings` with n items use between $\sim 8n$ and $\sim 32n$ bytes of memory.

- Always between 25% and 100% full.
- $\sim 8n$ when full. [array length = n]
- $\sim 32n$ when one-quarter full. [array length = $4n$]

```
public class ResizableArrayStackOfStrings {
   private String[] a;
   private int n = 0;

   ⋮
}
```

← 8 bytes × array length

Remark. This counts the memory for the stack itself, including the string references.

[but not the memory for the string objects, which the client allocates]

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AMORTIZATION

Worst-case analysis

Worst-case running time. Longest running time for an individual operation.

- Gold standard in analysis of algorithms.
  - applies to all inputs (of a given size)
  - provides an ironclad performance guarantee
  - standardizes way to compare different algorithms
- Can be unduly pessimistic.

e.g., when an expensive operation is rare

operation	worst
construct	$\Theta(1)$
push	$\Theta(n)$
pop	$\Theta(n)$

resizable-array stack with n items

```
stack = new ResizableArrayStackOfInts();
for (int i = 0; i < n; i++) {
   stack.push(i);
}
```

$\Theta(n)$ *worst case*

takes $\Theta(n)$ time in the worst case, not $\Theta(n^2)$

Amortized analysis

Amortized analysis. Provides a worst-case running time for a sequence of operations.

- Let $T(m)$ denote worst-case running time of sequence of m operations.
- Amortized cost per operation = $T(m) / m$. ← *on average, each operation costs at most this*
- Provides more robust and realistic analysis.

starting from an empty data structure

Bob Tarjan
(1986 Turing award)

Ex. Starting from an empty stack, any sequence of m push/pop operations takes $\Theta(m)$ time.

operation	worst	amortized
construct	$\Theta(1)$	$\Theta(1)$
push	$\Theta(n)$	$\Theta(1)$
pop	$\Theta(n)$	$\Theta(1)$

constant amortized time

resizable-array stack with n items

```
stack = new ResizableArrayStackOfInts();
for (int i = 0; i < n; i++) {
   stack.push(i);
}
```

$\Theta(n)$ *worst case*
$\Theta(1)$ *amortized*

takes $\Theta(n)$ time in the worst case, not $\Theta(n^2)$

Stacks and queues I: poll 3

Suppose that `QuickUnionPathCompressionUF` has the following performance properties.

What is the worst-case running time the following code fragment?

A. $\Theta(\log n)$

B. $\Theta(n)$

C. $\Theta(n \log n)$

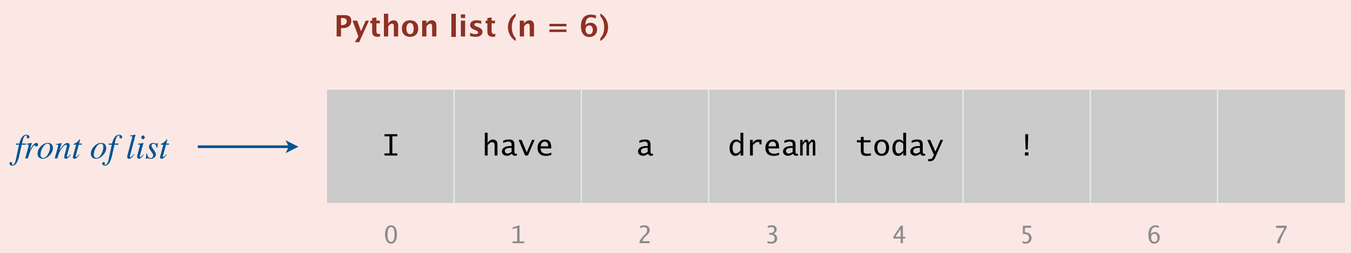
D. $\Theta(n^2)$

```
uf = new QuickUnionPathCompressionUF(n);
for (int i = 0; i < n; i++) {
   if (uf.find(x[i]) != uf.find(y[i]))
      uf.union(x[i], y[i]);
}
StdOut.println(uf.count());
```

operation	worst	amortized
construct	$\Theta(n)$	$\Theta(n)$
union	$\Theta(n)$	$\Theta(\log n)$
find	$\Theta(n)$	$\Theta(\log n)$
count	$\Theta(1)$	$\Theta(1)$

Stacks and queues I: poll 4

Python implements a `list` as a resizable array (with the first element always at index 0). Which of the following can you infer about the worst-case running times of various operations, where n is the length of the list?



A. Adding an element to front of list takes $\Theta(1)$ time.

B. Adding an element to back of list takes $\Theta(1)$ time.

C. Replacing element i in the list with a new value takes $\Theta(1)$ time.

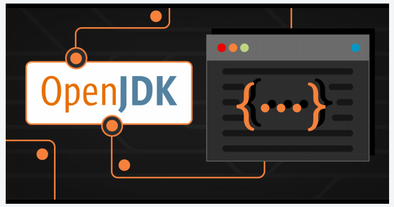
D. *None of the above*.

Worst-case performance guarantees in Java

Java. Rarely provides worst-case performance guarantees.

- Garbage collector: automatically deallocate memory no longer in use.
- Just-in-time compiler: compile bytecode to native machine code at runtime.
- Thread scheduler: determine which thread to execute next.

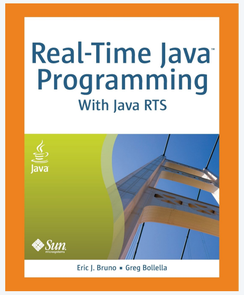
← *operations are expensive, but run infrequently*



Real-time Java. Provides worst-case performance guarantees.

- Pacemakers.
- Industrial robots.
- Air-traffic control.

← *systems with hard deadlines*



This course. We ignore such issues in our analysis.

Queue with two stacks

Problem. Implement a queue with two stacks so that:

- $\Theta(1)$ extra memory (besides two stacks).
- Starting from an empty queue, any sequence of m queue operations makes $\Theta(m)$ stack operations.

⟵ *amortized analysis*
(worst case bound on sequence of operations)

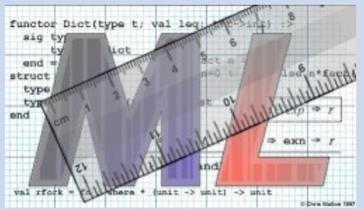
Applications.

- Job interview.
- Implement an immutable or persistent queue.
- Implement a queue in a purely functional programming language.

Haskell

Lisp

OCaml



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Parameterized stack

We implemented: `StackOfStrings`.

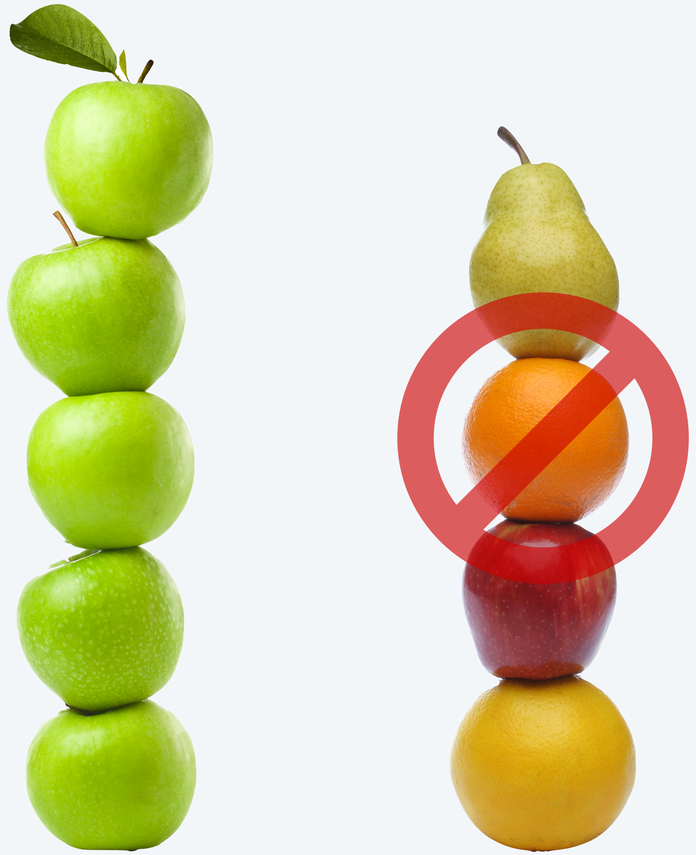
We also want: `StackOfURLs`, `StackOfInts`, `StackOfApples`, `StackOfOranges`, ...

Solution in Java: generics.

Guiding principle: prefer compile-time errors to run-time errors.

type argument
(use to specify type and invoke constructor)

```
Stack<Apple> stack = new Stack<Apple>();
Apple apple = new Apple();
stack.push(apple);
Orange orange = new Orange();
stack.push(orange);   <-- compile-time error
...
```



Generic stack: array implementation

The way it should be.

```
public class FixedCapacityStackOfStrings {
   private String[] a;
   private int n = 0;

   public Fixed...OfStrings(int capacity)
   {  a = new String[capacity];  }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(String item)
   {  a[n++] = item;  }

   public String pop()
   {  return a[--n];  }

}
```

stack of strings (fixed-length array)

```
public class FixedCapacityStack<Item> {
   private Item[] a;
   private int n = 0;

   public FixedCapacityStack(int capacity)
   {  a = new Item[capacity];  }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(Item item)
   {  a[n++] = item;  }

   public Item pop()
   {  return a[--n];  }

}
```

@#$! generic array creation not allowed in Java*

type variable (name `Item` *is our convention)*

generic stack (fixed-length array) ???

Generic stack: array implementation

The way it is.

```
public class FixedCapacityStackOfStrings {
   private String[] a;
   private int n = 0;

   public Fixed...OfStrings(int capacity)
   {  a = new String[capacity];  }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(String item)
   {  a[n++] = item;  }

   public String pop()
   {  return a[--n];  }

}
```

stack of strings (fixed-length array)

```
public class FixedCapacityStack<Item> {
   private Item[] a;
   private int n = 0;

   public FixedCapacityStack(int capacity)
   {  a = (Item[]) new Object[capacity]; }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(Item item)
   {  a[n++] = item;  }

   public Item pop()
   {  return a[--n];  }

}
```

the ugly cast

generic stack (fixed-length array)

Unchecked cast

```
~/cos226/queues> javac -Xlint:unchecked FixedCapacityStack.java
FixedCapacityStack.java:26: warning: [unchecked] unchecked cast
        a = (Item[]) new Object[capacity];
                     ^
  required: Item[]
  found:    Object[]
  where Item is a type-variable:
    Item extends Object declared in class FixedCapacityStack
1 warning
```

Q. Why does Java require a cast (or reflection)?

Short answer. Backward compatibility.

Long answer. Need to learn about type erasure and covariant arrays.

Stacks and queues I: poll 5

How to declare and initialize an empty stack of integers in Java?

A.
```
Stack stack1 = new Stack();
```

B.
```
Stack<int> stack2 = new Stack();
```

C.
```
Stack<int> stack3 = new Stack<int>();
```

D. *None of the above.*

Generic data types: autoboxing and unboxing

Q. What to do about primitive types?

Wrapper type.

- Each primitive type has an associated "wrapper" reference type.
- Ex: `Integer` is wrapper type associated with `int`.

primitive	wrapper
`int`	`Integer`
`double`	`Double`
`boolean`	`Boolean`
`char`	`Character`

Autoboxing. Automatic cast from primitive type to wrapper type.

Unboxing. Automatic cast from wrapper type to primitive type.

```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);          // stack.push(Integer.valueOf(17));
int x = stack.pop();     // int x = stack.pop().intValue();
```

Bottom line. Client code can use generic stack with any data type.

Caveat. Performance overhead for primitive types.

Stacks and queues summary

Fundamental data types.

- Value: collection of objects.
- Operations: add, remove, iterate, size, test if empty.

next lecture

Stack. [LIFO] Remove the item most recently added.

Queue. [FIFO] Remove the item least recently added.

Efficient implementations.

- Resizable array.
- Singly linked list. ← *next lecture*

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