Consistency

Spring 2025





Strict Serializability

- Transactions: operations that span multiple objects (e.g., keys in KV store) atomically commit (or abort).
- Total order: There exists some legal total ordering of transactions.
 - Legal (intuitively defined for strict serializability): in the total ordering, read operations "see" the latest write operation.
- Preserves real-time commit order: if *txn A* commits before *txn B* begins, then *txn A* occurs before *txn B* in the total order.
 - Write ops in a committed txn are visible to all future txns' read ops.
 - Intuition: once a read "sees" a txn and commits, all future reads must also "see" that txn.

Pros: applications can easily reason about correctness of transactions. **Cons:** strict serializability imposes high read and write latencies on system.

Strict Serializability Example

Strictly S	erializable?	Yes	Strictly	Serializable?	No
P1:	{W(x)b, W(y)b}		P1:	{W(x)b, W(y)b}	
P2: {W(x)a	}		P2: {W(x)a}	
P3:	{R(x)a}	$\{R(x)b\}$	P3:	{R(y)b}	{R(x)a}
P4:	{R(x)b}	{R(y)b}	P4:	{R(x)b}	{R(y)b}



Linearizability

- Total order: There exists some legal total order of operations (not txns).
- Difference from *strict serializability*?
 - Single-object operations! No transactions!
- Preserves real-time ordering: if an operation *A* completes before operation *B* begins, then op A occurs before op *B* in the total order.
 - A completed write op is visible to all future read ops.
 - Intuition: once a read "sees" a new write, all future reads must also "see" that write.

Pros: Easy to reason about correctness **Cons:** High read and write latencies

Linearizability Example

Linearizable?			No		Line	Linearizable?			
P1:	W(x)a				P1:	W(x)a			
P2:		W(x)b			P2:	W(x)b			
P3:			R(x)b	R(x)a	P3:	R(x)	a R(x)b		
P4:			R(x)b	R(x)a	P4:	R(x)	a R(x)b		



Sequential Consistency

- **Total order**: there exists some legal total order of operations.
- Preserves process ordering: total order respects order of each process's operations.
- Difference from *linearizability*?
 - Order of ops across processes not determined by real-time

Pros: Can allow more orderings than linearizability \rightarrow better performance **Cons:** Many possible sequential executions \rightarrow increased application complexity

Sequential Consistency Example

Seq	uential	ly Con	sistent	? Ye	s Sequ	Sequentially Consistent?				
P1:	W(x)a				P1:	W(x)a				
P2:		W(x)b			P2:	V	V(x)b			
P3:			R(x)b	R(x)a	P3:			R(x)b	R(x)a	
P4:			R(x)b	R(x)a	P4:			R(x)a	R(x)b	



Causal+ Consistency

- Partial order: order causally related ops the same way across all processes
- +: replicas' total order eventually converges.
- Difference from *sequential consistency*?
 - Only causally related ops need to be ordered: **no guaranteed total order**.
 - Concurrent ops may be ordered differently across different processes.

Pros: preserves causality while improving efficiency.

Cons: harder to reason about concurrency.

Ops	Concurrent	P1	P2	
a,b	No			
a,e	Yes	а		
a,g	No	h		е
c,e	Yes			
c,d	No			f
d,g	No	C		
d,f	No			g
e,g	No	d		
a,d	No	u ↓	Ļ	

Causal+ Consistency Example

Causally+ Consistent? Causally+ Consistent? Yes No P1: W(x)a P1: W(x)a P2: W(x)b P2: R(x)a W(x)bP3: R(x)b R(x)a P3: R(x)a R(x)b P4: R(x)a P4: R(x)a



Eventual Consistency

- Eventual convergence: If no more writes, all replicas eventually agree.
- Difference from *causal consistency*?
 - Does not preserve causal relationships
 - Is the "+" in causal+.
- Frequently used with application conflict resolution, anti-entropy

Pros: highly available; think Bayou.

Cons: no safety guarantees, need conflict resolution.

In a nutshell...

Strict Serializability: total order + real time guarantees over *transactions* Linearizability: total order + real time guarantees over *operations* Sequential consistency: total order + process order Causal+ consistency: causally ordered + replicas eventually converge Eventual consistency: eventually, everyone should agree on state

Exercise 1:

Consistency Model:

Sequential

Strictly Serializable Yes

Linearizable	Yes

Yes

P1:	{W(x) 1, W(y) 2}	{R(y) 4}	
P2:	{W(x) 1, R(y) 4}		

Causal+ Yes Eventual Yes

P3: {W(x) 0, W(y) 4}

P4: {R(x) 0} {R(x) 1}

Exercise 2:

P1:

P2:

W(x) 1

Consistency Model:

Linearizable	Yes
Sequential	Yes
Causal+	Yes
Eventual	Yes
	Linearizable Sequential Causal+ Eventual

P3: R(x) 1 W(y) 4

R(x) 1

P4: R(x) 1 R(y) 4

Exercise 3:

							Linearizable	No
P1:	W(x) 3				W(y) 7		Sequential	Yes
P2:		W(x) 1					Causal+	Yes
P3:			R(x) 1	R(x) 3		R(y) 7	Eventual	Yes
P4:			R(x) 1	R(x) 3		R(y) 7		
P5:			R(x) 1	R(x) 3		R(y) 7		

Exercise 4:

							Linearizable	No
P1:	W(x) 3				W(y) 7		Sequential	No
P2:		W(x) 1					Causal+	Yes
P3:			R(x) 1	R(x) 3		R(y) 7	Eventual	Yes
P4:			R(x) 3	R(x) 1		R(y) 7		
P5:			R(x) 1	R(x) 3		R(y) 7		

Exercise 5:

							Linearizable	No
P1:	W(x) 1						Sequential	No
P2:		W(x) 3					Causal+	Yes
P3:			W(x) 7				Eventual	Yes
P4:				R(x) 3	R(x) 7	R(x) 1		
P5:				R(x) 3	R(x) 1	R(x) 7		

Exercise 6:

								Linearizable	No
P1:	W(x) 1							Sequential	No
P2:		W(x) 3						Causal+	Yes
P3:			R(x) 3	W(x) 7				Eventual	Yes
P4:					R(x) 3	R(x) 7	R(x) 1		
P5:					R(x) 3	R(x) 1	R(x) 7		

Exercise 7:

								Linearizable	No
								Sequential	No
P1:	W(x) 1							Causal+	No
P2:		R(x) 1	W(x) 3					Eventual	Yes
P3:				R(x) 3	W(x) 7				
P4:						R(x) 3	R(x) 7	R(x) 1	
P5:						R(x) 3	R(x) 1	R(x) 7	