Subdivision Surfaces

COS 426, Spring 2022
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Princeton University

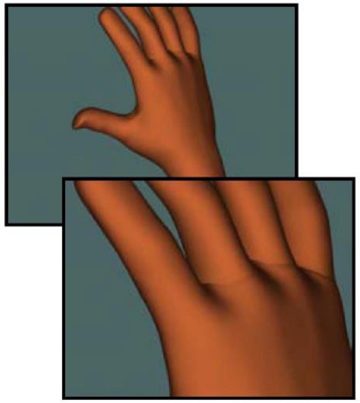
3D Object Representations

- Raw data
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Parametric
  - ➢ Subdivision
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

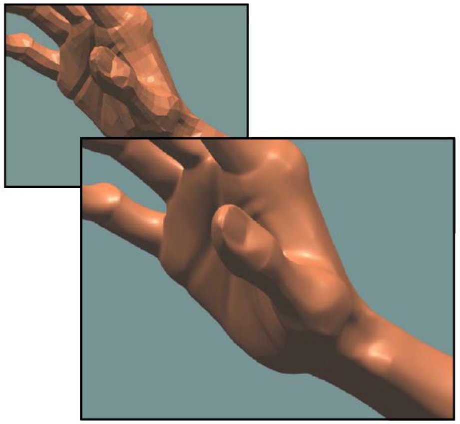
Subdivision Surfaces

- Alternative to parametric surfaces, overcoming:
  - Many patches
  - Difficult to mark sharp features
  - Irregularities after deformation

Woody's hand (NURBS)

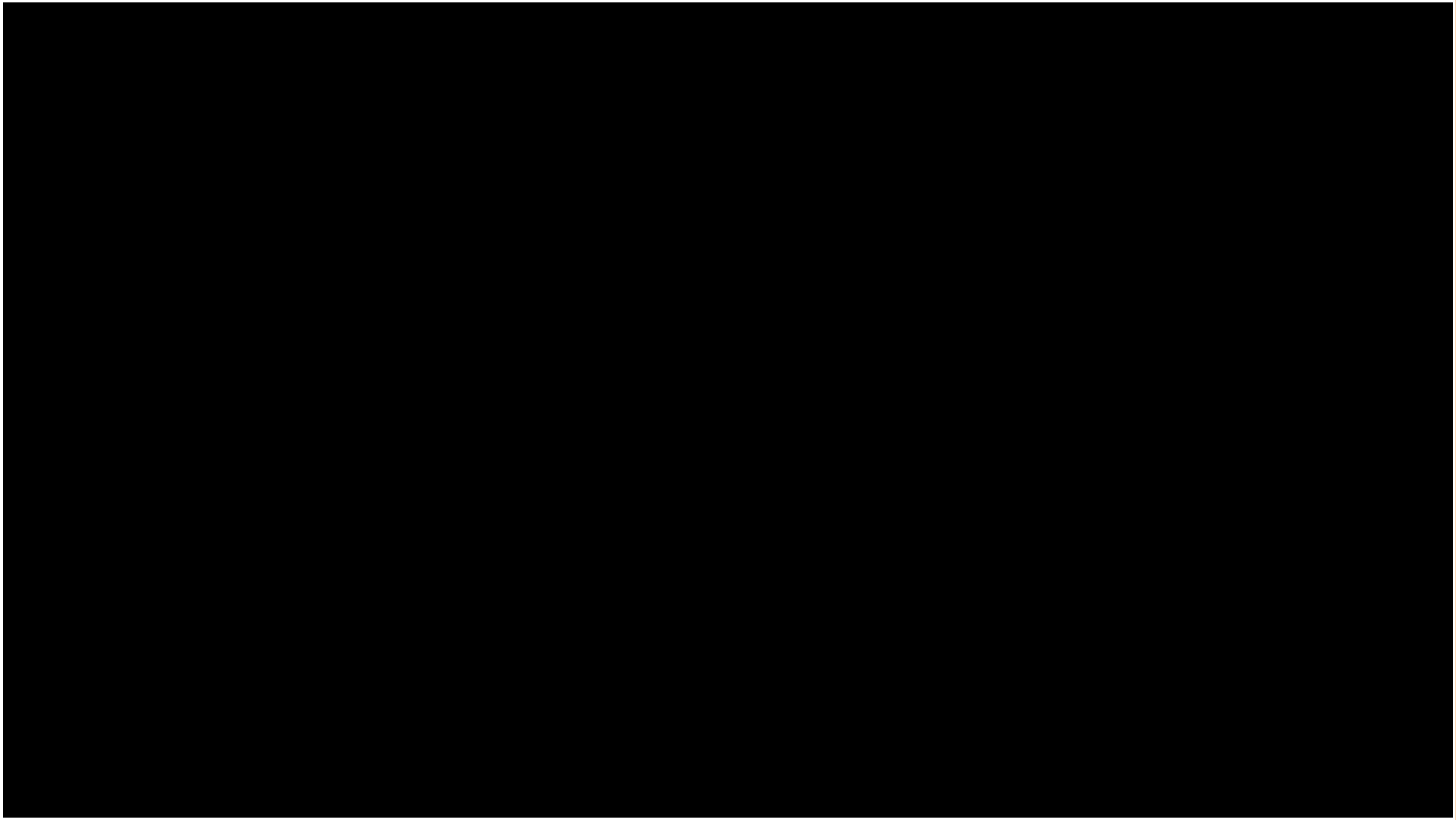


Geri's hand (subdivision)



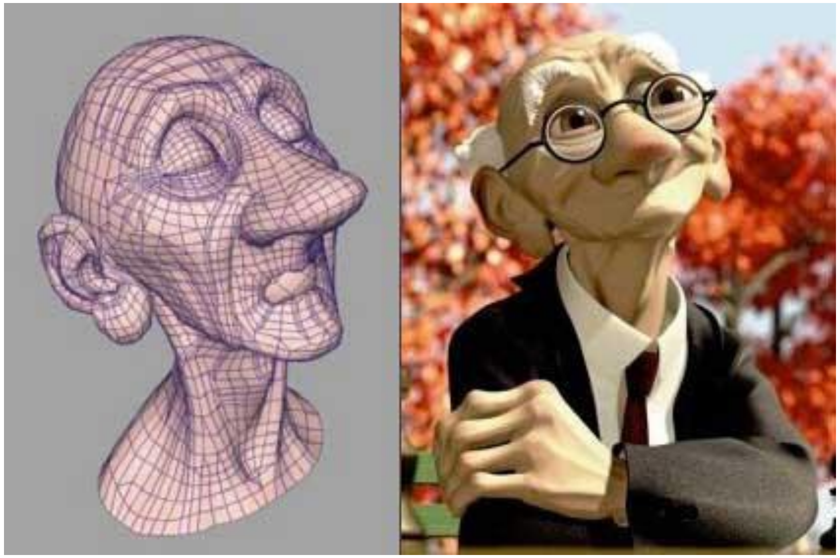
Stanford Graphics course notes

Geri’s Game



Geri's Game

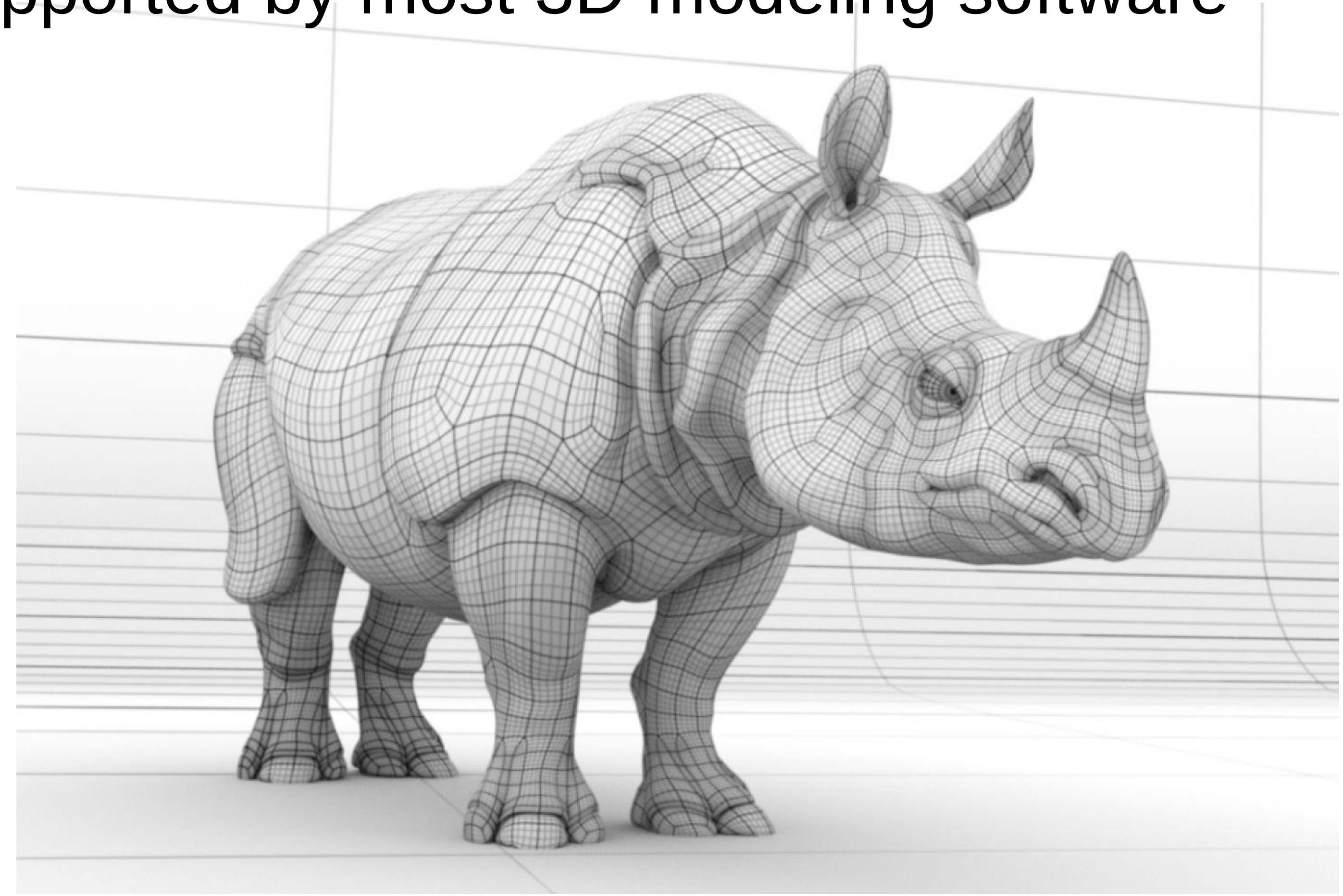
- "served as a demonstration of a new animation tool called subdivision surfaces" (Wikipedia)
- Subdivision used for head, hands & clothing
- Academy Award winner



Geri's Game © Pixar Animation Studios

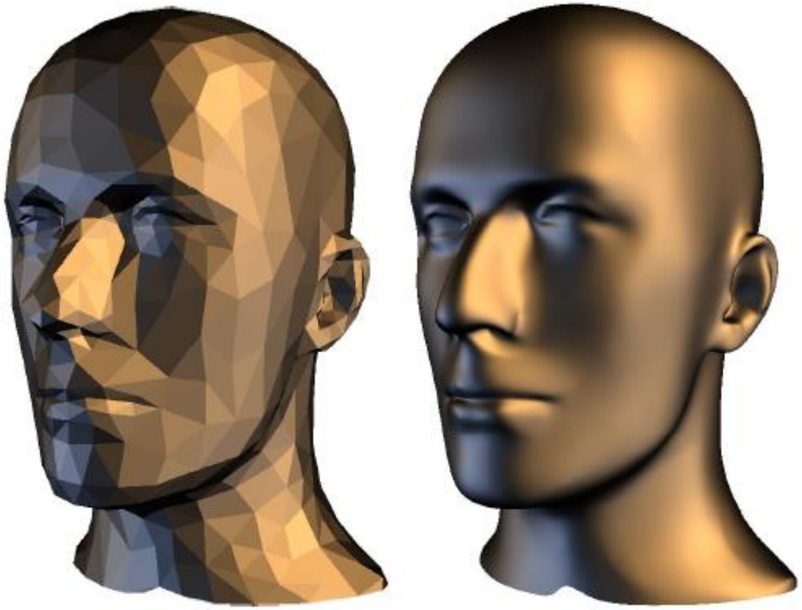
Subdivision Surfaces

- Used in movie and game industries
- Supported by most 3D modeling software



Subdivision Surfaces

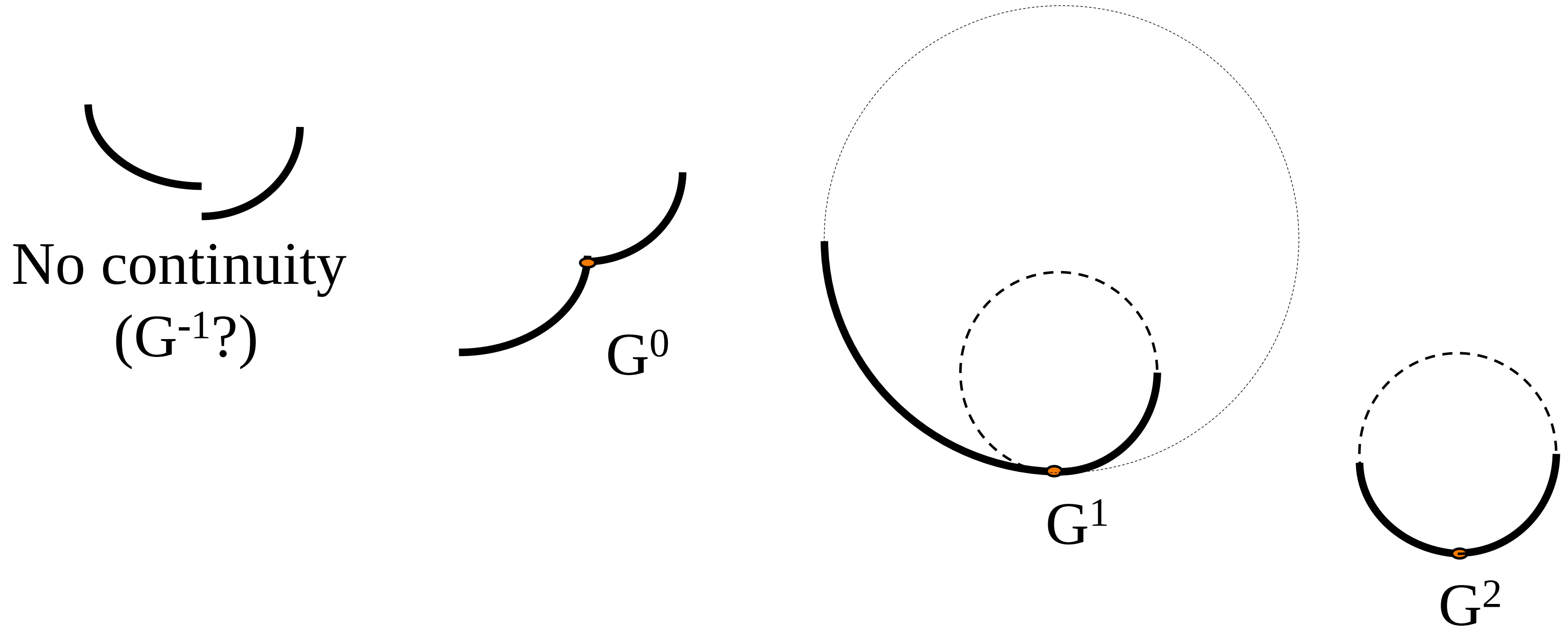
- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - ➢ Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections



Reif & Schroeder 2000

Review on Continuity

A curve / surface with G^k continuity has a continuous k-th derivative, geometrically.

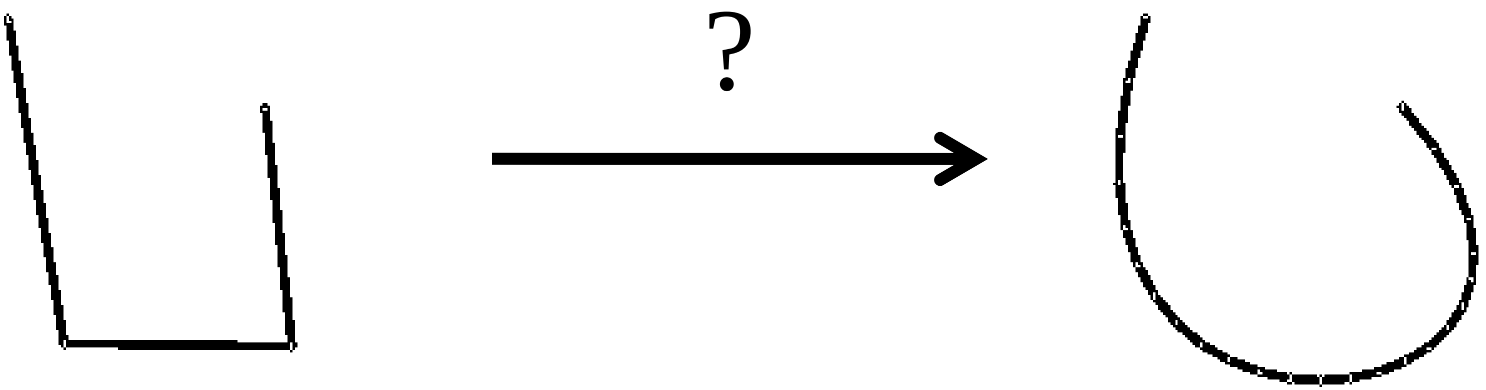


Similar to (but not the same as) C^k continuity, which refers to continuity with respect to parameter

e.g.: $f_x(u) = r_x \cos(2\pi u)$ *(but we're going to say C^k from now on...)*

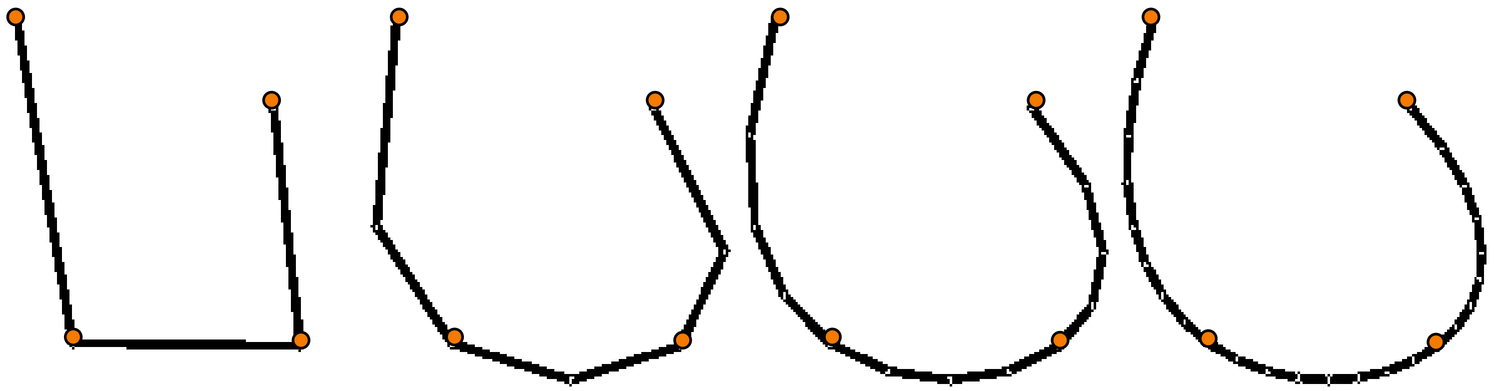
Subdivision

- How do you make a curve with guaranteed continuity?



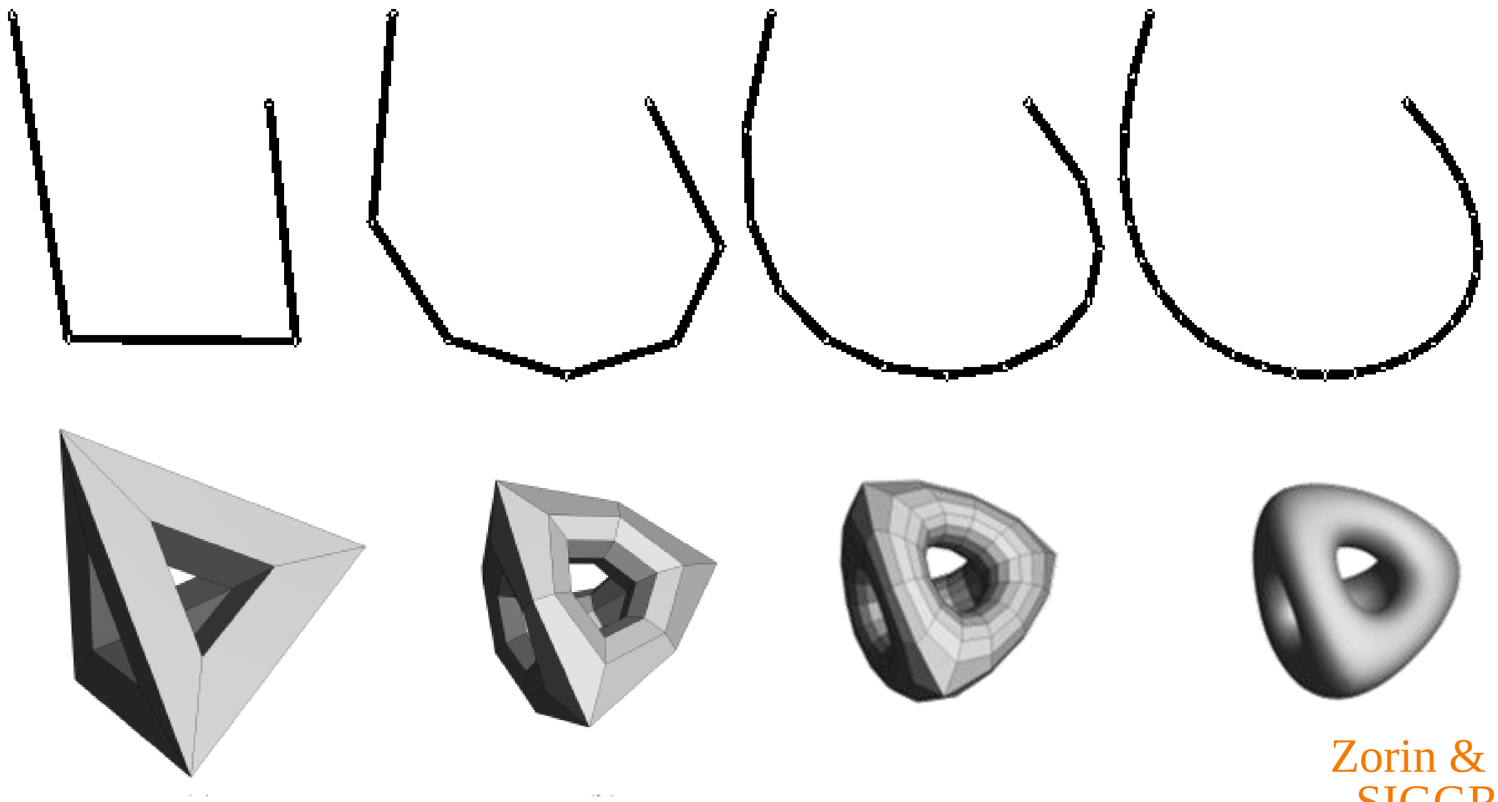
Subdivision

- How do you make a curve with guaranteed continuity? …



Subdivision

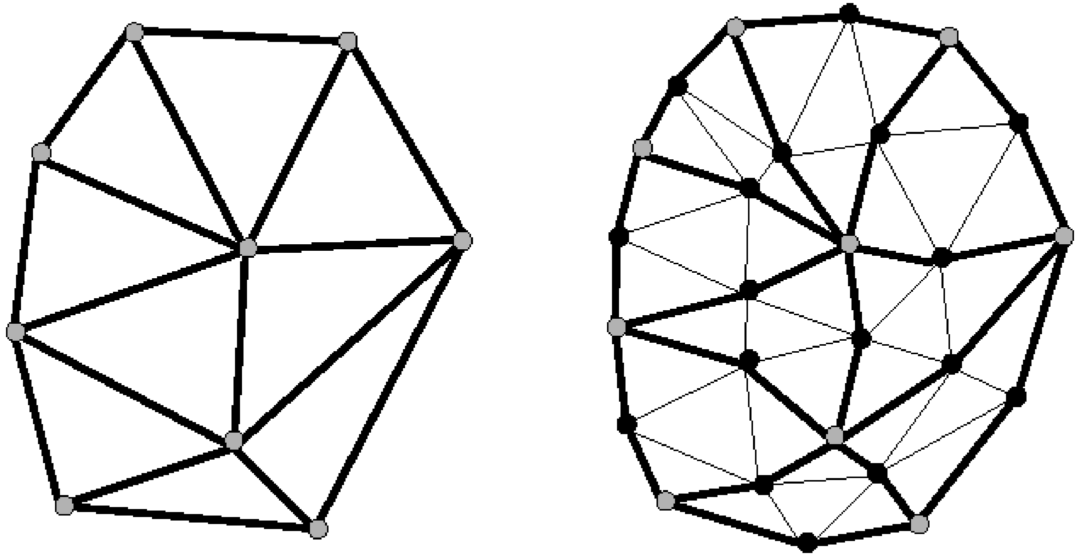
- How do you make a surface with guaranteed continuity?



Zorin & Schroeder
SIGGRAPH 99
Course Notes

Subdivision Surfaces

- Repeated application of
  1. **Topology refinement (splitting faces)**
  2. **Geometry refinement (weighted averaging)**



Zorin & Schroeder
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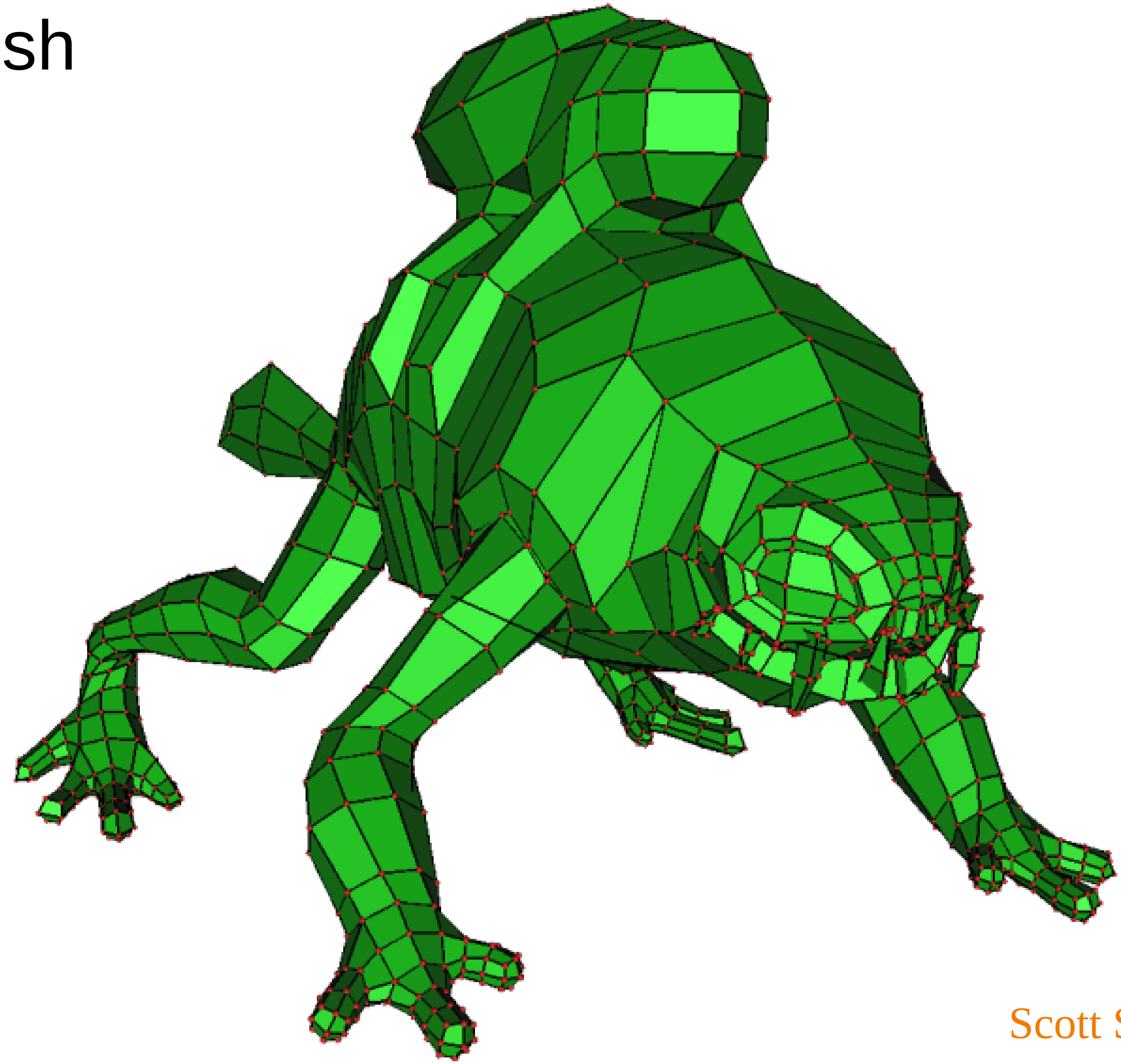
Course Notes

Subdivision Surfaces – Examples

- Base mesh



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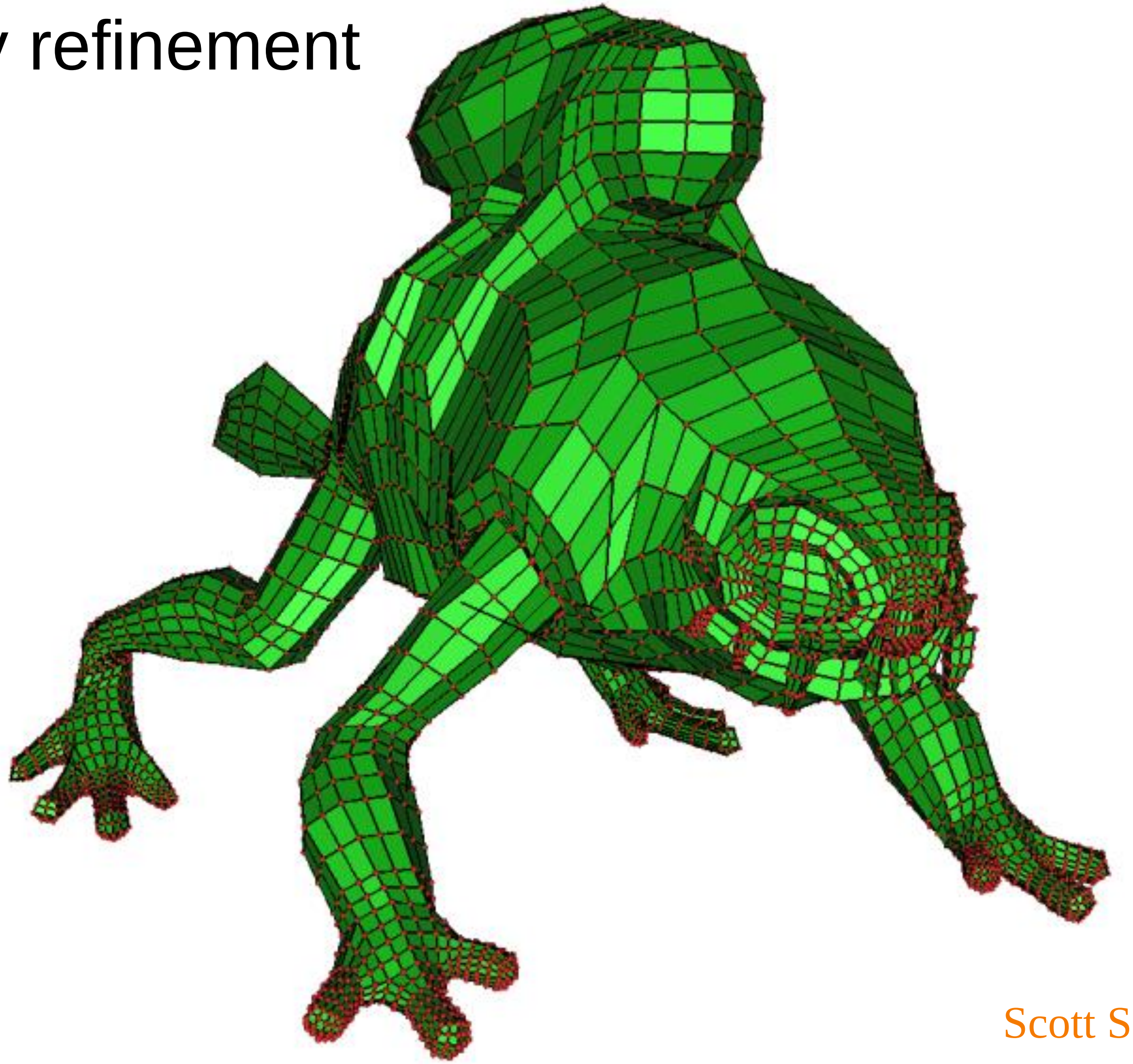
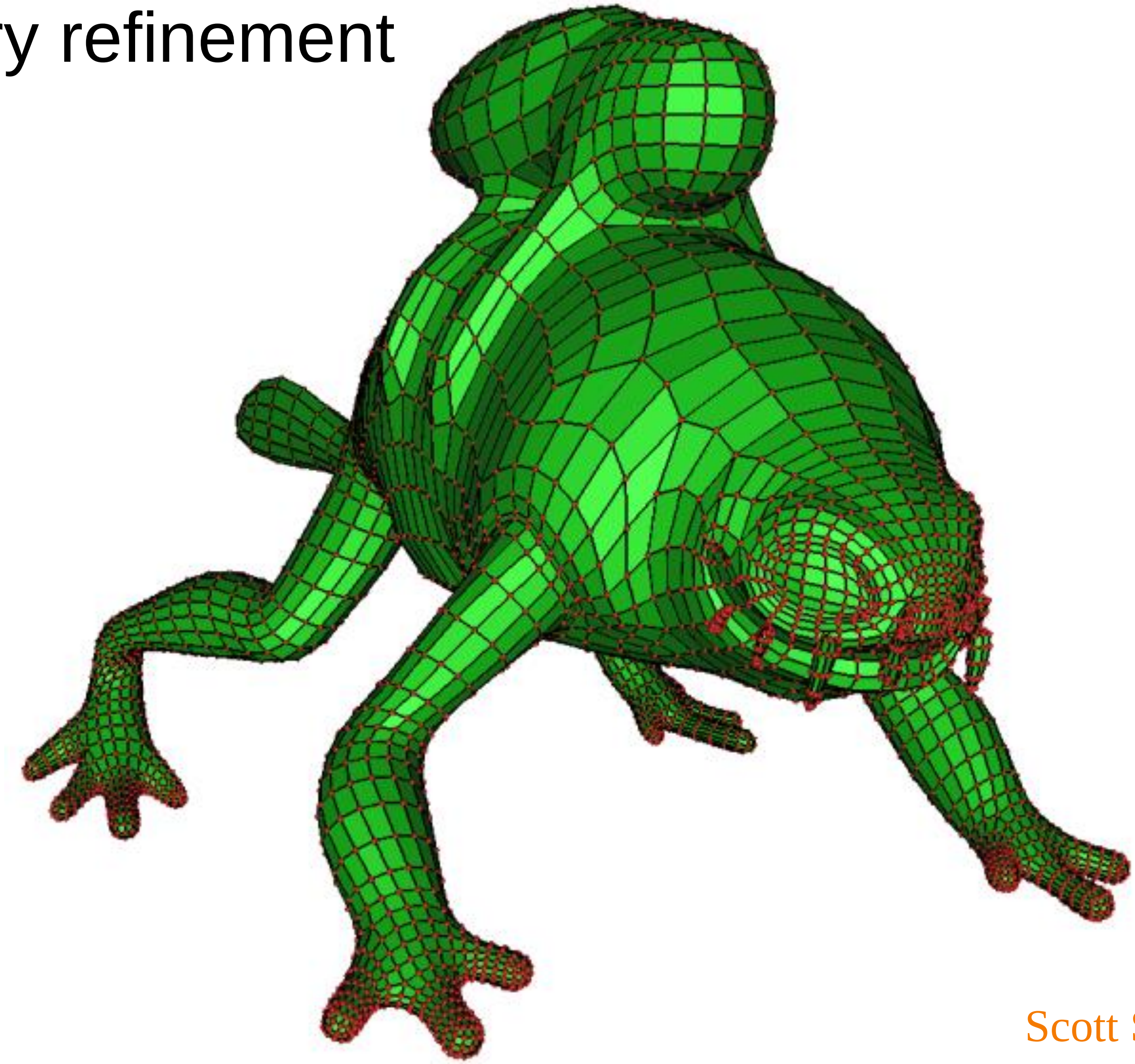
Subdivision Surfaces – Examples

- Topology refinement



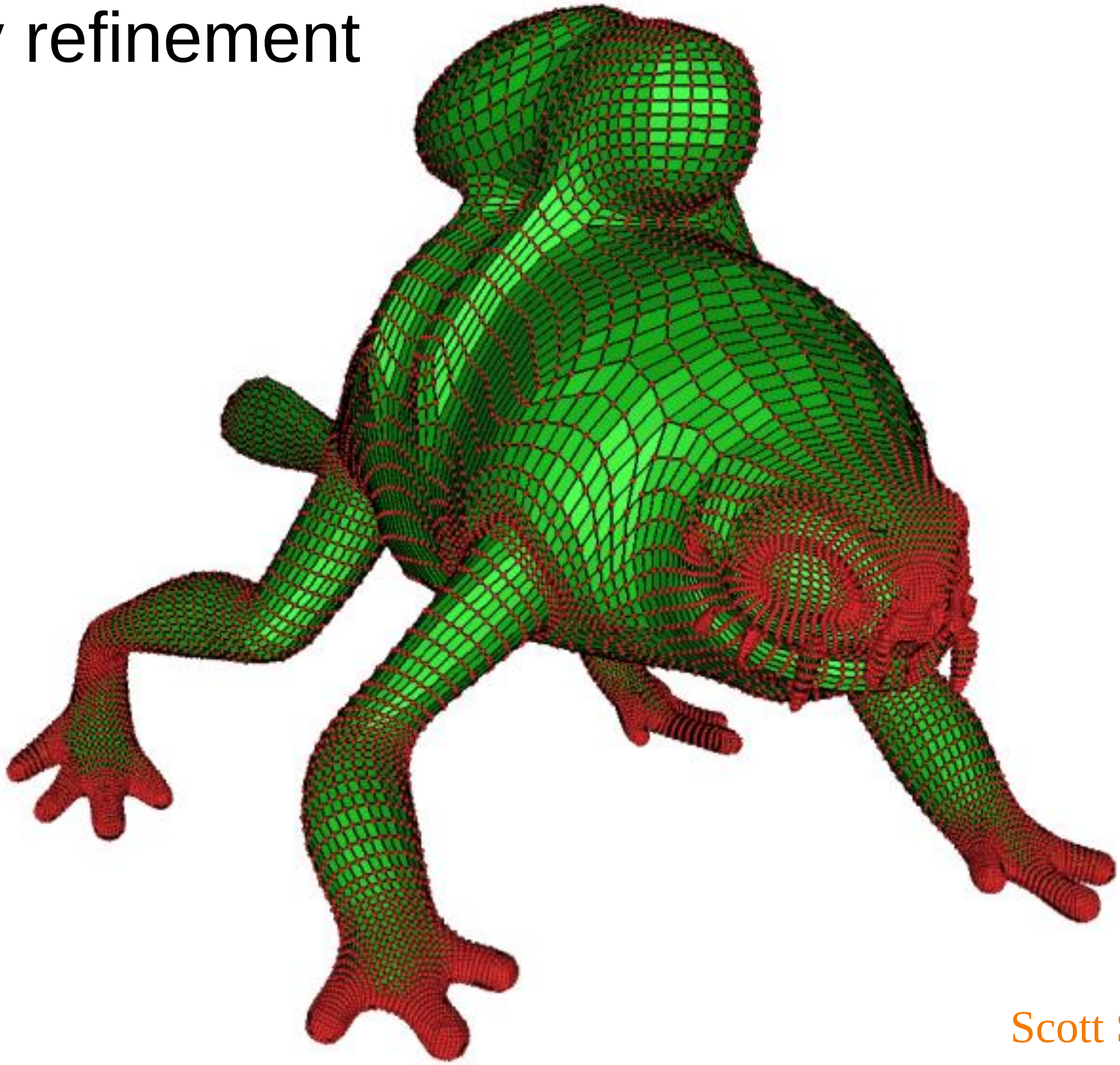
Subdivision Surfaces – Examples

- Geometry refinement



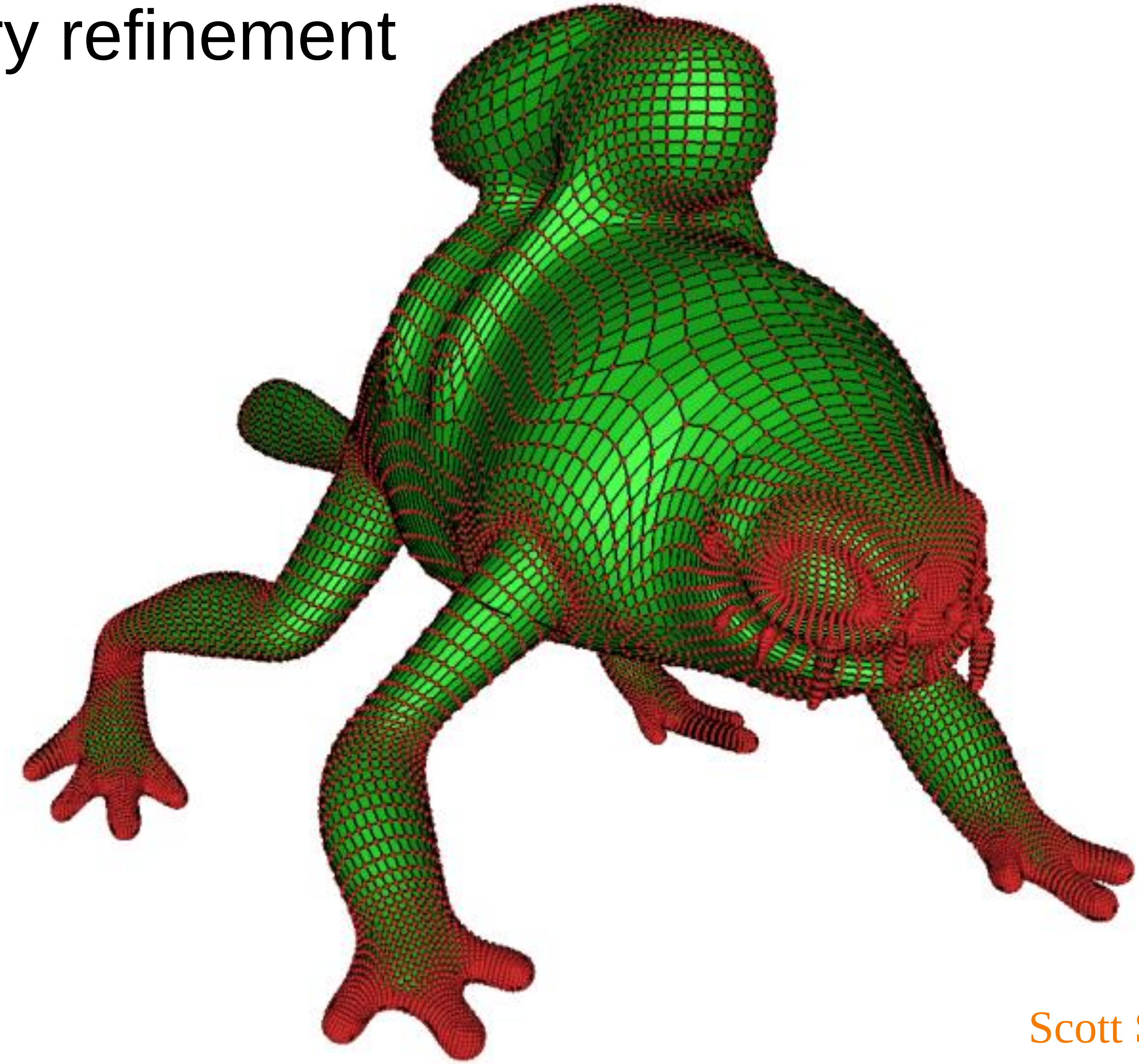
Subdivision Surfaces – Examples

- Topology refinement



Subdivision Surfaces – Examples

- Geometry refinement



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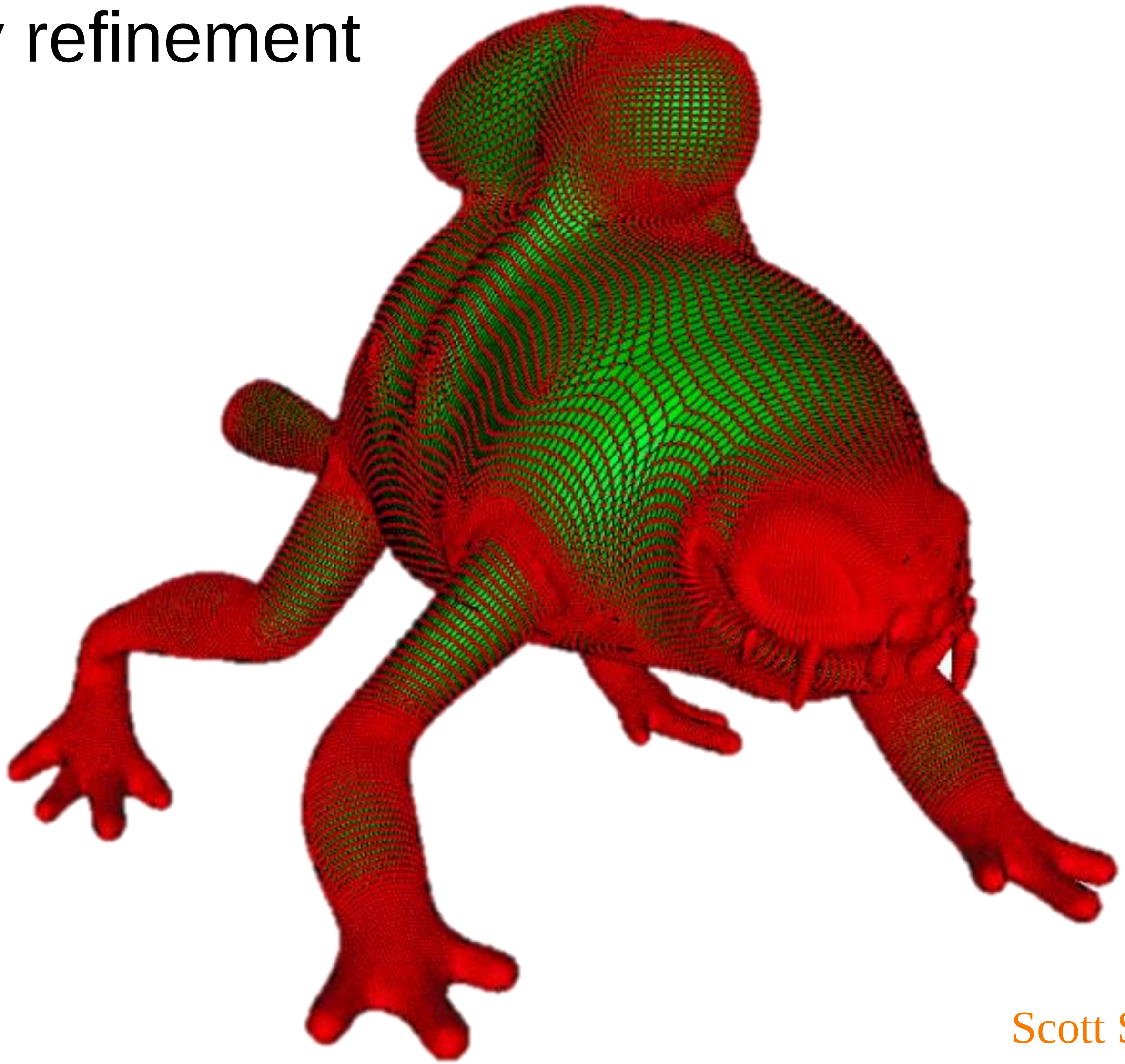
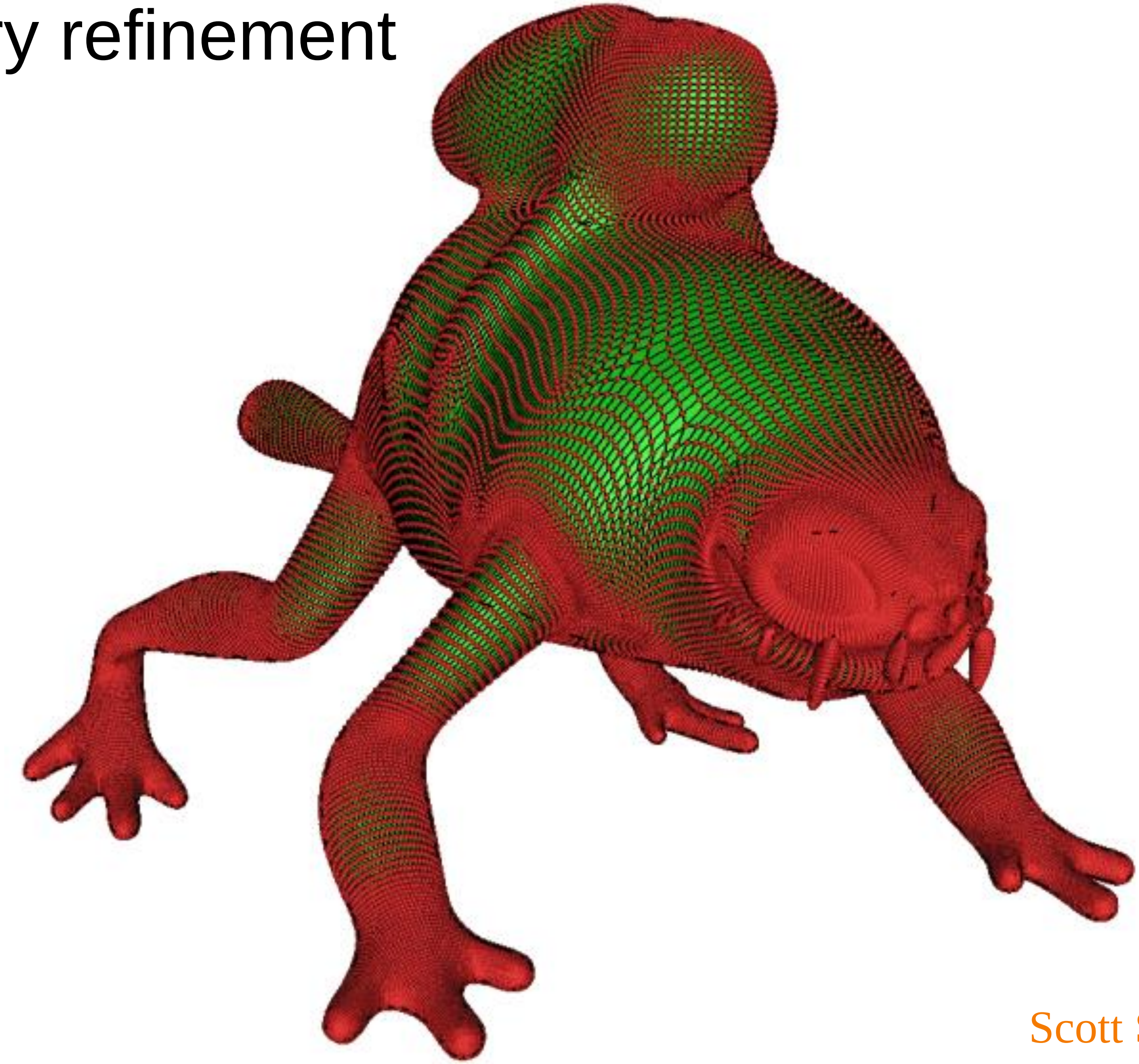
Subdivision Surfaces – Examples

- Topology refinement



Subdivision Surfaces – Examples

- Geometry refinement



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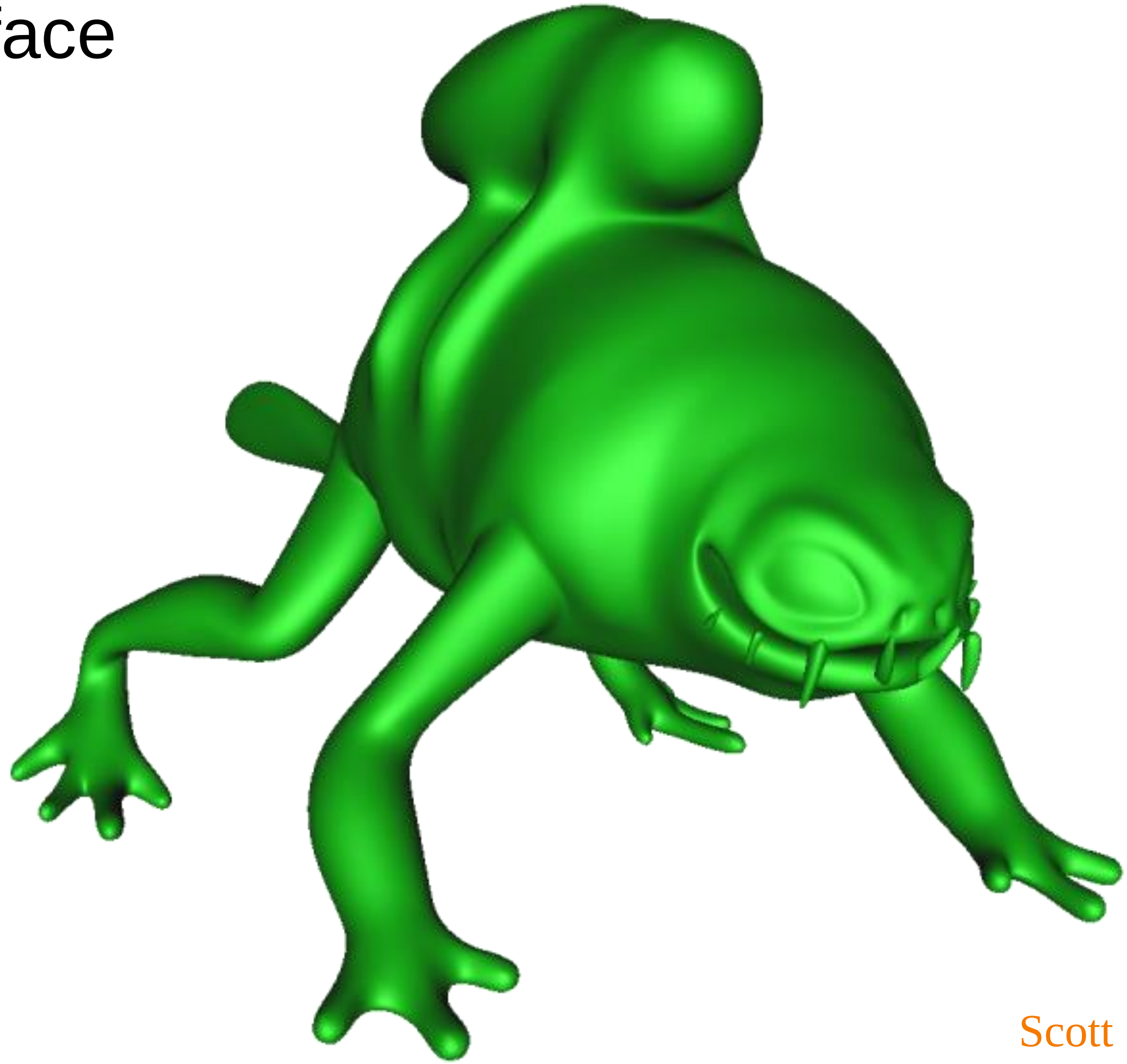
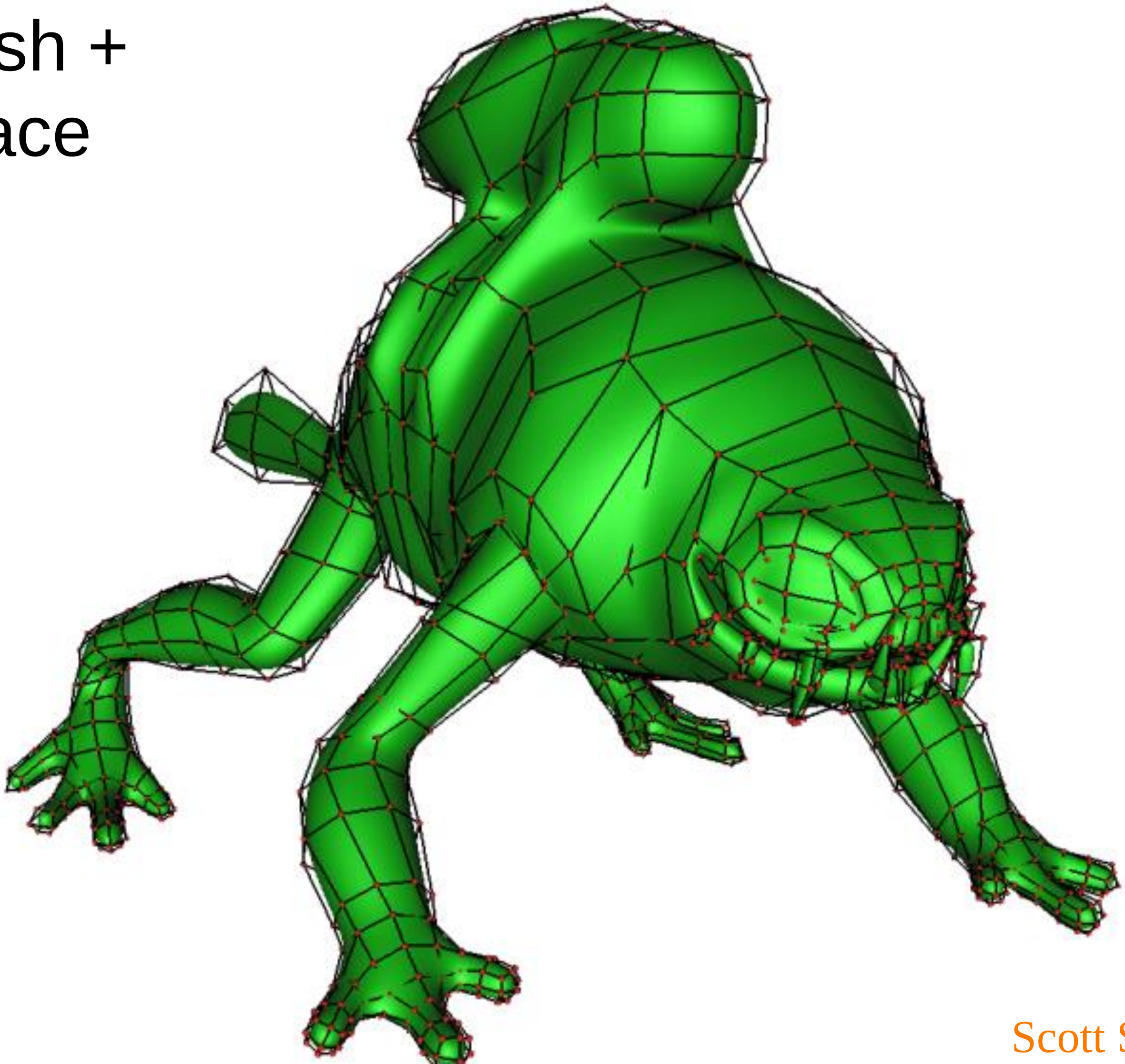
Subdivision Surfaces – Examples

- Limit surface



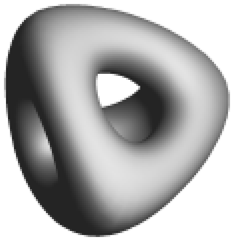
Subdivision Surfaces – Examples

- Base mesh + limit surface



Design of Subdivision Rules

- What types of input?
  - Quad meshes, triangle meshes, etc.
- How to refine topology?
  - Simple implementations
- How to refine geometry?
  - Smoothness guarantees in limit surface
    - » Continuity (C^0, C^1, C^2, …?)
  - Provable relationships between limit surface and original control mesh
    - » Interpolation of vertices?
    - » Surface within their convex hull?

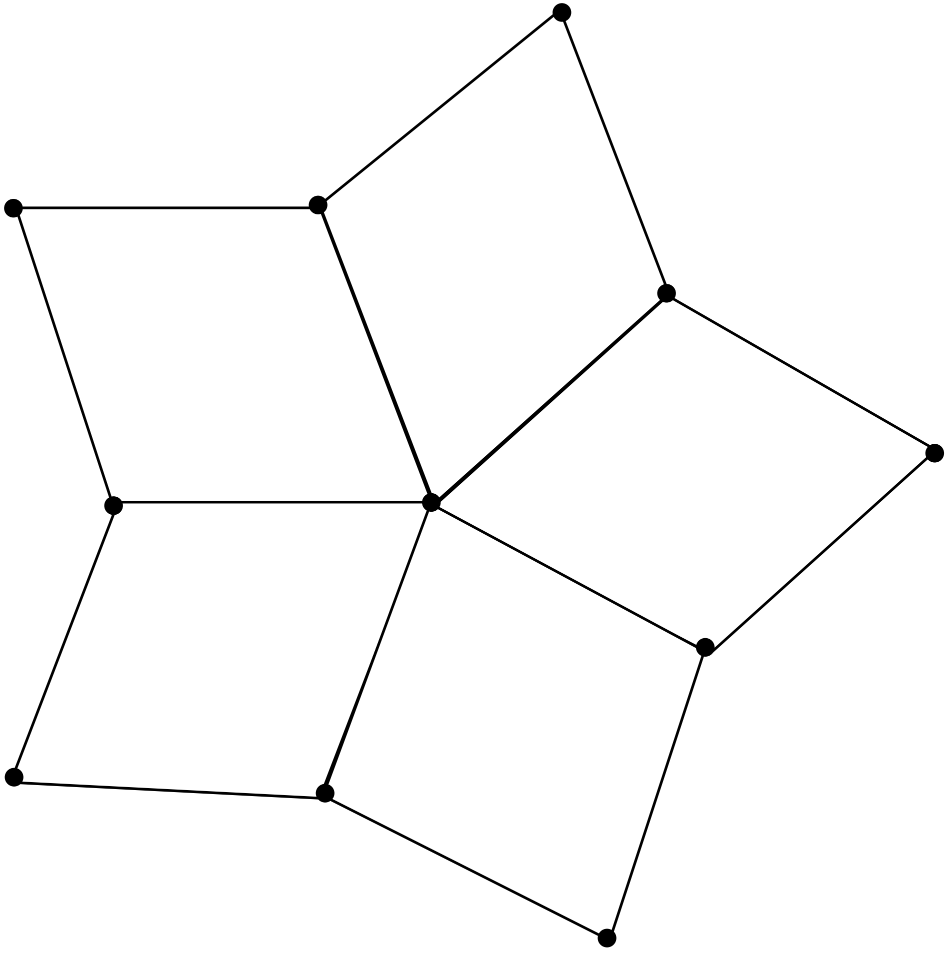


Linear Subdivision

- Type of input
  - Quad mesh -- four-sided polygons (*quads*)
- Topology refinement rule
  - Split every quad into four at midpoints
- Geometry refinement rule
  - Average vertex positions

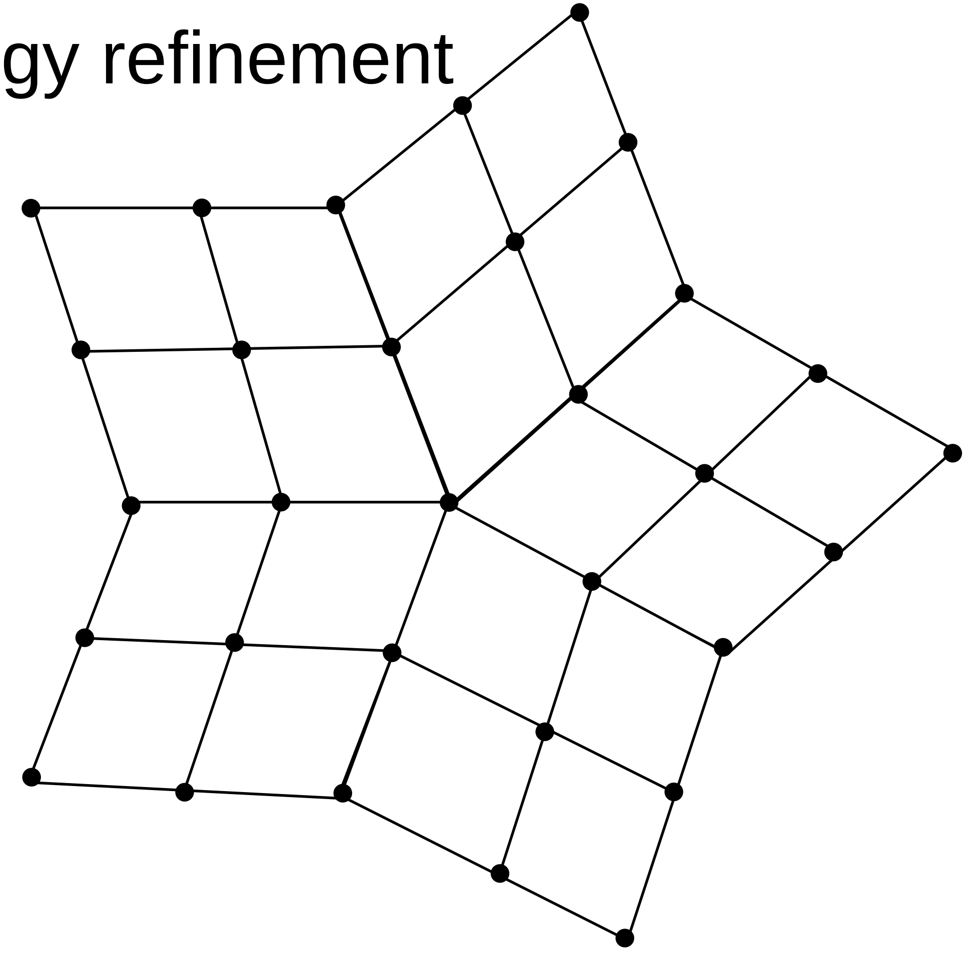
Note: simple example to demonstrate how such schemes work, but not the best scheme…

Linear Subdivision



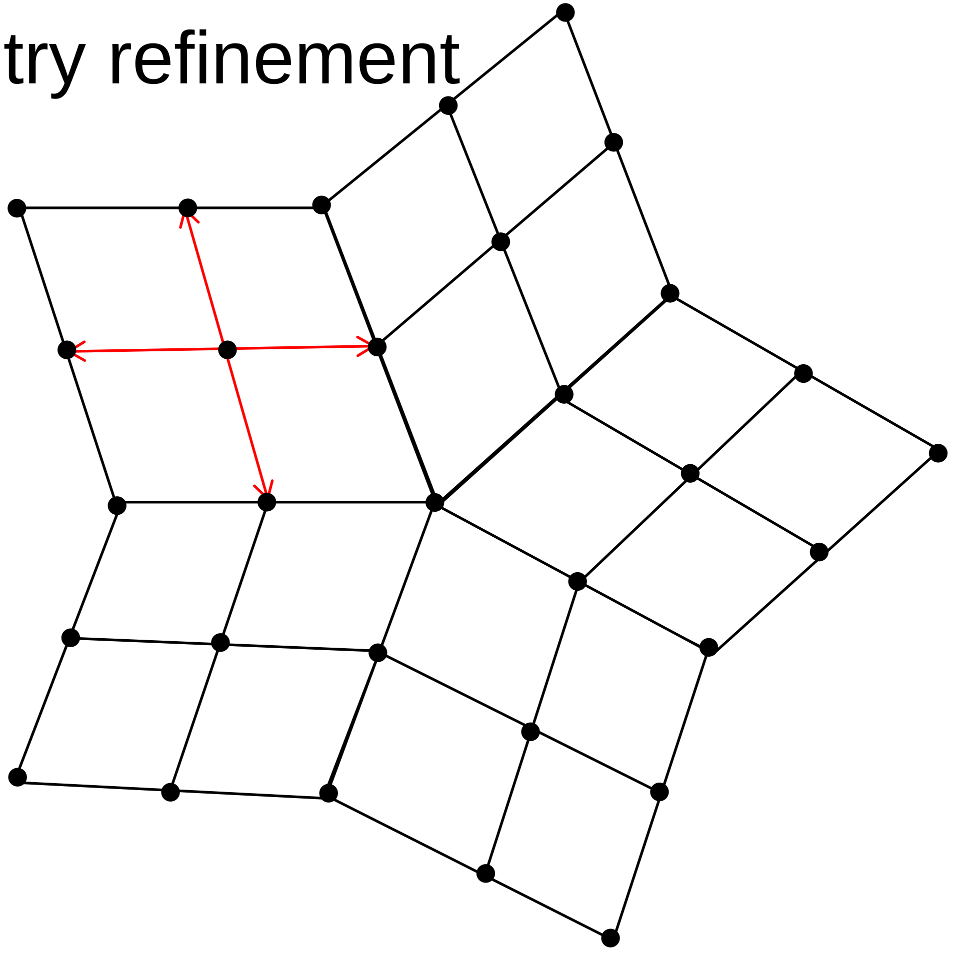
Linear Subdivision

- Topology refinement



Linear Subdivision

- Geometry refinement



Linear Subdivision

LinearSubivision (F_0, V_0, k)
 for i = 1 …k levels
  (F_i, V_i) = RefineTopology(F_{i-1}, V_{i-1})
  RefineGeometry(F_i, V_i)
 return (F_k, V_k)

Linear Subdivision

RefineTopology (F, V)

$newV = V$

$newF = \{\}$

for each face F_i

Insert new vertex c at centroid of F_i into $newV$

return ($newF$, $newV$)

Linear Subdivision

RefineTopology (F, V)

$newV = V$

$newF = \{\}$

for each face F_i

Insert new vertex c at centroid of F_i into $newV$

for j = 1 to 4

Insert in $newV$ new vertex e_j at centroid of each edge ($F_{i,j}$, $F_{i,j+1}$)

return ($newF$, $newV$)

Linear Subdivision

RefineTopology (F, V)

$newV = V$

$newF = \{\}$

for each face F_i

Insert new vertex c at centroid of F_i into $newV$

for j = 1 to 4

Insert in $newV$ new vertex e_j at centroid of each edge ($F_{i,j}$, $F_{i,j+1}$)

for j = 1 to 4

Insert new face ($F_{i,j}$, e_j, c, e_{j-1}) into $newF$

return ($newF$, $newV$)

Linear Subdivision

RefineGeometry(F, V)

$newV = V$

$newF = F$

for each vertex V_i in $newV$

$weight = 0;$

$newV[i] = (0,0,0)$

return ($newF$, $newV$)

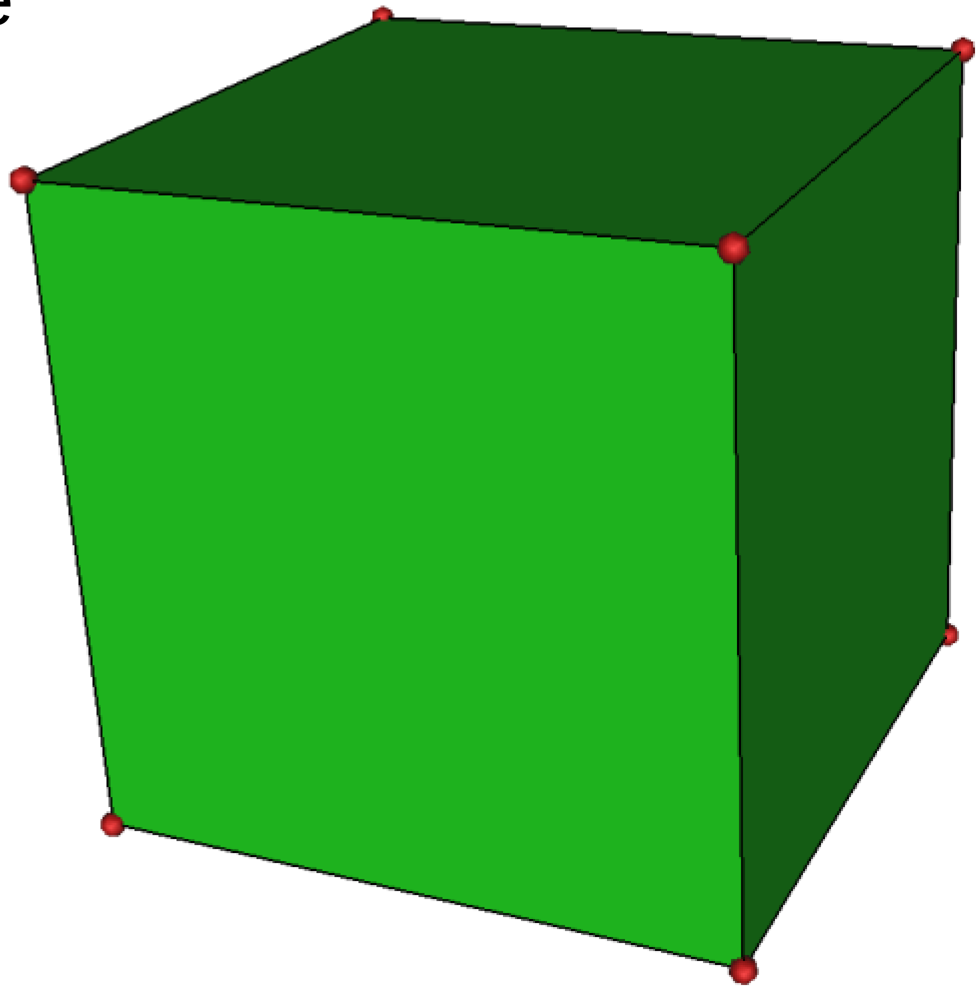
Linear Subdivision

RefineGeometry(F, V)

```
newV = V
newF = F
for each vertex V_i in newV
    weight = 0;
    newV[i] = (0,0,0)
    for each face F_j connected to V_i
        newV[i] += centroid of F_j
        weight += 1.0;
    newV[i] /= weight
return (newF, newV)
```

Linear Subdivision

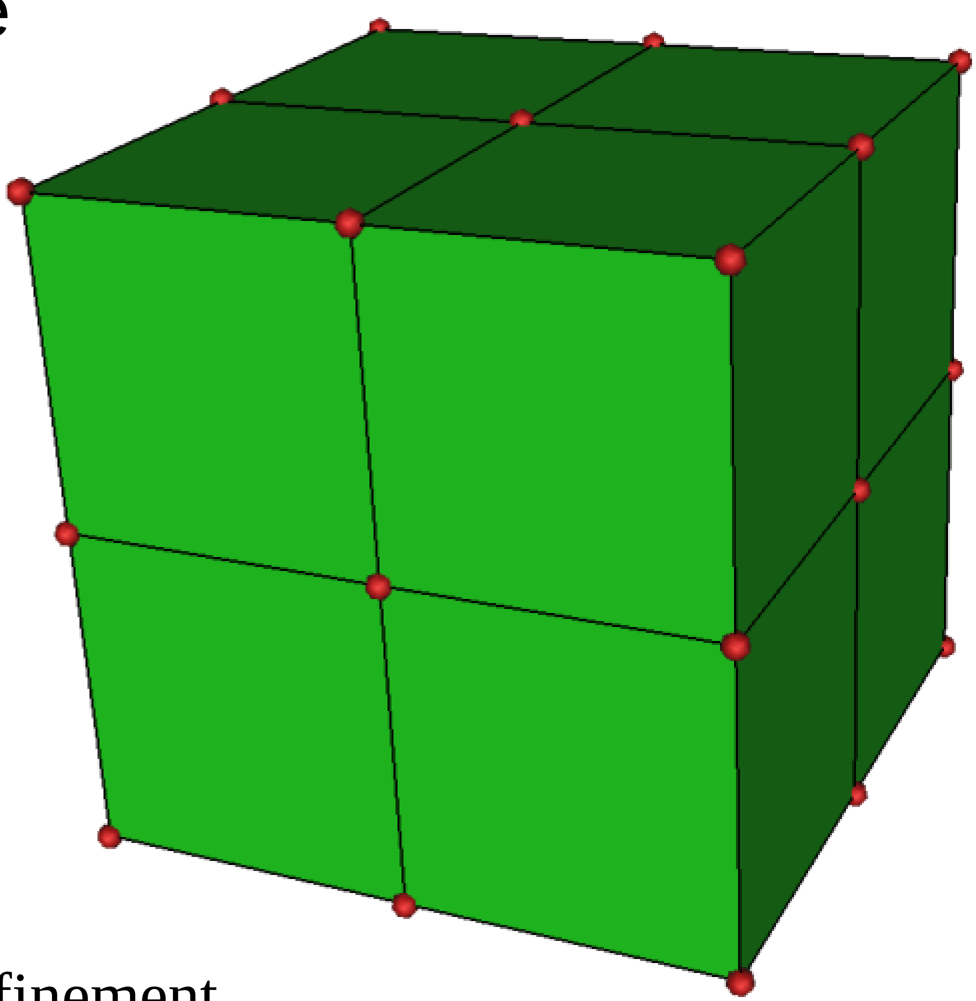
- Example



Input mesh

Linear Subdivision

- Example



Topology refinement

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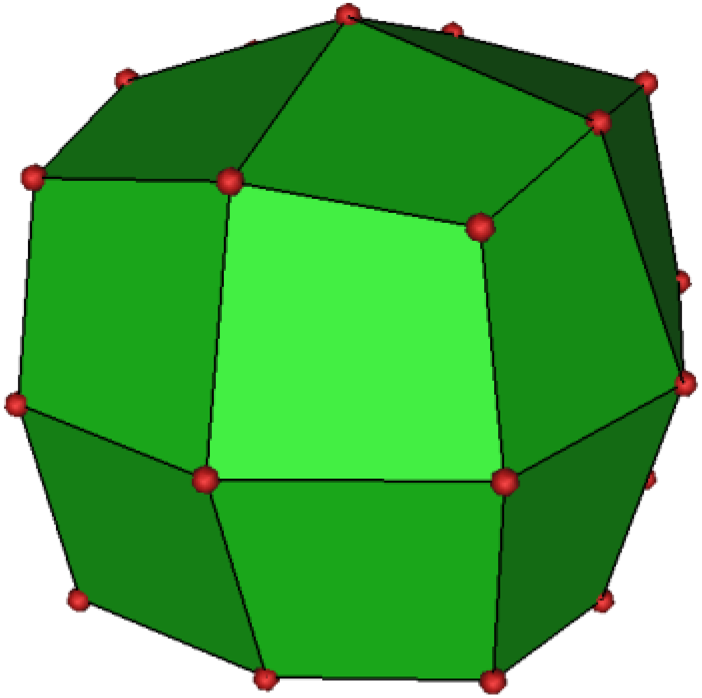
Linear Subdivision

- Example



Geometry refinement

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Linear Subdivision

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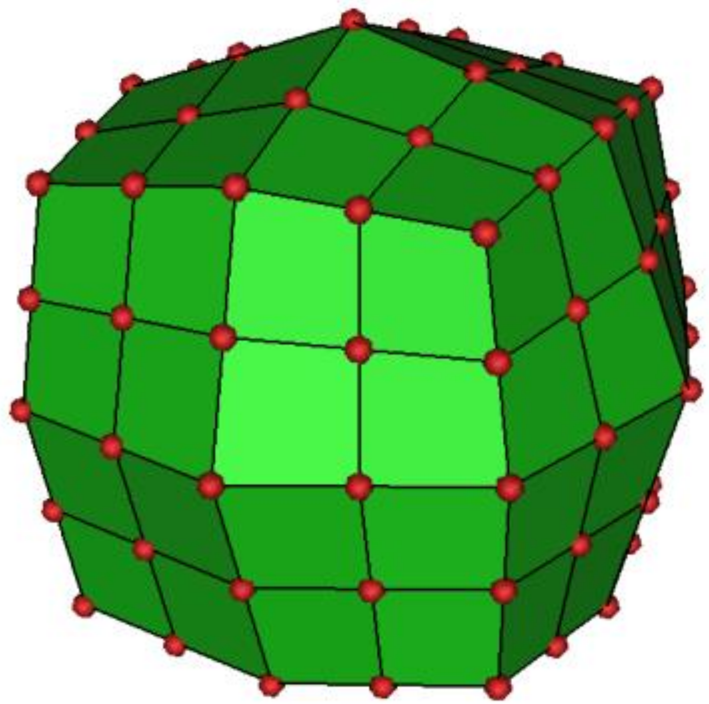

Topology refinement

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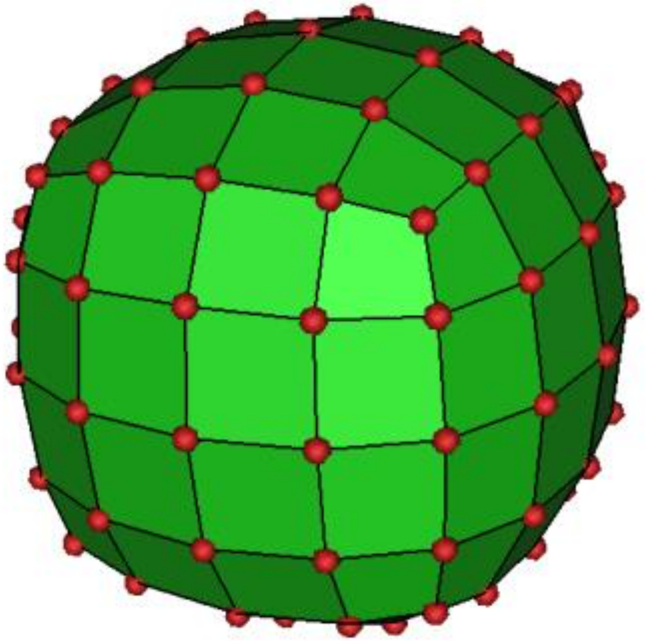
Linear Subdivision

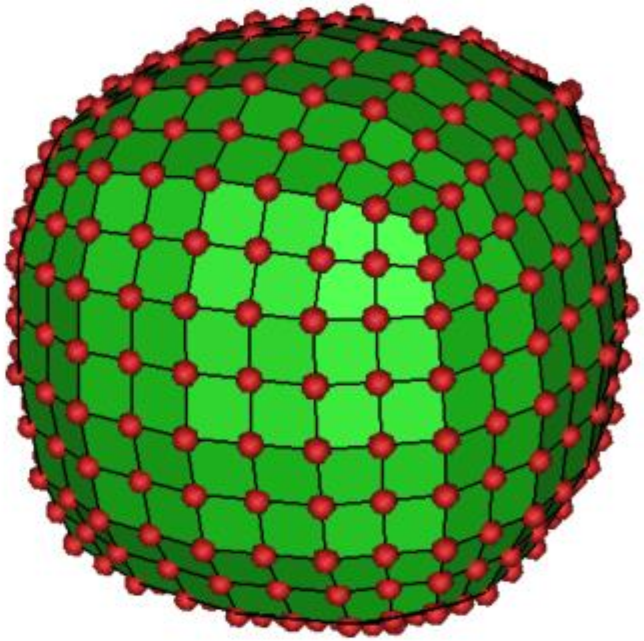
- Example



Geometry refinement

Linear Subdivision

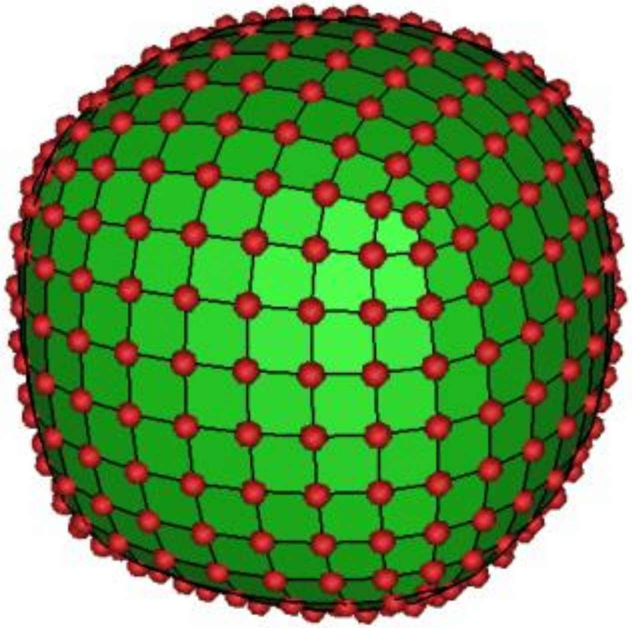
- Example



Topology refinement

Linear Subdivision

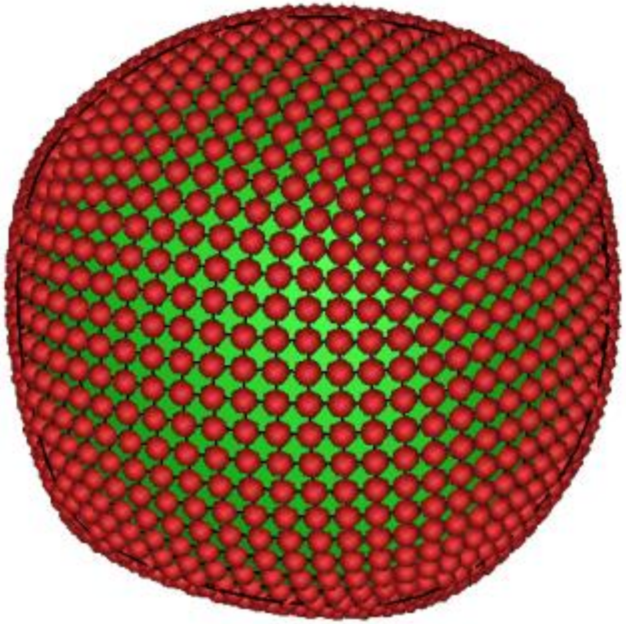
- Example



Geometry refinement

Linear Subdivision

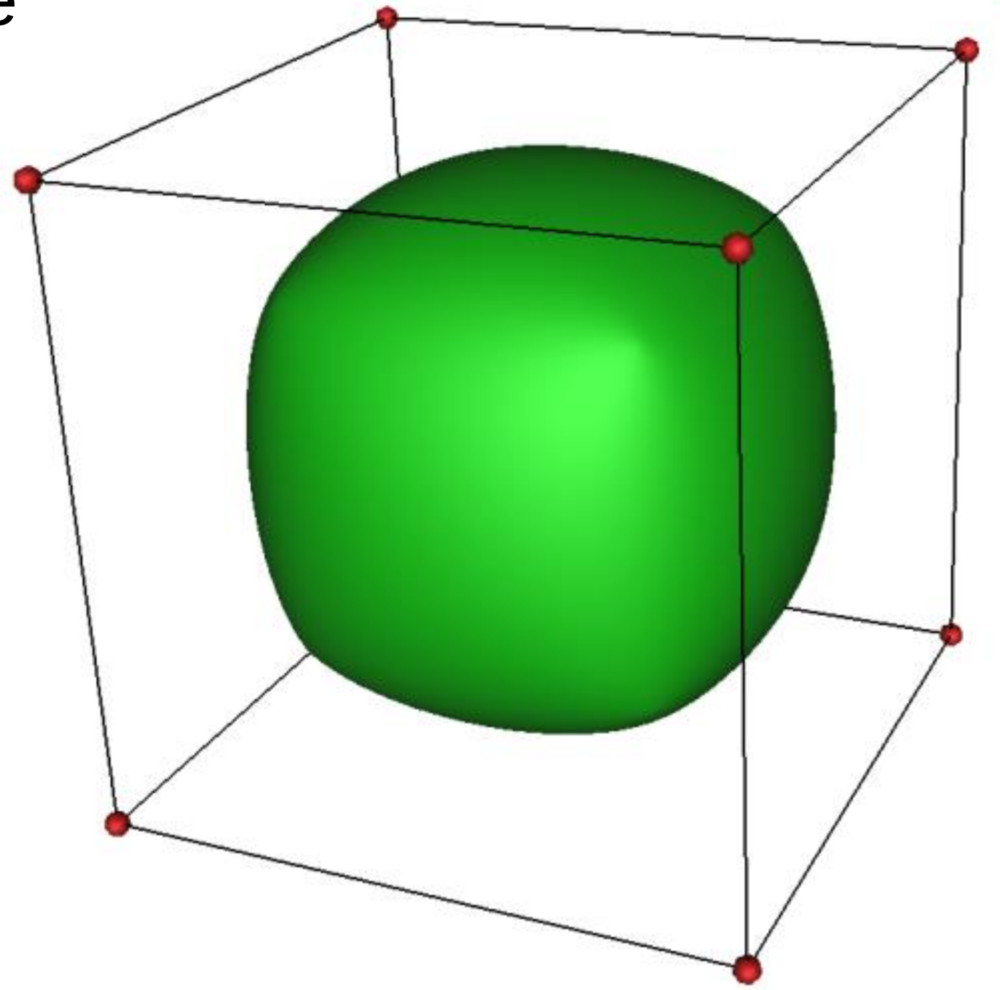
- Example



Topology refinement

Linear Subdivision

- Example



Final result

Subdivision Demo

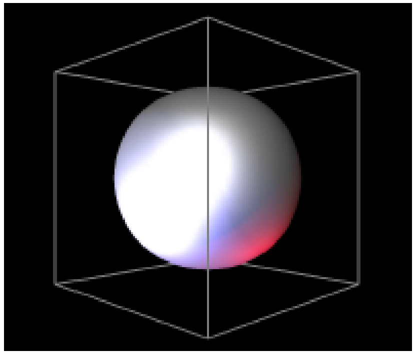
https://threejs.org/examples/webgl_modifier_subdivision.html

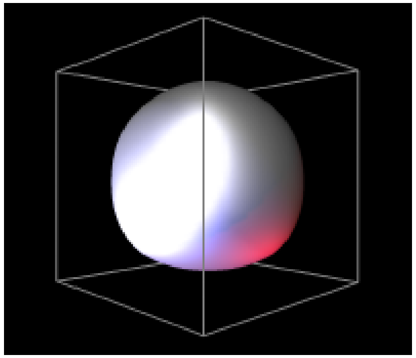
Subdivision Schemes

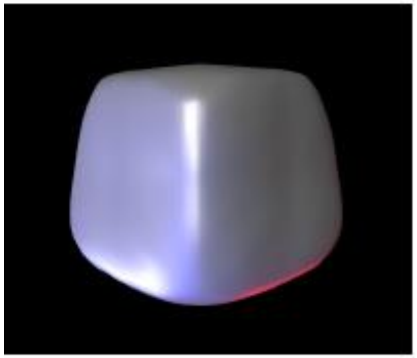
- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others
- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

… which makes differences in …

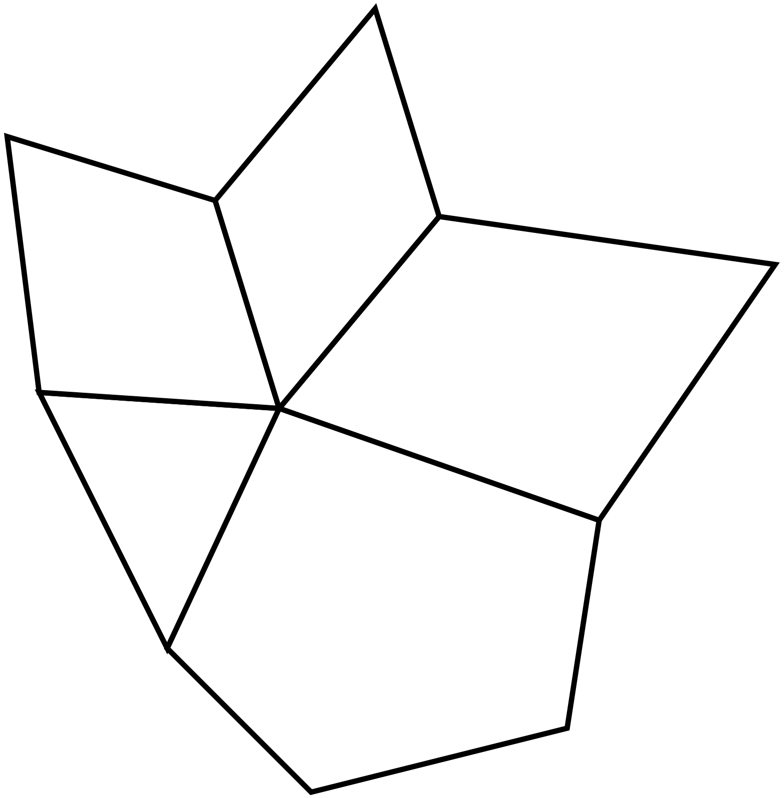
  - Provable properties







Catmull-Clark Subdivision



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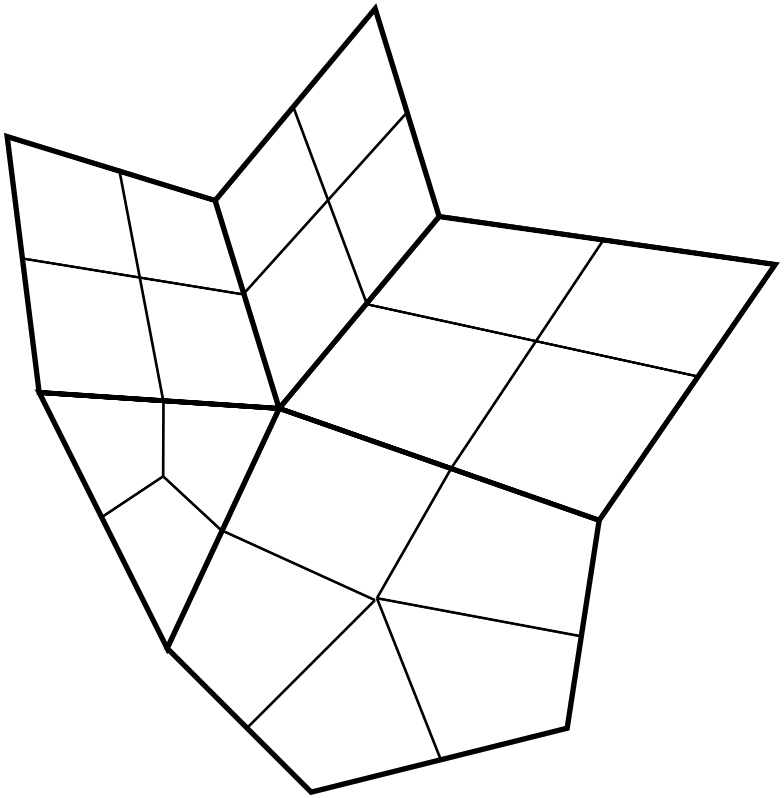

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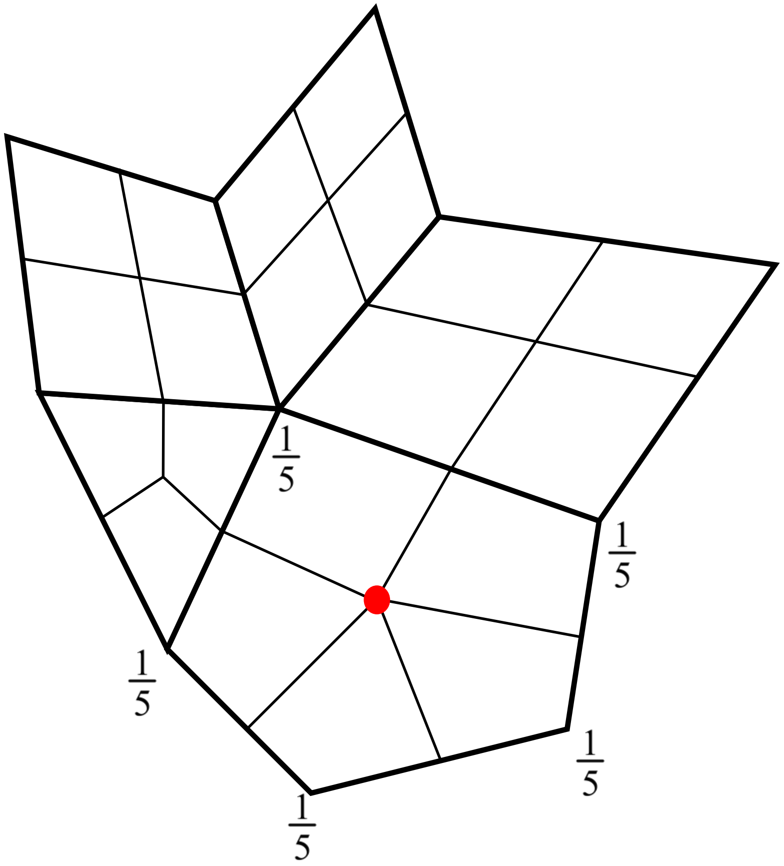

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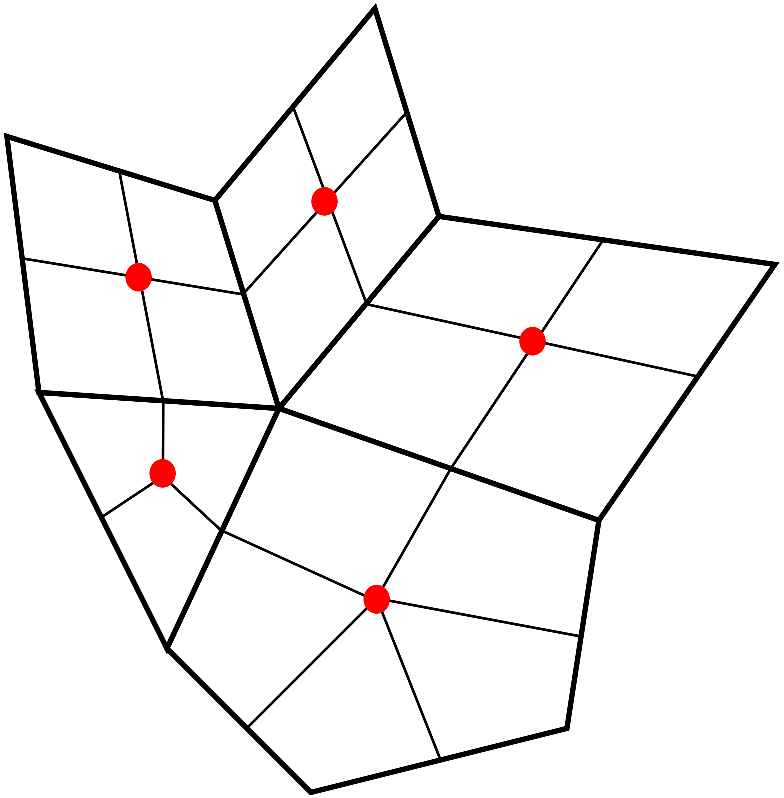

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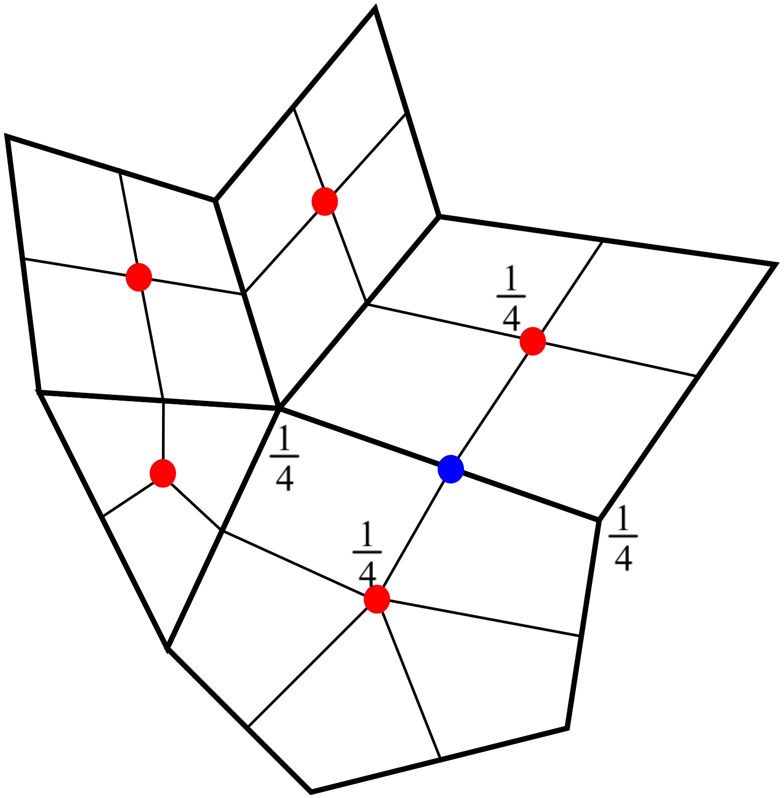
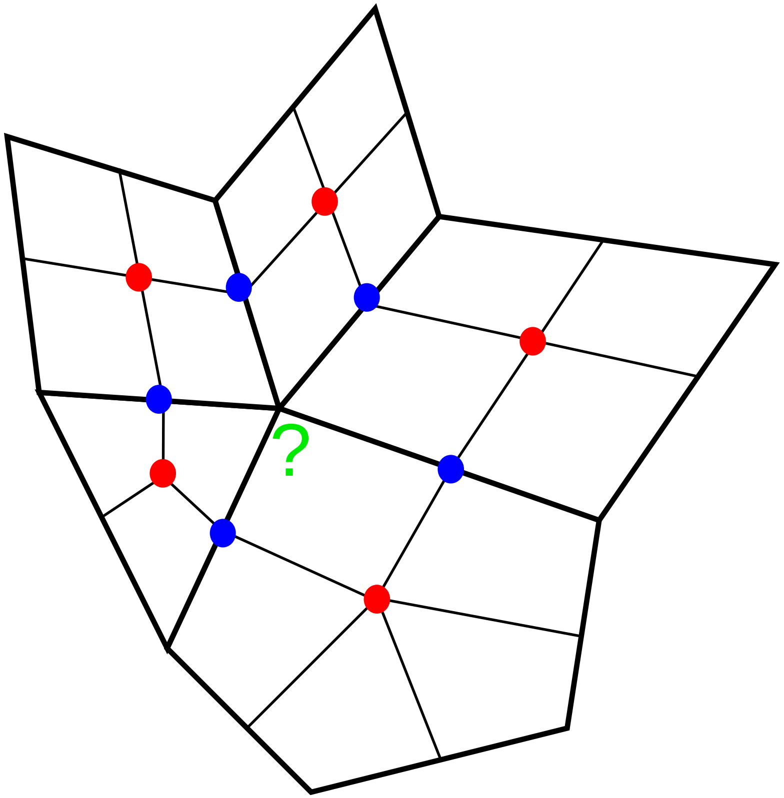
Catmull-Clark Subdivision

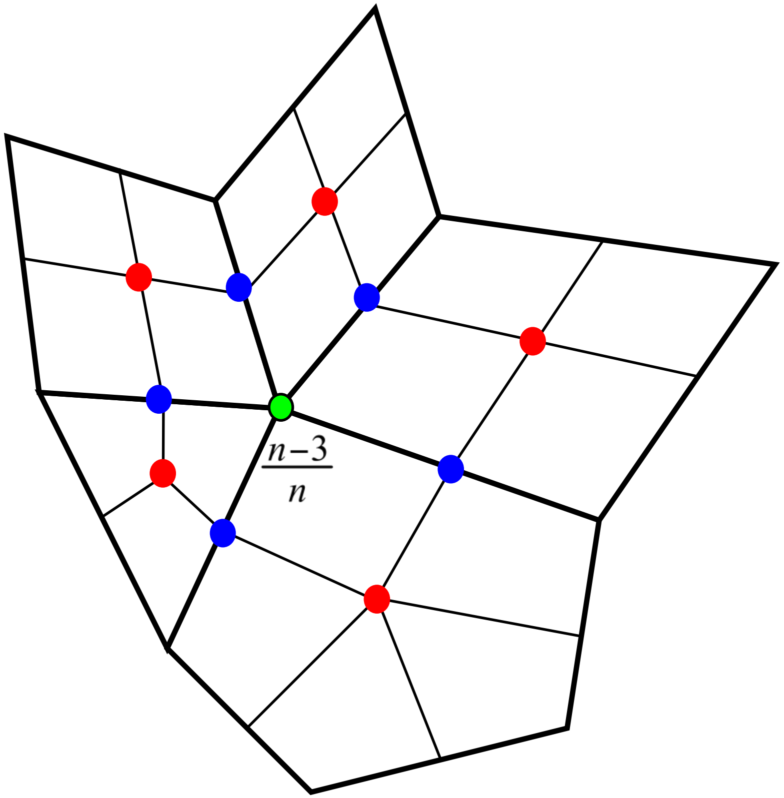


Catmull-Clark Subdivision



Catmull-Clark Subdivision

New ● = (● – ● + (n-3) * ●) / n



n = #faces a point belongs to.

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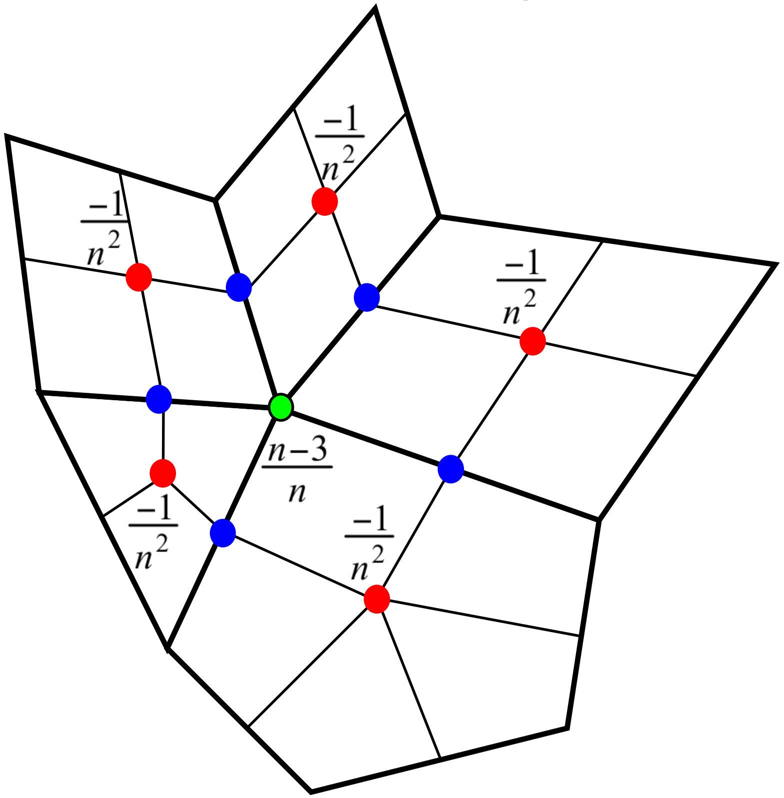
Catmull-Clark Subdivision

New (green point) = ((blue points) –1 * avg of (red points) + (n-3) * (green point)) / n



n = #faces a point belongs to.

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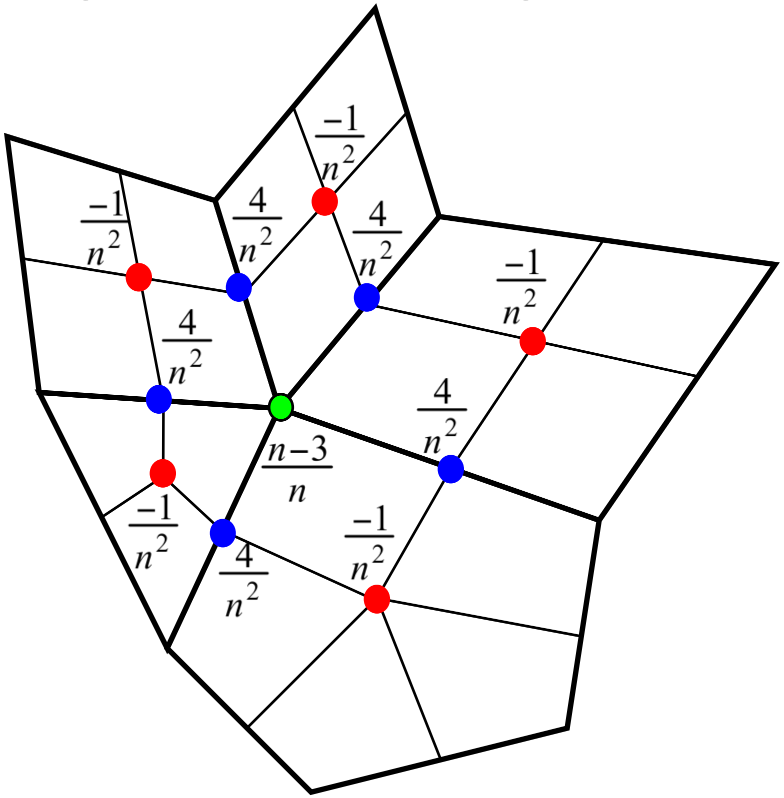
Catmull-Clark Subdivision

New ● = (4 * avg of ● –1 * avg of ● + (n-3) * ●) / n



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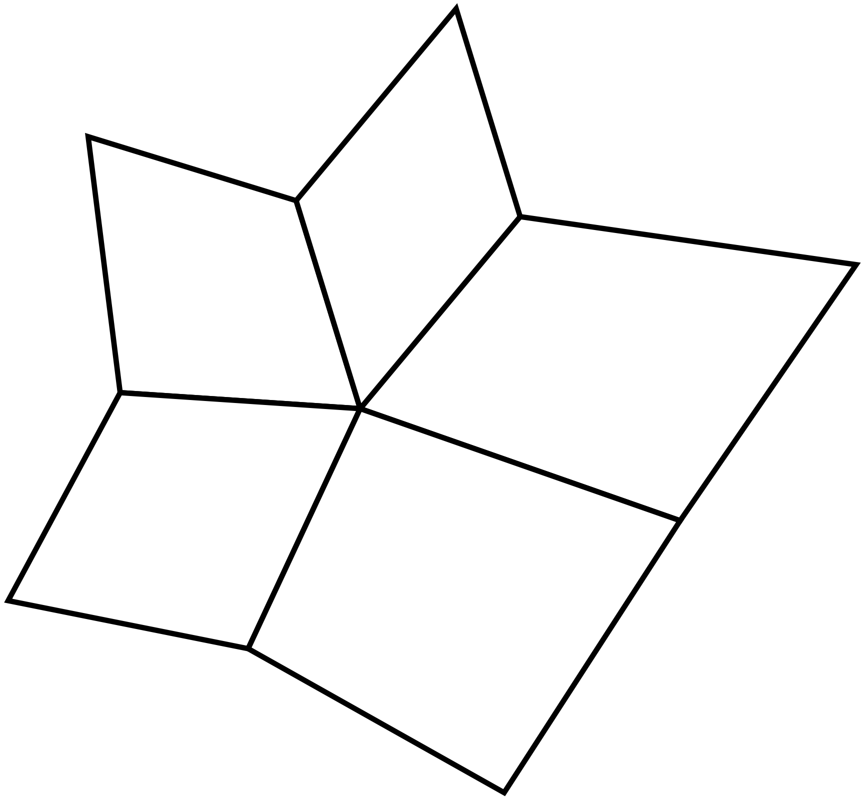
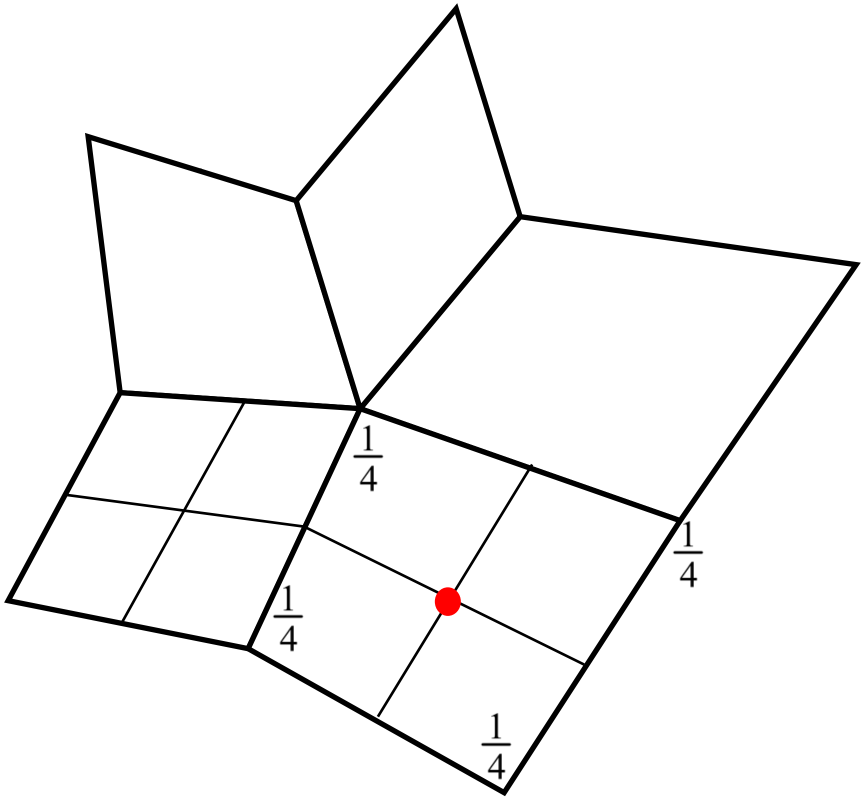
Catmull-Clark Subdivision



Catmull-Clark Subdivision



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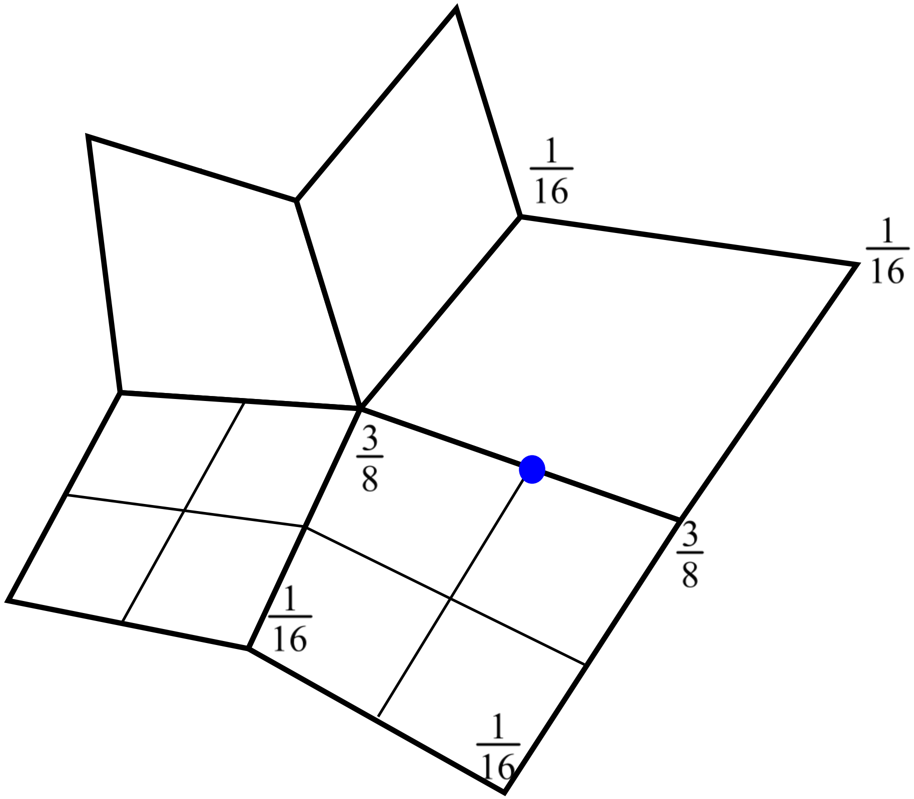

Catmull-Clark Subdivision

Linear
Subdivision

Catmull-Clark
Subdivision

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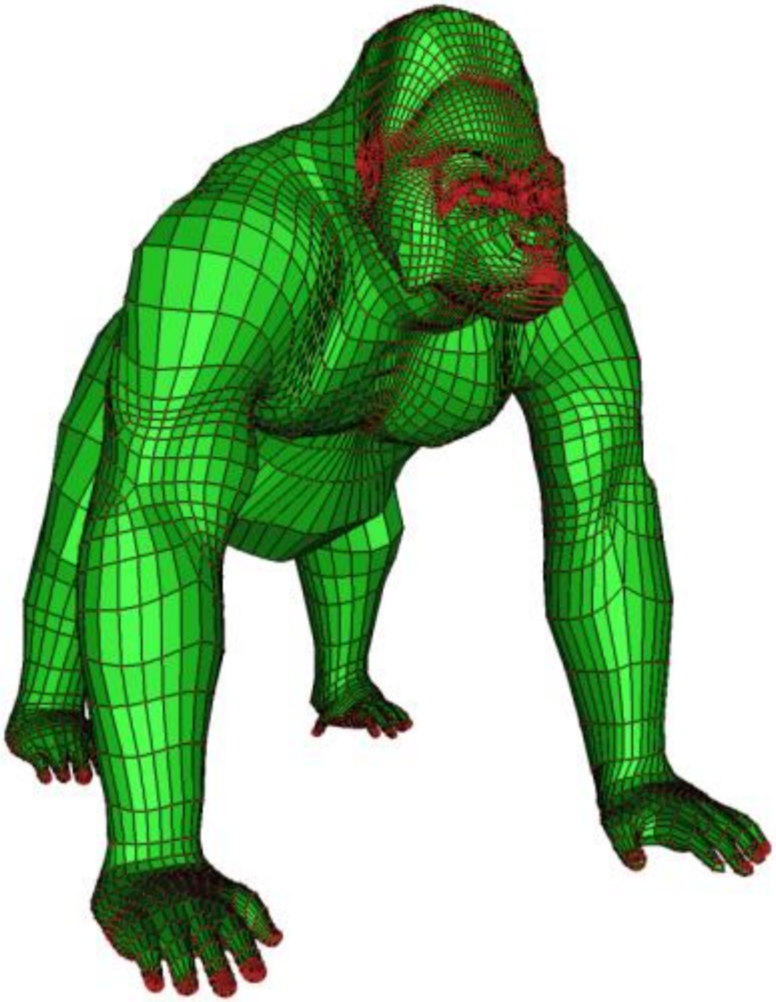
Catmull-Clark Subdivision



Catmull-Clark Subdivision

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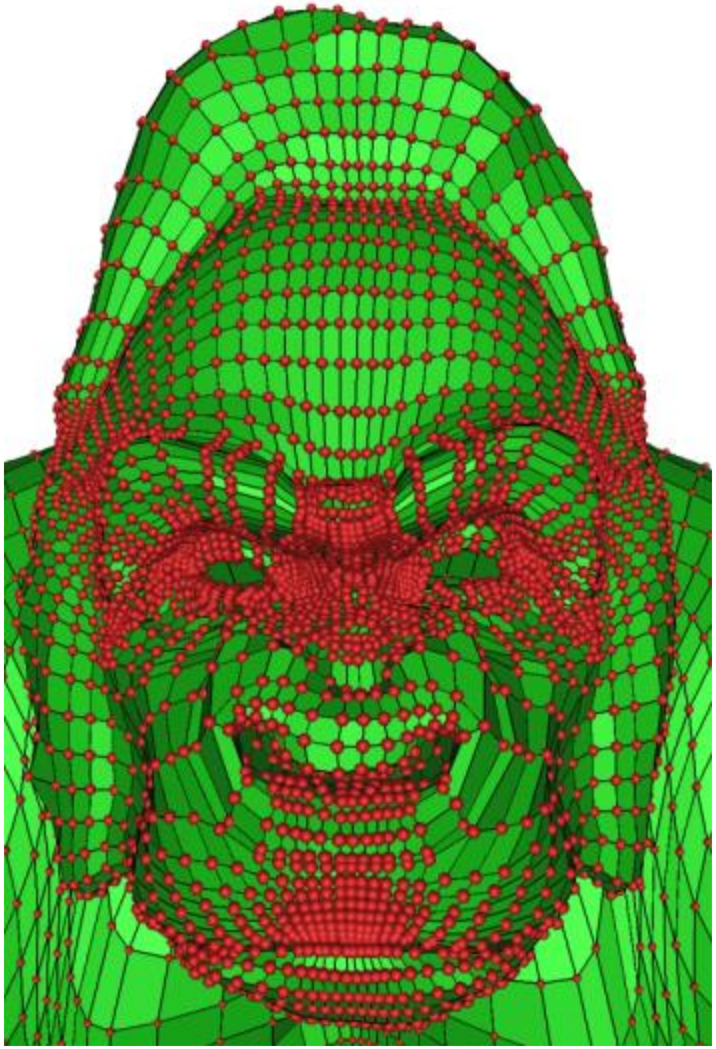
Catmull-Clark Subdivision



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Catmull-Clark Subdivision

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Catmull-Clark Subdivision

- One round of subdivision produces all quads
- Smoothness of limit surface
  - C^2 almost everywhere
  - C^1 at vertices with valence $\neq$ 4
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull
- Most commonly used subdivision scheme in the movies…

Pixar

Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - ➢ Loop
  - Many others
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  - Input topology
  - How refine topology
  - How refine geometry

… which makes differences in …

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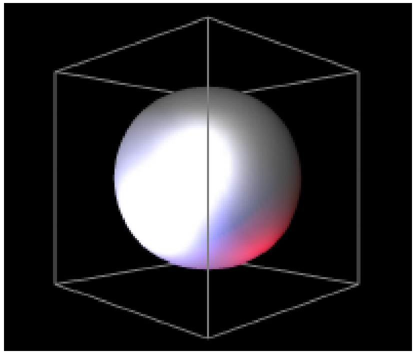
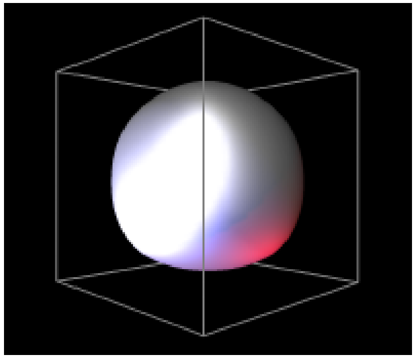
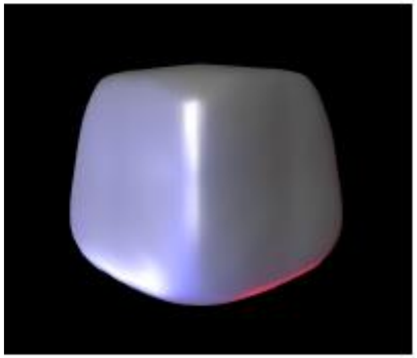
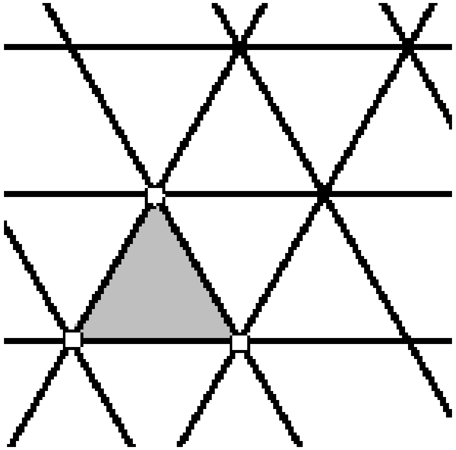






Loop Subdivision

- Operates on pure triangle meshes
- Subdivision rules
  - Linear subdivision
  - Averaging rules for “even / odd” (white / black) vertices



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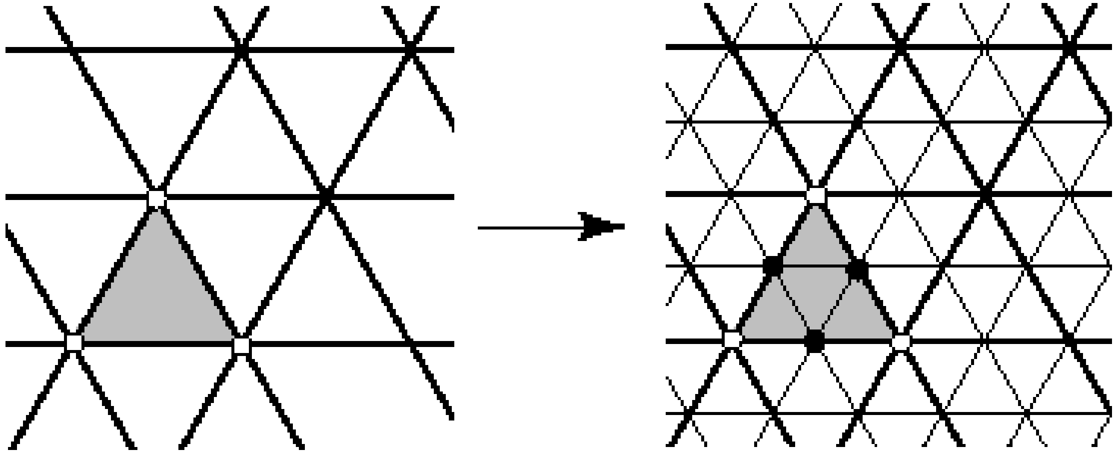
  - Linear subdivision
  - Averaging rules for “even / odd” (white / black) vertices



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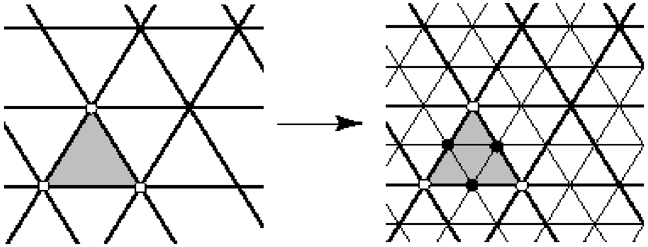
Course Notes

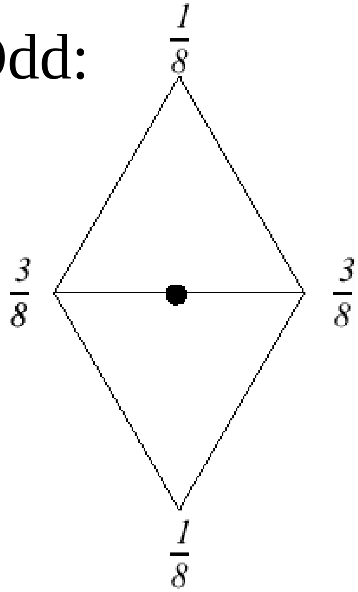
Loop Subdivision

Averaging rules

- Weights for “odd” and “even” vertices



Odd:



… but what about vertices with valence ≠ 6 ?

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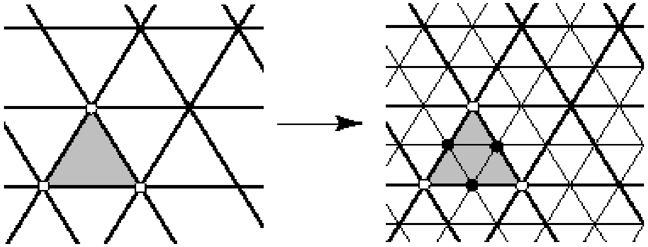
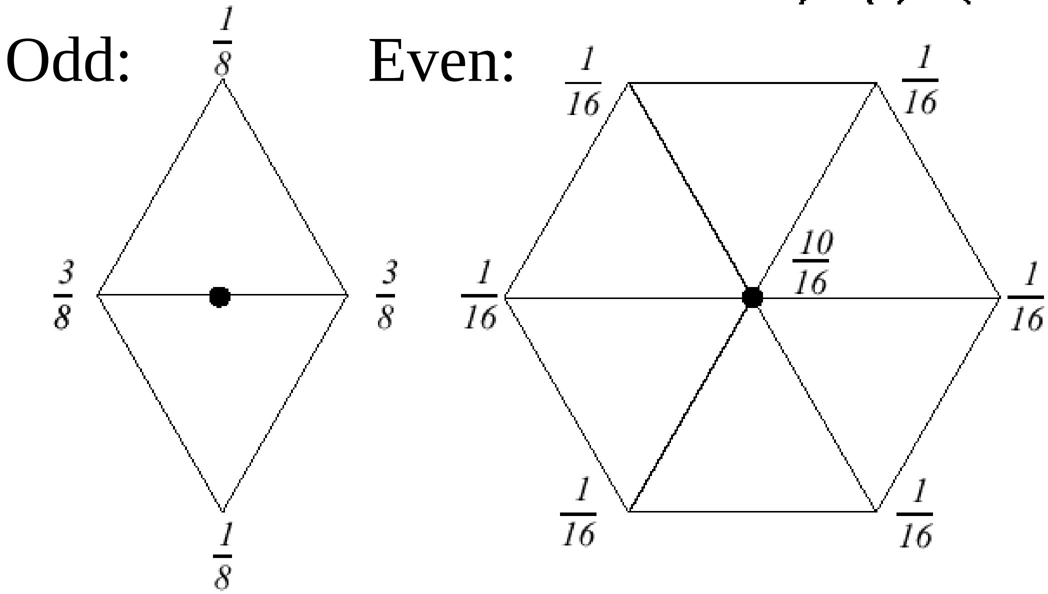
Loop Subdivision

Averaging rules

- Weights for “odd” and “even” vertices





… but what about vertices with valence ≠ 6 ?

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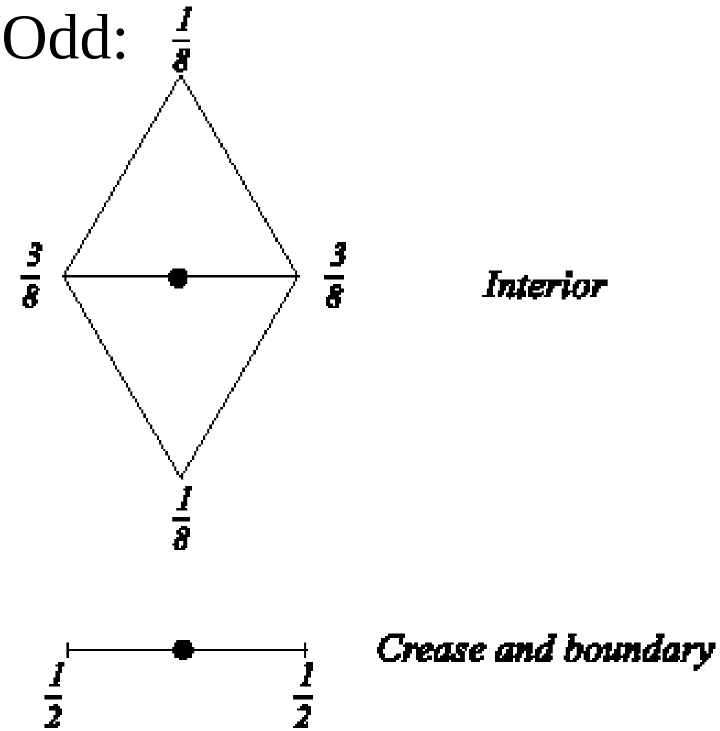
Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:

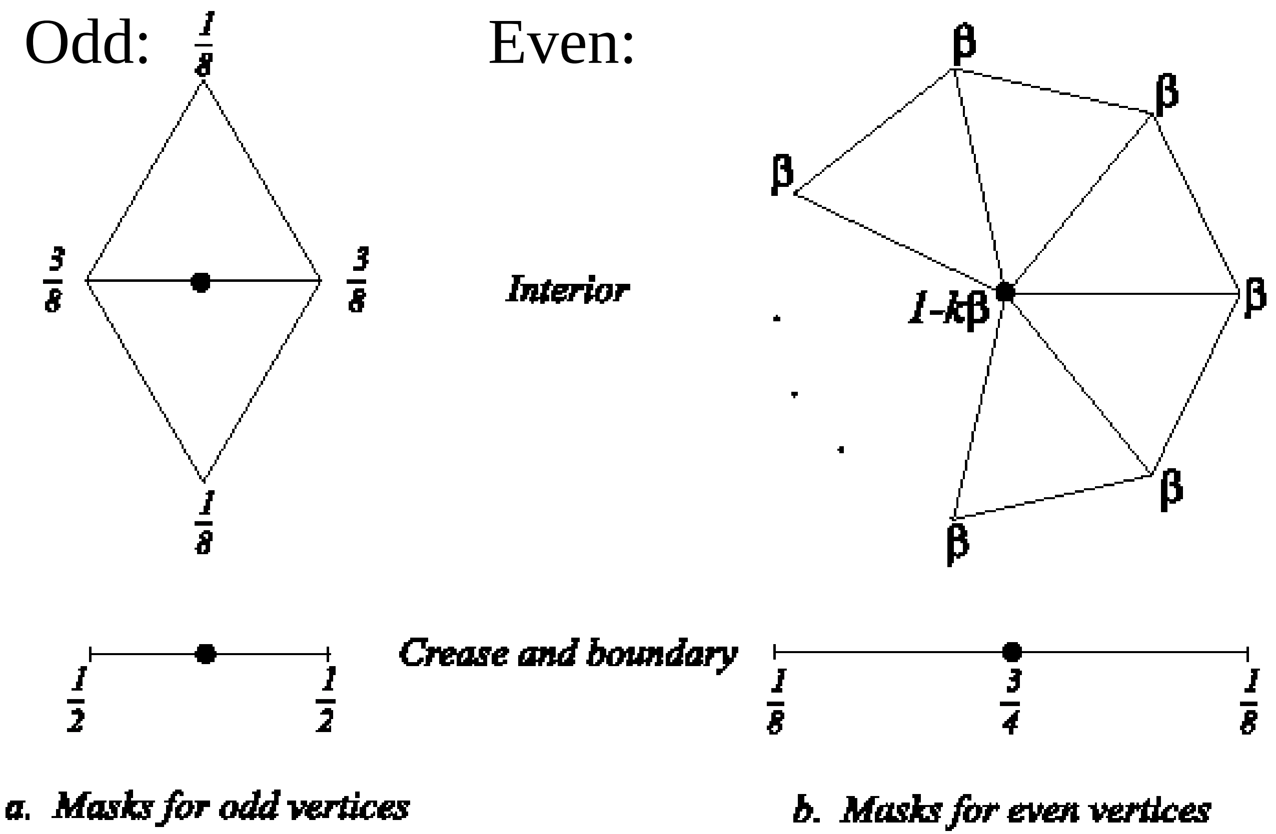
Odd:



a. Masks for odd vertices

Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:



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Loop Subdivision

- How to choose β?
  - Analyze properties of limit surface
  - Interested in continuity of surface and smoothness

» Original Loop

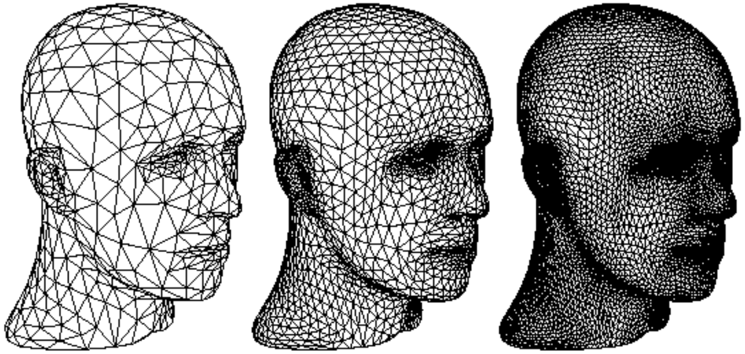
$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

» Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

Loop Subdivision

- Operates only on triangle meshes
- Smoothness of limit surface
  - C^2 almost everywhere
  - C^1 at vertices with valence $\neq 6$
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull



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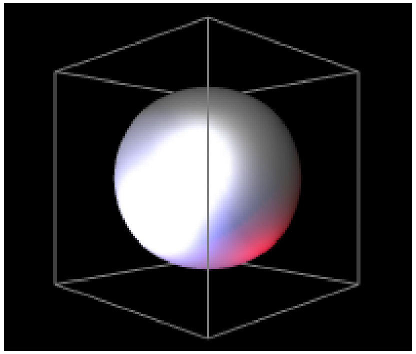
Subdivision Schemes

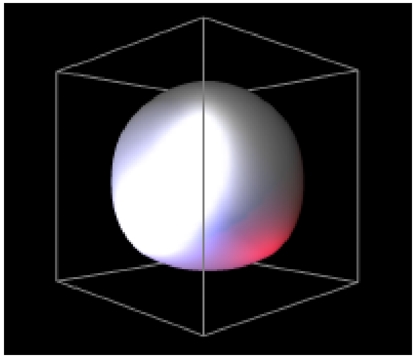
- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others

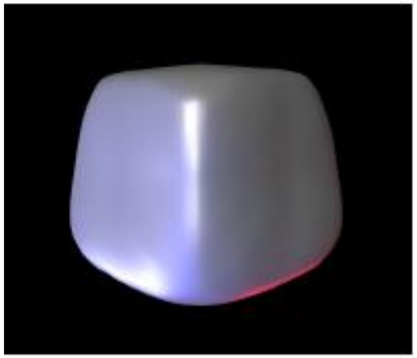
- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

… which makes differences in …

  - Provable properties

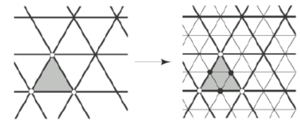
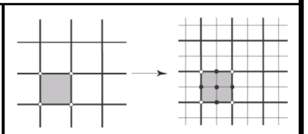


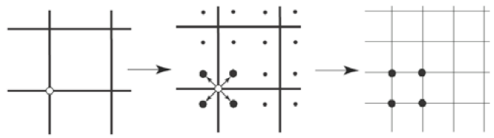




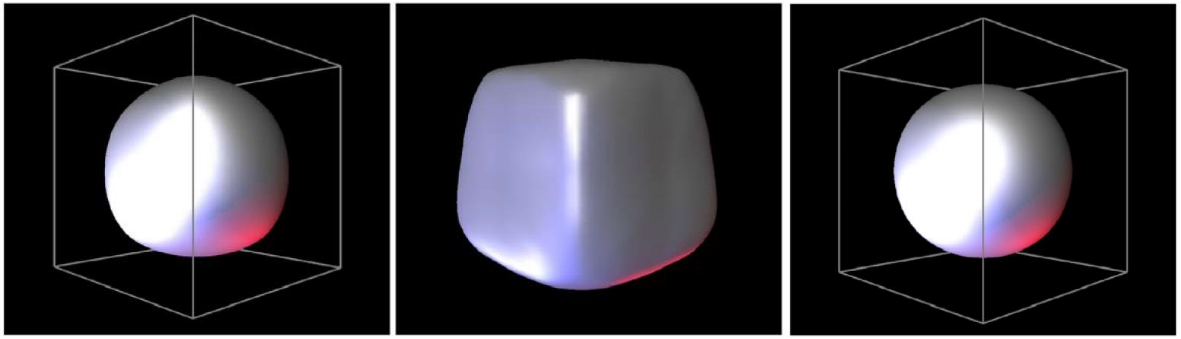
Subdivision Zoo

- Other subdivision schemes

Primal (face split)		
	 *Triangular meshes*	 *Quad Meshes*
Approximating	Loop(C^2)	Catmull-Clark(C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)



Dual (vertex split)
Doo-Sabin, Midedge(C^1)
Biquartic (C^2)



Loop *Butterfly* *Catmull-Clark*

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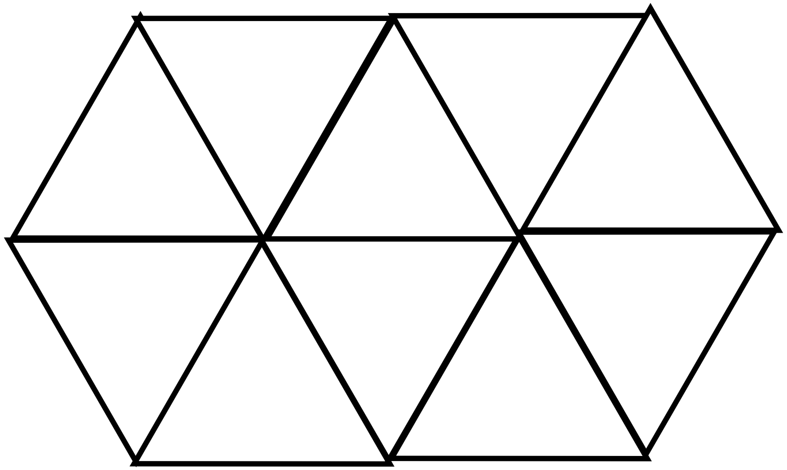
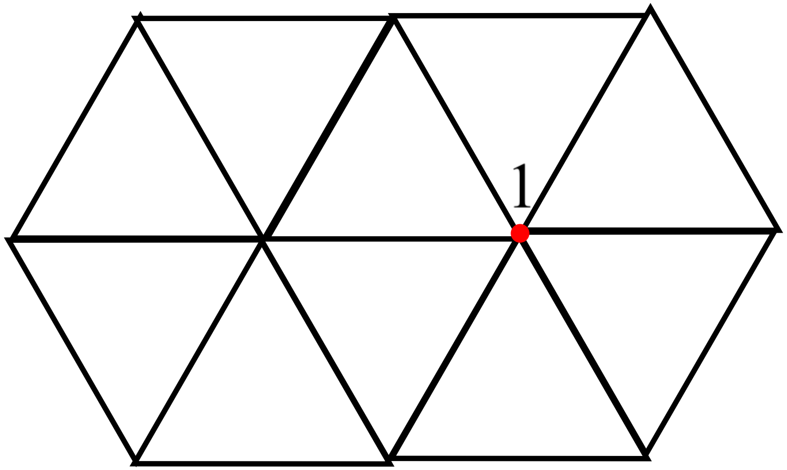
Other Subdivision Schemes

- Butterfly subdivision



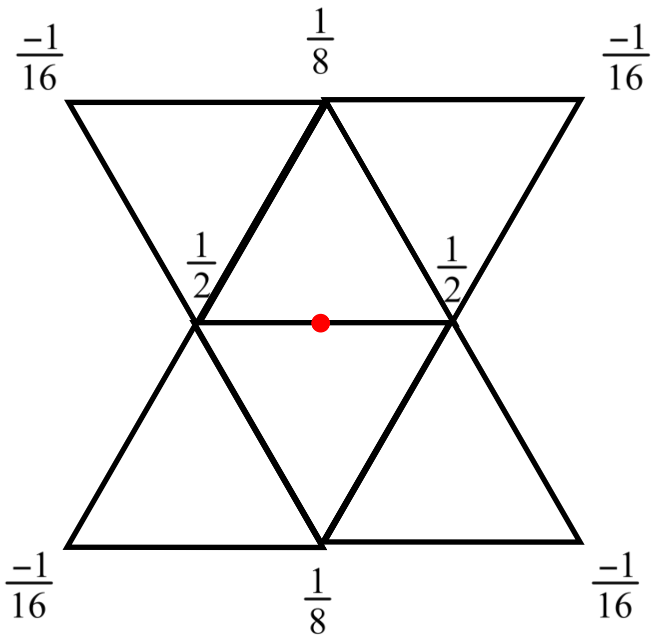
Other Subdivision Schemes

- Butterfly subdivision



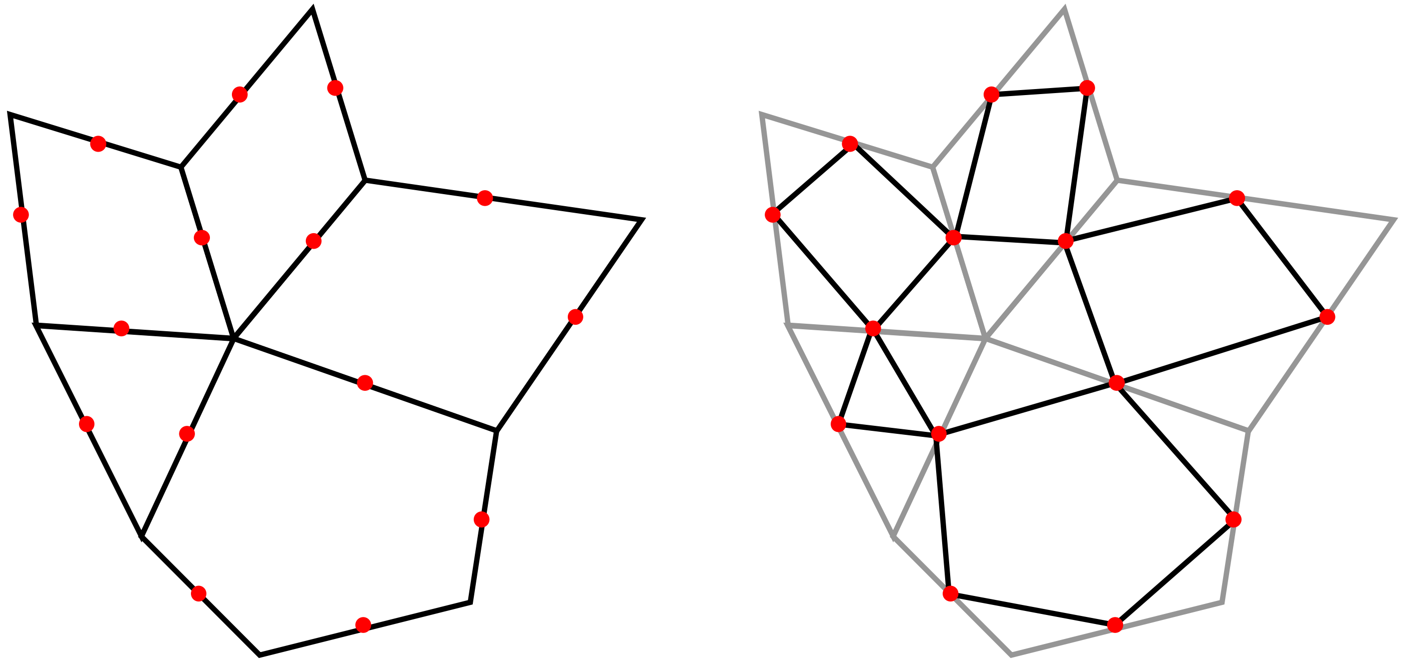
Other Subdivision Schemes

- Butterfly subdivision



Other Subdivision Schemes

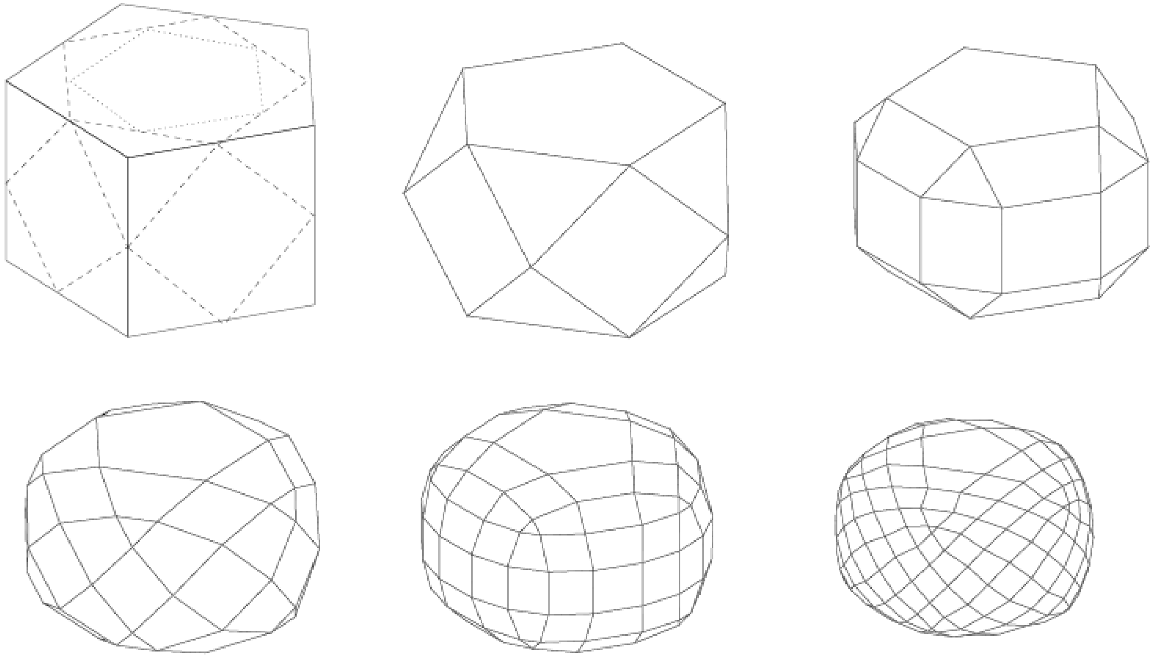
- Vertex-split subdivision
  (Doo-Sabin, Midedge, Biquartic)



One step of Midegde subdivision

Other Subdivision Schemes

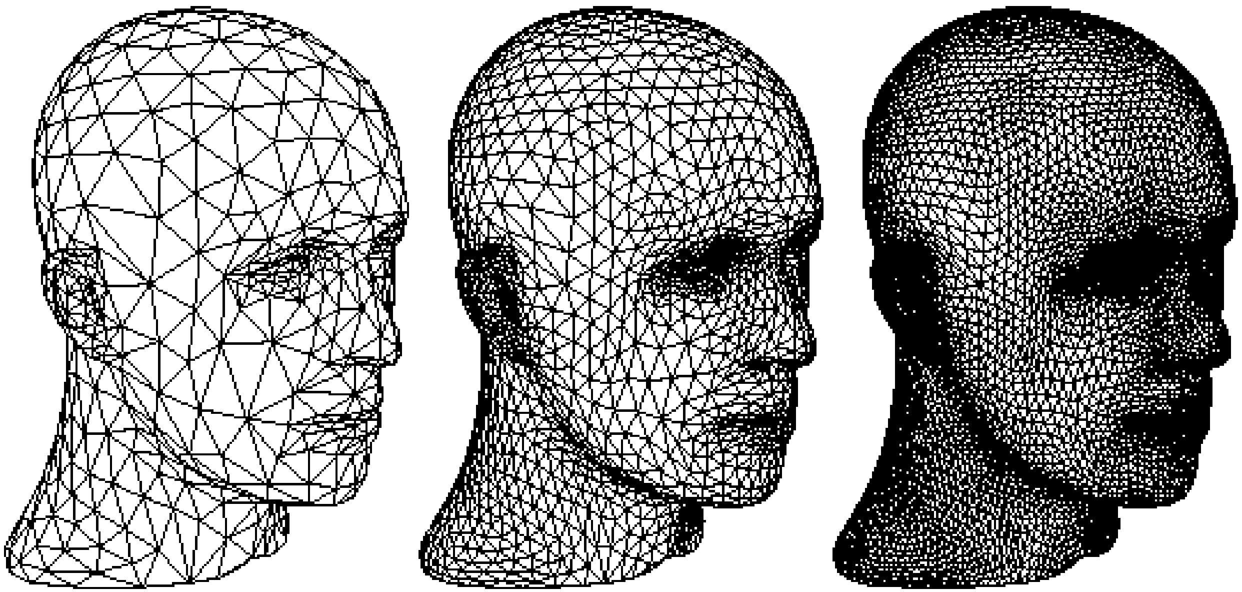
- Vertex-split subdivision
  (Doo-Sabin, Midedge, Biquartic)



Multiple steps of Midedge subdivision

Drawing Subdivision Surfaces

- Goal:
  - Draw best approximation of smooth limit surface
  - **With limited triangle budget**



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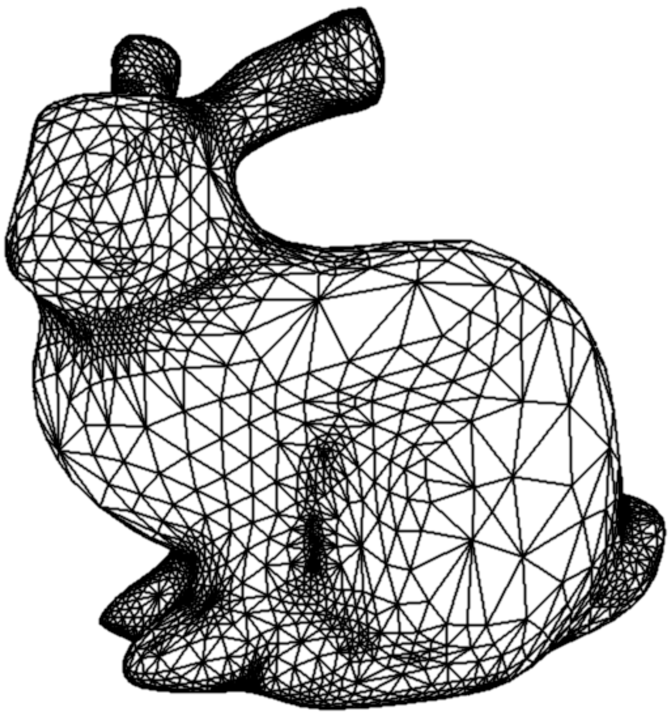
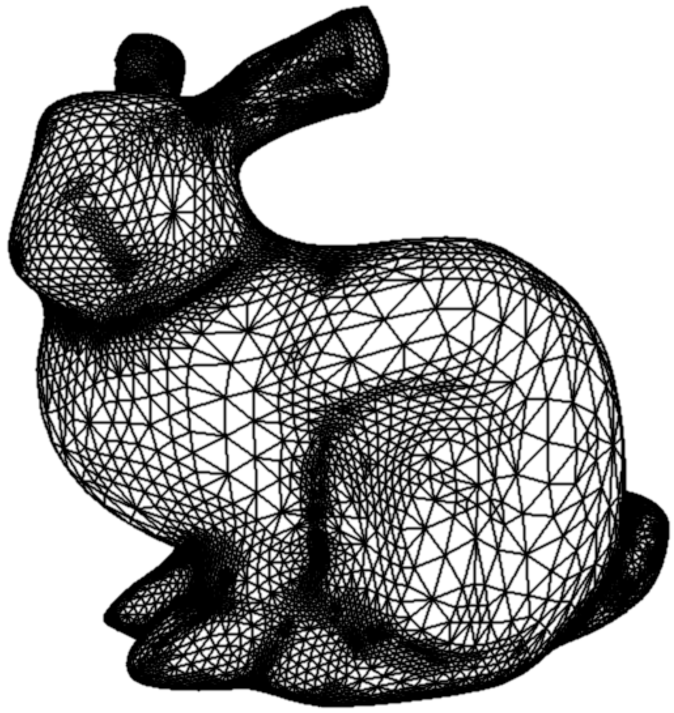
Drawing Subdivision Surfaces

- Goal:
  - Draw best approximation of smooth limit surface
  - With limited triangle budget
- Solution:
  - Stop subdivision at different levels across the surface
  - Stop-criterion depending on quality measure
- Quality of approximation can be defined by
  - Projected (screen) area of final triangles
  - Local surface curvature

Adaptive Subdivision



10072 Triangles

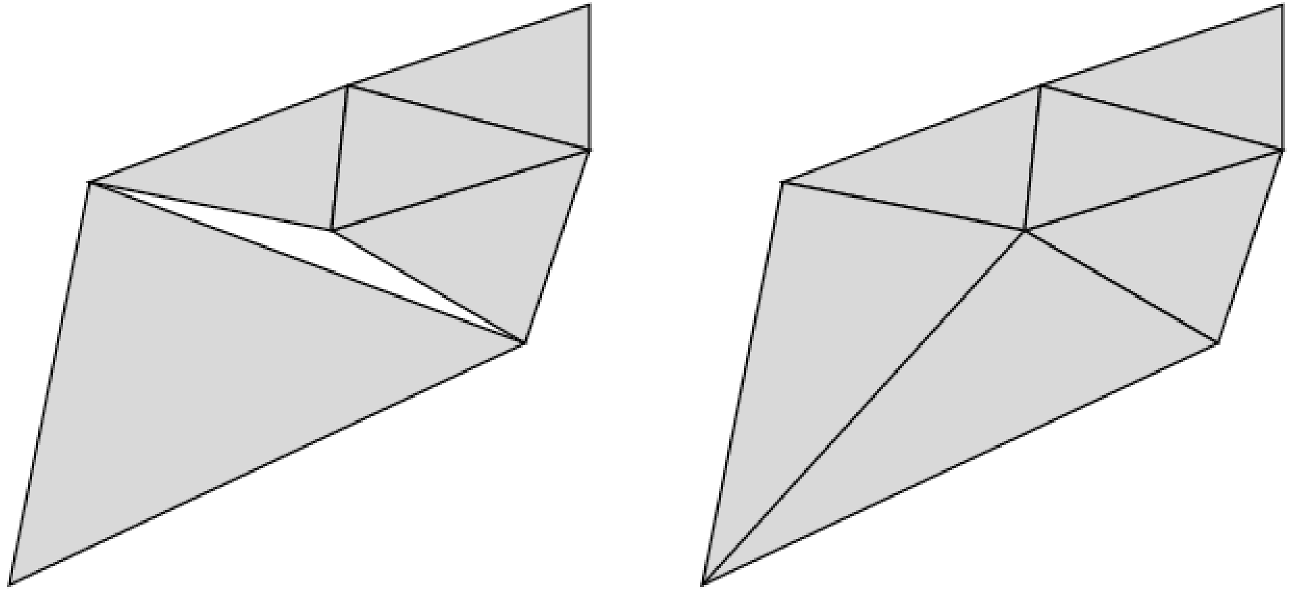


228654 Triangles

[Kobbelt 2000]

Adaptive Subdivision

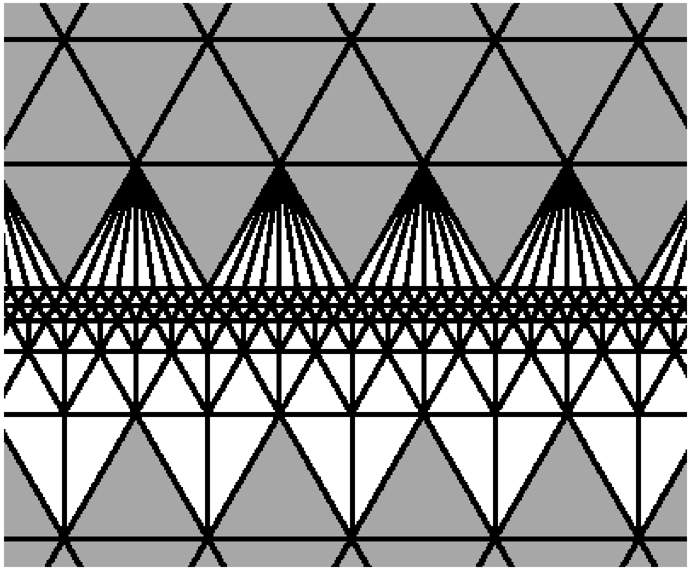
- Problem:
  - Different levels of subdivision may lead to gaps in the surface



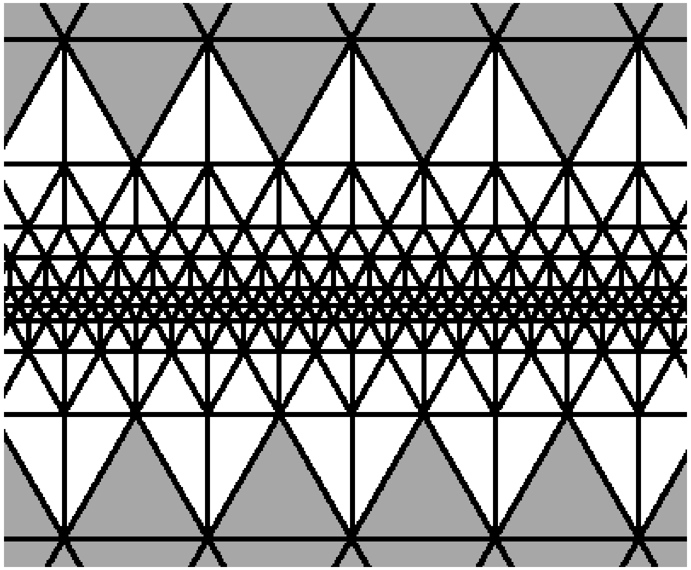
[Kobbelt 2000]

Adaptive Subdivision

- Solution:
  - Replacing incompatible coarse triangles by *triangle fan*
  - Balanced subdivision: neighboring subdivision levels must not differ by more than one



Unbalanced

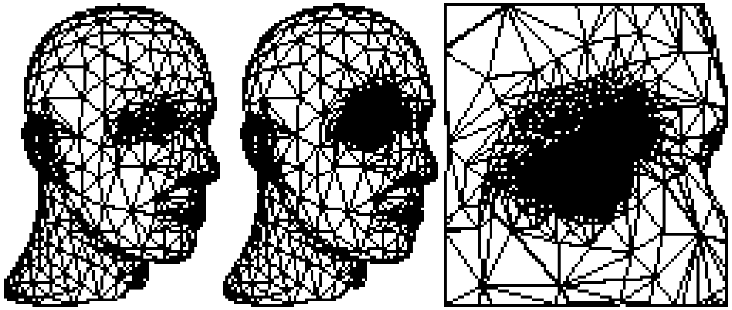


Balanced

[Kobbelt 2000]

Subdivision Surface Summary

- Advantages:
  - Simple method for describing complex surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Intuitive specification
  - Local support
  - Guaranteed continuity
  - Multiresolution

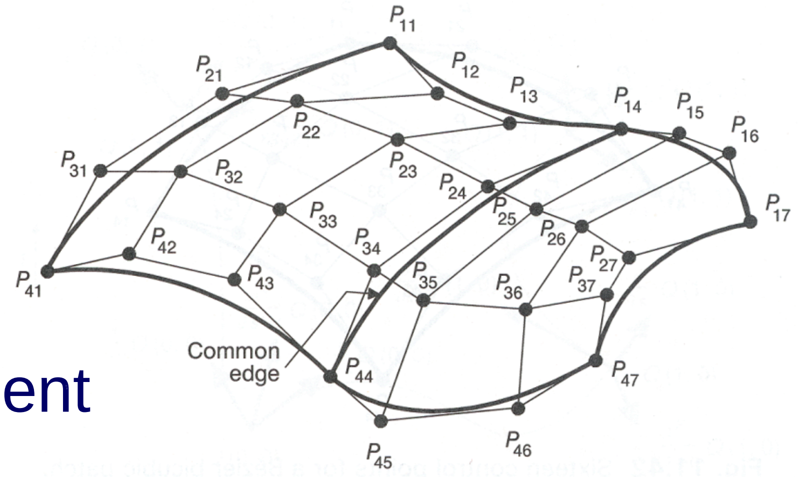


- Difficulties:
  - Parameterization
  - Intersections

Comparison

Parametric surfaces

- Provide parameterization
- More restriction on topology of control mesh
- Some require careful placement of control mesh vertices to guarantee continuity (e.g., Bezier)



Subdivision surfaces

- No parameterization
- Subdivision rules can be defined for arbitrary topologies
- Provable continuity for all placements of control mesh vertices

Comparison

Feature	Polygonal Mesh	Parametric Surface	Subdivision Surface
Accurate	No	Yes	Yes
Concise	No	Yes	Yes
Intuitive specification	No	Yes	Yes
Local support	Yes	Yes	Yes
Affine invariant	Yes	Yes	Yes
Arbitrary topology	Yes	No	Yes
Guaranteed continuity	No	Yes	Yes
Natural parameterization	No	Yes	No
Efficient display	Yes	Yes	Yes
Efficient intersections	No	No	No