# 3D Modeling

COS 426, Spring 2022

Princeton University

Felix Heide

Blender demo (musimduit)

# Syllabus

I. Image processing
II. Modeling
III. Rendering
IV. Animation

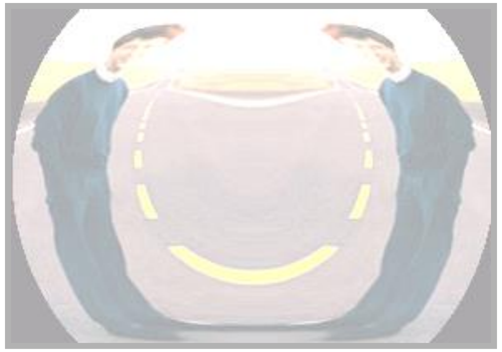
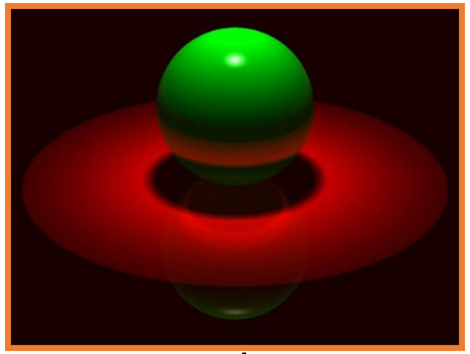
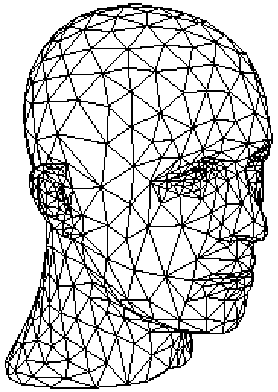


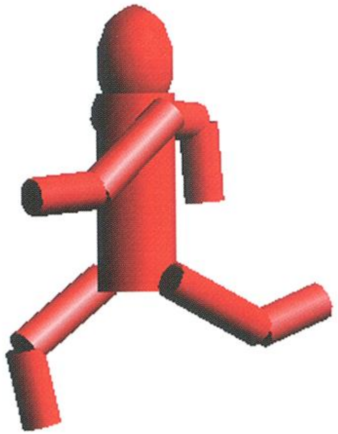
Image Processing
*(Rusty Coleman, CS426, Fall99)*



Rendering
*(Michael Bostock, CS426, Fall99)*



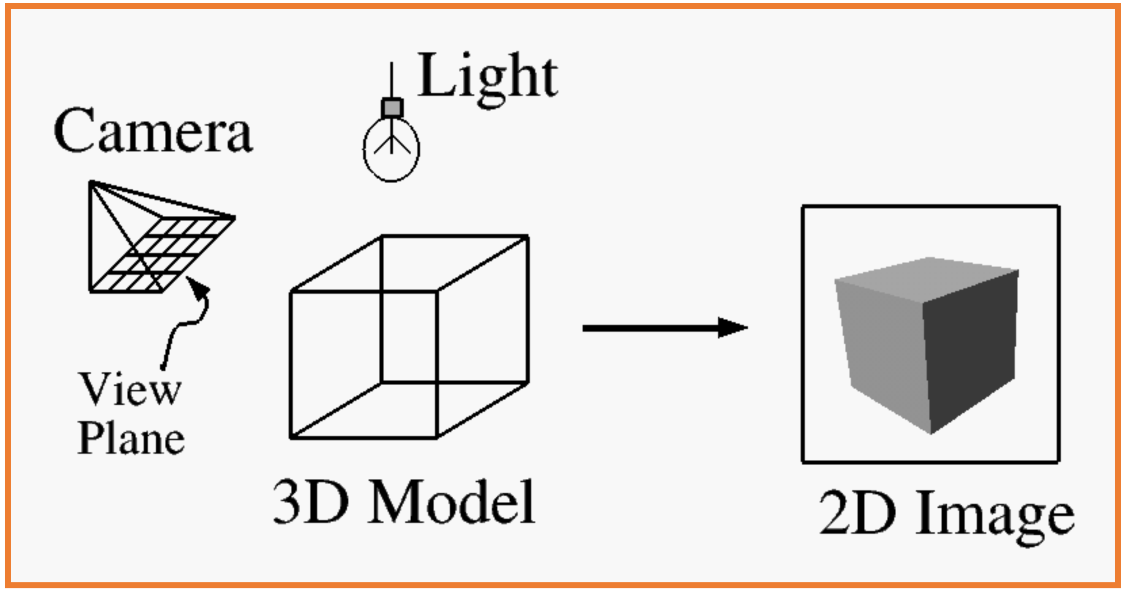
Modeling
*(Denis Zorin, CalTech)*



Animation
*(Angel, Plate 1)*

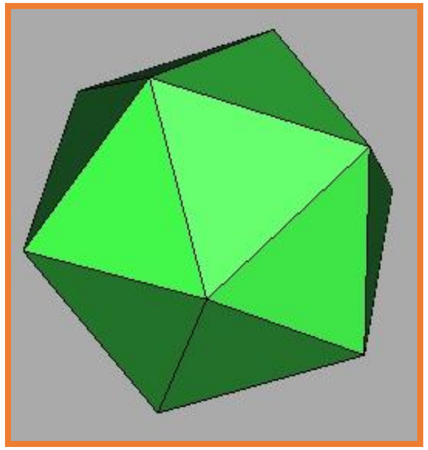
# What is 3D Modeling?

- Topics in computer graphics
  - Imaging = *representing 2D images*
  - Modeling = ***representing 3D objects***
  - Rendering = *constructing 2D images from 3D models*
  - Animation = *simulating changes over time*

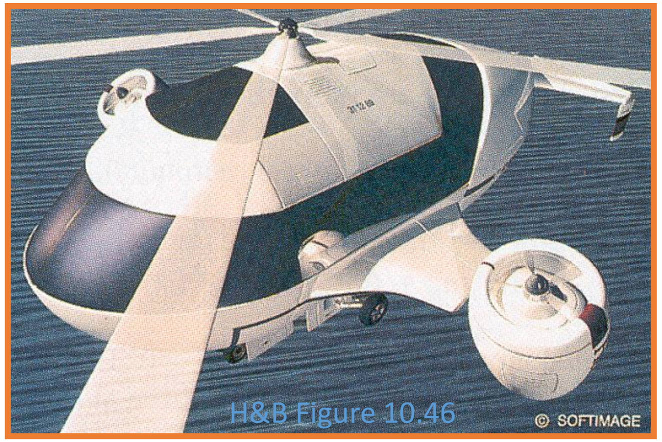


# Modeling

- How do we ...
  - Represent 3D objects in a computer?
  - Acquire computer representations of 3D objects?
  - Manipulate these representations?



Stanford Graphics Laboratory




H&B Figure 10.46

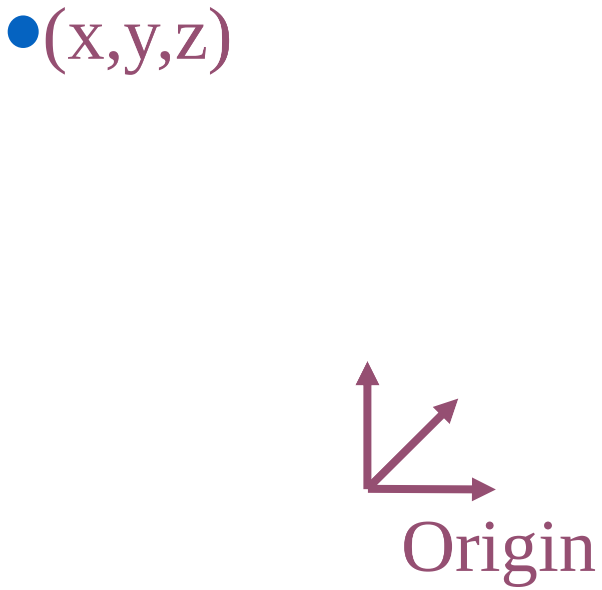
# Modeling Background

- Scene is usually approximated by 3D primitives
  - Point
  - Vector
  - Line segment
  - Ray
  - Line
  - Plane
  - Polygon

# 3D Point

- Specifies a location
  - Represented by three coordinates
  - Infinitely small

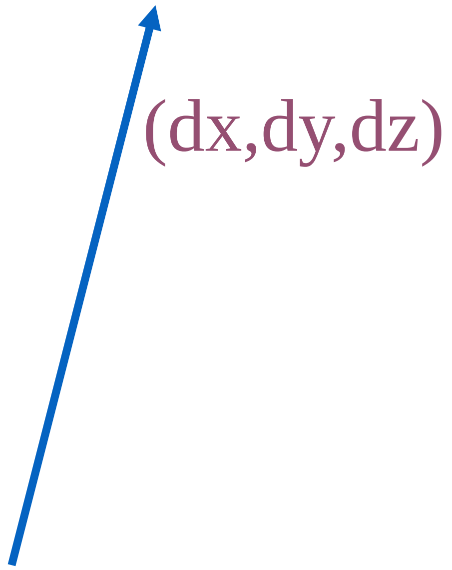
```
typedef struct {
    Coordinate x;
    Coordinate y;
    Coordinate z;
} Point;
```



# 3D Vector

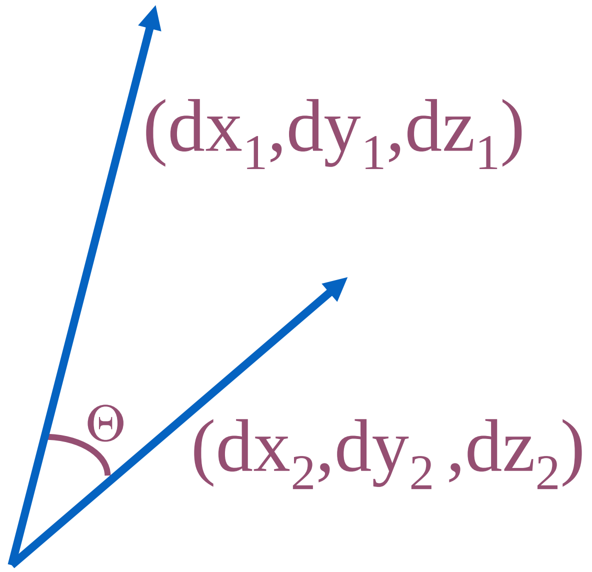
- Specifies a direction and a magnitude
  - Represented by three coordinates
  - Magnitude ||V|| = sqrt(dx · dx + dy · dy + dz · dz)
  - Has no location

```
typedef struct {
    Coordinate dx;
    Coordinate dy;
    Coordinate dz;
} Vector;
```



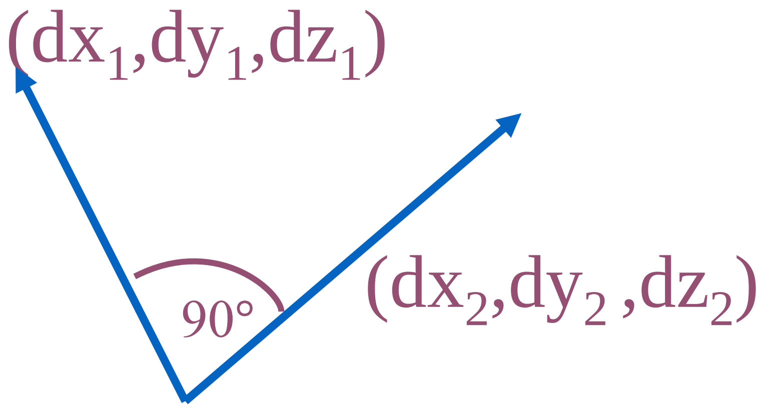
# 3D Vector

- Dot product of two 3D vectors
  - $V_1 \cdot V_2 = \|V_1\| \, \|V_2\| \cos(\Theta)$



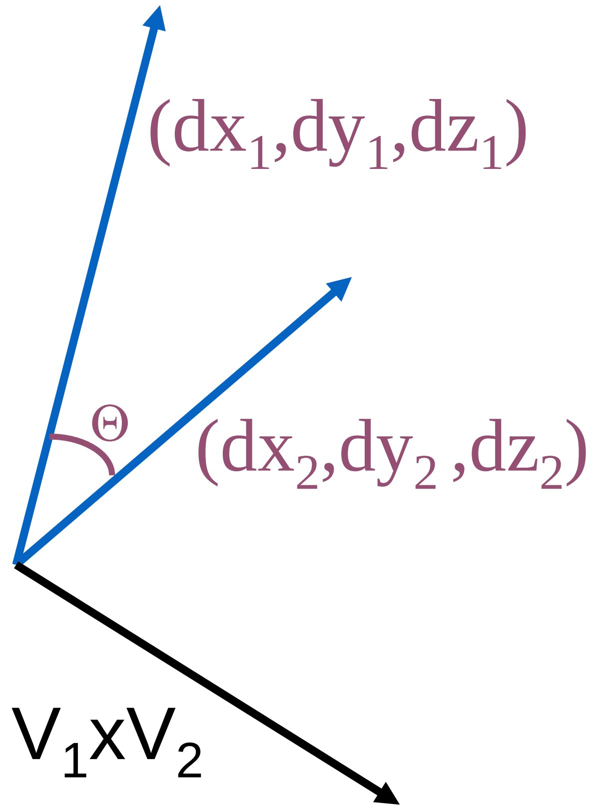
# 3D Orthogonality

- Dot product of two 3D vectors
  - $V_1 \cdot V_2 = \|V_1\| \, \|V_2\| \cos(\pi/2) = 0$



# 3D Vector

- Cross product of two 3D vectors
  - $V_1$ x $V_2$ = vector perpendicular to both $V_1$ and $V_2$
  - $||V_1 \times V_2|| = ||V_1|| \, ||V_2|| \sin(\Theta)$



# 3D Vector

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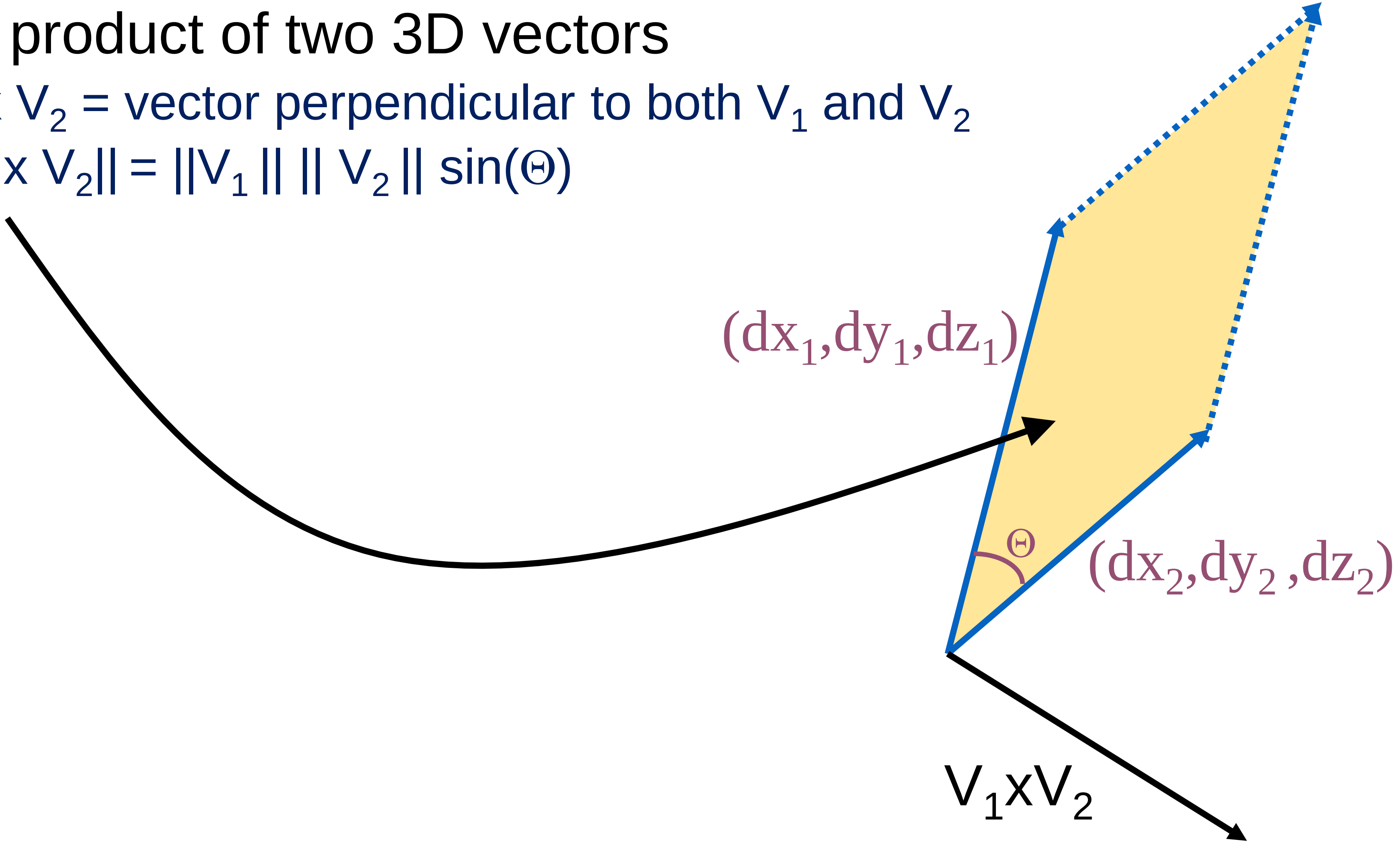
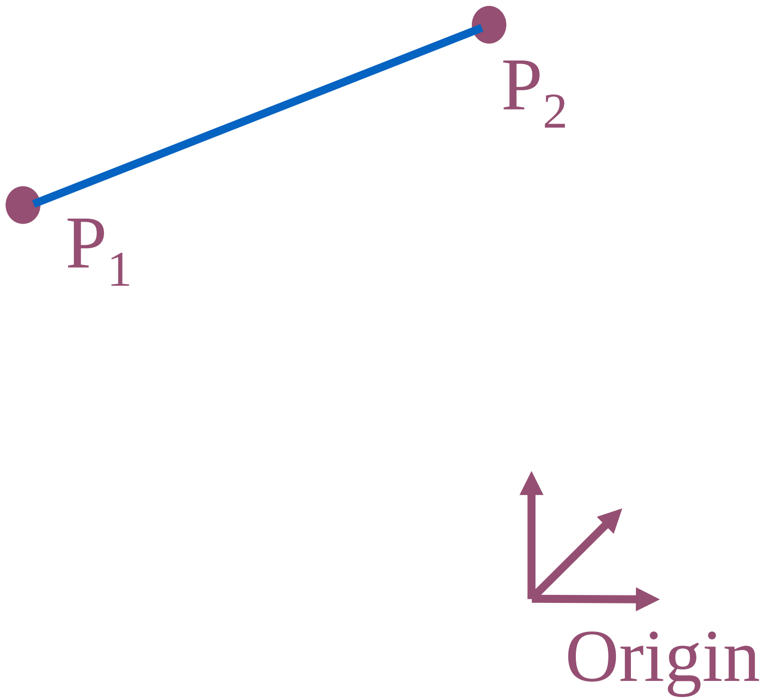


# 3D Line Segment

- Linear path between two points
  - Parametric representation:
    - $P = P_1 + t\,(P_2 - P_1), \quad (0 \leq t \leq 1)$

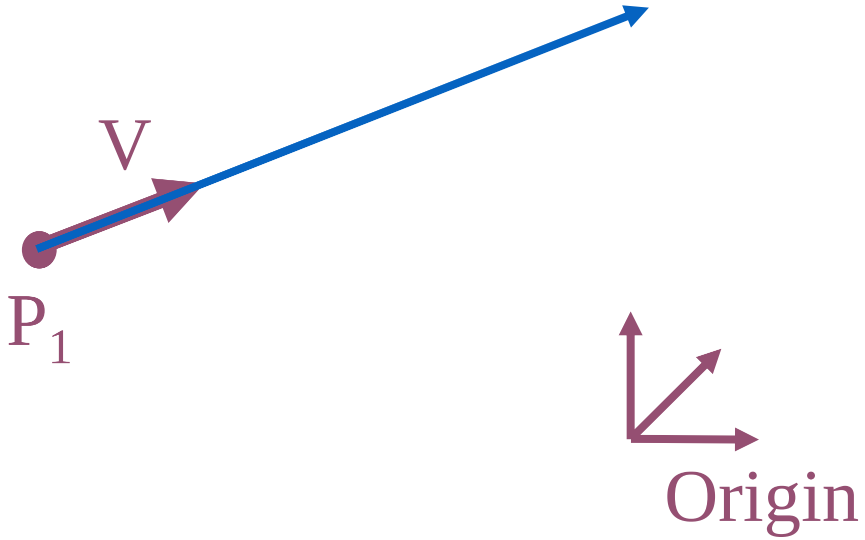
```
typedef struct {
    Point P1;
    Point P2;
} Segment;
```



# 3D Ray

- Line segment with one endpoint at infinity
  - Parametric representation:
    - $P = P_1 + t\,V, \quad (0 <= t < \infty)$

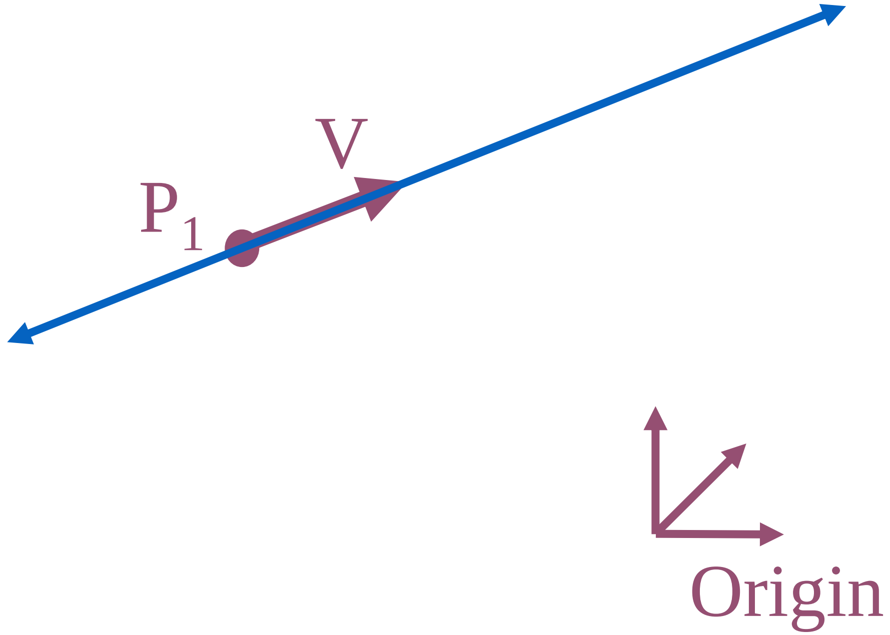
```
typedef struct {
    Point P1;
    Vector V;
} Ray;
```



# 3D Line

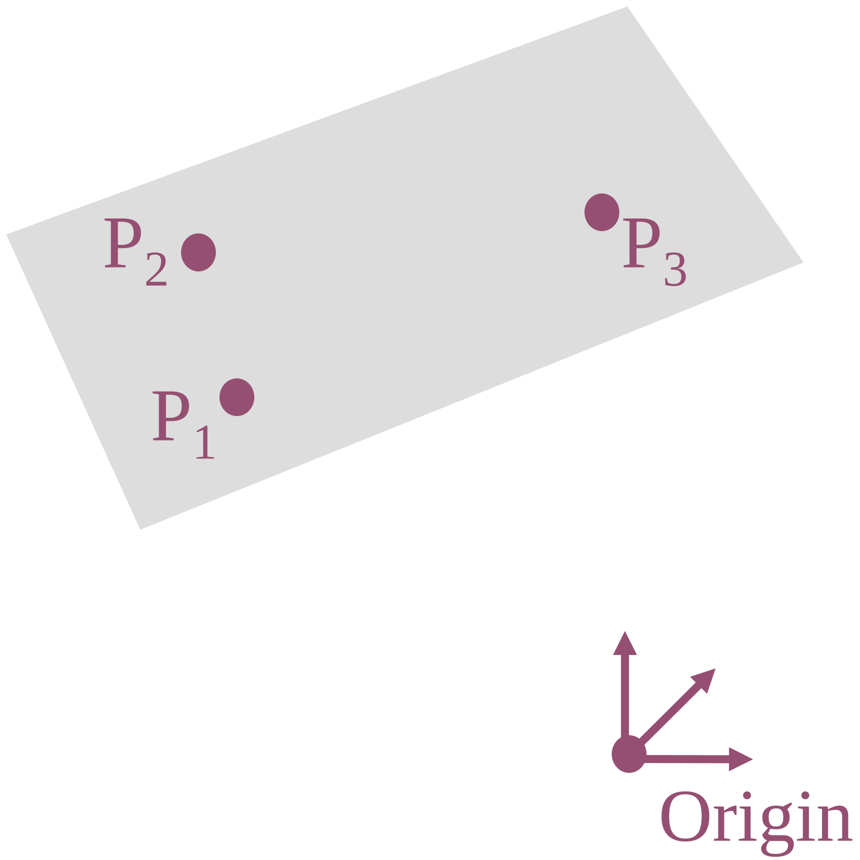
- Line segment with both endpoints at infinity
  - Parametric representation:
    - $P = P_1 + t\,V, \quad (-\infty < t < \infty)$

```
typedef struct {
    Point P1;
    Vector V;
} Line;
```



# 3D Plane

- Defined by three points in 3D space

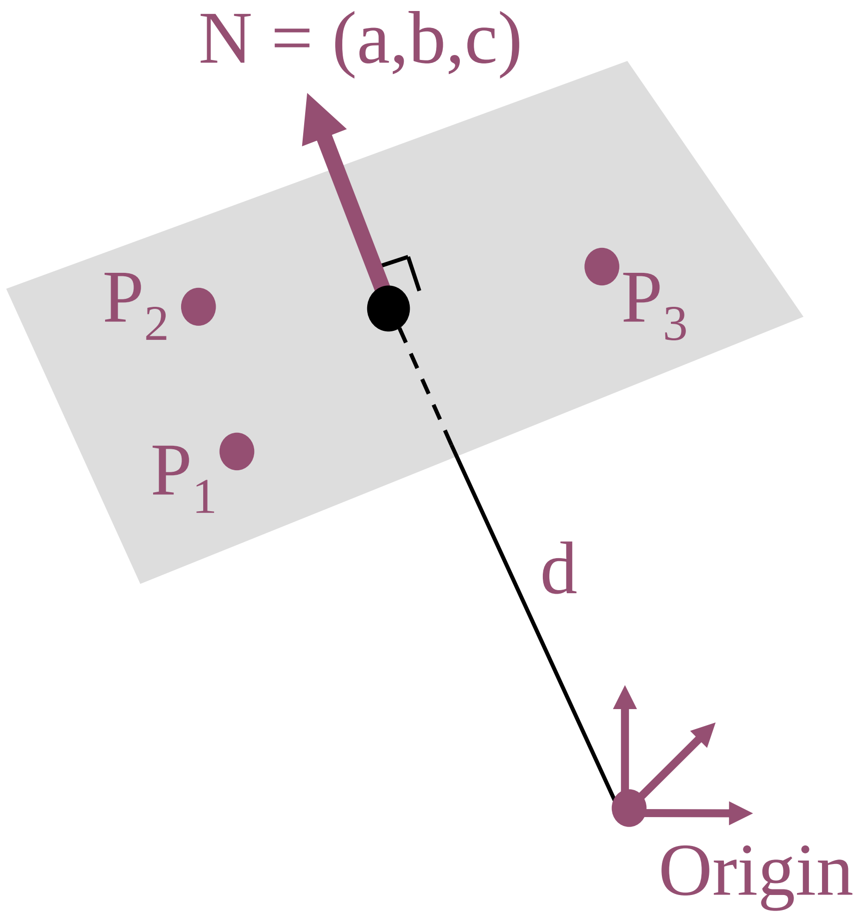


# 3D Plane

- A linear combination of three points
  - Implicit representation:
    - $P \cdot N - d = 0$, or
    - $N \cdot (P - P_1) = 0$, or
    - $ax + by + cz + d = 0$

```
typedef struct {
    Vector N;
    Distance d;
} Plane;
```

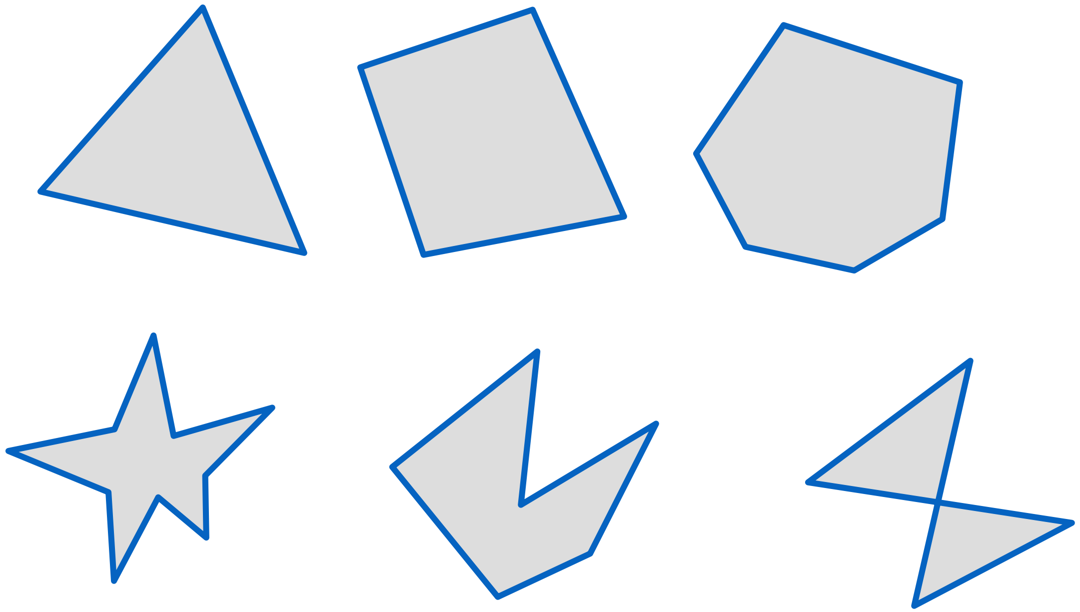
  - N is the plane “normal”
    - Unit-length vector
    - Perpendicular to plane



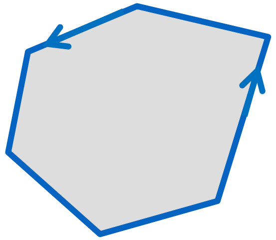
# 3D Polygon

- Set of points "inside" a sequence of coplanar points

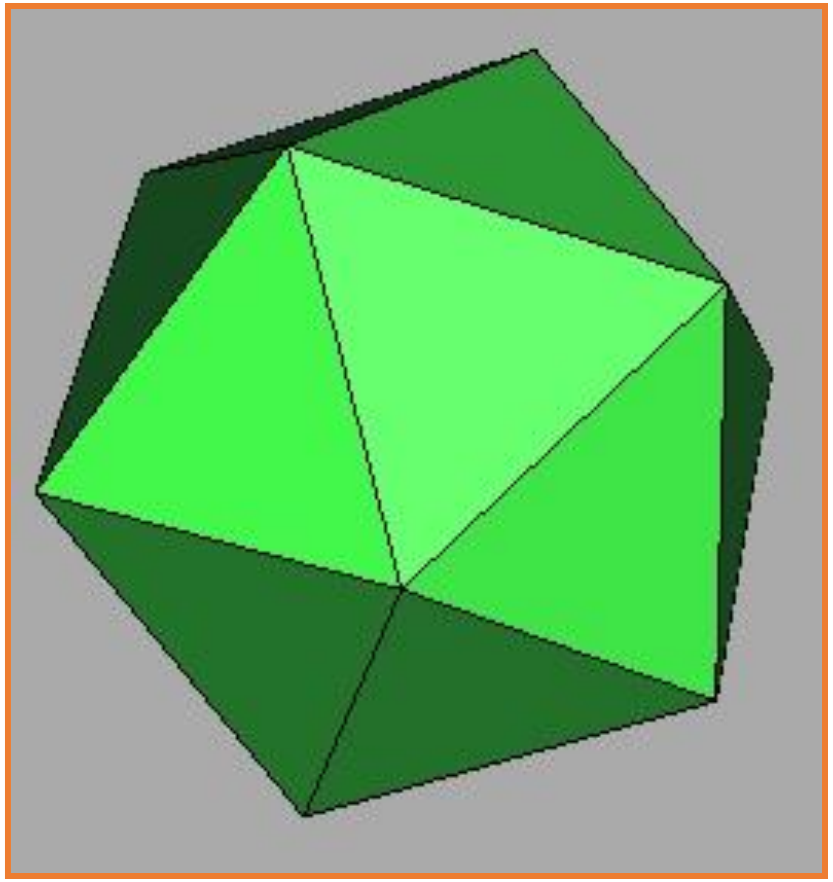
```
typedef struct {
    Point *points;
    int npoints;
} Polygon;
```



Points are in counter-clockwise order

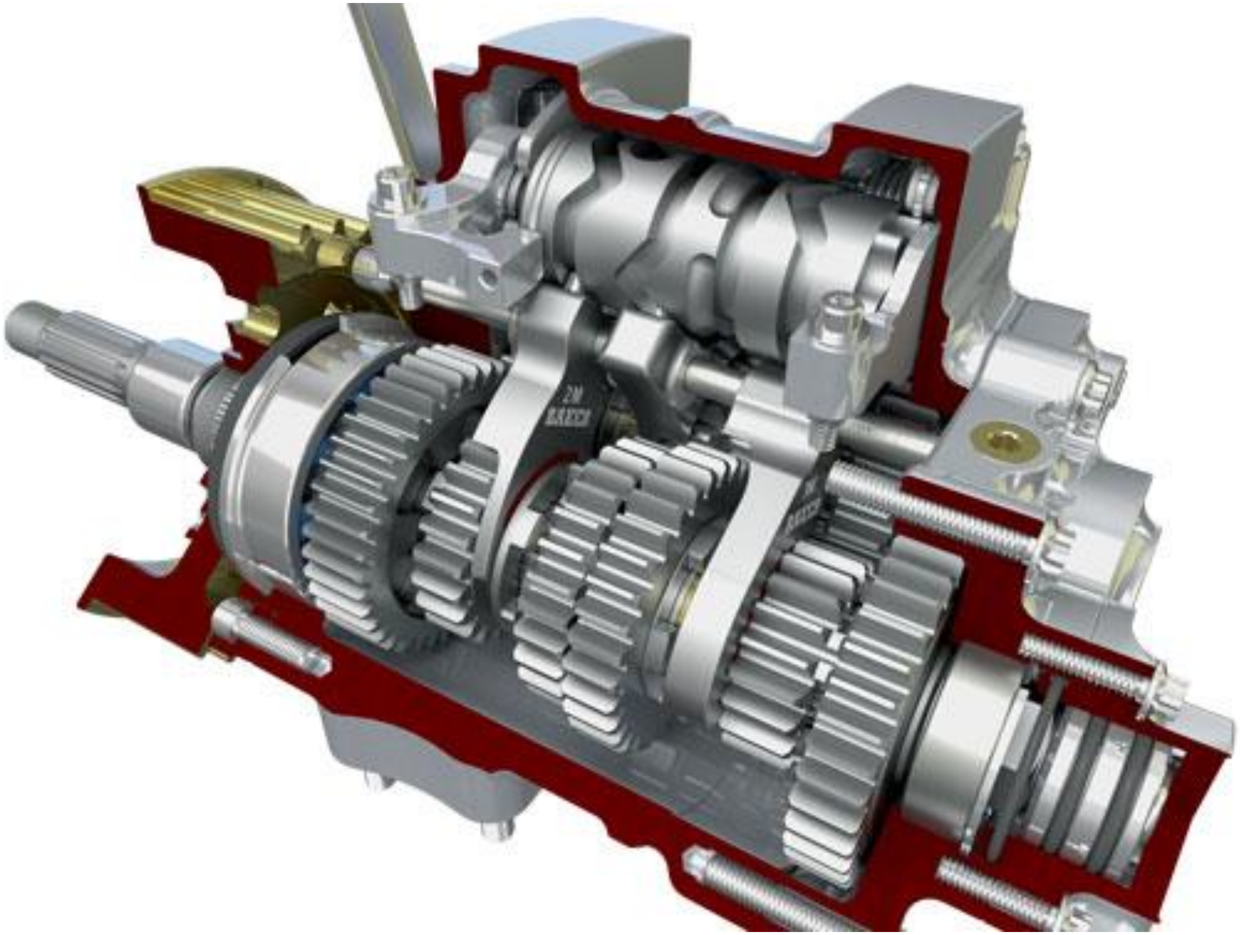


# 3D Object Representations



How can this object be represented in a computer?

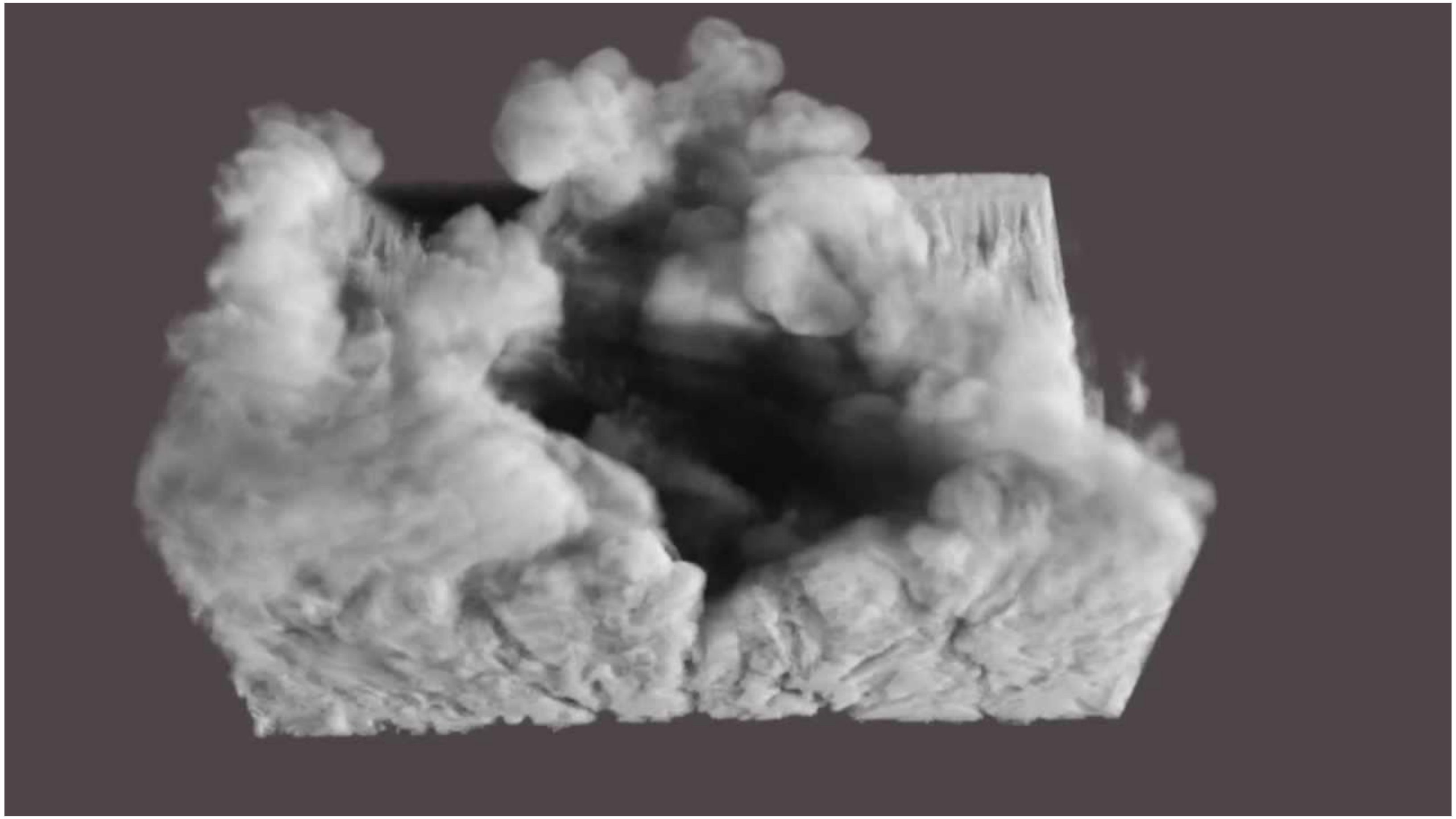
# 3D Object Representations



Solidworks

How about this one?

# 3D Object Representations



This one?

FumeFx

# 3D Object Representations

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# Equivalence of Representations

- Thesis:
  - Each representation has enough expressive power to model the shape of any geometric object
  - It is possible to perform all geometric operations with any representation
- Analogous to Turing-equivalence
  - Computers and programming languages are Turing-equivalent, but each has its benefits…

# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation

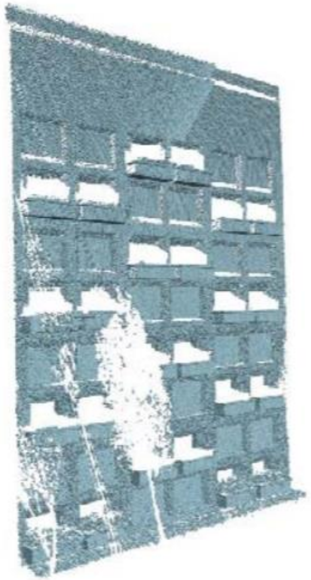
→ Data structures determine algorithms

# Why Different Representations?

Efficiency for different tasks

- Acquisition
  - Range Scanning
- Rendering
- Analysis
- Manipulation
- Animation







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# Why Different Representations?

Efficiency for different tasks

- Acquisition
    - Computer Vision
- Rendering
- Analysis
- Manipulation
- Animation

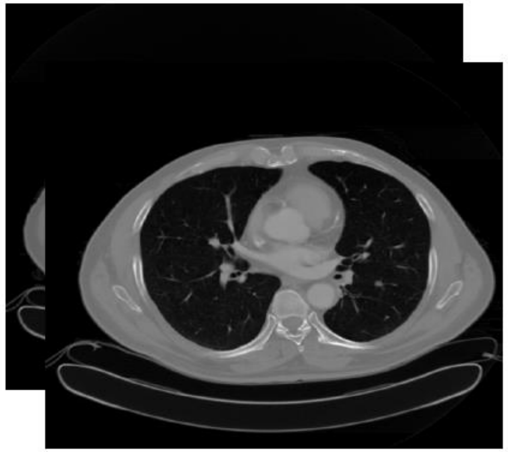

Indiana University


USC

# Why Different Representations?

Efficiency for different tasks

- Acquisition
  - Tomography
- Rendering
- Analysis
- Manipulation
- Animation



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# Why Different Representations?

Efficiency for different tasks

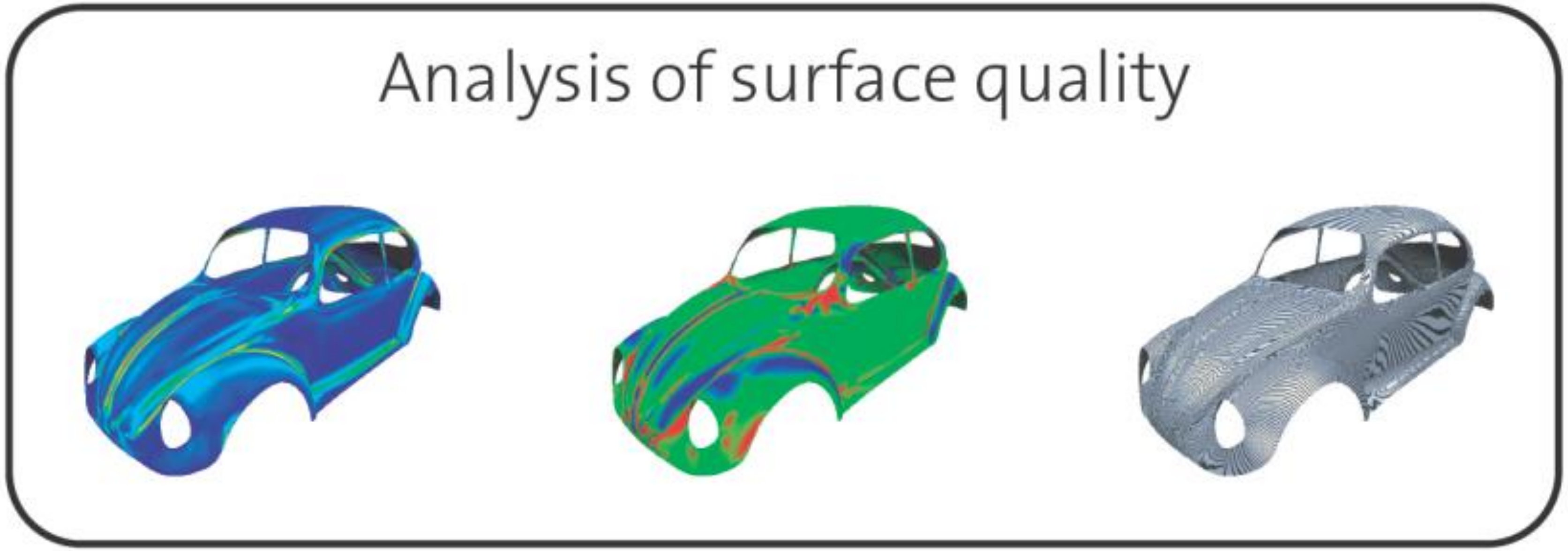
- Acquisition
- Rendering
  - Intersection
- Analysis
- Manipulation
- Animation


Autodesk

# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Curvature, smoothness
- Manipulation
- Animation

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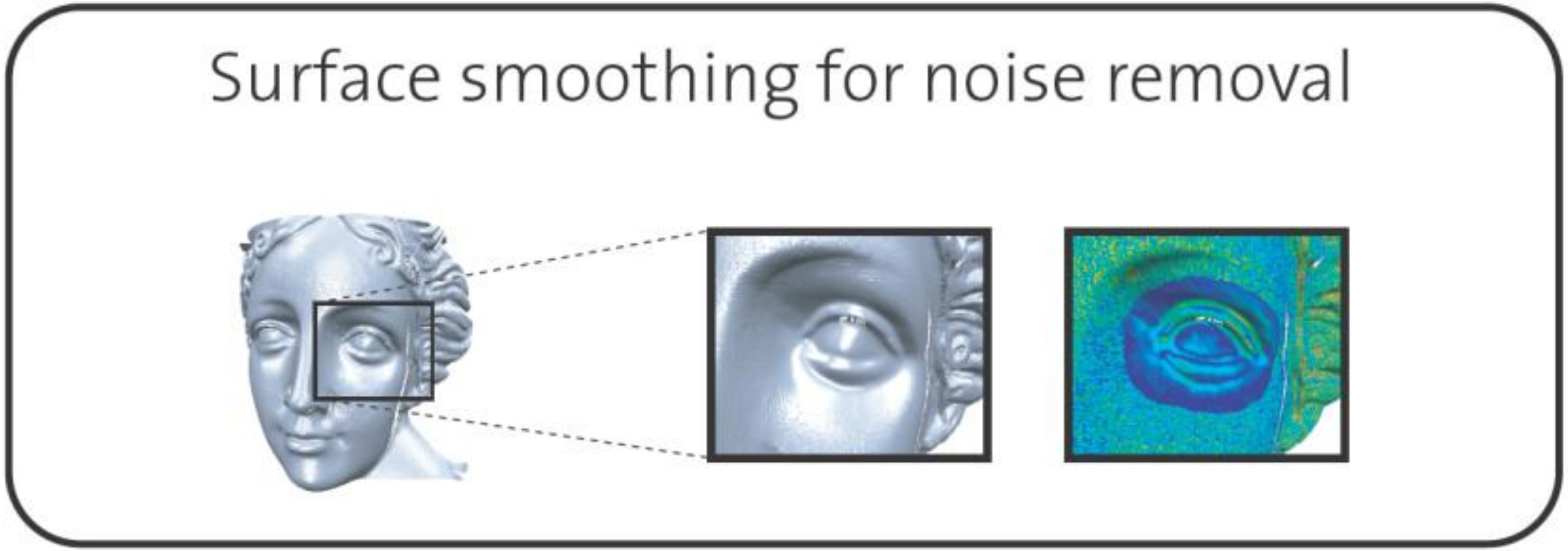
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Fairing
- Manipulation
- Animation

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# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Texture mapping
- Manipulation
- Animation

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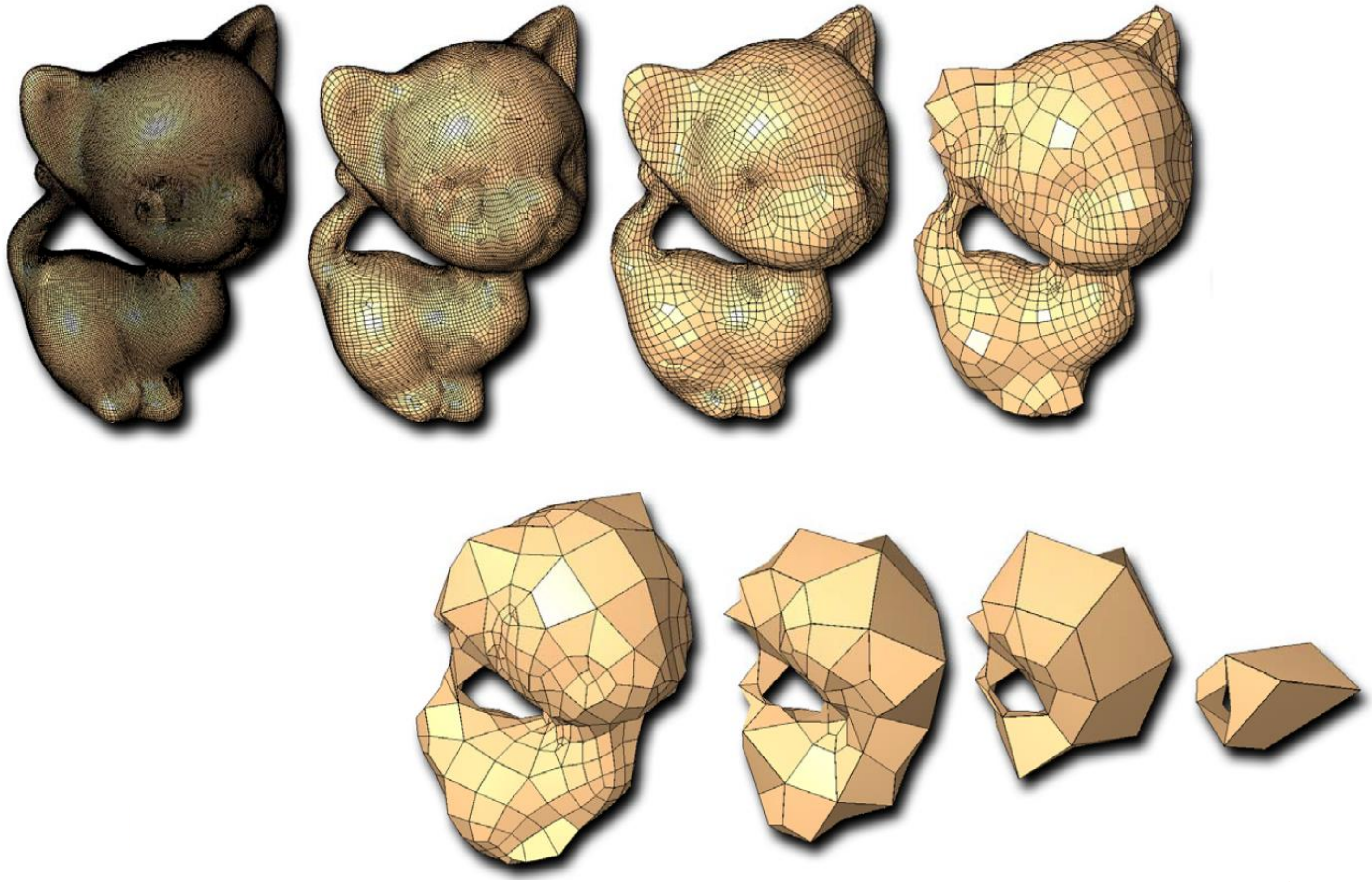
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Reduction
- Manipulation
- Animation



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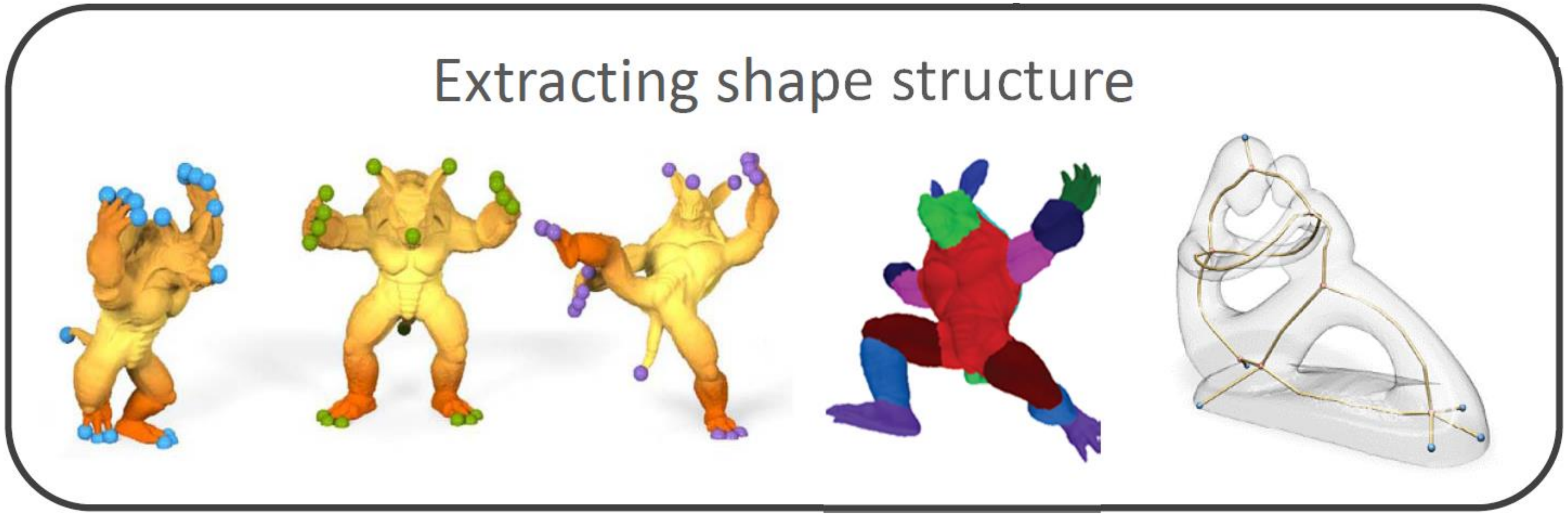
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
    - Structure
- Manipulation
- Animation

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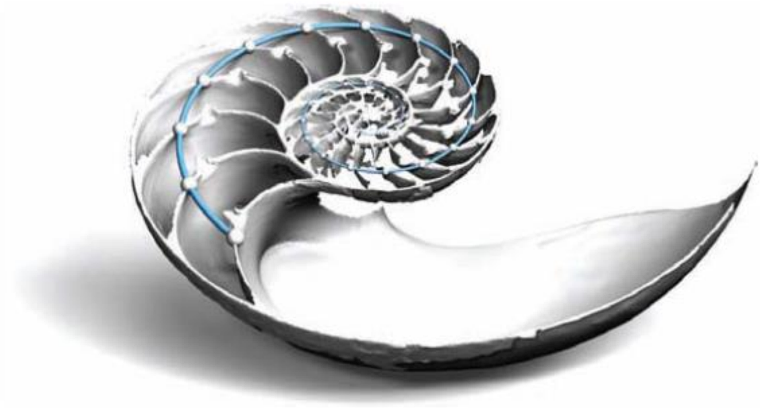
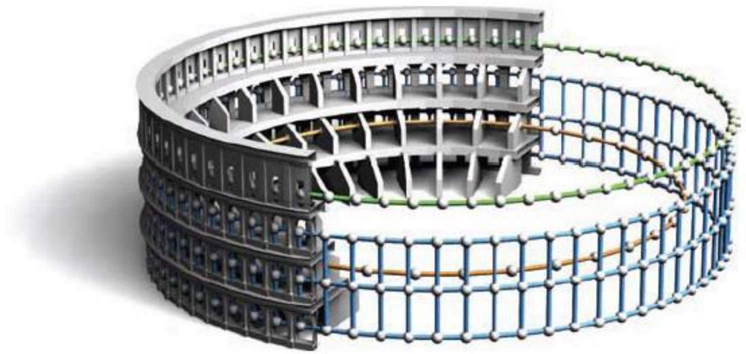
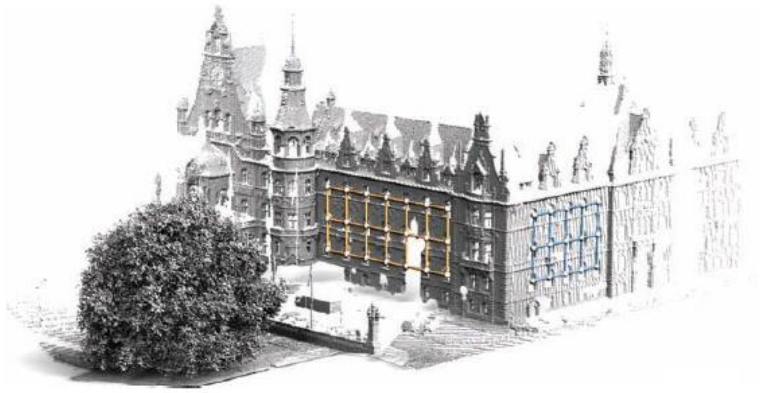
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Symmetry detection
- Manipulation
- Animation



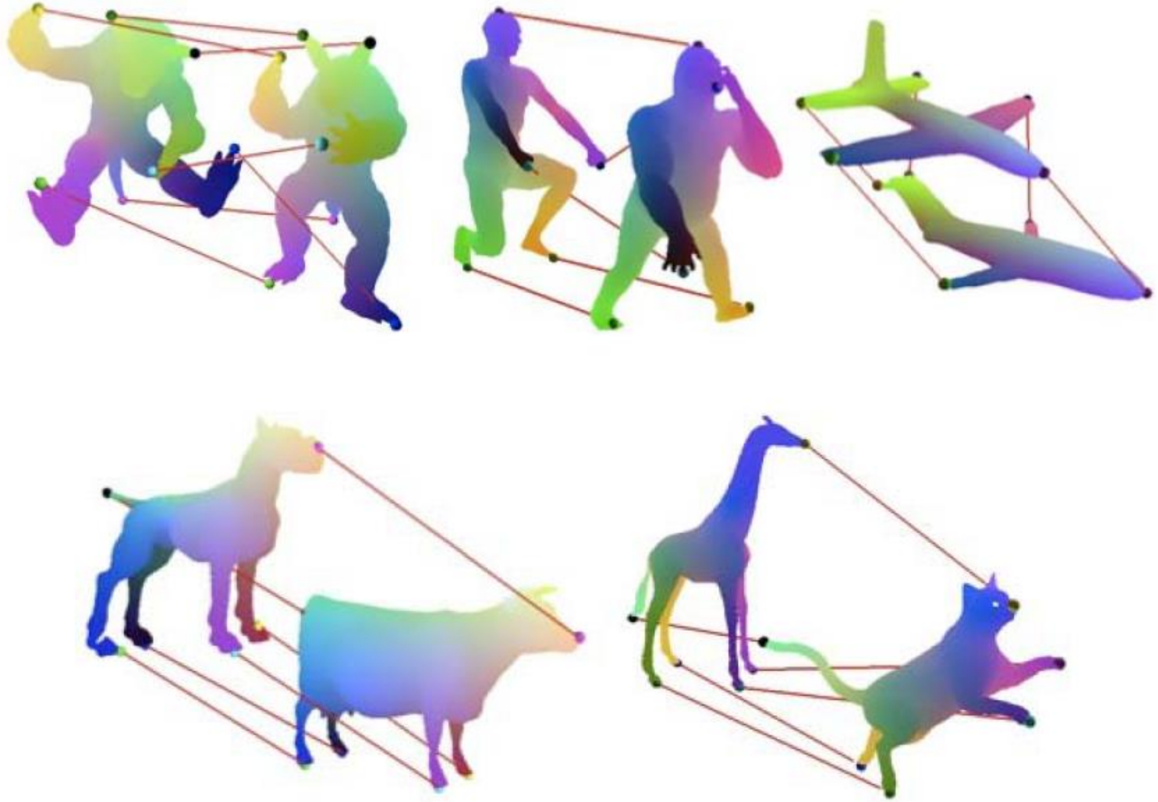




# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Correspondence
- Manipulation
- Animation



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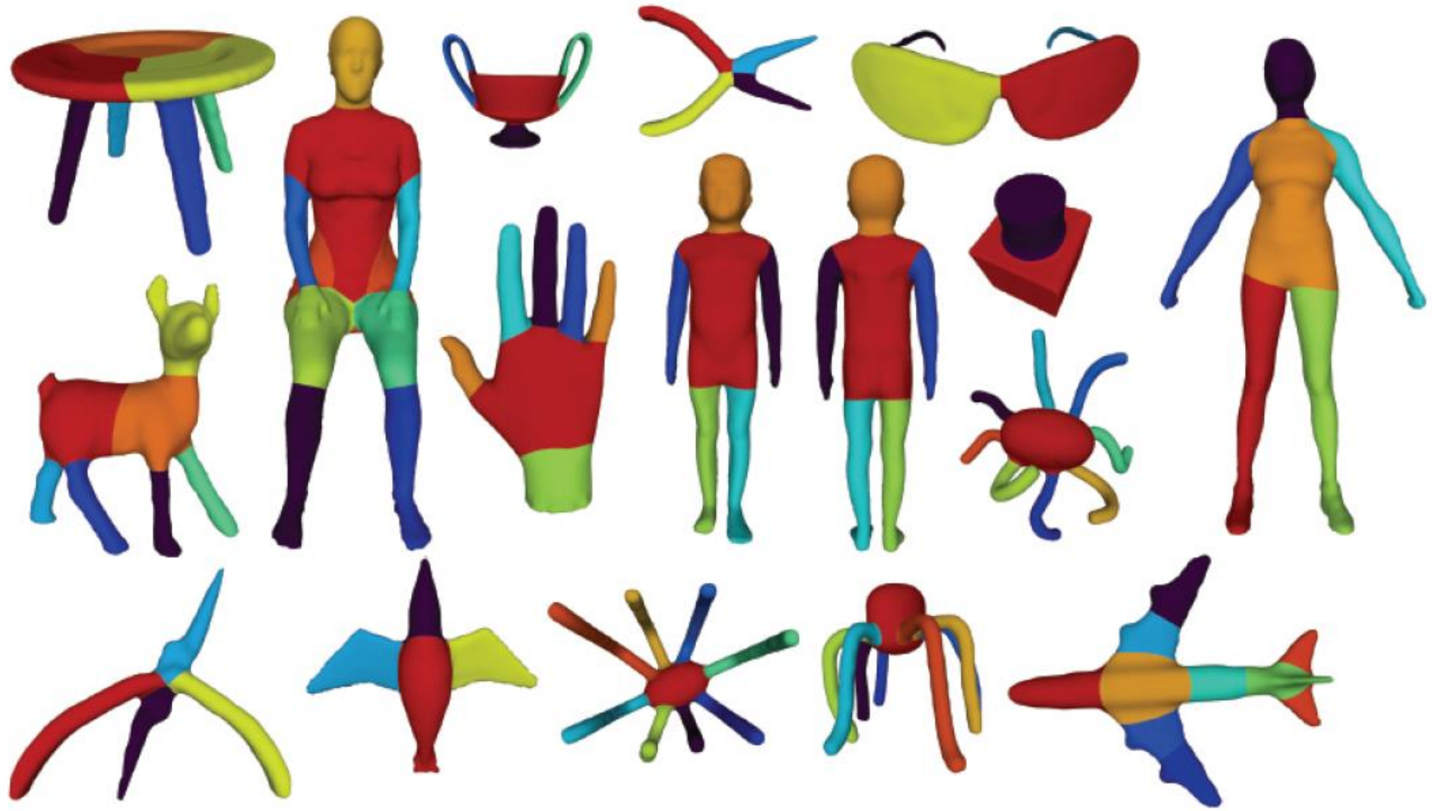
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Segmentation
- Manipulation
- Animation



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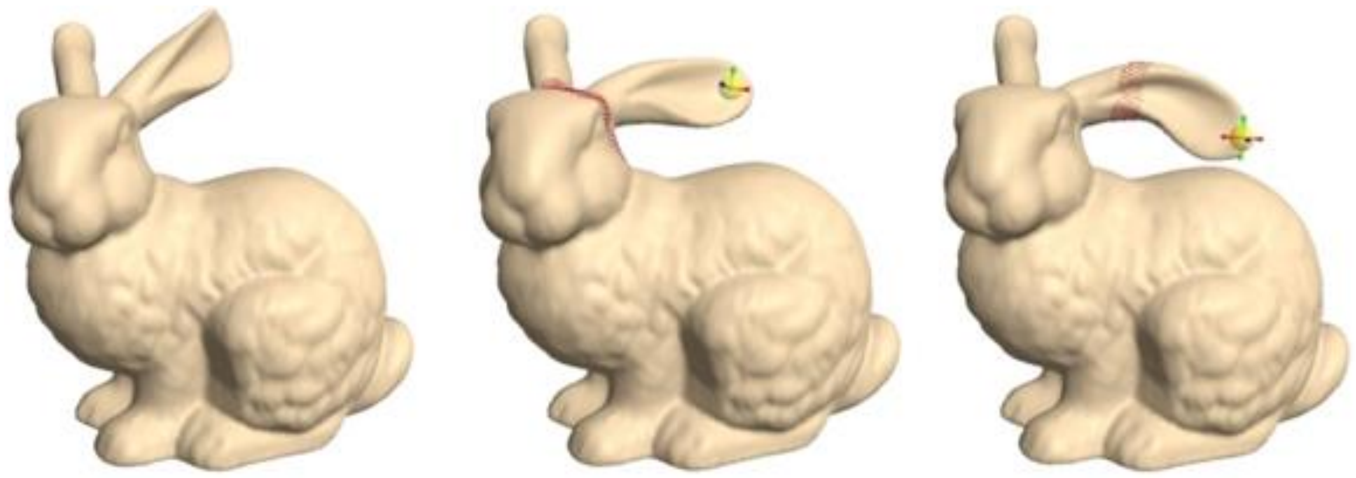
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
  - Deformation
- Animation

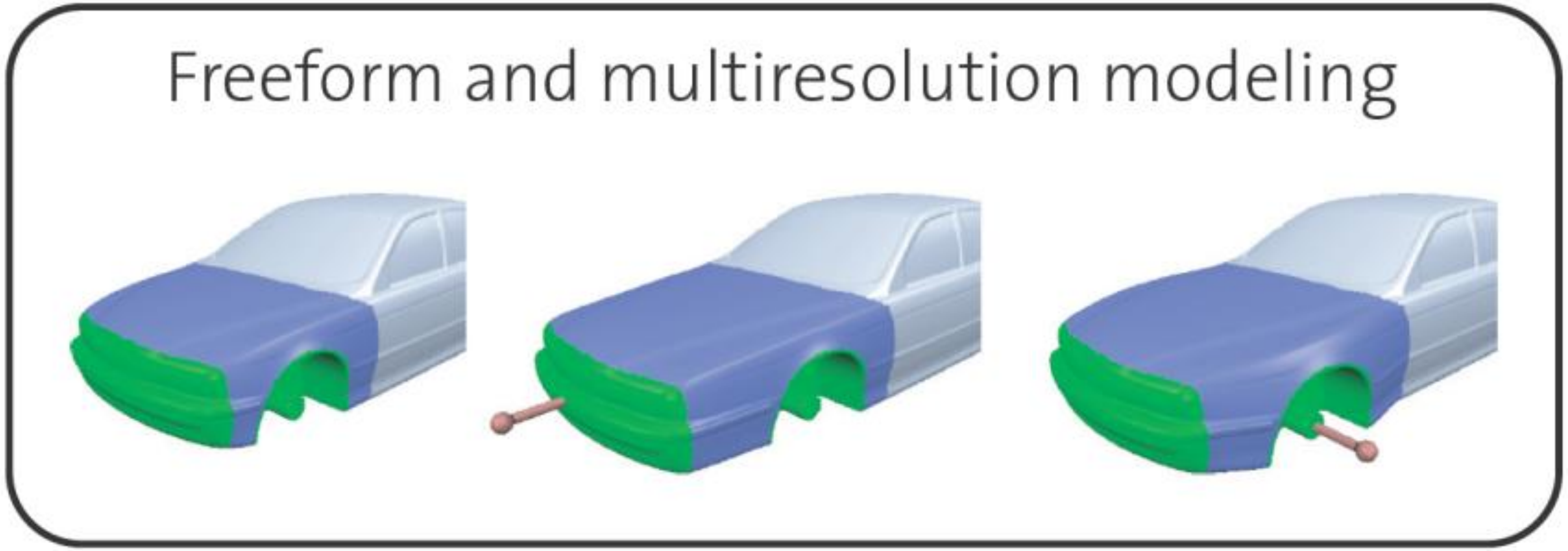


IGL

# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
  - Deformation
- Animation

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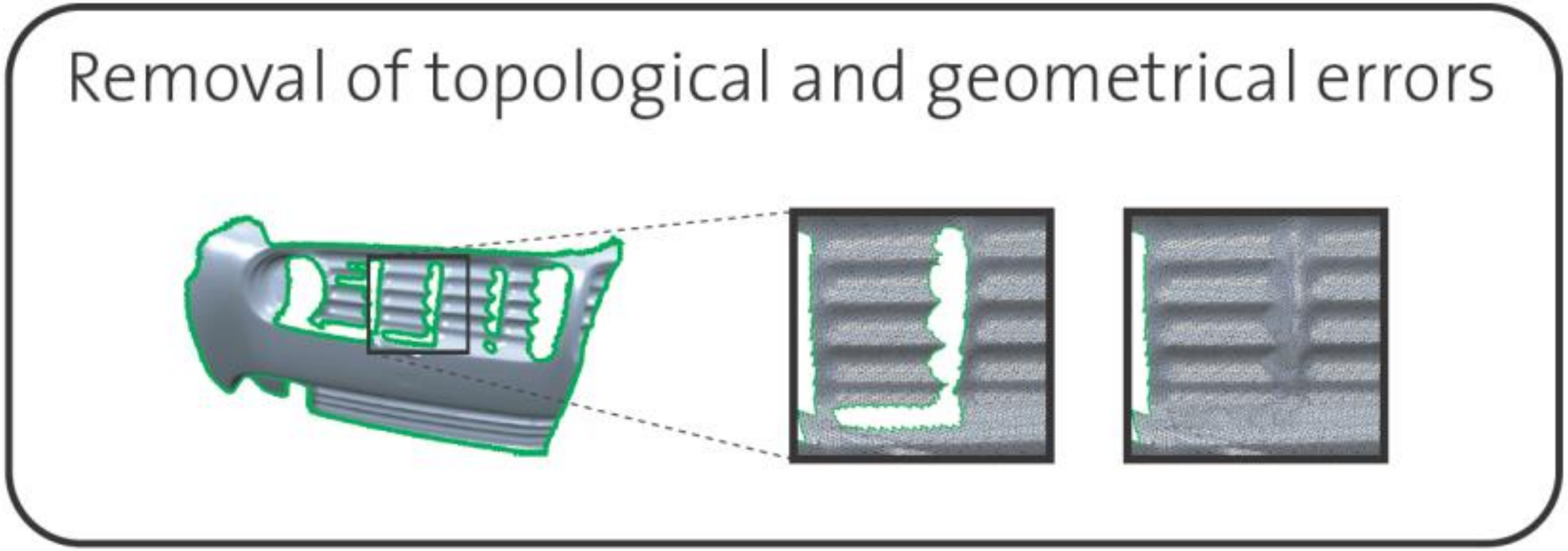
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
  - Healing
- Animation

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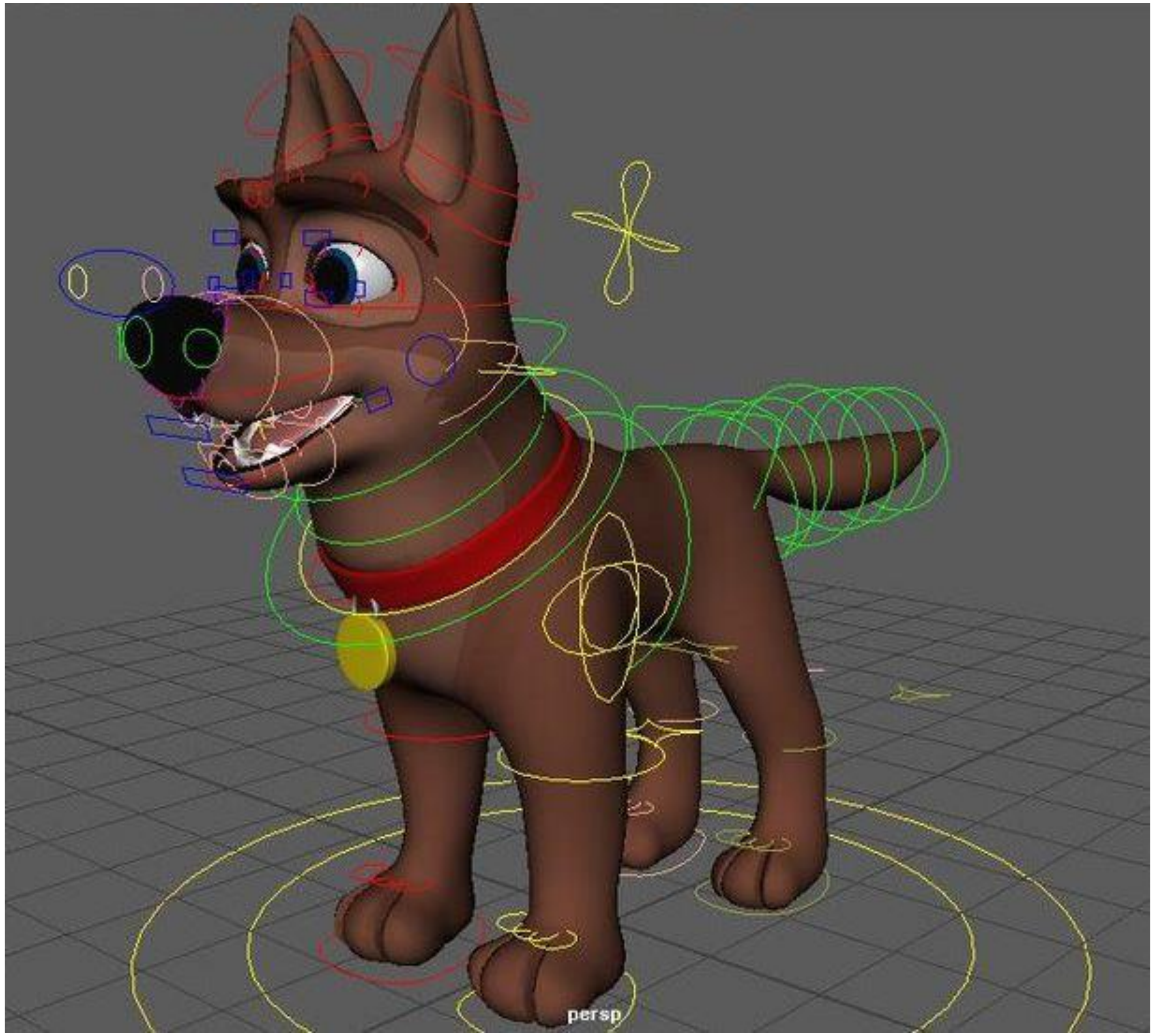
# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
  - Rigging

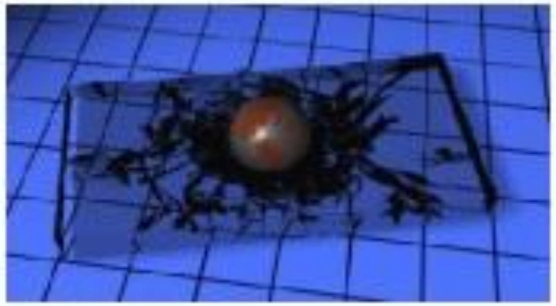


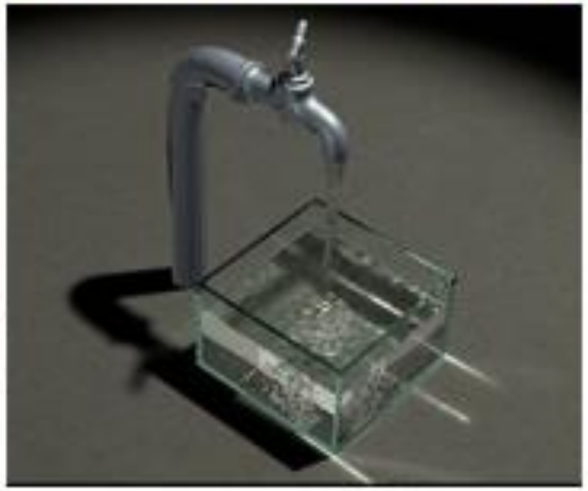

Animation Buffet

# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
    - Simulation



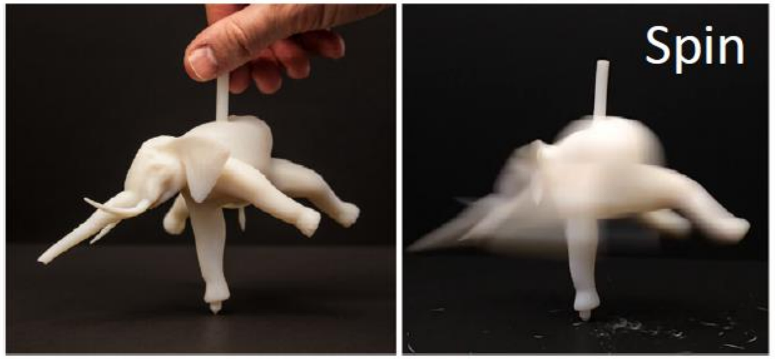


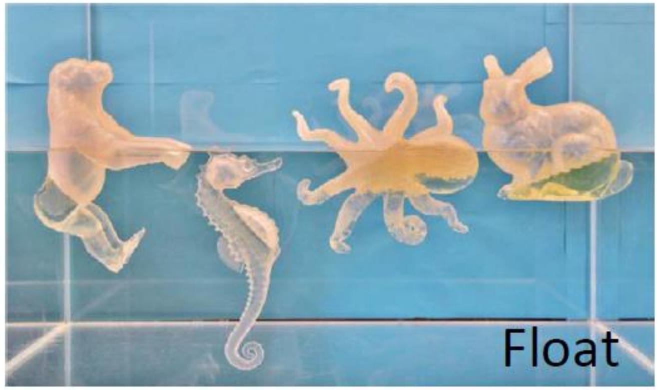
Physically Based Modelling course notes, USC

# Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
  - Fabrication

# 3D Object Representations

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

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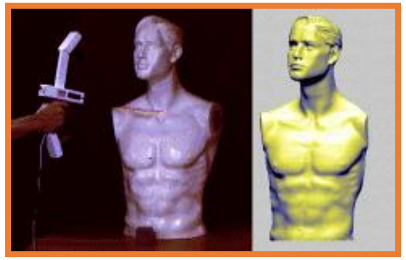
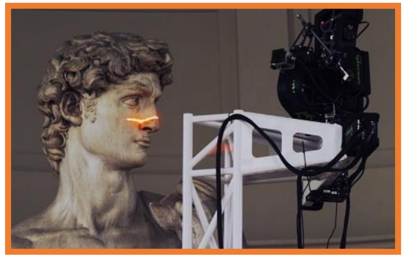
# Range Image

Set of 3D points mapping to pixels of depth image

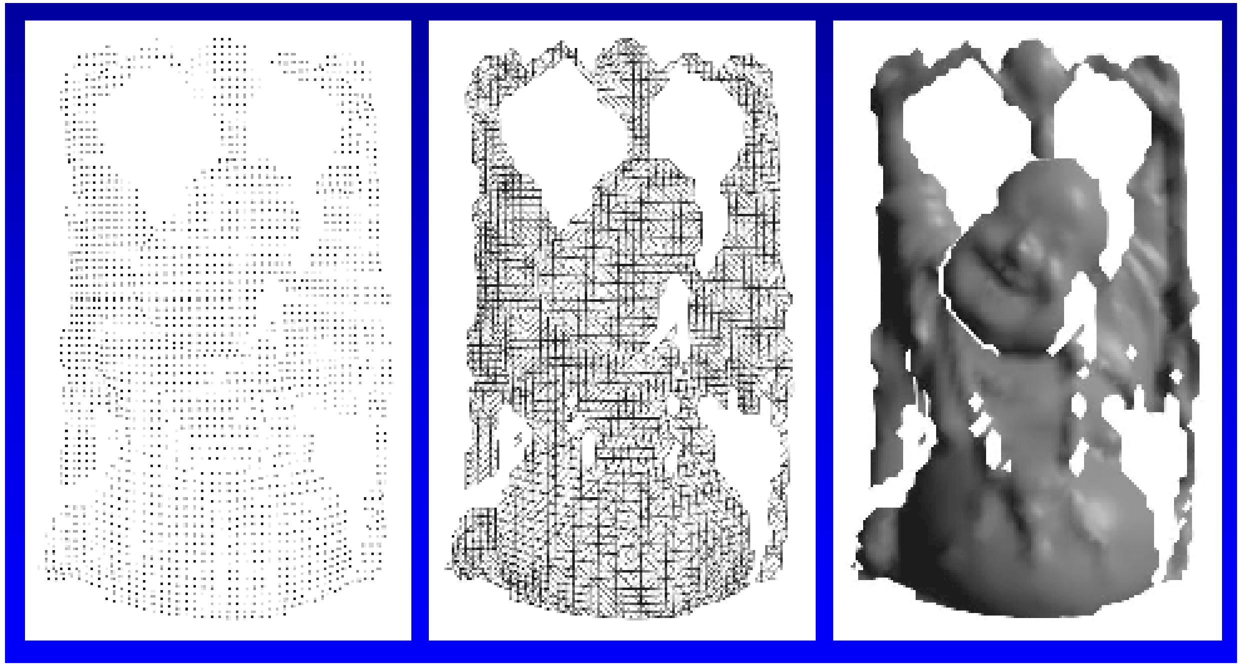
- Can be acquired from range scanner



Cyberware



Stanford



Range Image Tesselation Range Surface

Brian Curless
SIGGRAPH 99
Course #4 Notes

# Point Cloud

Unstructured set of 3D point samples

- Acquired from range finder, computer vision, etc


Velodyne Lidar Scan

# 3D Object Representations

- Points
  - Range image
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- Surfaces
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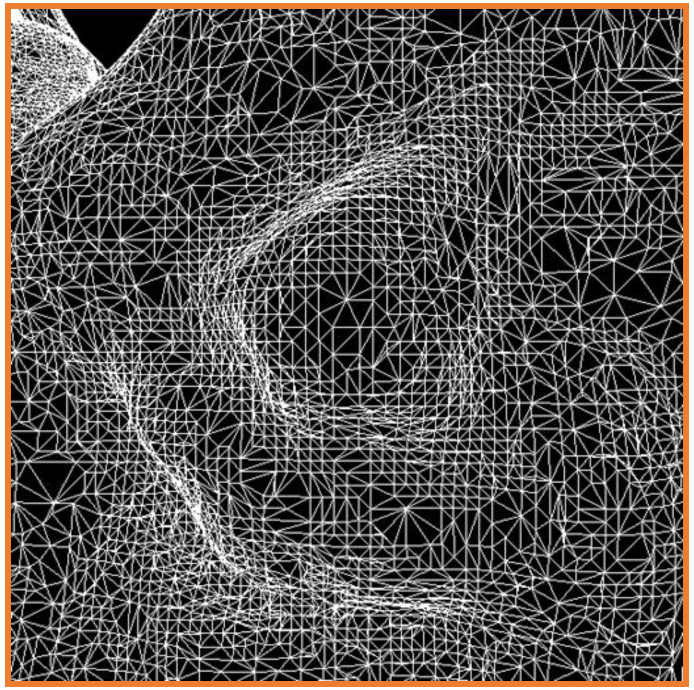
# Polygonal Mesh

Connected set of polygons (often triangles)

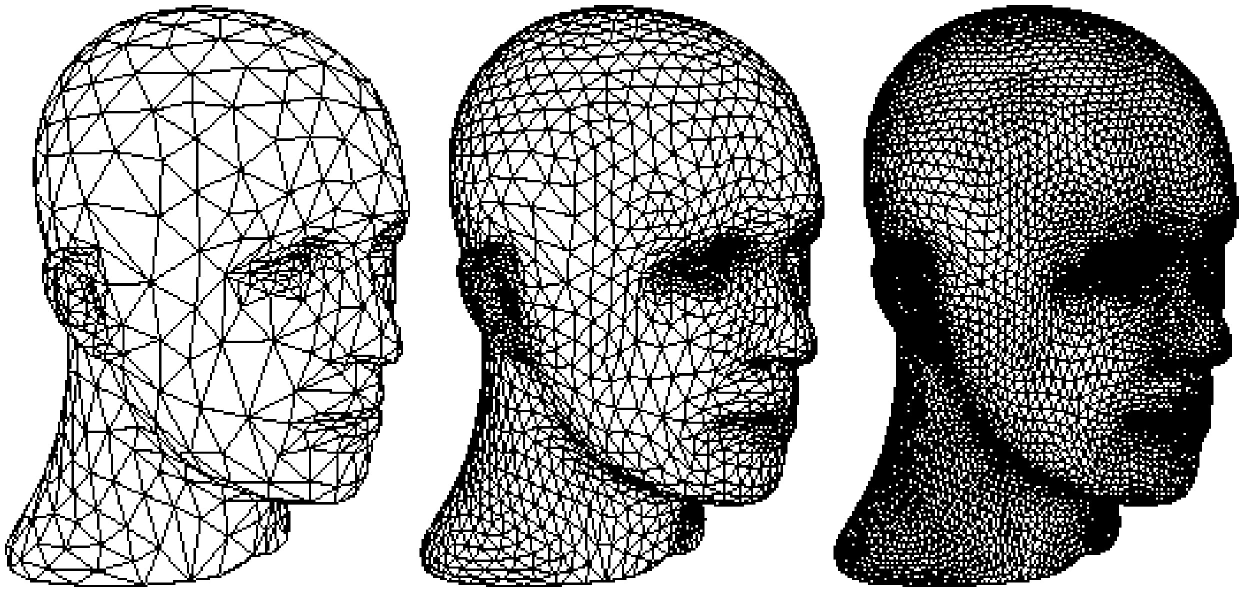


Stanford Graphics Laboratory

# Subdivision Surface

Coarse mesh & subdivision rule

- Smooth surface is limit of sequence of refinements

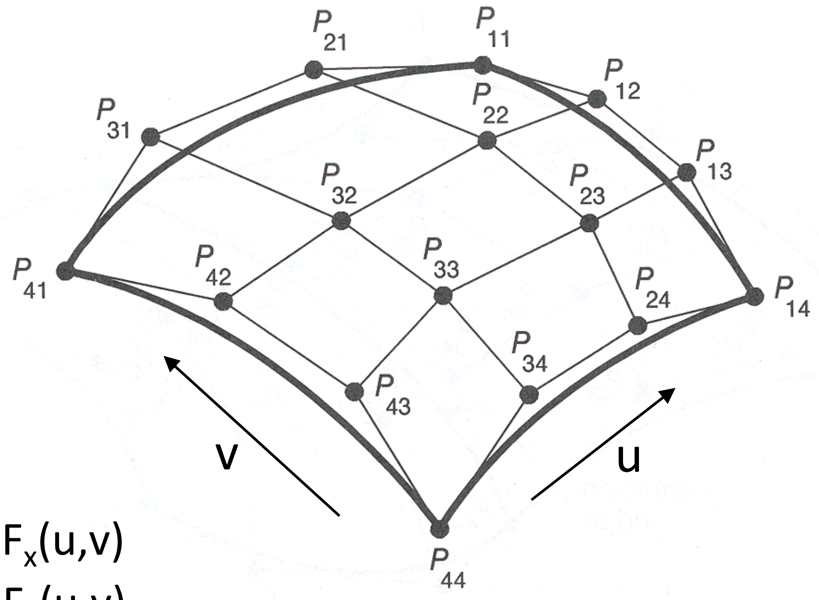


Zorin & Schroeder
SIGGRAPH 99
Course Notes

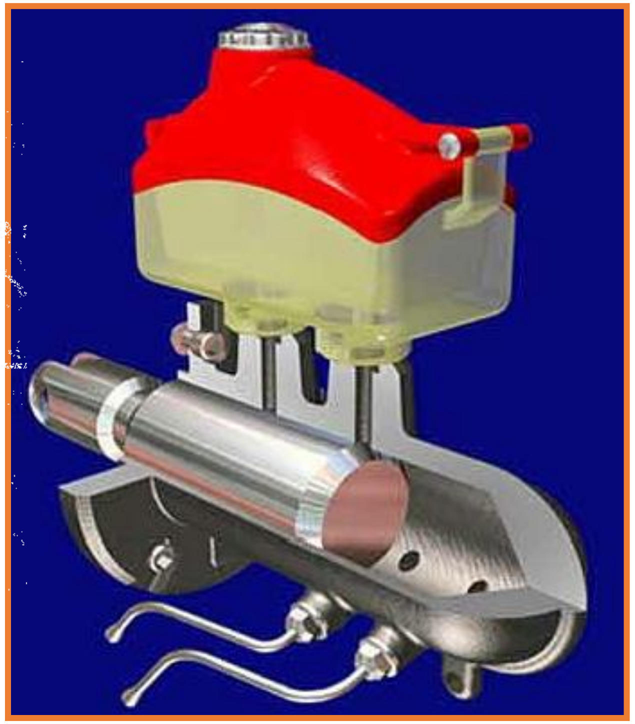
# Parametric Surface

Tensor-product spline patches

- Each patch is parametric function
- Careful constraints to maintain continuity



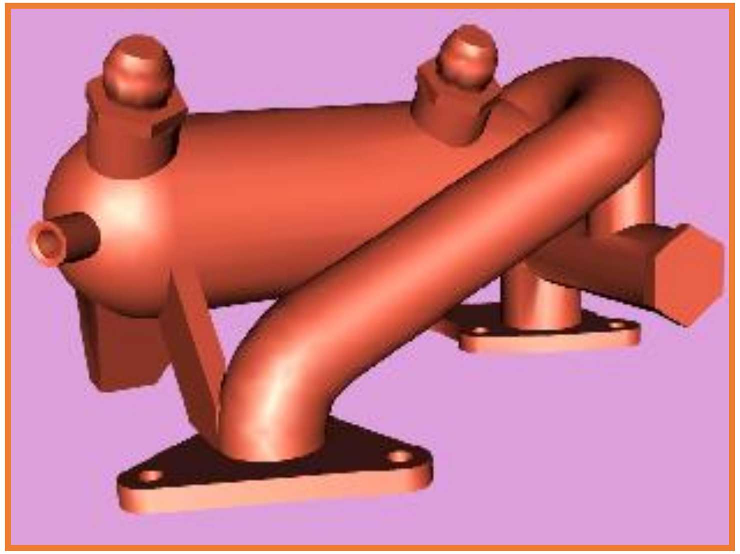

$x = F_x(u,v)$
$y = F_y(u,v)$
$z = F_z(u,v)$



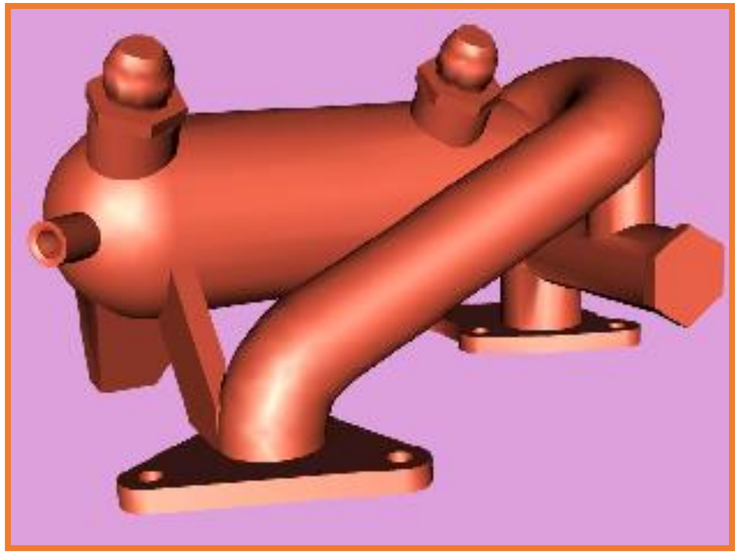
FvDFH Figure 11.44

# Implicit Surface

Set of all points satisfying: F(x,y,z) = 0



Polygonal Model



Implicit Model

Bill Lorensen
SIGGRAPH 99
Course #4 Notes

# 3D Object Representations

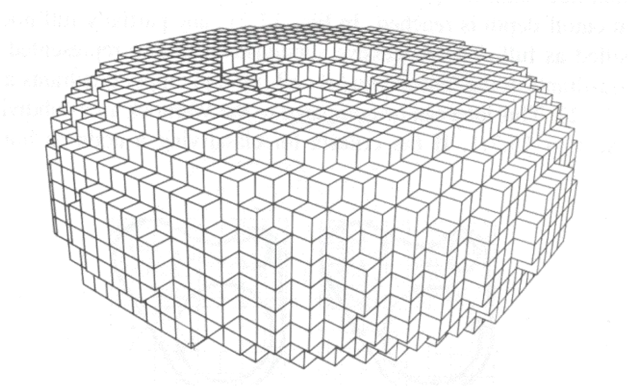
- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# Voxel grid

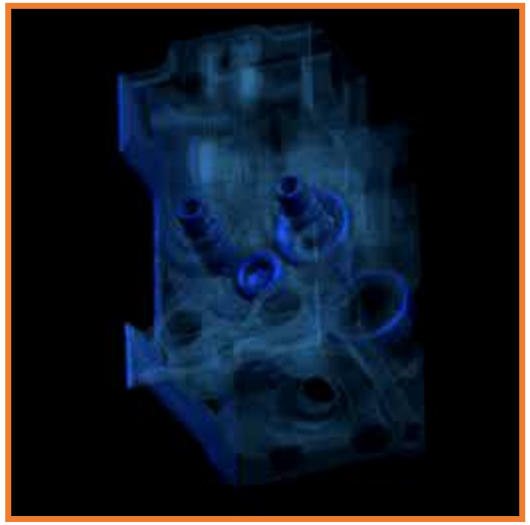
Uniform volumetric grid of samples:

- Occupancy
  (object vs. empty space)
- Density
- Color
- Other function
  (speed, temperature, etc.)

- Often acquired via
  simulation or from
  CAT, MRI, etc.



FvDFH Figure 12.20

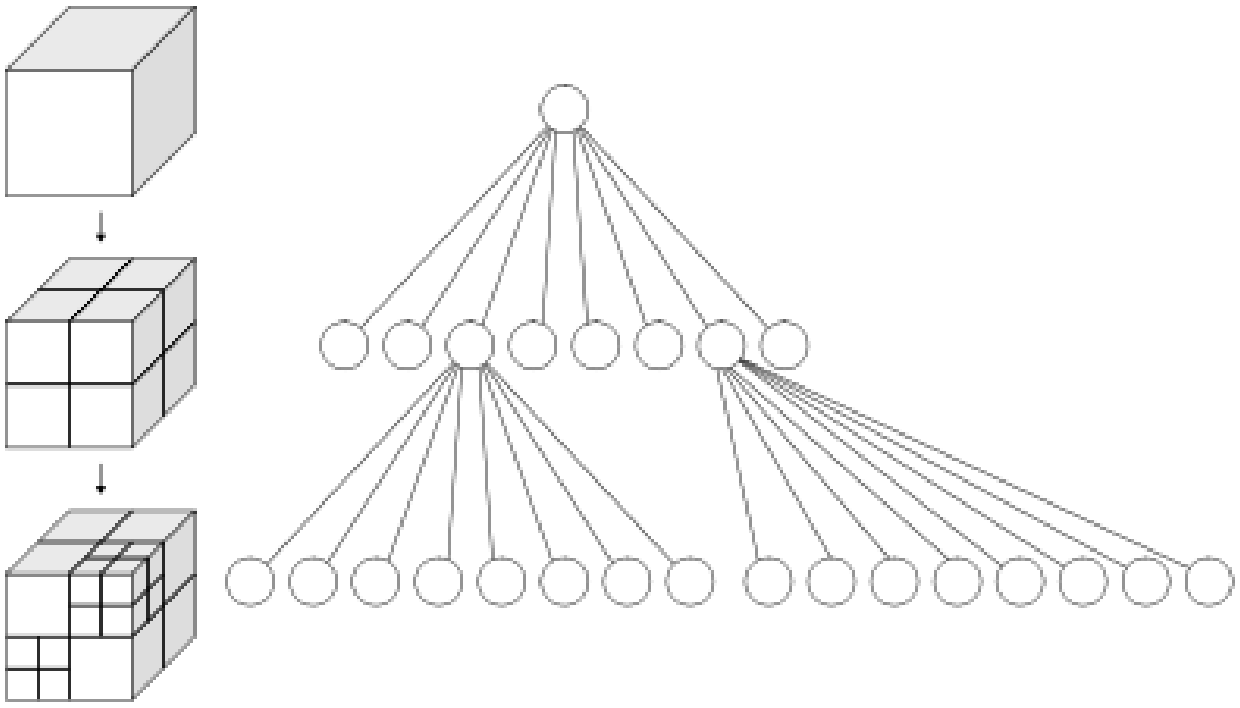


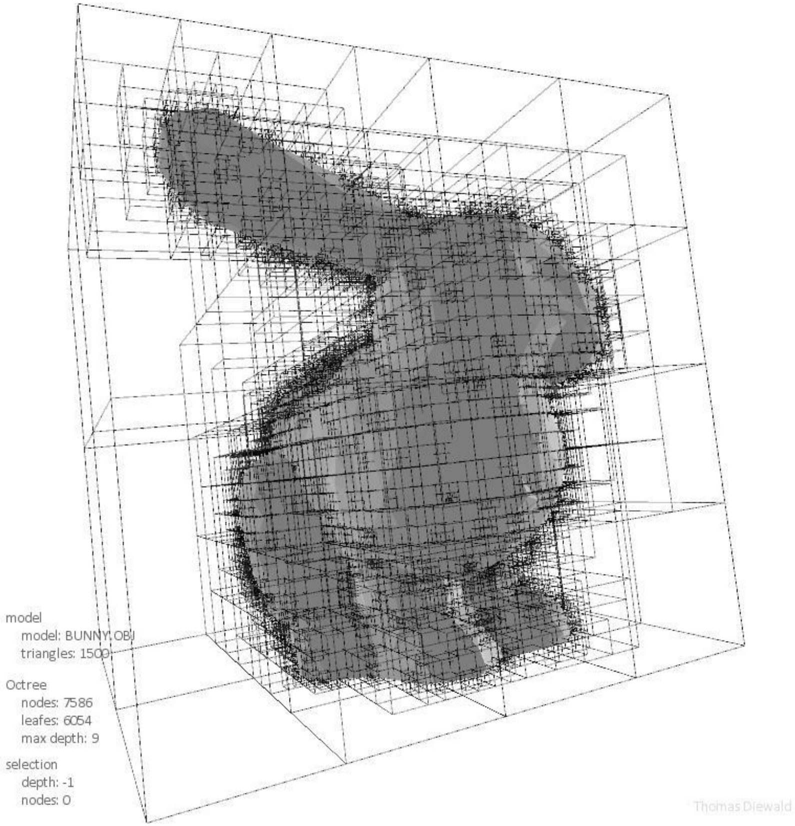
Stanford Graphics Laboratory

# Octree

The adaptive version of the voxel grid

- Significantly more space efficient
- Makes operations more cumbersome

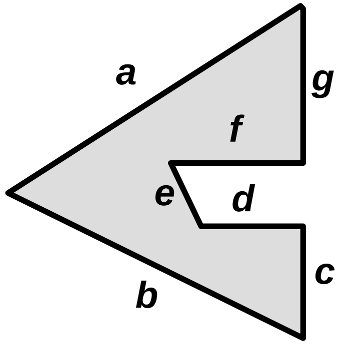




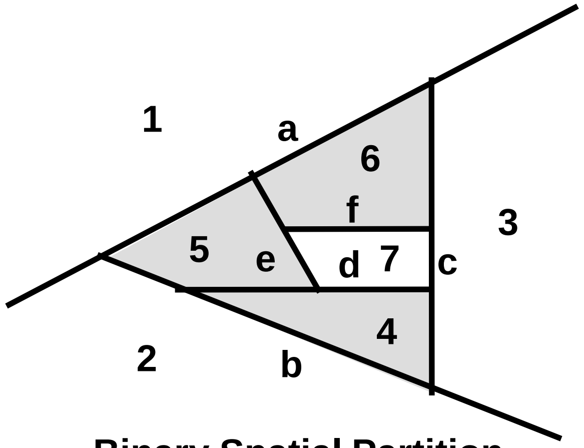
# BSP Tree

Hierarchical **B**inary **S**pace **P**artition with solid/empty cells labeled

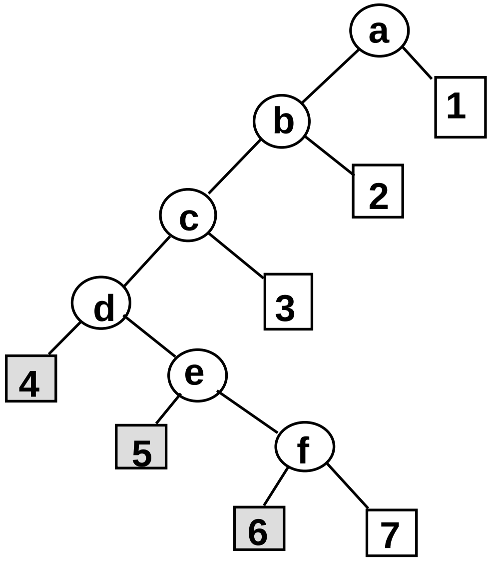
- Constructed from polygonal representations




**Object**




**Binary Spatial Partition**

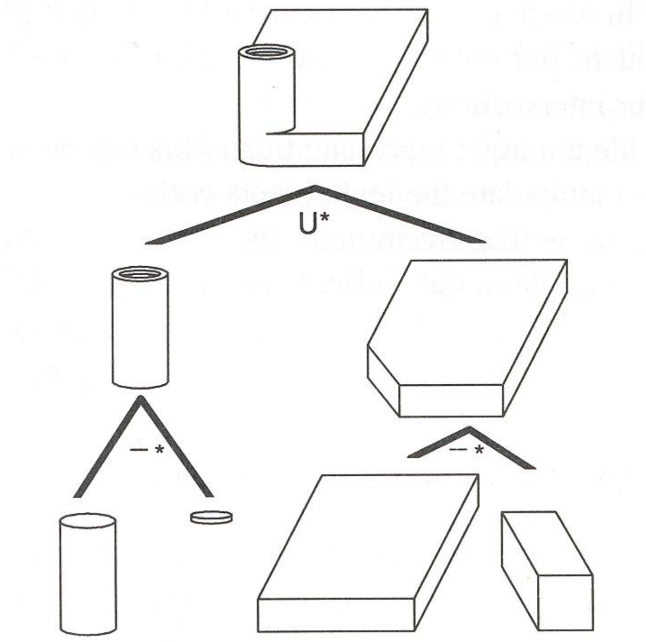



**Binary Tree**

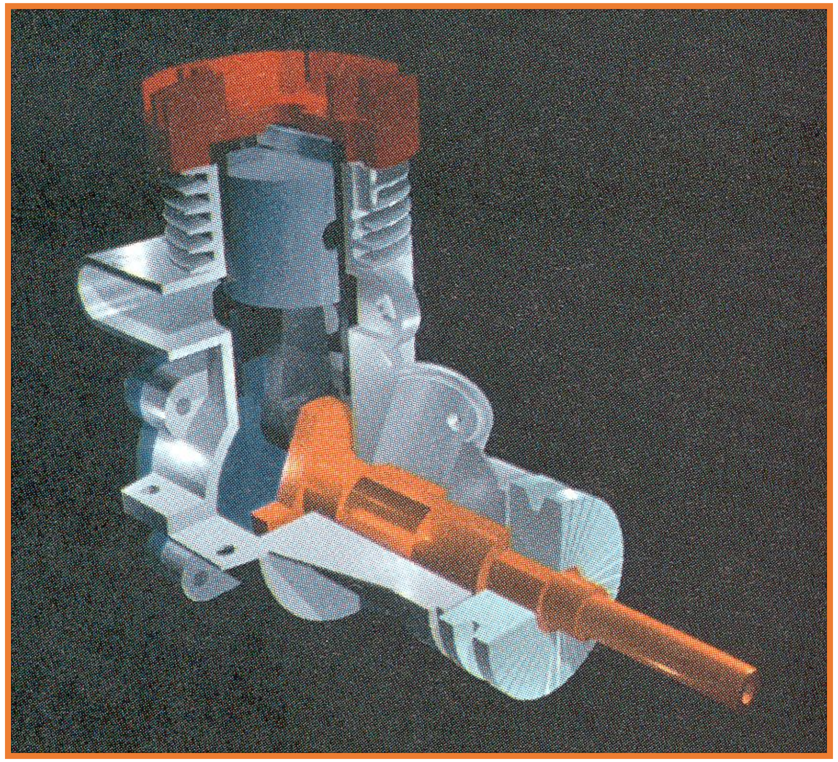
Naylor

# CSG

Constructive Solid Geometry: set operations (union, difference, intersection) applied to simple shapes




FvDFH Figure 12.27



H&B Figure 9.9

# 3D Object Representations

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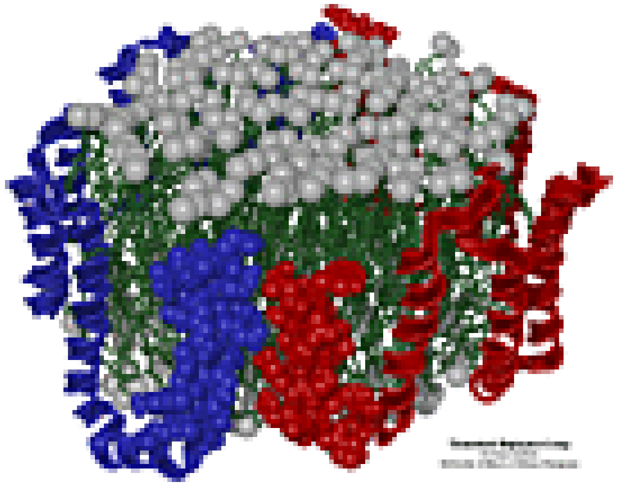
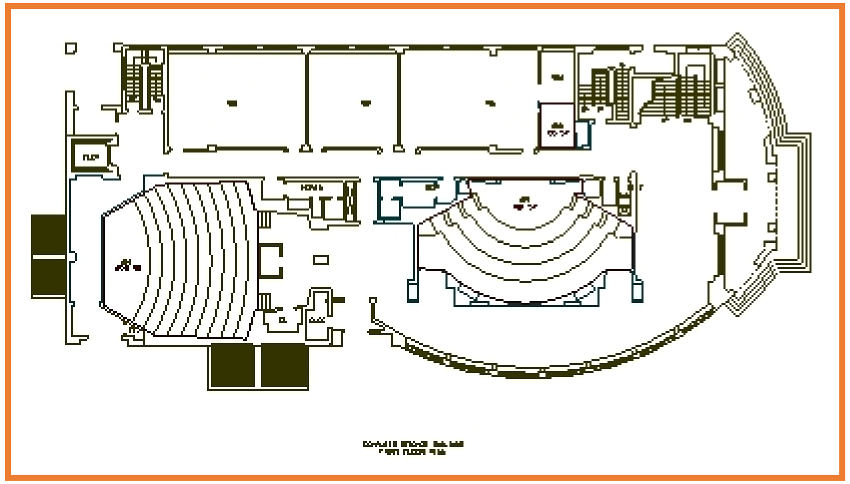
# Application Specific



Apo A-1

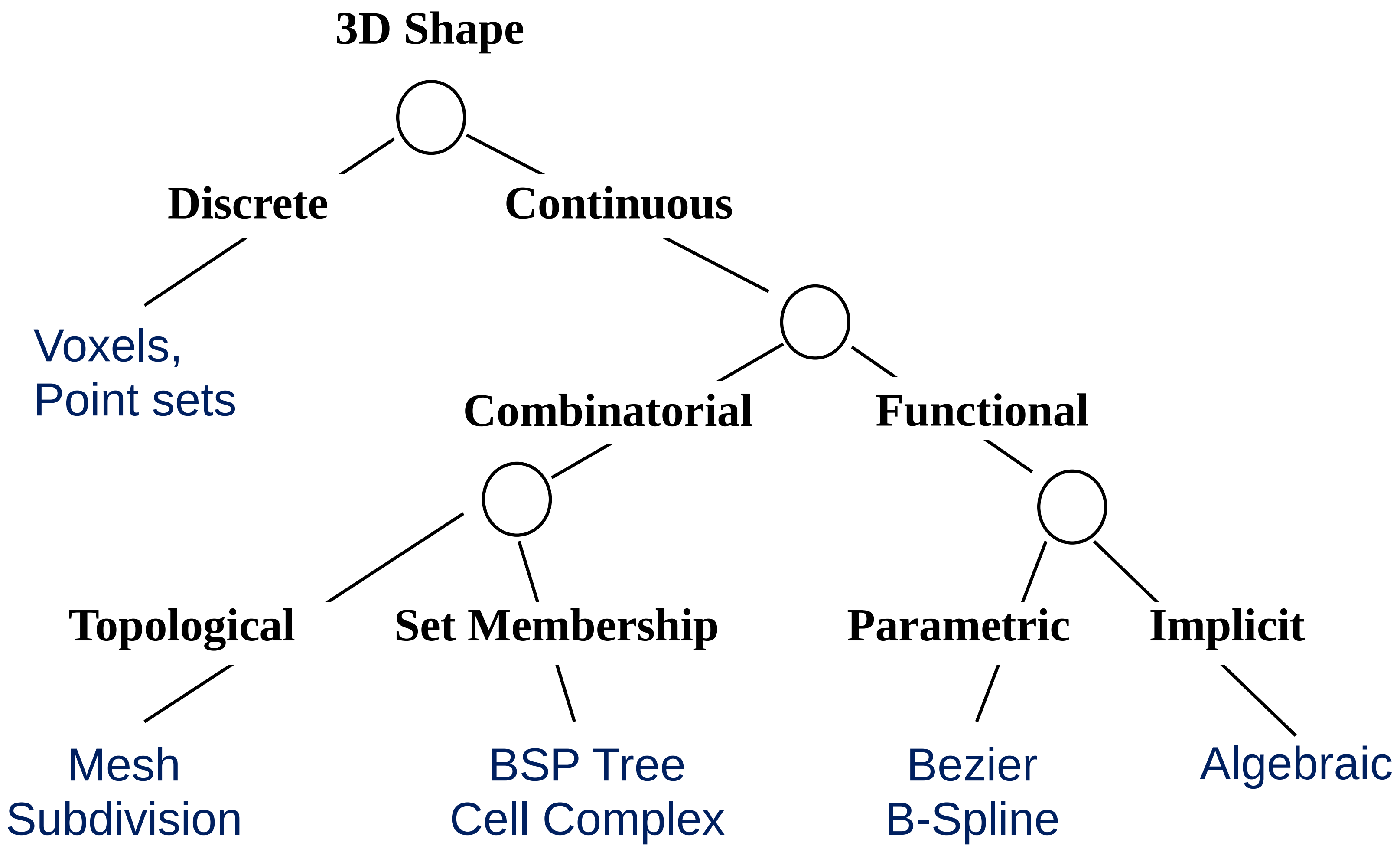
*(Theoretical Biophysics Group, University of Illinois at Urbana-Champaign)*



Architectural Floorplan

*(CS Building, Princeton University)*

# Taxonomy of 3D Representations




Naylor

# Equivalence of Representations

- Thesis:
  - Each representation has enough expressive power to model the shape of any geometric object
  - It is possible to perform all geometric operations with any representation
- Analogous to Turing-equivalence
  - Computers and programming languages are Turing-equivalent, but each has its benefits…

# Computational Differences

- Efficiency
  - Representational complexity (e.g. surface vs. volume)
  - Computational complexity (e.g. $O(n^2)$ vs $O(n^3)$ )
  - Space/time trade-offs  (e.g. tree data structures)
  - Numerical accuracy/stability (e.g. degree of polynomial)
- Simplicity
  - Ease of acquisition
  - Hardware acceleration
  - Software creation and maintenance
- Usability
  - Designer interface vs. computational engine

# Upcoming Lectures

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
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