

# Consistency Models



COS 418: Distributed Systems  
Lecture 15

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## Consistency Models

- Contract between a distributed system and the applications that run on it
- A consistency model is a set of **guarantees** made by the distributed system

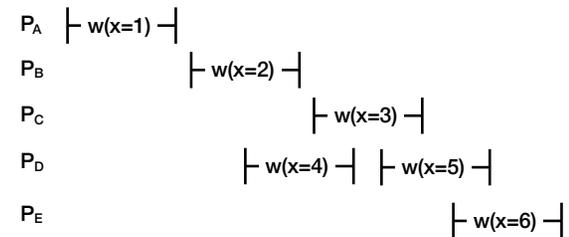
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## Linearizability

- All replicas execute operations in **some** total order
- That total order preserves the **real-time ordering** between operations
  - If operation A **completes** before operation B **begins**, then A is ordered before B in real-time
  - If neither A nor B completes before the other begins, then there is no real-time order. But there must be *some* total order.

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## Real-Time Ordering Examples



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Linearizable?

$P_A \vdash w(x=1) \dashv$   
 $P_B \quad \vdash w(x=2) \dashv$   
 $P_C \quad \quad \vdash w(x=3) \dashv$   
 $P_D \quad \quad \vdash w(x=4) \dashv \vdash w(x=5) \dashv$   
 $P_E \quad \quad \quad \vdash w(x=6) \dashv$

$P_F \vdash r(x)=1 \dashv \vdash r(x)=2 \dashv \vdash r(x)=3 \dashv \vdash r(x)=6 \dashv \vdash r(x)=5 \dashv \quad \checkmark$

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Linearizable?

$P_A \vdash w(x=1) \dashv$   
 $P_B \quad \vdash w(x=2) \dashv$   
 $P_C \quad \quad \vdash w(x=3) \dashv$   
 $P_D \quad \quad \vdash w(x=4) \dashv \vdash w(x=5) \dashv$   
 $P_E \quad \quad \quad \vdash w(x=6) \dashv$

$P_G \vdash r(x)=1 \dashv \vdash r(x)=2 \dashv \vdash r(x)=5 \dashv \vdash r(x)=6 \dashv \vdash r(x)=5 \dashv \quad \times$

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Linearizable?

$P_A \vdash w(x=1) \dashv$   
 $P_B \quad \vdash w(x=2) \dashv$   
 $P_C \quad \quad \vdash w(x=3) \dashv$   
 $P_D \quad \quad \vdash w(x=4) \dashv \vdash w(x=5) \dashv$   
 $P_E \quad \quad \quad \vdash w(x=6) \dashv$

$P_H \vdash r(x)=1 \dashv \vdash r(x)=4 \dashv \vdash r(x)=2 \dashv \vdash r(x)=3 \dashv \vdash r(x)=6 \dashv \quad \checkmark$

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Linearizable?

$P_A \vdash w(x=1) \dashv$   
 $P_B \quad \vdash w(x=2) \dashv$   
 $P_C \quad \quad \vdash w(x=3) \dashv$   
 $P_D \quad \quad \vdash w(x=4) \dashv \vdash w(x=5) \dashv$   
 $P_E \quad \quad \quad \vdash w(x=6) \dashv$

$P_I \vdash r(x)=1 \dashv \vdash r(x)=4 \dashv \vdash r(x)=5 \dashv \vdash r(x)=6 \dashv \vdash r(x)=3 \dashv \quad \times$

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## Linearizability

== “Appears to be a Single Machine”

- Single machine processes requests one by one in the order it receives them
  - Will receive requests ordered by real-time in that order
  - Will receive all requests in some order
- Atomic Multicast, Viewstamped Replication, Paxos, and RAFT provide Linearizability
- Single machine processing incoming requests one at a time also provide Linearizability ☺

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## Linearizability is Ideal?

- Hides the complexity of the underlying distributed system from applications!
  - Easier to write applications
  - Easier to write correct applications
- But, performance trade-offs

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## Stronger vs Weaker Consistency

- Stronger consistency models
  - + Easier to write applications
  - More guarantees for the system to ensure  
( Results in performance tradeoffs )
- Weaker consistency models
  - Harder to write applications
  - + Fewer guarantees for the system to ensure

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## Strictly Stronger Consistency

- A consistency model *A* is strictly stronger than *B* if it allows a strict subset of the behaviors of *B*
  - Guarantees are strictly stronger

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## Linearizability

- All replicas execute operations in **some** total order
- That total order preserves the **real-time ordering** between operations
  - If operation A **completes** before operation B **begins**, then A is ordered before B in real-time
  - If neither A nor B completes before the other begins, then there is no real-time order. But there must be *some* total order.

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## Sequential Consistency

- All replicas execute operations in **some** total order
- That total order preserves the **process ordering** between operations
  - If process P **issues** operation A before operation B, then A is ordered before B by the process order (i.e., preserves local ordering)
  - If operations A and B are done by different processes then there is no process order between them. But there must be *some* total order.

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## Sequential Consistency ≈ “Appears to be a Single Machine”

- Single machine processes requests one by one in the order it receives them
  - Will receive requests ordered by process order in that order
  - Will receive all requests in some order

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## Linearizability is strictly stronger than Sequential Consistency

Linearizability:  $\exists$  total order + real-time ordering

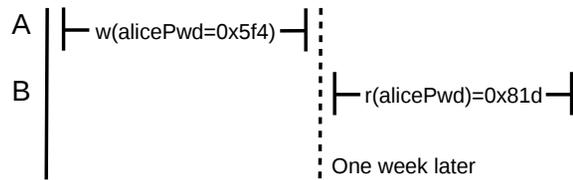
Sequential:  $\exists$  total order + process ordering

where Process ordering  $\subseteq$  Real-time ordering

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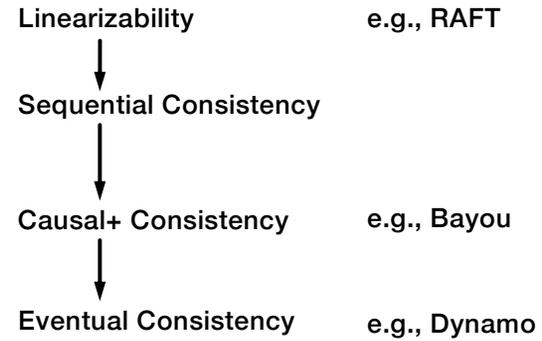
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### Sequential But Not Linearizable



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### Consistency Hierarchy



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### Causal+ Consistency

- Partially orders all operations, does not totally order them
  - Does not look like a single machine
  
- Guarantees
  - For each process,  $\exists$  an order of all writes + that process's reads
  - Order respects the happens-before ( $\rightarrow$ ) ordering of operations
  - + replicas converge to the same state
    - Skip details, makes it stronger than eventual consistency

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### Causal Consistency

1. Writes that are **potentially** causally related must be seen by all processes in same order.
  
2. Concurrent writes may be seen in a different order on different processes.

**Concurrent: Ops not causally related**

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### Causal Consistency

- Writes that are **potentially** causally related must be seen by all processes in same order.
- Concurrent writes may be seen in a different order on different processes.

Concurrent: Ops not causally related

Physical time ↓

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### Causal Consistency

Operations	Concurrent?
a, b	
b, f	
c, f	
e, f	
e, g	
a, c	
a, e	

Physical time ↓

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### Causal Consistency

Operations	Concurrent?
a, b	N
b, f	Y
c, f	Y
e, f	Y
e, g	N
a, c	Y
a, e	N

Physical time ↓

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### Causal+ But Not Sequential

$$P_A \quad | \text{w}(x=1) \quad | \quad | \text{r}(y)=0 \quad |$$

$$P_B \quad | \text{w}(y=1) \quad | \quad | \text{r}(x)=0 \quad |$$

✓ Casual+

Happens Before Order:  $w(x=1) \rightarrow r(y)=0$   
 $w(y=1) \rightarrow r(x)=0$

$P_A$  Order:  $w(x=1), r(y)=0, w(y=1)$   
 $P_B$  Order:  $w(y=1), r(x)=0, w(x=1)$

✗ Sequential

Process Ordering:  $w(x=1) \rightarrow r(y)=0$   
 $w(y=1) \rightarrow r(x)=0$

No Total Order:  $w(x=1) \rightarrow r(y)=0$   
 $w(y=1) \rightarrow r(x)=0$

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### Eventual But Not Causal+

$P_A \vdash w(x=1) \dashv \vdash \vdash w(y)=1 \dashv \vdash$   
 $P_B \vdash r(y)=1 \dashv \vdash \vdash r(x)=0 \dashv \vdash$

✓ **Eventual**      ✗ **Causal+**

As long as  $P_B$  eventually would see  $r(x)=1$  this is fine

Happens Before Ordering:  $w(x=1) \rightarrow w(y)=1$   
 $r(y)=1 \rightarrow r(x)=0$

No Order for  $P_B$ :  $w(x=1) \rightarrow w(y)=1$   
 $r(y)=1 \rightarrow r(x)=0$

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### Consistency Hierarchy

Linearizability      e.g., RAFT  
 ↓  
 Sequential Consistency  
 ↓  
 Causal+ Consistency      e.g., Bayou  
 ↓  
 Eventual Consistency      e.g., Dynamo

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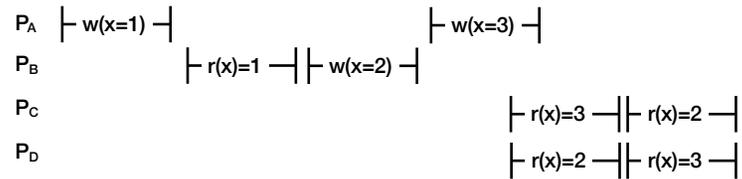
### Causal Consistency: Quiz

$P_A \vdash w(x=1) \dashv \vdash \vdash w(x=3) \dashv \vdash$   
 $P_B \vdash r(x)=1 \dashv \vdash \vdash w(x=2) \dashv \vdash$   
 $P_C \vdash r(x)=3 \dashv \vdash \vdash r(x)=2 \dashv \vdash$   
 $P_D \vdash r(x)=2 \dashv \vdash \vdash r(x)=3 \dashv \vdash$

- Valid under causal consistency
- **Why?**  $w(x=3)$  and  $w(x=2)$  are concurrent
  - So all processes don't (need to) see them in same order
- $P_C$  and  $P_D$  read the values '1' and '2' in order as potentially causally related. No 'causality' for '3'.

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### Sequential Consistency: Quiz

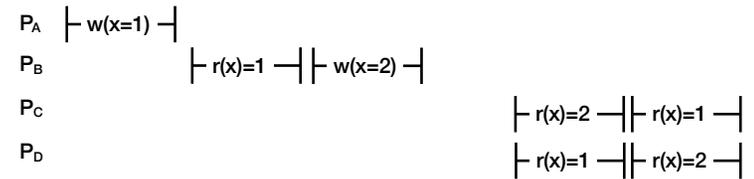


- Invalid under sequential consistency
- Why?  $P_C$  and  $P_D$  see 2 and 3 in different order
- But fine for causal consistency: 2 and 3 are not causally related

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### Causal Consistency: Quiz

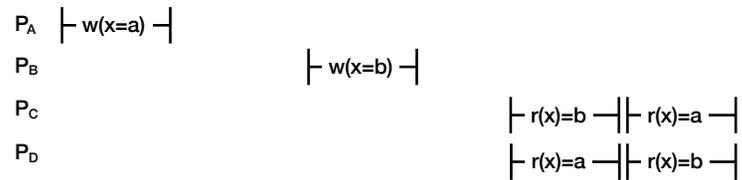


**X**  $x=2$  happens after  $x=1$

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### Causal Consistency: Quiz



✓  $P_B$  doesn't read value of 1 before writing 2

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