COS320: Compiling Techniques

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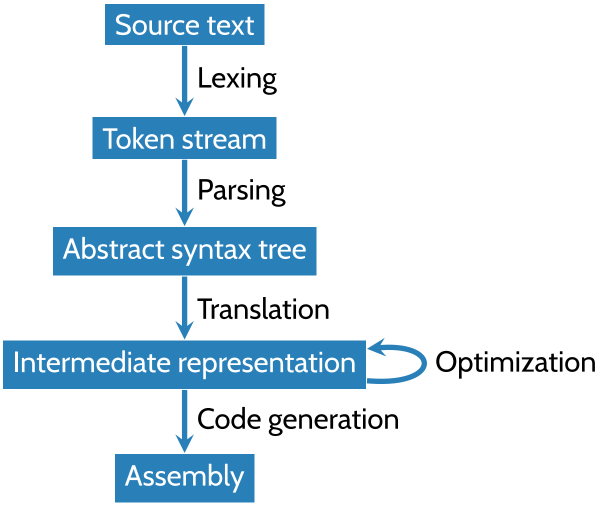
Lexing

Compiler phases (simplified)



- The *lexing* (or *lexical analysis*) phase of a compiler breaks a stream of characters (source text) into a stream of *tokens*.
  - Whitespace and comments often discarded
- A *token* is a sequence of characters treated as a unit (a *lexeme*) along with an *token type*:
  - *identifier tokens*: `x`, `y`, `foo`, ...
  - *integer tokens*: `0`, `1`, `-14`, `512`, ...
  - *if tokens*: `if`
  - ...
- Algebraic datatypes are a convenient representation for tokens

$$
\begin{aligned}
\textbf{type}\ \ token\ =\ & IDENT\ \textbf{of}\ \ string \\
\mid\ & INT\ \textbf{of}\ \ int \\
\mid\ & IF \\
\mid\ & \ldots
\end{aligned}
$$

```
// compute absolute value
if (x < 0) {
  return -x;
} else {
  return x;
}
```

$$\downarrow \text{Lexer}$$

IF, *LPAREN*, *IDENT* "x", *LT*, *INT* 0, *RPAREN*, *LBRACE*,
RETURN, *MINUS*, *IDENT* "x", *SEMI*,
RBRACE, *ELSE*, *LBRACE*,
RETURN, *IDENT* "x", *SEMI*,
RBRACE

Implementing a lexer

- Option 1: write by hand
- Option 2: use a *lexer generator*
  - Write a *lexical specification* in a domain-specific language
  - Lexer generator compiles specification to a lexer (in language of choice)
- Many lexer generators available
  - lex, flex, ocamllex, jflex, …

Formal Languages

- An *alphabet* Σ is a finite set of symbols (e.g., $\{0, 1\}$, ASCII, unicode, tokens).
- A *word* (or *string*) over Σ is a finite sequence $w = w_1 w_2 w_3 \ldots w_n$, with each $w_i \in \Sigma$.
  - The *empty word* ϵ is a word over any alphabet
  - The set of all words over Σ is typically denoted Σ^*
  - E.g., $01001 \in \{0, 1\}^*$, *embiggen* $\in \{a, ..., z\}^*$
- A *language* over Σ is a set of words over Σ
  - Integer literals form a language over $\{0, ..., 9, -\}$
  - The keywords of OCaml form a (finite) language over ASCII
  - Syntactically-valid Java programs forms an (infinite) language over Unicode

Regular expressions (regex)

- Regular expressions are one mechanism for describing languages
- Abstract syntax of regular expressions:

$$
\begin{aligned}
\text{<RegExp>} ::= {} & \epsilon && \text{Empty word}\\
\mid {} & \Sigma && \text{Letter}\\
\mid {} & \text{<RegExp><RegExp>} && \text{Concatenation}\\
\mid {} & \text{<RegExp>} \mid \text{<RegExp>} && \text{Alternative}\\
\mid {} & \text{<RegExp>}^{*} && \text{Repetition}
\end{aligned}
$$

Regular expressions (regex)

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- Abstract syntax of regular expressions:

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	$\mid \Sigma$	Letter
	$\mid$ <RegExp><RegExp>	Concatenation
	$\mid$ <RegExp>$\mid$<RegExp>	Alternative
	$\mid$ <RegExp>*	Repetition

- Meaning of regular expressions:

$$
\begin{aligned}
\mathcal{L}(\epsilon) &= \{\epsilon\} \\
\mathcal{L}(a) &= \{a\} \\
\mathcal{L}(R_1 R_2) &= \{uv : u \in \mathcal{L}(R_1) \wedge v \in \mathcal{L}(R_2)\} \\
\mathcal{L}(R_1 | R_2) &= \mathcal{L}(R_1) \cup \mathcal{L}(R_2) \\
\mathcal{L}(R^*) &= \{\epsilon\} \cup \mathcal{L}(R) \cup \mathcal{L}(RR) \cup \mathcal{L}(RRR) \cup \ldots
\end{aligned}
$$

`ocamllex` regex concrete syntax

- 'a': letter
- "abc": string (equiv. `'a''b''c'`)
- R+: one or more repetitions of R (equiv. `RR*`)
- R?: zero or one R (equiv. `R|`ϵ)
- `['a'-'z']`: character range (equiv. `'a'|'b'|...|'z'`)
- `R as x`: bind string matched by `R` to variable `x`

Lexer generators

Lexer generators take as input a lexical specification, and output code that tokenizes a character stream w.r.t. that specification

Example lexical specification:

$$\overbrace{\textbf{identifier}}^{\text{token type}} = \overbrace{[a-zA-Z][a-zA-Z0-9]^*}^{\text{pattern}}$$

$$\textbf{integer} = [1-9][0-9]^*$$

$$\textbf{plus} = +$$

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- "foo+42+bar" $\rightarrow$ $\underbrace{\textbf{identifier}}_{\text{token type}}$ $\underbrace{\text{“foo”}}_{\text{lexeme}}$, **plus** "+", **integer** "42", **plus** "+", **identifier** "bar"

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- "foo+42+bar" $\rightarrow$ $\underbrace{\textbf{identifier}}_{\text{token type}}$ $\underbrace{\text{"foo"}}_{\text{lexeme}}$, **plus** "+", **integer** "42", **plus** "+", **identifier** "bar"
- Typically, lexical spec associates an *action* to each token type, which is code that is evaluted on the lexeme (often: produce a token value)

Disambiguation

- May be more than one way to lex a string:

$$
\begin{aligned}
IF &= \texttt{if} \\
IDENT &= \texttt{[a-zA-Z][a-zA-Z0-9]}^* \\
INT &= \texttt{[1-9][0-9]}^* \\
LT &= \texttt{<} \\
&\ldots
\end{aligned}
$$

  - Input string `ifx<10`: $\boxed{\texttt{IDENT “ifx”, LT, INT 10}}$ *or* $\boxed{\texttt{IF, IDENT “x”, LT, INT 10}}$?
  - Input string `if x<9`: $\boxed{\texttt{IF, IDENT “x”, LT, INT 9}}$ *or* $\boxed{\texttt{IDENT “if”, IDENT “x”, LT, INT 9}}$?

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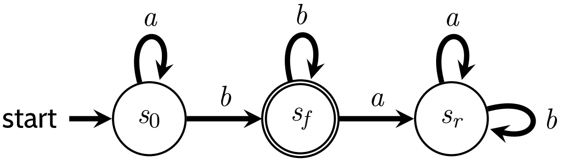
  - Input string `ifx<10`: $\boxed{\texttt{IDENT “ifx”, LT, INT 10}}$ *or* $\boxed{\texttt{IF, IDENT “x”, LT, INT 10}}$?
  - Input string `if x<9`: $\boxed{\texttt{IF, IDENT “x”, LT, INT 9}}$ *or* $\boxed{\texttt{IDENT “if”, IDENT “x”, LT, INT 9}}$?
- Two rules sufficient to disambiguate (remember these!)
  1. The lexer is greedy: always prefer longest match
  2. Order matters: prefer earlier patterns

Lexer generator pipeline

- Lexical specification is compiled to a *deterministic finite automaton* (DFA), which can be executed efficiently
- Typical pipeline: lexical specification $\rightarrow$ *non*deterministic FA $\rightarrow$ DFA
- Kleene's theorem: regular expressions, NFAs, and DFAs describe the same class of languages
    - A language is *regular* if it is accepted by a regular expression (equiv., NFA, DFA).

Deterministic finite automata (DFA)

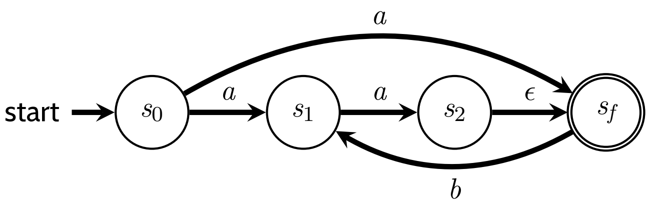


A *deterministic finite automaton* (DFA) $A = (Q, \Sigma, \delta, s, F)$ consists of

- Q: finite set of states
- Σ: finite alphabet
- $\delta : Q \times \Sigma \to Q$: transition function
  - Every state has *exactly* one outgoing edge per letter
- $s \in Q$: initial state
- $F \subseteq Q$: final (accepting) states

DFA accepts a string $w = w_1 \ldots w_n \in \Sigma^*$ iff $\delta(\ldots\delta(\delta(s, w_1), w_2), \ldots, w_n) \in F$.

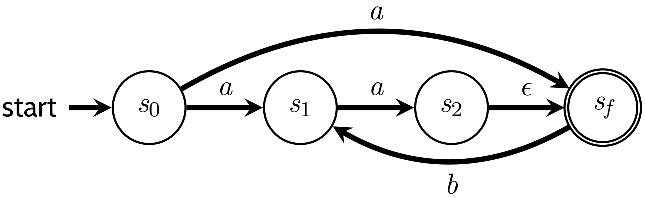
Non-deterministic finite automata



A *non-deterministic finite automaton* (NFA) $A = (Q, \Sigma, \Delta, s, F)$ generalization of a DFA, where

- $\Delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times Q$: transition *relation*
  - A state can have *more than one* outgoing edge for a given letter
  - A state can have *no* outgoing edges for a given letter
  - A state can have ϵ-transitions (read no input, but change state)

Non-deterministic finite automata



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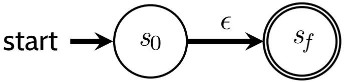
  - A state can have *more than one* outgoing edge for a given letter
  - A state can have *no* outgoing edges for a given letter
  - A state can have ϵ-transitions (read no input, but change state)

NFA accepts a string $w = w_1 \dots w_n \in \Sigma^*$ iff there exists a w-labeled path from s to an final state (i.e., there is some sequence $(q_0, u_1, q_1), (q_1, u_2, q_2), \dots, (q_{m-1}, u_m, q_m)$ with $q_0 = s$, $q_m \in F$, and $u_1 u_2 \dots u_m = w$.

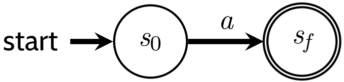
Regex $\rightarrow$ NFA

Case: ϵ (empty word)



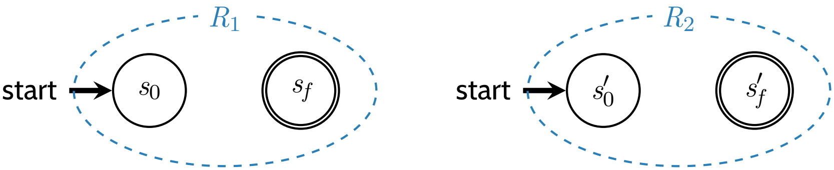
Regex $\rightarrow$ NFA

Case: a (letter)



Regex $\rightarrow$ NFA

Case: $R_1 R_2$ (concatenation)



Regex $\rightarrow$ NFA

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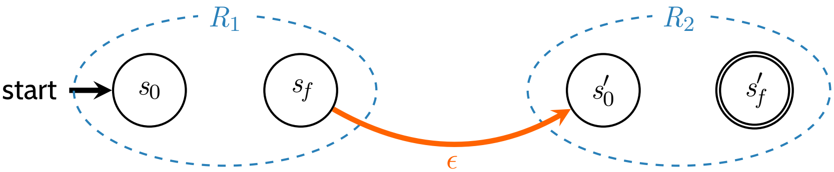
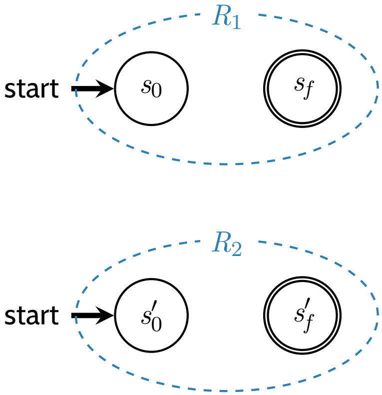


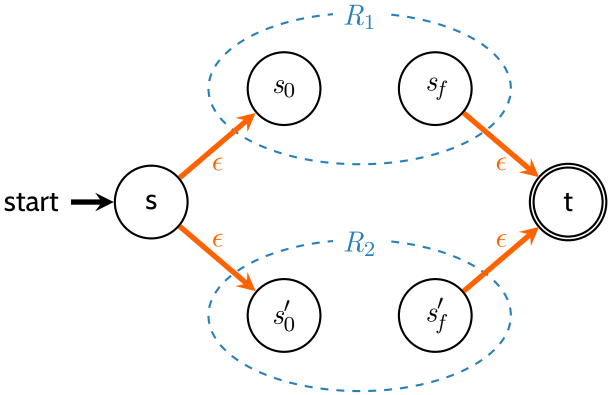
Regex $\rightarrow$ NFA

Case: $R_1 | R_2$ (alternative)



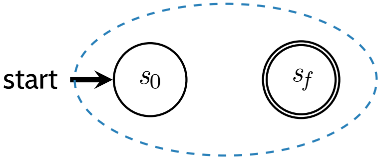
Regex $\rightarrow$ NFA

Case: $R_1 | R_2$ (alternative)



Regex $\to$ NFA

Case: R^* (iteration)



Regex $\rightarrow$ NFA

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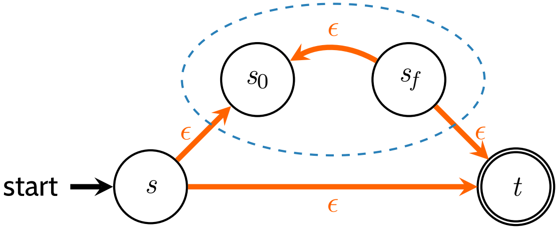
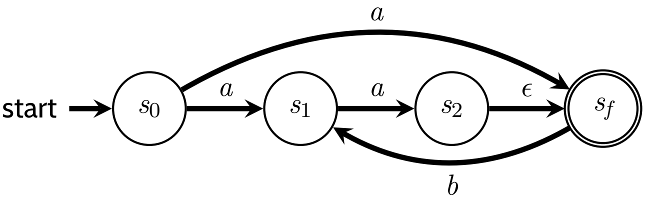
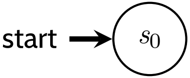


NFA $\rightarrow$ DFA

- For any NFA, there is a DFA that recognizes the same language
- **Intuition**: the DFA simulates all possible paths of the NFA simultaneously
  - There is an unbounded number of paths *but* we only care about the "end state" of each path, not its history
  - States of the DFA track the set of possible states the NFA could be in
  - DFA accepts when *some* path accepts

NFA $\rightarrow$ DFA





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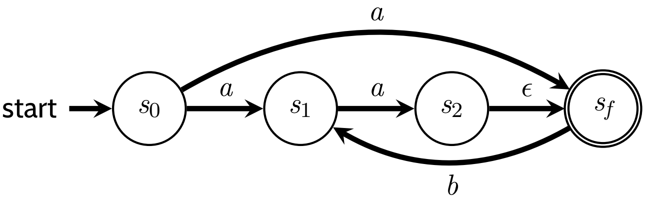
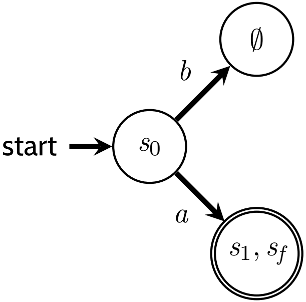
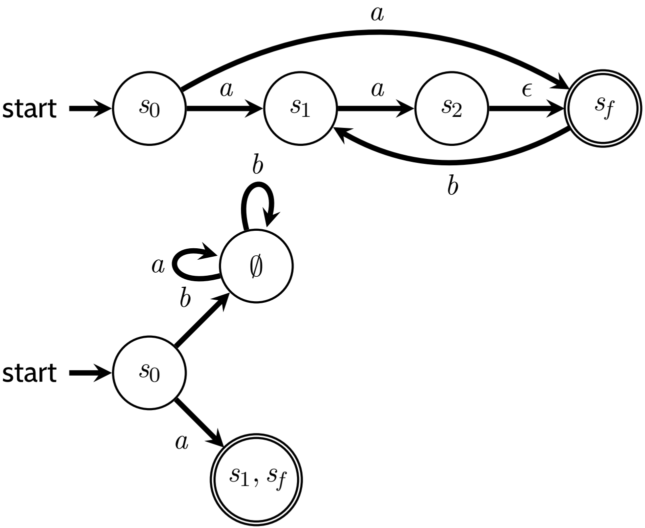




NFA → DFA



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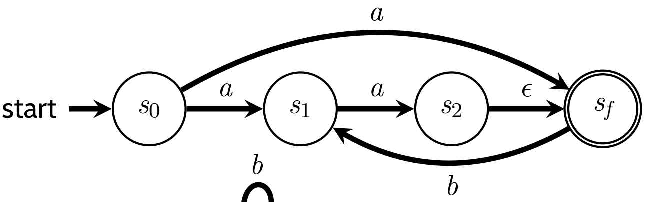
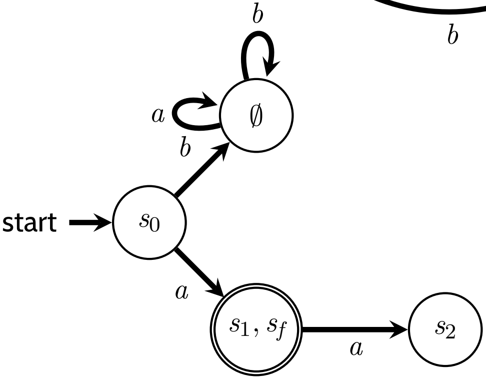
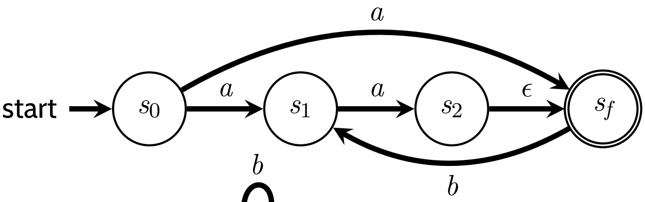
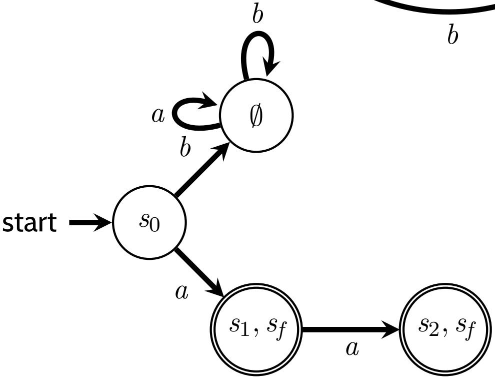

start
s_0
a
s_1
a
s_2
ϵ
s_f
a
b


start
s_0
$\emptyset$
a
b
b
a
s_1, s_f
a
s_2

NFA $\rightarrow$ DFA





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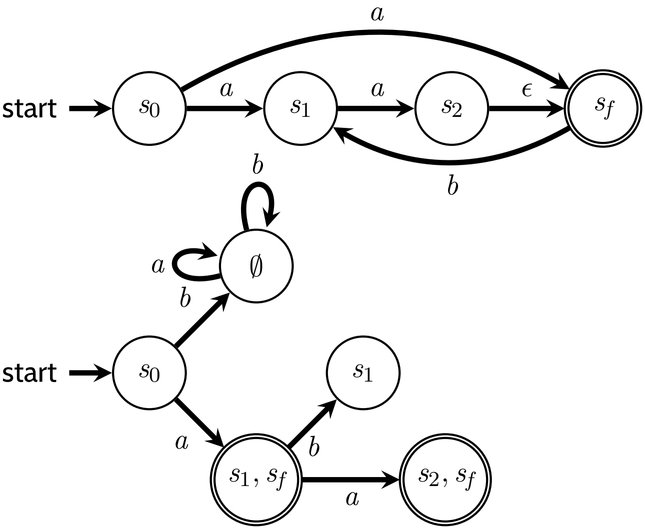

start
s_0
s_1
s_2
s_f
a
a
a
ϵ
b
$\emptyset$
a
b
b
start
s_0
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s_1, s_f
b
s_1
a
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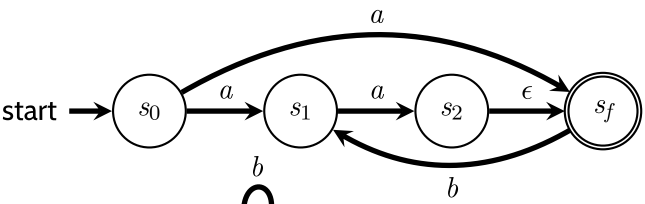
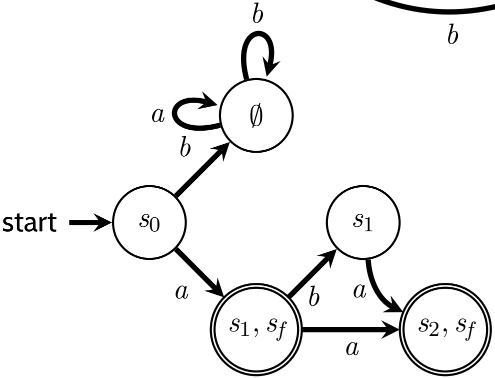
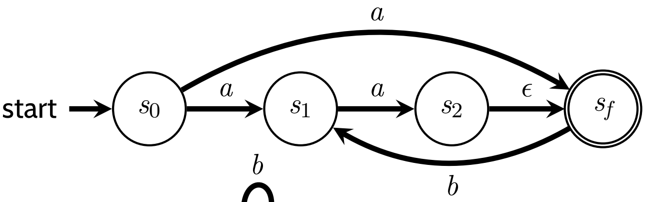
NFA $\rightarrow$ DFA





NFA → DFA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ϵ
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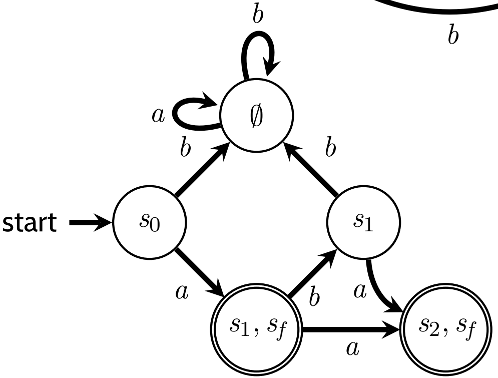
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s_2, s_f
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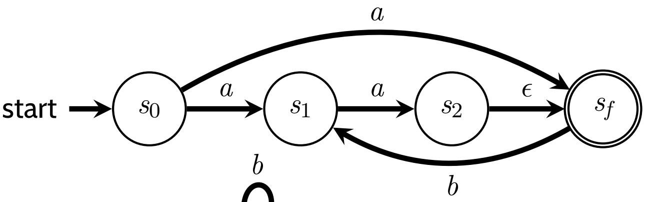
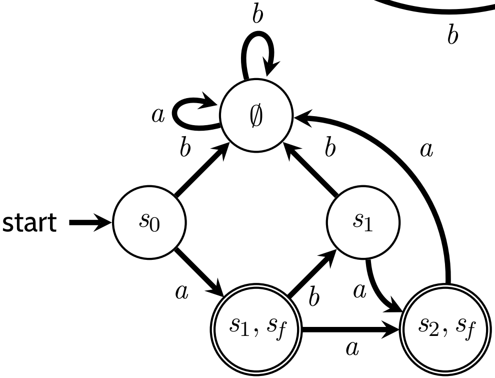
b
b
b
a
b
a
a

NFA $\rightarrow$ DFA





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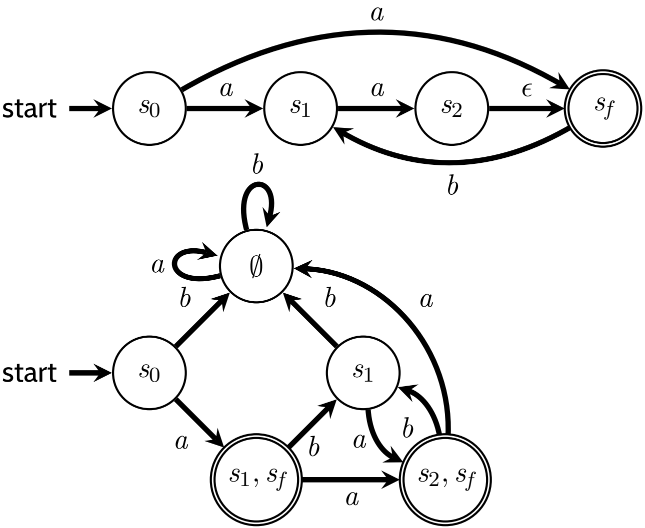
start
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s_1
s_2
s_f
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a
ϵ
b
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s_1
s_1, s_f
s_2, s_f
a
b
b
a
b
a
a
b
b
a
a

NFA $\rightarrow$ DFA, formally

- Have: NFA $A = (Q, \Sigma, \delta, s, F)$. Want: DFA $A' = (Q', \Sigma, \delta', s', F')$ that accepts same language.
- For any $S \subseteq Q$, define the ϵ-closure of S to be the set of states reachable from S by ϵ transitions (incl. S)
  ϵ-$cl(S)$ = smallest set that contains S and such that $\forall(q, \epsilon, q') \in \Delta$, $q \in S \Rightarrow q' \in S$

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- Construct DFA as follows:
  - $Q' =$ set of all ϵ-closed subsets of Q
  - $\delta'(S, a) = \epsilon$-closure of $\{q_2 : \exists q_1 \in S.(q_1, a, q_2) \in \Delta\}$
  - $s' = \epsilon$-closure of $\{s\}$
  - $F' = \{S \in Q' : S \cap F \neq \emptyset\}$

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- Crucial optimization: only construct states that are reachable from s'

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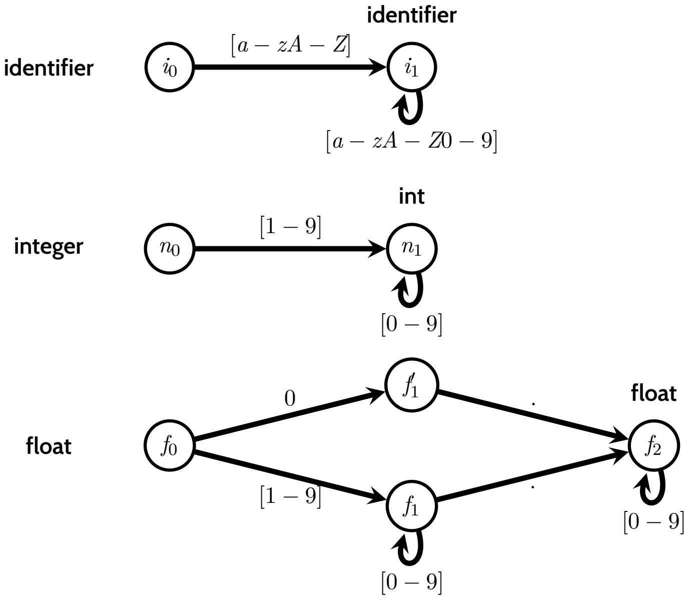
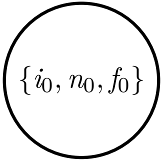
- Crucial optimization: only construct states that are reachable from s'
- Less crucial, still important: minimize DFA (Hopcroft's algorithm, $O(n \log n)$)

Lexical specification $\to$ String classifier

- Want: partial function *match* mapping strings to token types
  - *match*(s) = highest-priority token type whose pattern matches s (undef otherwise)
- Process:
  1. Convert each pattern to an NFA. Label accepting states w/ token types.
  2. Take the union of all NFAs
  3. Convert to DFA
     - States of the DFA labeled with *sets* of token types.
     - Take highest priority.

$$\textbf{identifier} = [a - zA - Z][a - zA - Z0 - 9]^*$$
$$\textbf{integer} = [1 - 9][0 - 9]^*$$
$$\textbf{float} = ([1 - 9][0 - 9]^*|0).[0 - 9]^+$$


identifier
identifier
[a − zA − Z]
i0
i1
[a − zA − Z0 − 9]
int
integer
[1 − 9]
n0
n1
[0 − 9]
float
float
0
f'1
.
[1 − 9]
f0
f1
.
f2
[0 − 9]
[0 − 9]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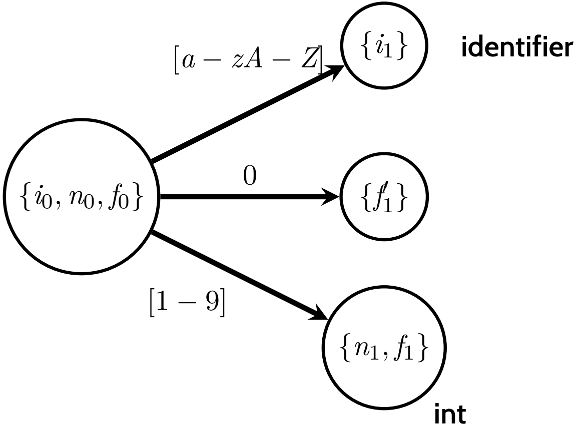
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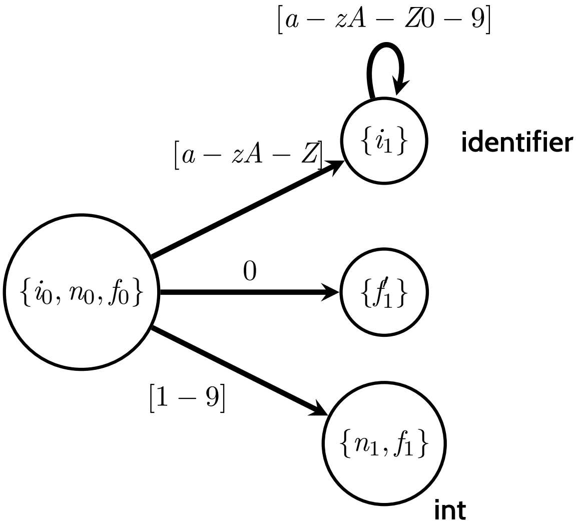
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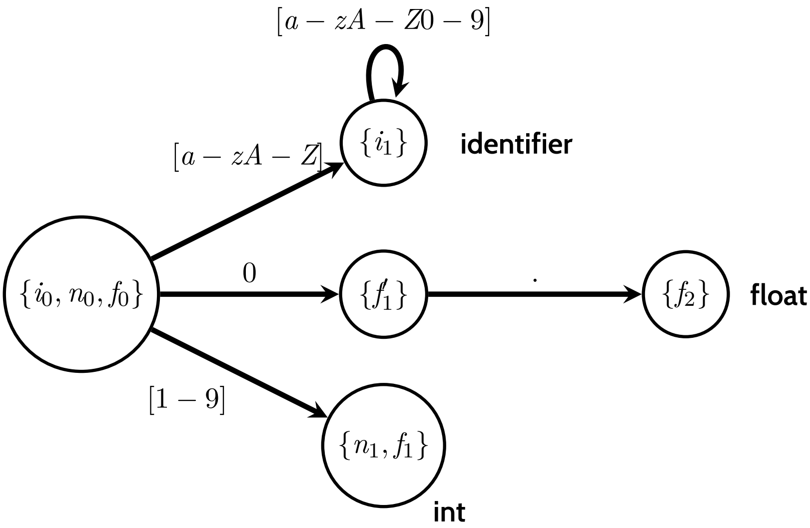
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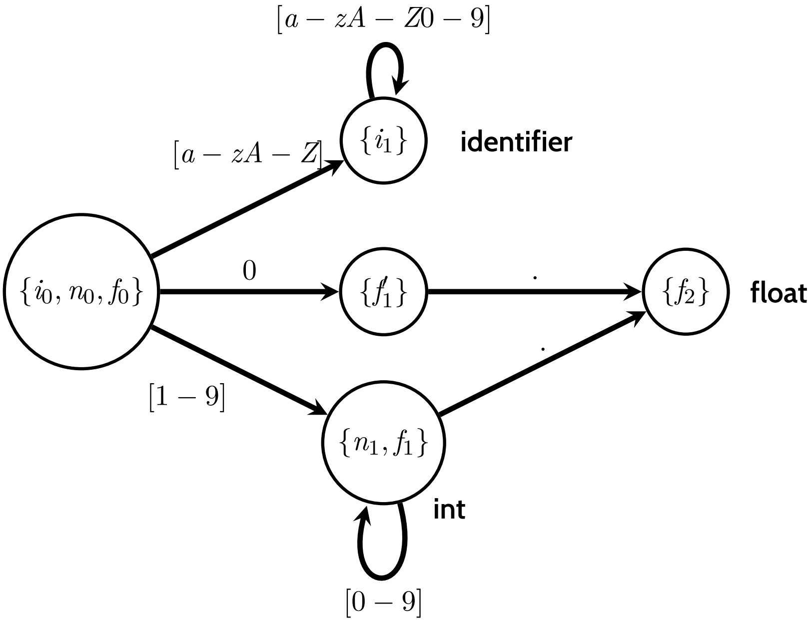
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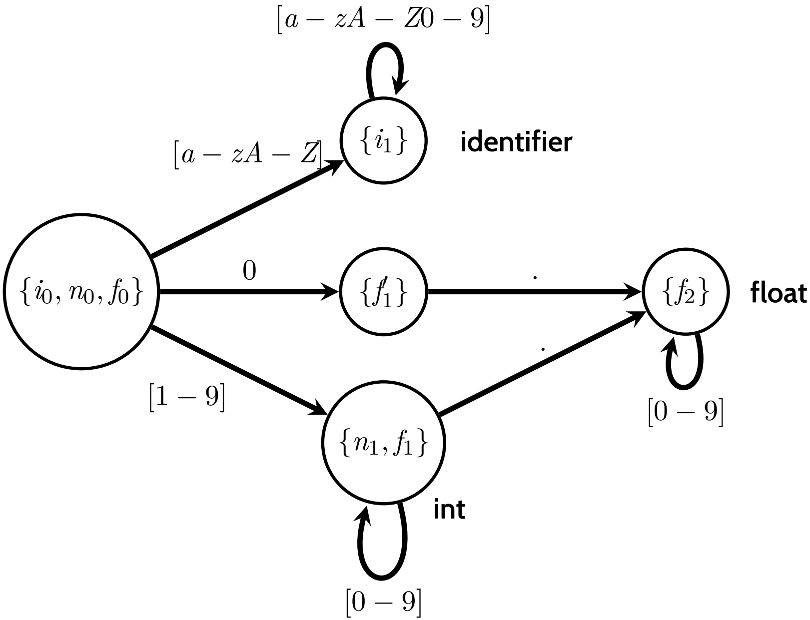

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int
$[0-9]$