DENOTATIONAL SEMANTICS

Syntax

Num $n ::= 0 \mid 1 \mid 2 \mid 3 \mid \ldots$

Var $x ::= x \mid y \mid z \mid \ldots$

Exp $e ::= n \mid x \mid e_1 + e_2 \mid \text{let } x := e_1 \text{ in } e_2$

Prog $p ::= e$

$\text{Val} = \mathbb{Z}$

$\text{Env} = \text{Var} \to \text{Val}$

$\mathcal{N} : \text{Num} \to \text{Val}$

$\mathcal{E} : \text{Exp} \to \text{Env} \to \text{Val}$

$\mathcal{P} : \text{Prog} \to \text{Val}$

$\mathcal{E}[\![n]\!] = \lambda\rho.\ \mathcal{N}[\![n]\!]$

$\mathcal{E}[\![x]\!] = \lambda\rho.\ \rho\, x$

$\mathcal{E}[\![e_1 + e_2]\!] = \lambda\rho.\ \mathcal{E}[\![e_1]\!]\rho + \mathcal{E}[\![e_2]\!]\rho$

$\mathcal{E}[\![\text{let } x := e_1 \text{ in } e_2]\!] = \lambda\rho.\ \mathcal{E}[\![e_2]\!](\text{upd}(\rho, x, \mathcal{E}[\![e_1]\!]\rho))$

$\quad \text{upd}(\rho, x, v) = \lambda y.\ \text{if } x \equiv y \text{ then } v \text{ else } \rho\, y$

$\mathcal{P}[\![e]\!] = \mathcal{E}[\![e]\!](\lambda x.\ 0)$

$\mathcal{E}(\text{NUM } n) = \lambda\rho.\mathcal{N}(n)$

$\mathcal{E}(\text{VAR } x) = \lambda\rho.\ \rho\, x$

$\mathcal{E}(\text{PLUS } e_1\ e_2) = \ldots$

STRUCTURED CONTROL FLOW

$$\text{Cmd} \quad c ::= x := e \mid c_1 ; c_2 \mid \text{if } e \text{ then } c_1 \text{ else } c_2 \mid \text{while } e \text{ do } c$$

$$\mathcal{C} : \text{Cmd} \to (\text{Env} \to \text{Env})$$

$$\mathcal{C}[\![x := e]\!] \rho = \text{upd}(\rho, x\ \mathcal{E}[\![e]\!] \rho)$$

$$\mathcal{C}[\![c_1 ; c_2]\!] \rho = \mathcal{C}[\![c_2]\!] (\mathcal{C}[\![c_1]\!] \rho)$$

$$\mathcal{C}[\![\text{if } e \text{ then } c_1 \text{ else } c_2]\!] \rho = \text{if } \mathcal{E}[\![e]\!] \rho \neq 0 \text{ then } \mathcal{C}[\![c_1]\!] \rho \text{ else } \mathcal{C}[\![c_2]\!] \rho$$

$$\mathcal{C}[\![\text{while } e \text{ do } c]\!] \rho = (\text{rec } f(r) = \text{if } \mathcal{E}[\![e]\!] r \neq 0 \text{ then } f(\mathcal{C}[\![c]\!] r \text{ else } r)\ \rho$$

UNSTRUCTURED CONTROL FLOW

$$\text{Cmd} \quad c ::= x := e \mid c_1 ; c_2 \mid \text{if } e \text{ then } c \mid \text{loop } c \mid \text{continue} \mid \text{break}$$

$$A$$

$$K = \text{Env} \to A$$

$$\cancel{\mathcal{C} : K \to K \to K \to K} \quad \mathcal{C} : \text{Cmd} \to (K \to K \to K \to K)$$

$$\mathcal{C}[\![x := e]\!] k_1\, k_2\, k_3 = \lambda \rho .\ k_1(\text{upd}(\rho, x, \mathcal{E}[\![e]\!] \rho))$$

$$\mathcal{C}[\![\text{if } e \text{ then } c]\!] k_1\, k_2\, k_3 = \lambda \rho .\ \text{if } \mathcal{E}[\![e]\!] \neq 0 \text{ then } \mathcal{C}[\![c_1]\!] k_1\, k_2\, k_3\, \rho \text{ else } k_1\, \rho$$

$$\mathcal{C}[\![\text{loop } c]\!] k_1\, k_2\, k_3 = (\text{rec } f(k) = \mathcal{C}[\![c]\!] (f\, k)\, (f\, k)\, k_1)\ k_1$$

$$\mathcal{C}[\![\text{continue}]\!] k_1\, k_2\, k_3 = k_2$$

$$\mathcal{C}[\![\text{break}]\!] k_1\, k_2\, k_3 = k_3$$

$$A = \text{Val}$$

$$\mathcal{P} : \text{Cmd} \to A$$

$$\mathcal{P}[\![c]\!] = \mathcal{C}[\![c]\!] (\lambda \rho .\ \rho(x_0))\ (\lambda \rho . 0)\ (\lambda \rho . 0)\ (\lambda x . 0)$$