3D Modeling

COS 426, Spring 2021

Princeton University
Felix Heide

Syllabus

I. Image processing
II. Modeling
III. Rendering
IV. Animation

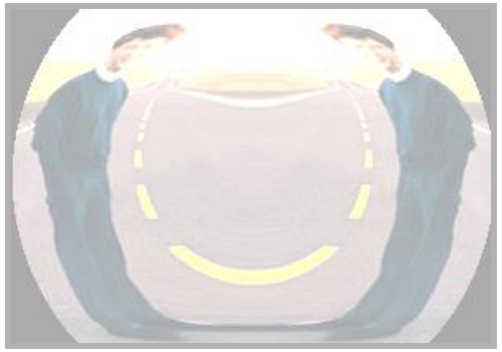
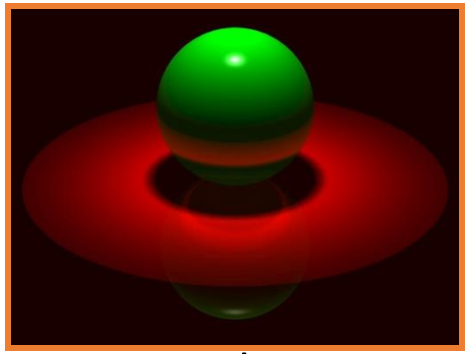
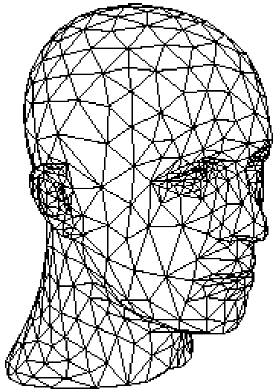
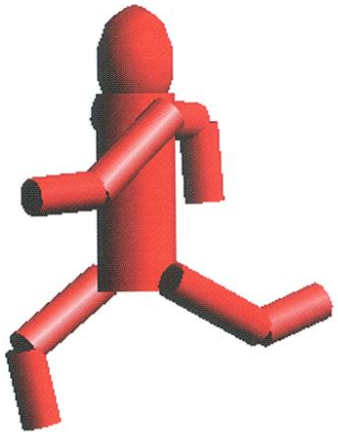


Image Processing
(Rusty Coleman, CS426, Fall99)

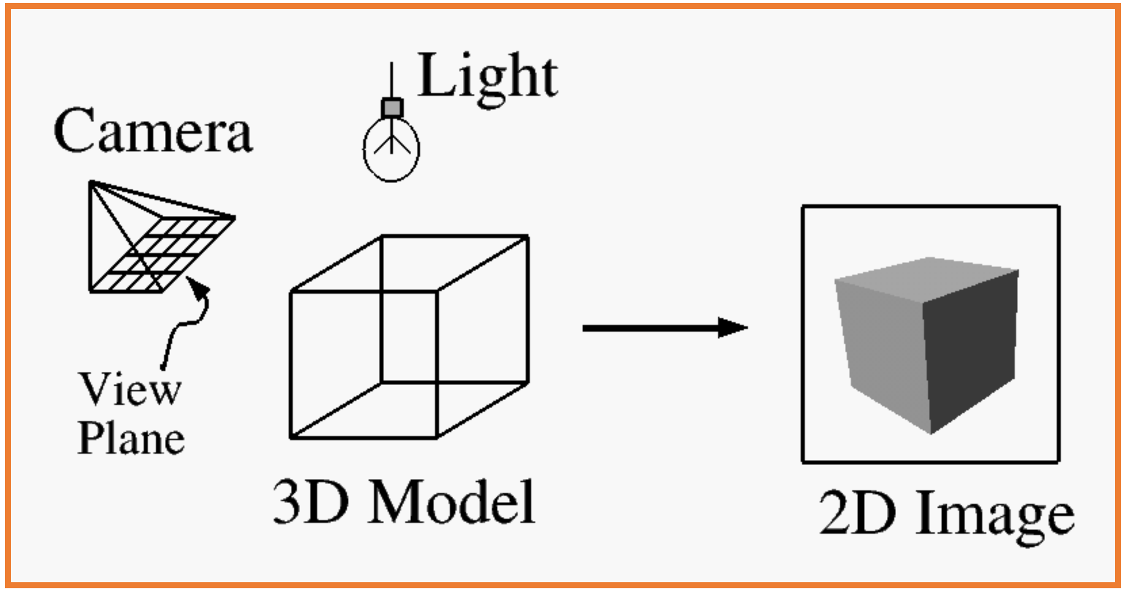

Rendering
(Michael Bostock, CS426, Fall99)


Modeling
(Denis Zorin, CalTech)


Animation
(Angel, Plate 1)

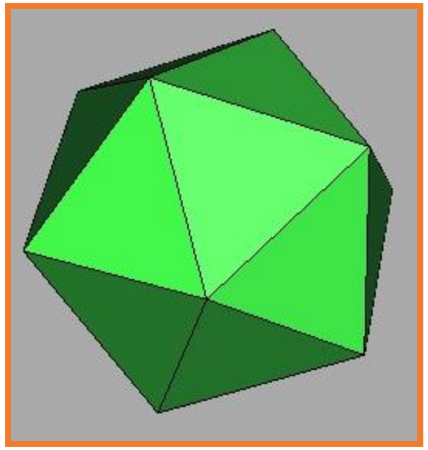
What is 3D Modeling?

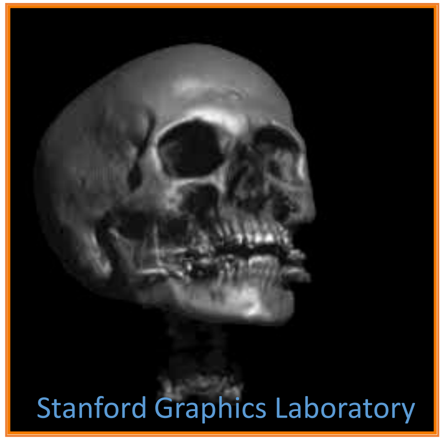
- Topics in computer graphics
  - Imaging = *representing 2D images*
  - Modeling = ***representing 3D objects***
  - Rendering = *constructing 2D images from 3D models*
  - Animation = *simulating changes over time*

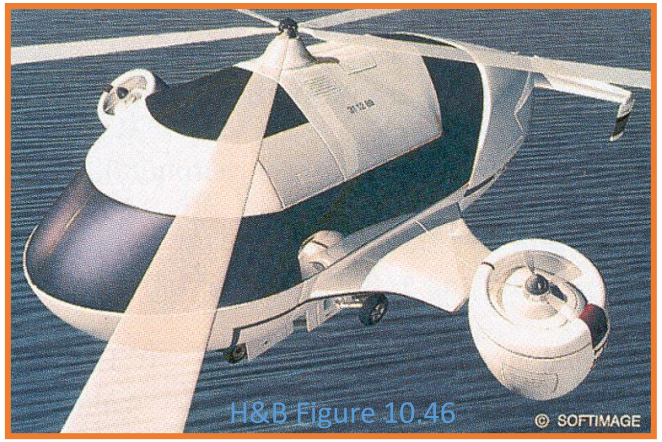


Modeling

- How do we ...
  - Represent 3D objects in a computer?
  - Acquire computer representations of 3D objects?
  - Manipulate these representations?







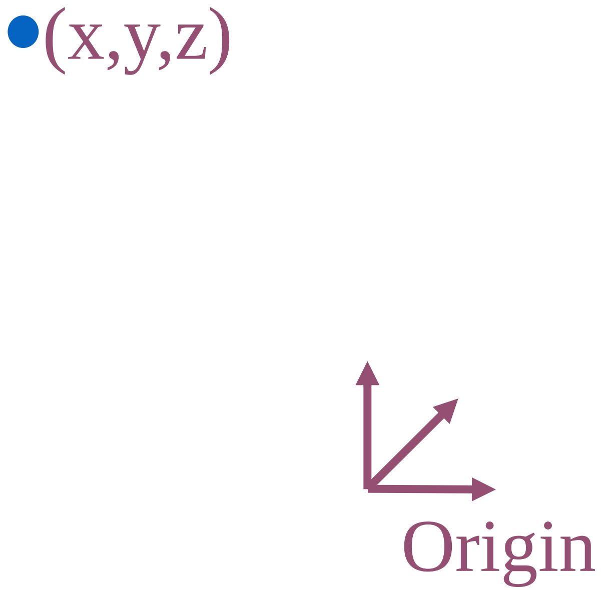
Modeling Background

- Scene is usually approximated by 3D primitives
  - Point
  - Vector
  - Line segment
  - Ray
  - Line
  - Plane
  - Polygon

3D Point

- Specifies a location
  - Represented by three coordinates
  - Infinitely small

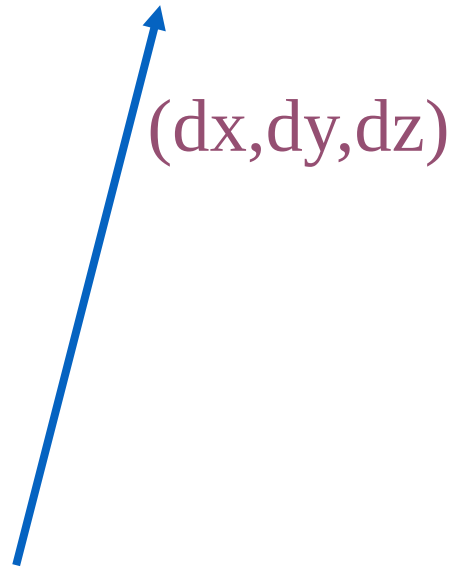
```
typedef struct {
    Coordinate x;
    Coordinate y;
    Coordinate z;
} Point;
```



3D Vector

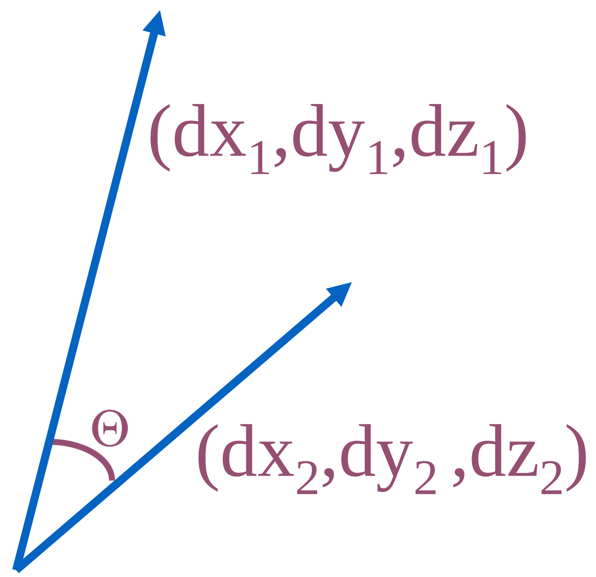
- Specifies a direction and a magnitude
  - Represented by three coordinates
  - Magnitude ||V|| = sqrt(dx dx + dy dy + dz dz)
  - Has no location

```
typedef struct {
    Coordinate dx;
    Coordinate dy;
    Coordinate dz;
} Vector;
```



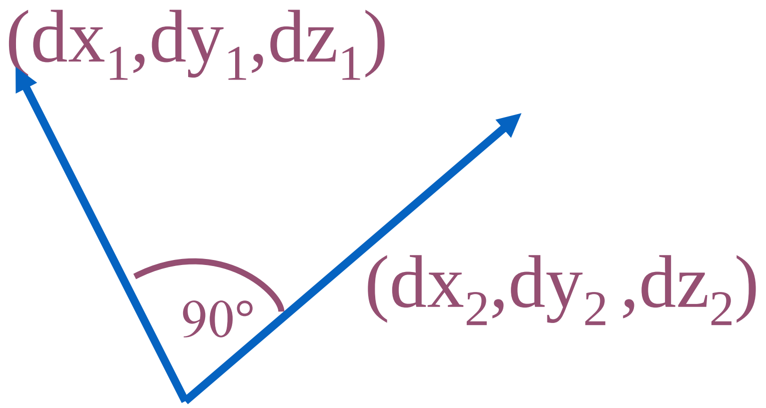
3D Vector

- Dot product of two 3D vectors
  - $V_1 \cdot V_2 = ||V_1|| \; || V_2 || \cos(\Theta)$



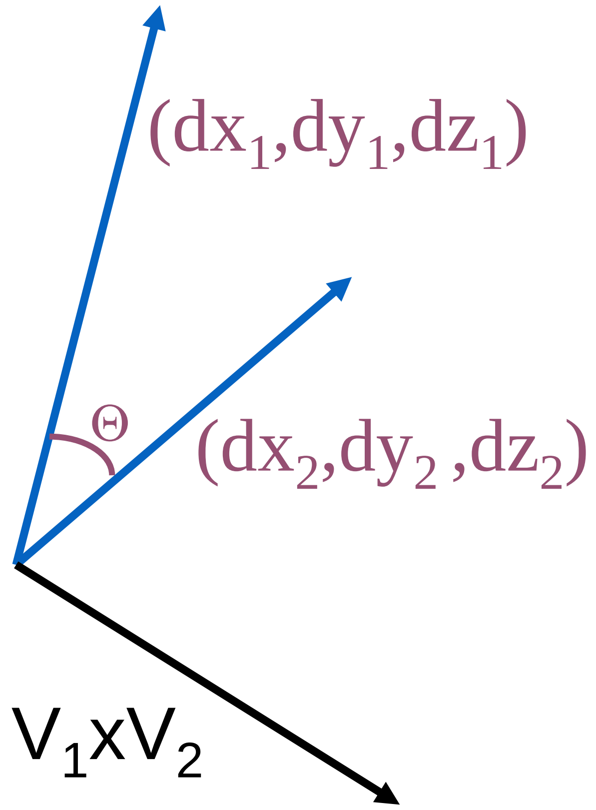
3D Orthogonality

- Dot product of two 3D vectors
  - $V_1 \cdot V_2 = ||V_1|| \; || V_2 || \cos(\pi/2) = 0$



3D Vector

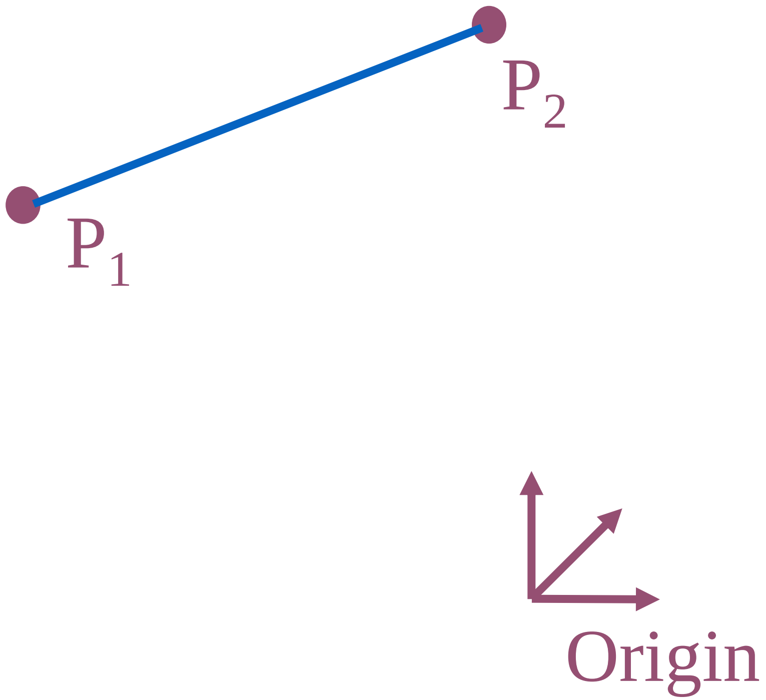
- Cross product of two 3D vectors
  - $V_1 x V_2$ = vector perpendicular to both V_1 and V_2
  - $||V_1 x V_2|| = ||V_1|| \; ||V_2|| \sin(\Theta)$



3D Line Segment

- Linear path between two points
  - Parametric representation:
    - $P = P_1 + t\,(P_2 - P_1), \quad (0 \leq t \leq 1)$

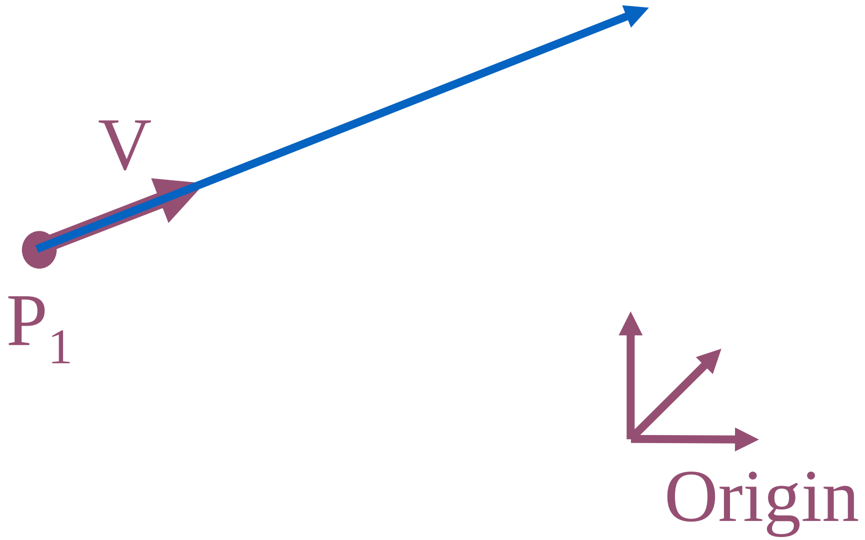
```
typedef struct {
    Point P1;
    Point P2;
} Segment;
```



3D Ray

- Line segment with one endpoint at infinity
  - Parametric representation:
    - $P = P_1 + t\,V, \quad (0 <= t < \infty)$

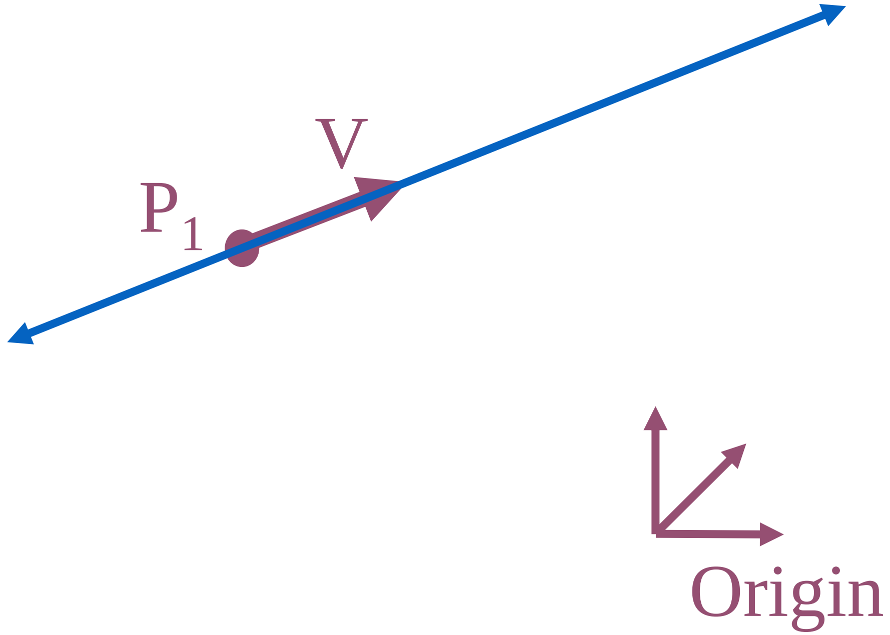
```
typedef struct {
    Point P1;
    Vector V;
} Ray;
```



3D Line

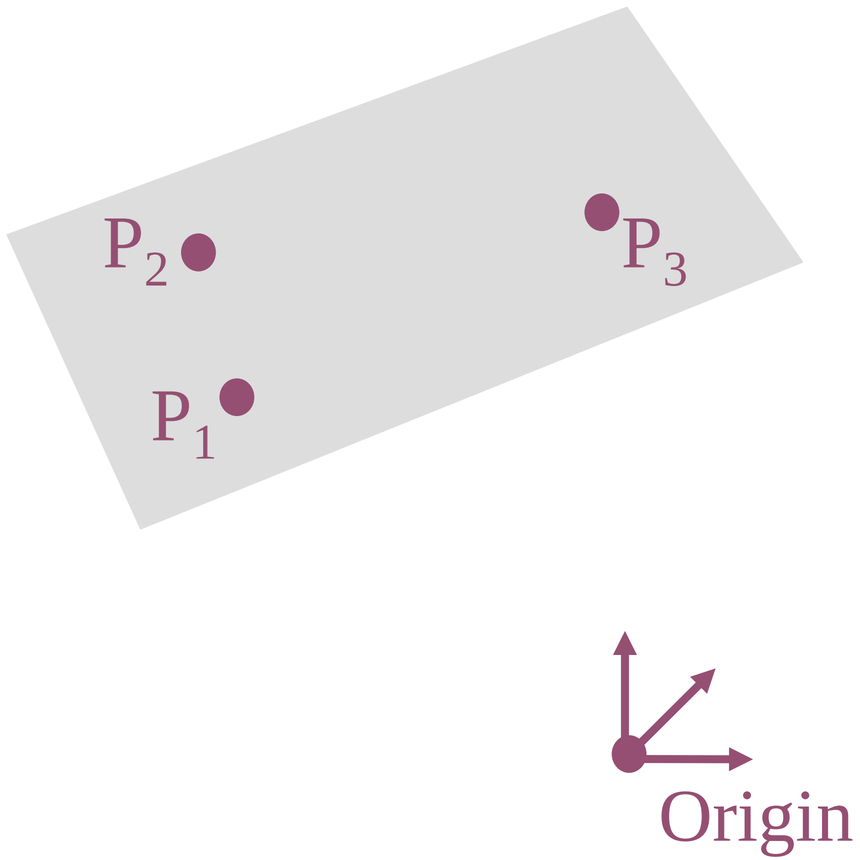
- Line segment with both endpoints at infinity
  - Parametric representation:
    - $P = P_1 + t\,V, \quad (-\infty < t < \infty)$

```
typedef struct {
    Point P1;
    Vector V;
} Line;
```



3D Plane

- Defined by three points in 3D space

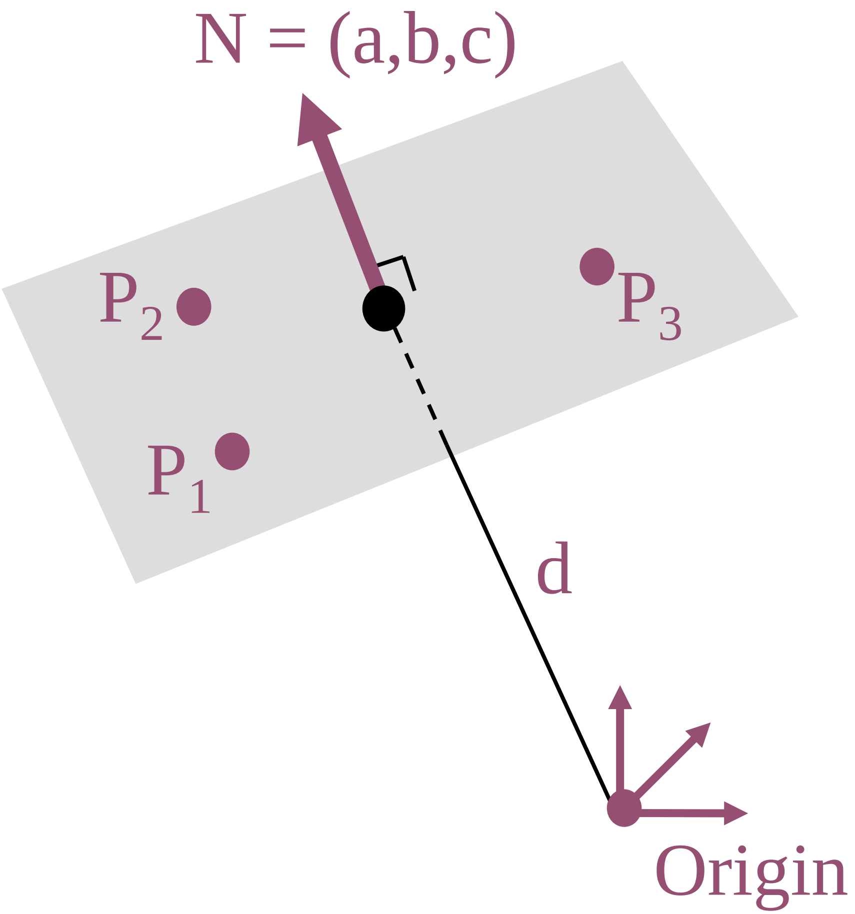


3D Plane

- A linear combination of three points
  - Implicit representation:
    - $P \cdot N - d = 0$, or
    - $N \cdot (P - P_1) = 0$, or
    - $ax + by + cz + d = 0$

```
typedef struct {
    Vector N;
    Distance d;
} Plane;
```

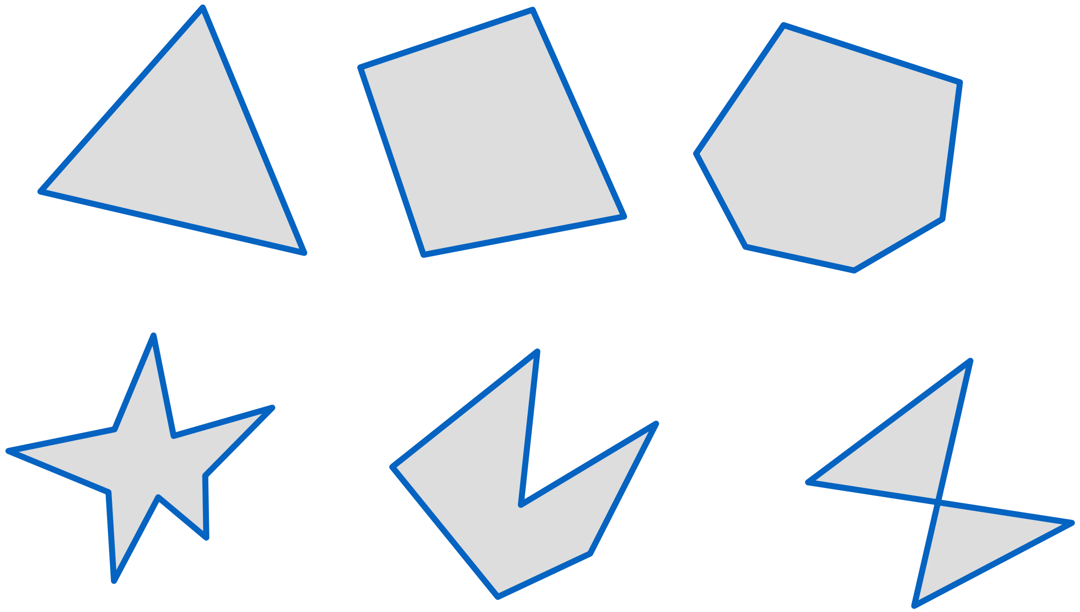
  - N is the plane “normal”
    - Unit-length vector
    - Perpendicular to plane



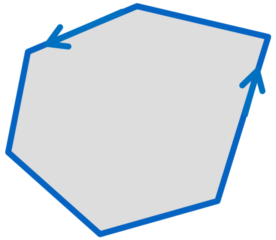
3D Polygon

- Set of points “inside” a sequence of coplanar points

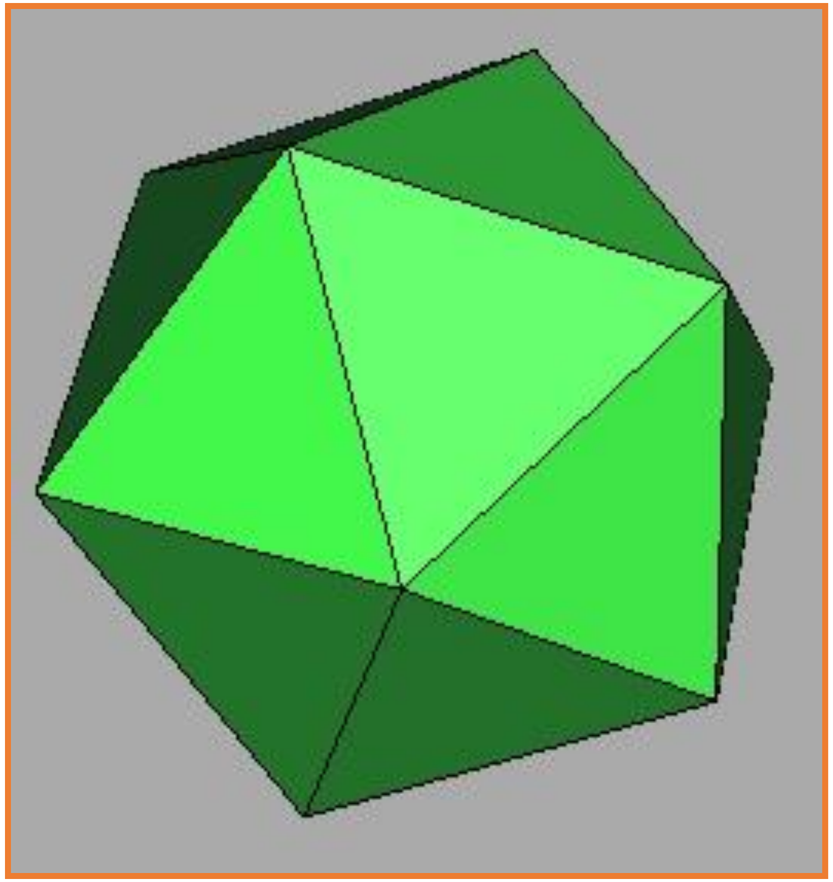
```
typedef struct {
      Point *points;
      int npoints;
} Polygon;
```



Points are in counter-clockwise order

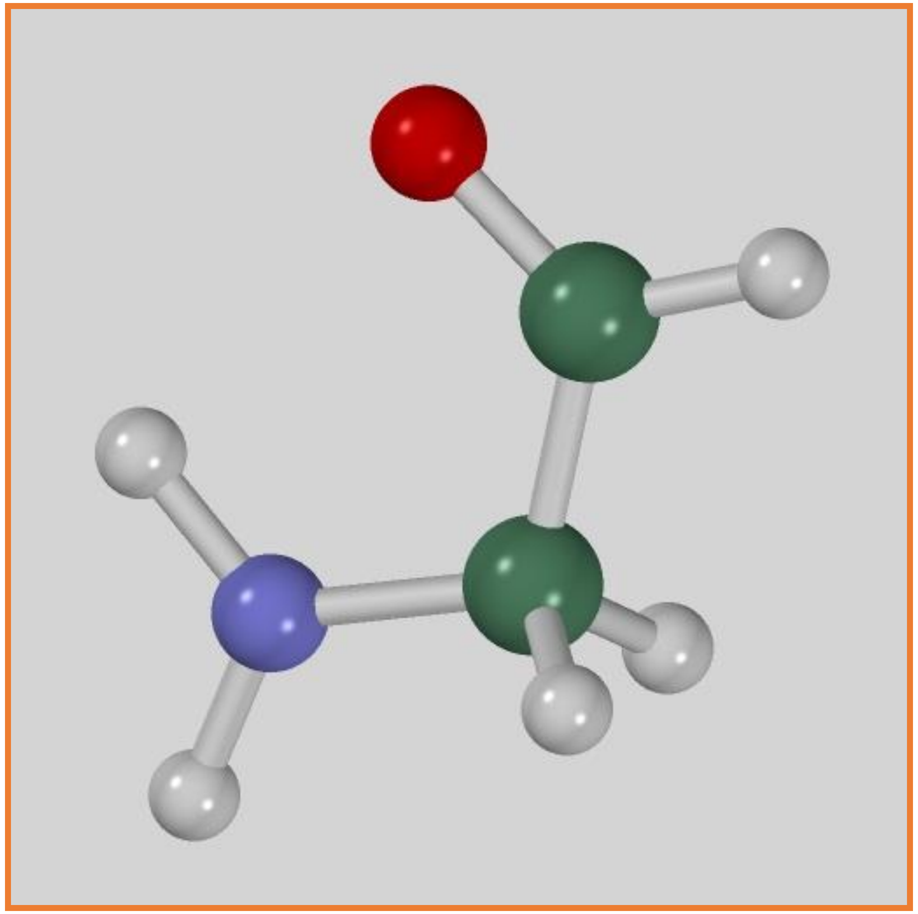


3D Object Representations



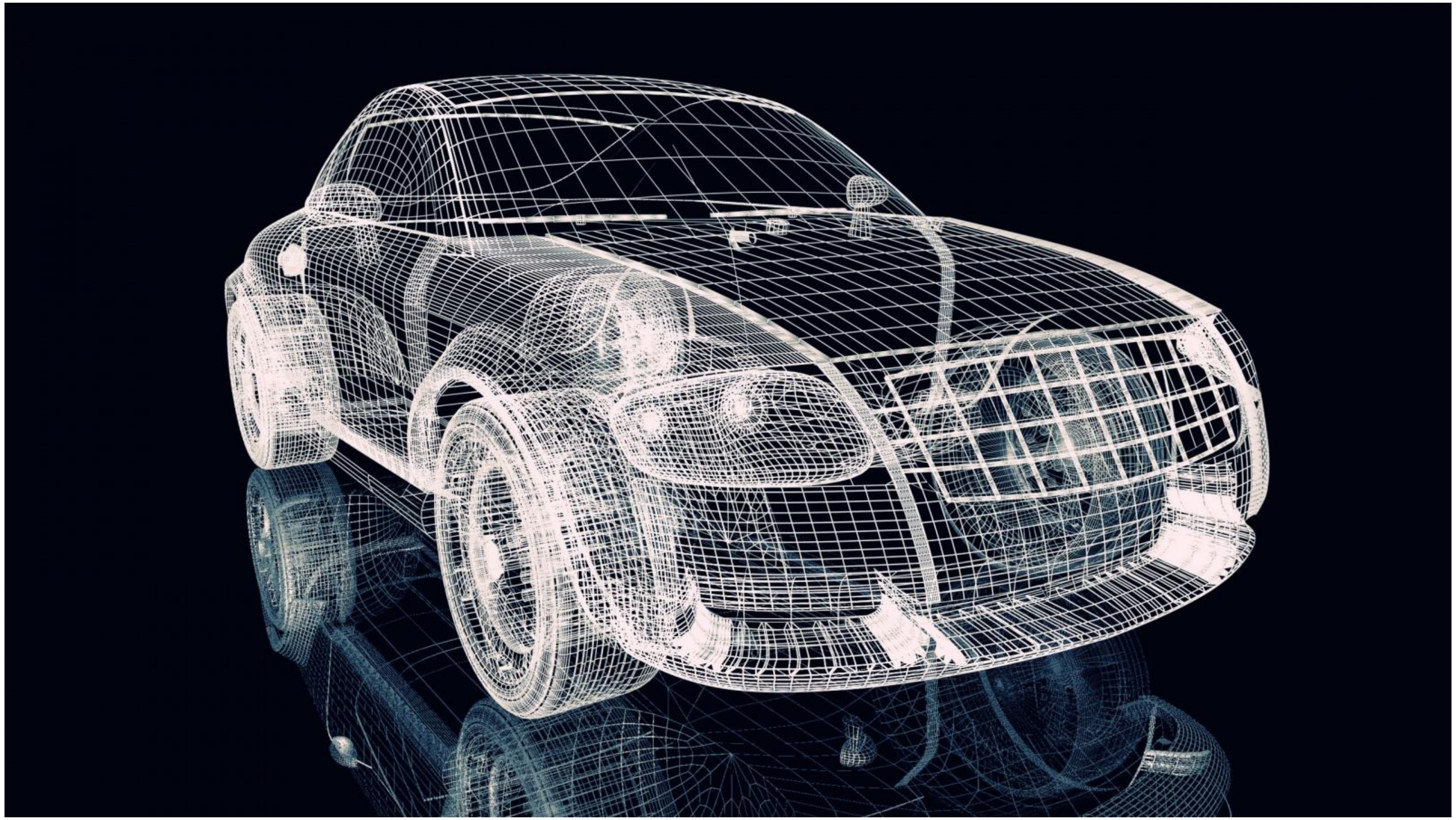
How can this object be represented in a computer?

3D Object Representations



How about this one?

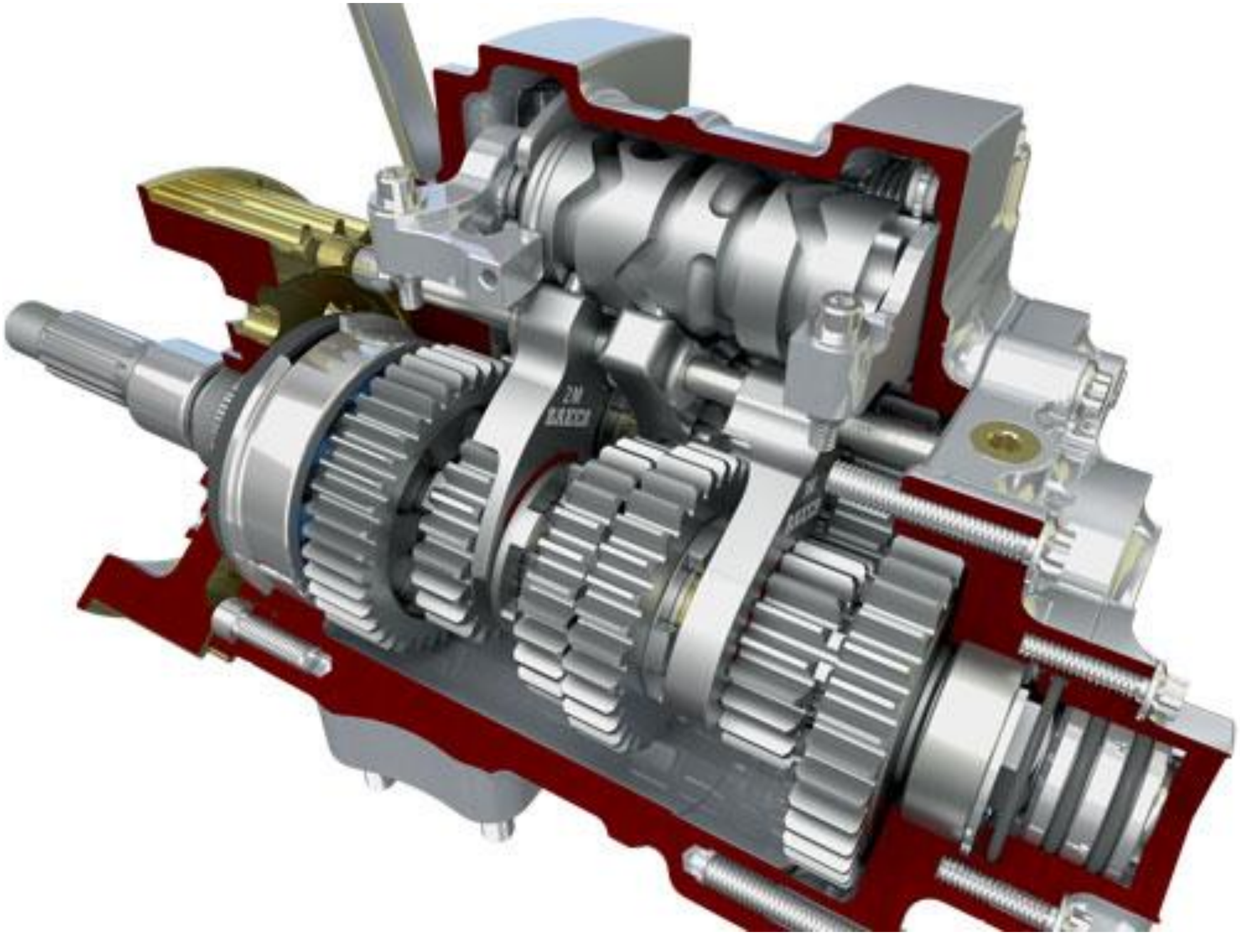
3D Object Representations



Wallpapersonly.net

This one?

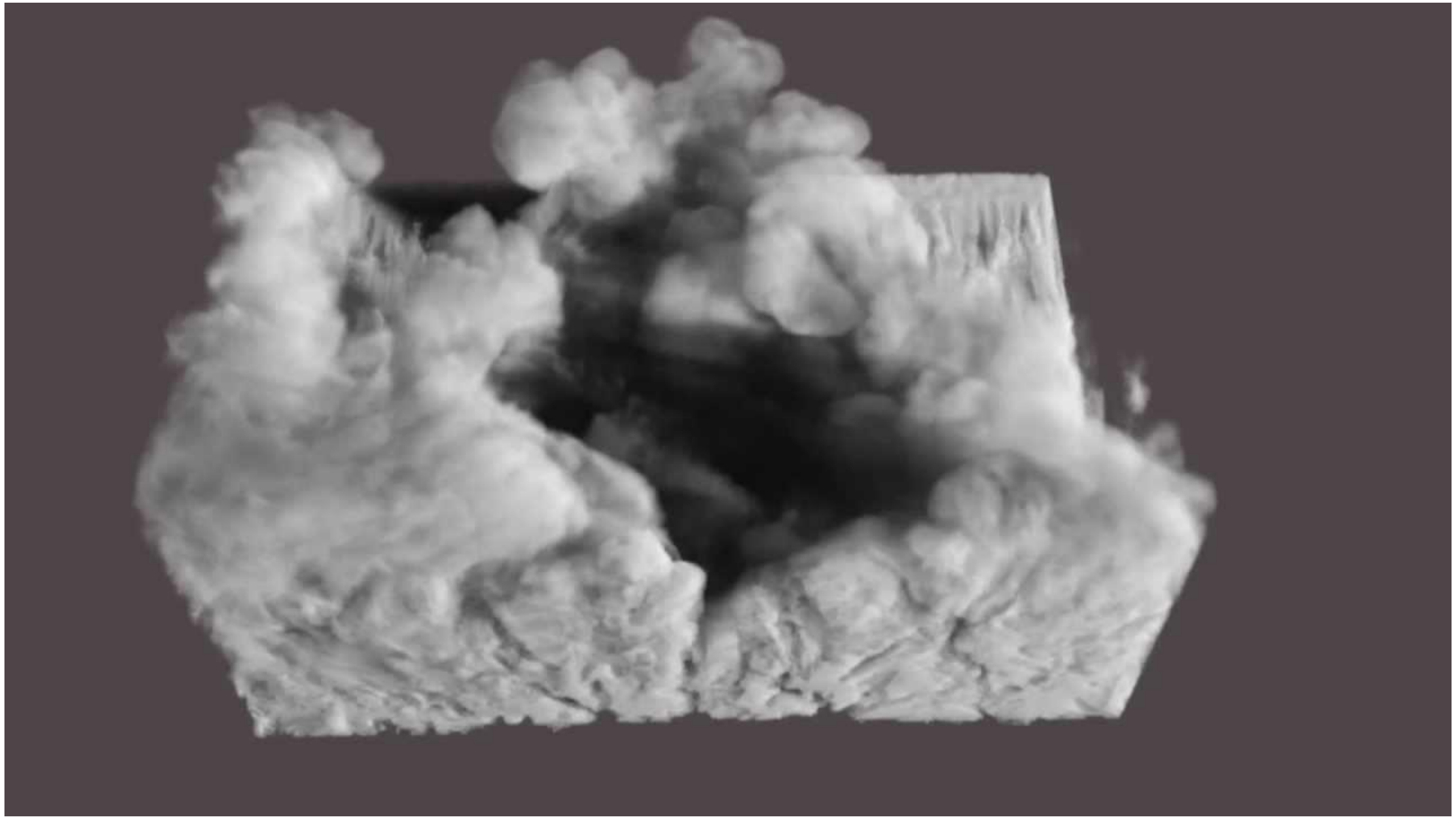
3D Object Representations



Solidworks

This one?

3D Object Representations



This one?

FumeFx

3D Object Representations

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

Equivalence of Representations

- Thesis:
  - Each representation has enough expressive power to model the shape of any geometric object
  - It is possible to perform all geometric operations with any fundamental representation
- Analogous to Turing-equivalence
  - Computers and programming languages are Turing-equivalent, but each has its benefits…

Why Different Representations?

Efficiency for different tasks

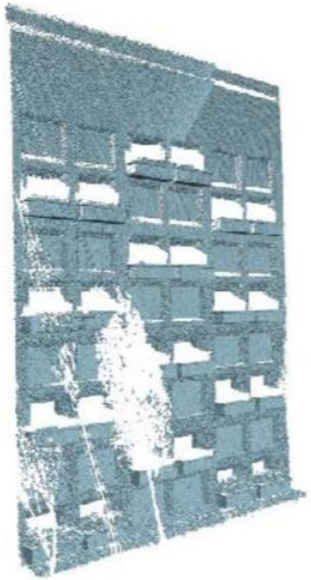
- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation

→ Data structures determine algorithms

Why Different Representations?

Efficiency for different tasks

- Acquisition
  - Range Scanning
- Rendering
- Analysis
- Manipulation
- Animation



DGP course notes, Technion

Why Different Representations?

Efficiency for different tasks

- Acquisition
  - Computer Vision
- Rendering
- Analysis
- Manipulation
- Animation

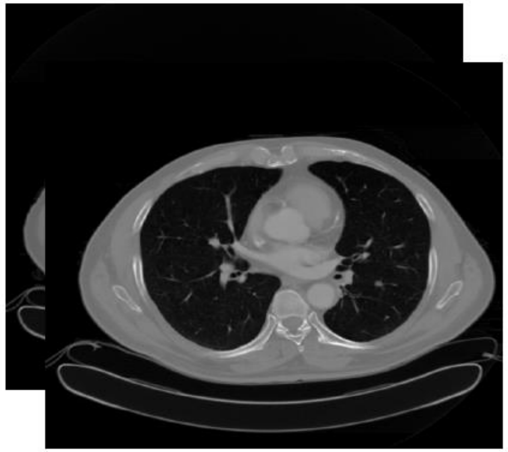
Indiana University

USC

Why Different Representations?

Efficiency for different tasks

- Acquisition
  - Tomography
- Rendering
- Analysis
- Manipulation
- Animation



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Why Different Representations?

Efficiency for different tasks

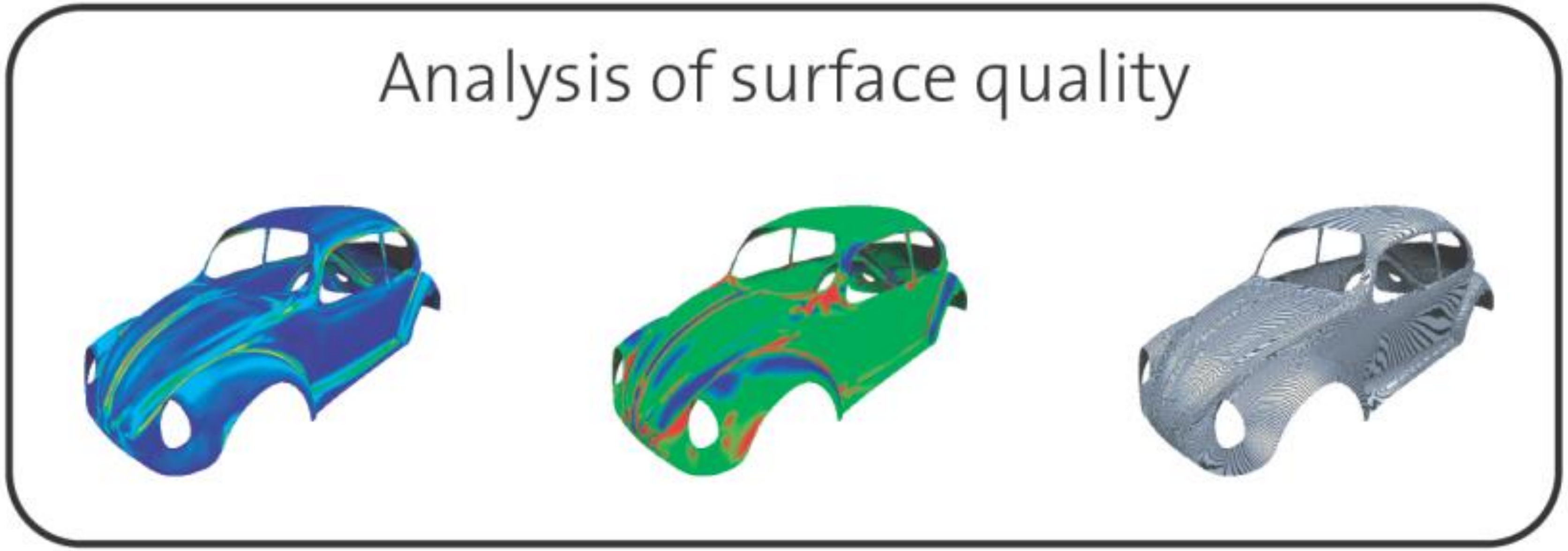
- Acquisition
- Rendering
  - Intersection
- Analysis
- Manipulation
- Animation

Autodesk

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Curvature, smoothness
- Manipulation
- Animation



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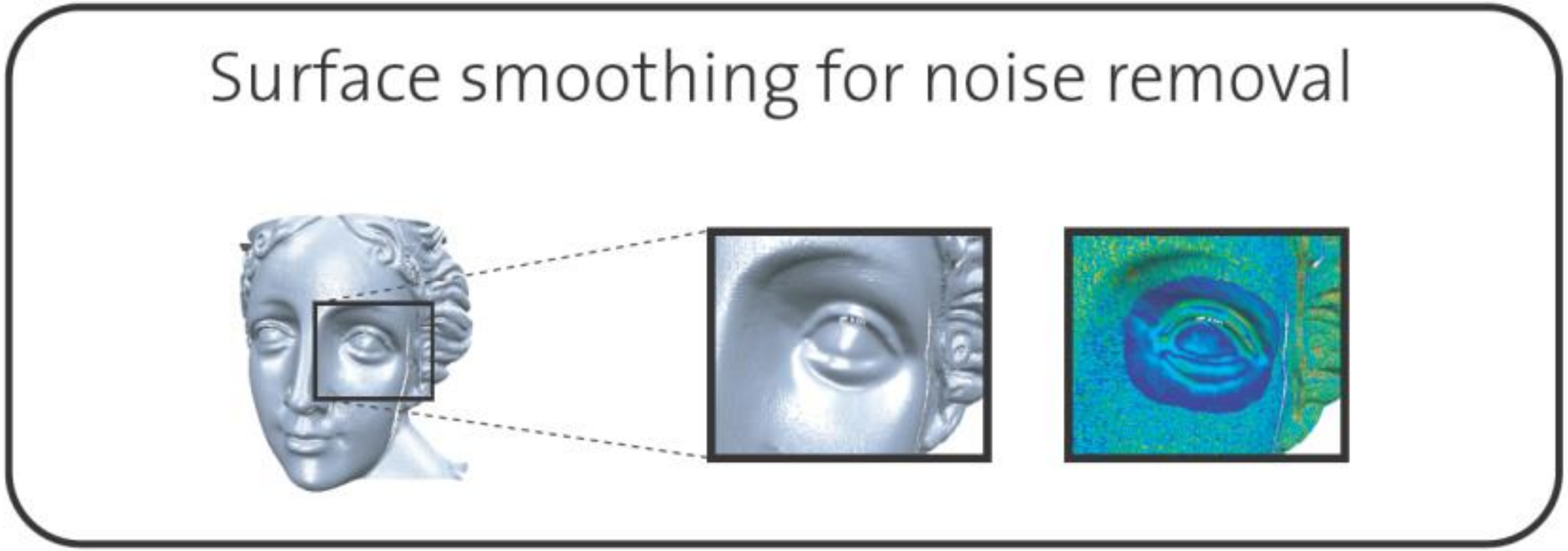
Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Fairing
- Manipulation
- Animation



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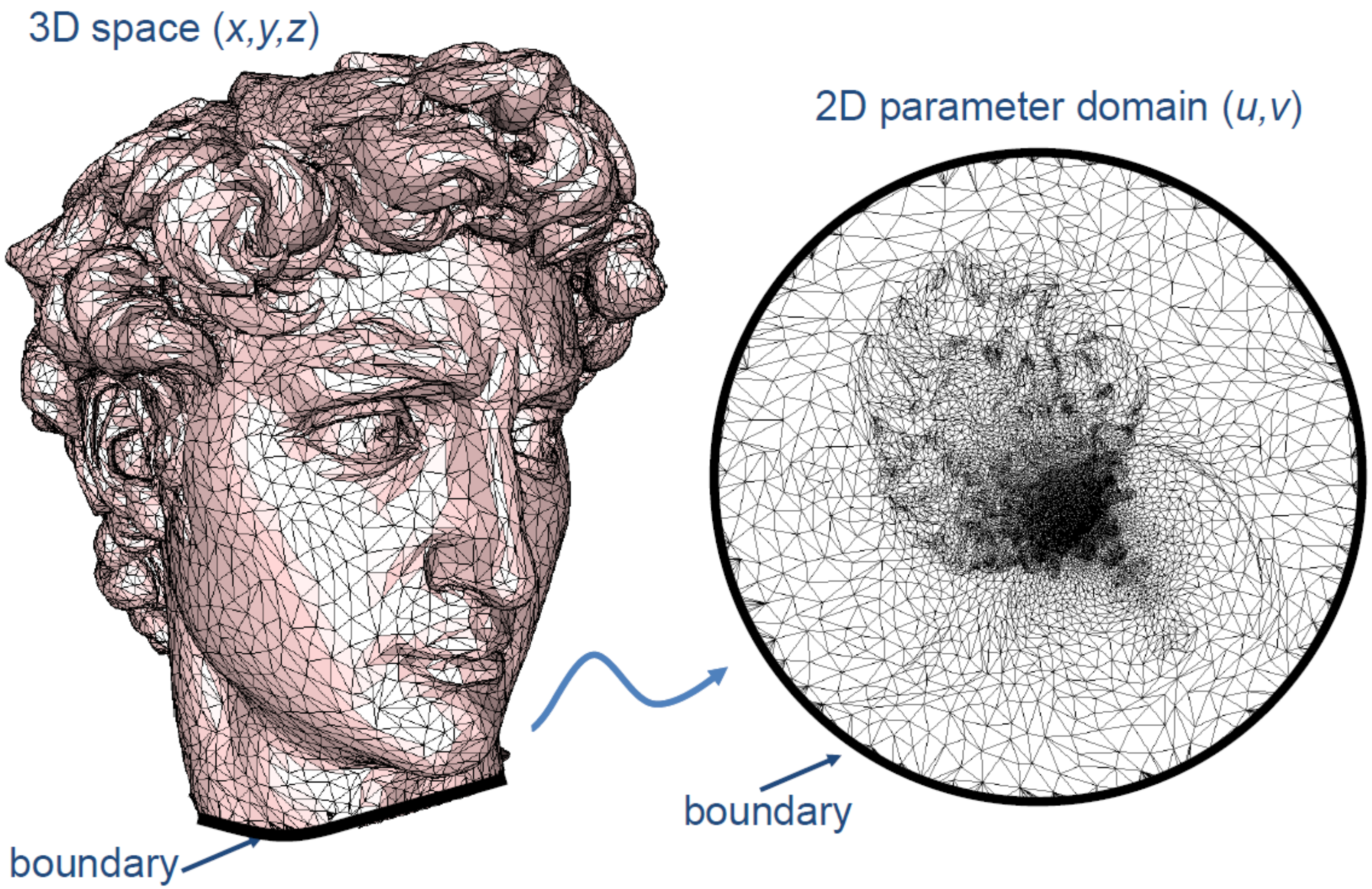
Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Parametrization
- Manipulation
- Animation



Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
    - Texture mapping
- Manipulation
- Animation

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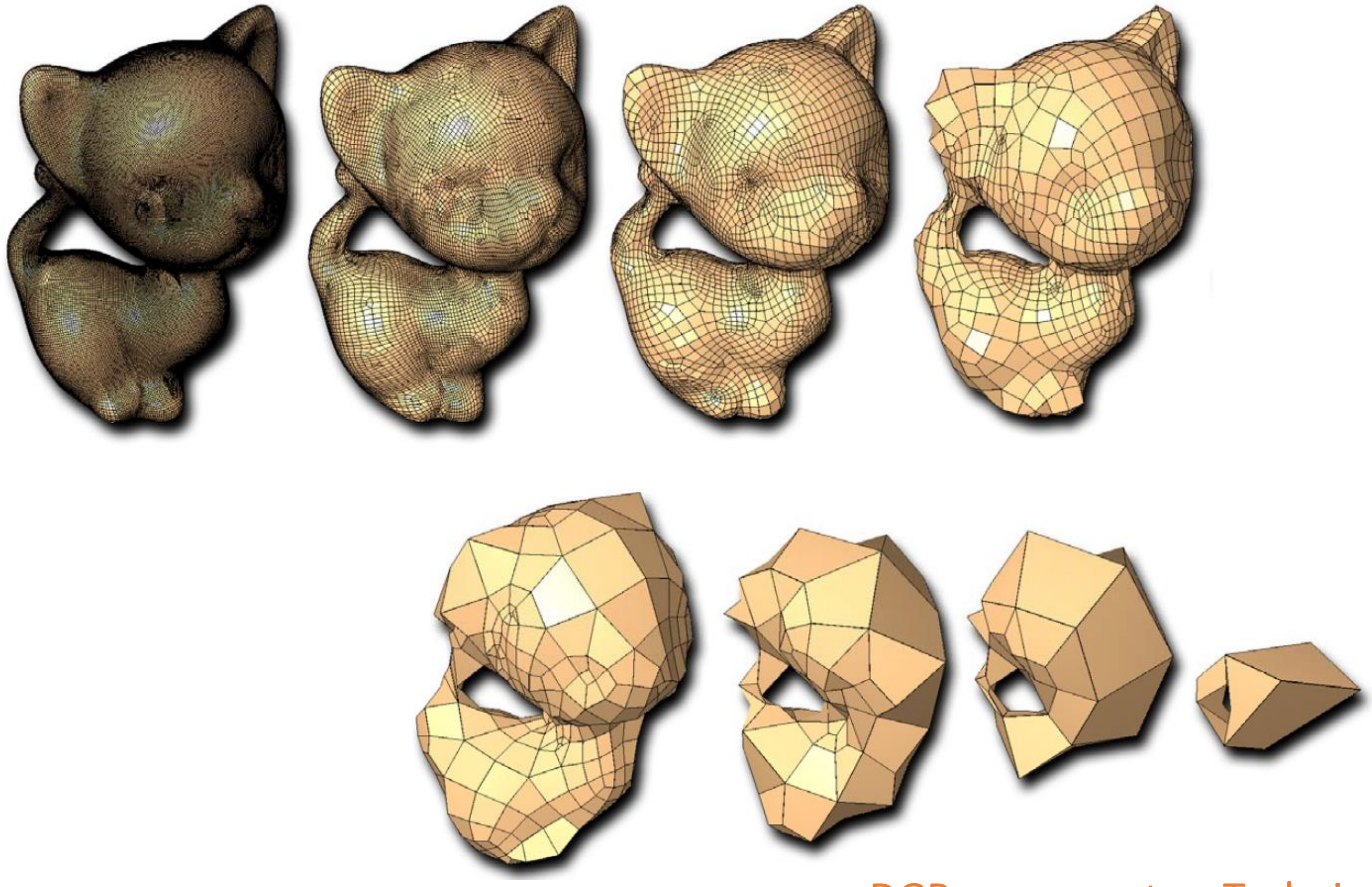
Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Reduction
- Manipulation
- Animation



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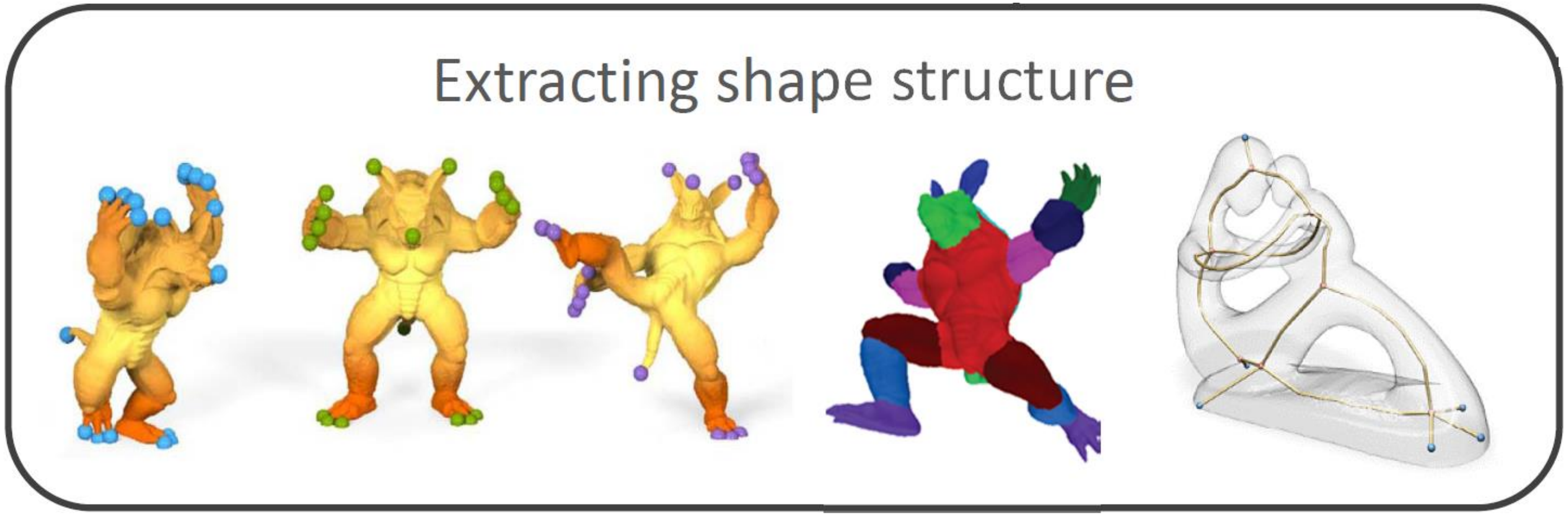
Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
    - Structure
- Manipulation
- Animation



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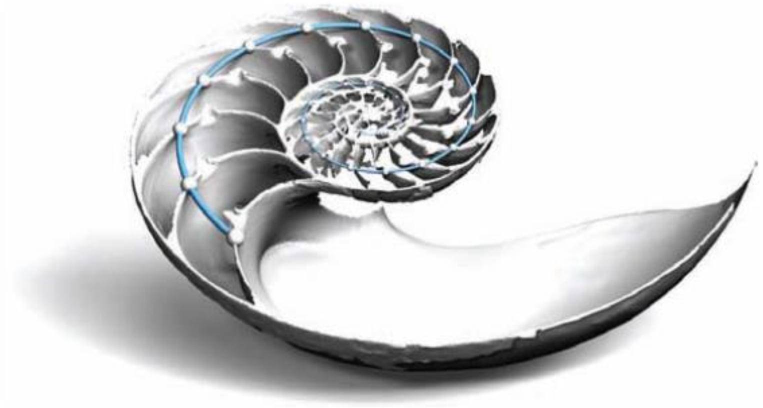
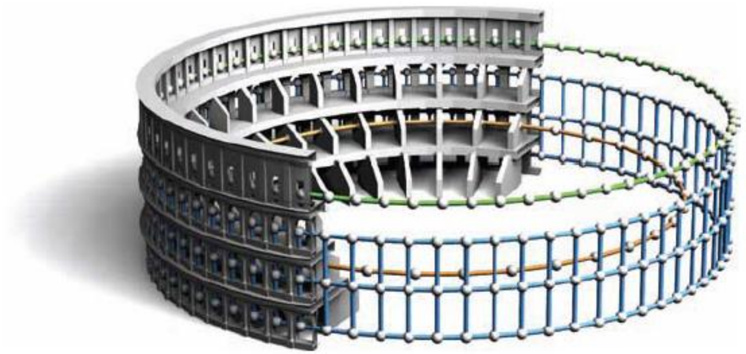
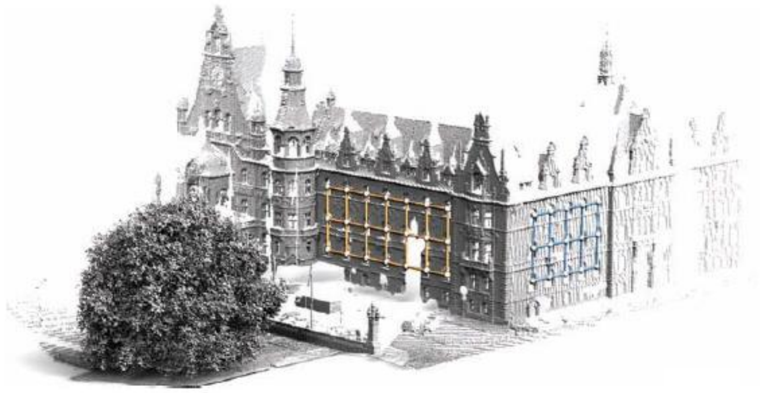
Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
    - Symmetry detection
- Manipulation
- Animation

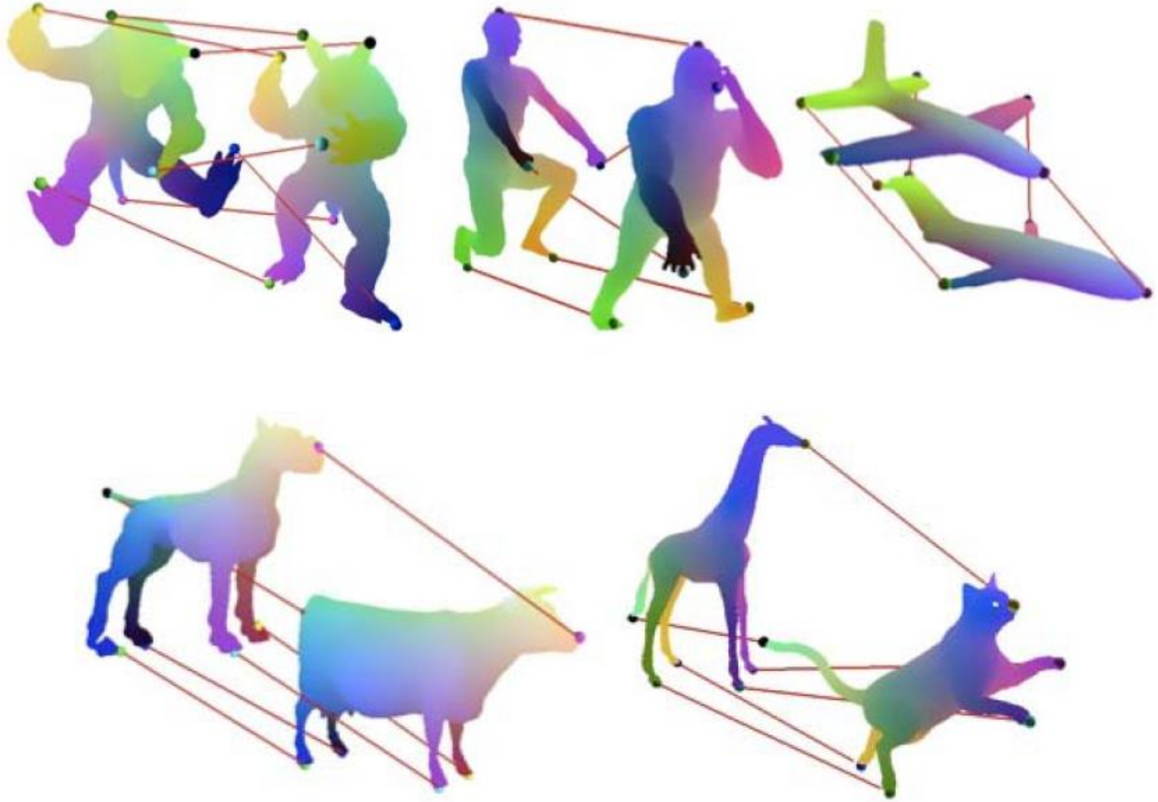


Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Correspondence
- Manipulation
- Animation



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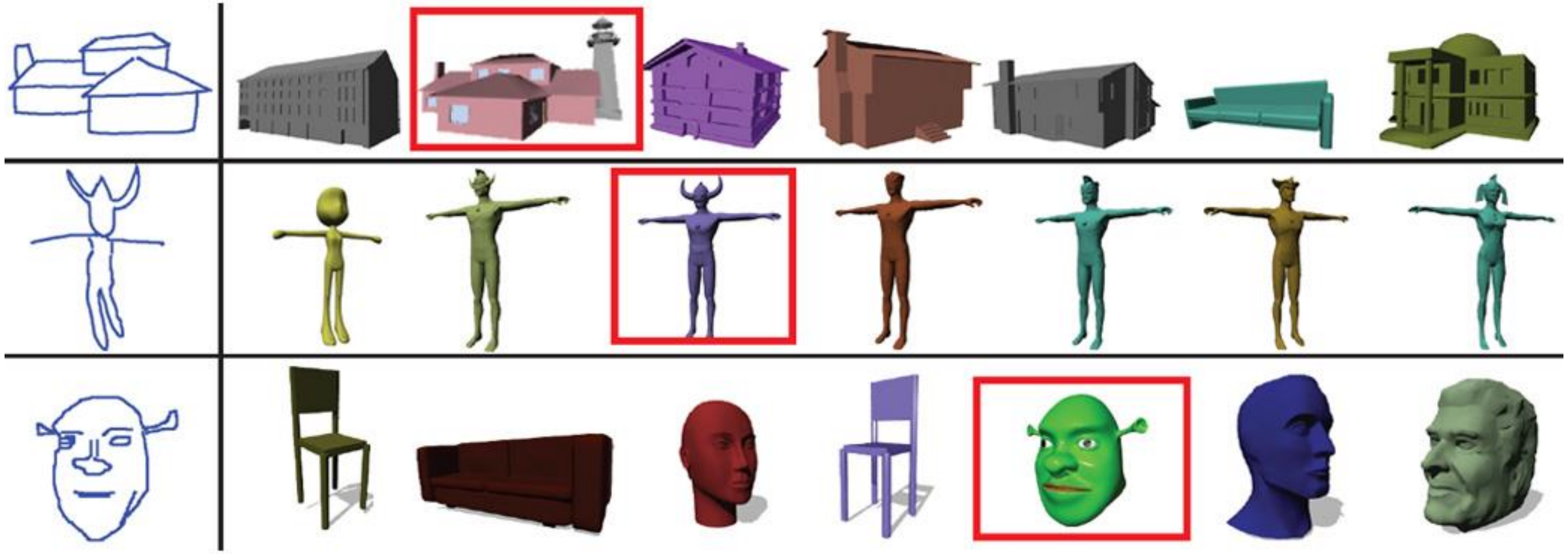
Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Shape retrieval
- Manipulation
- Animation

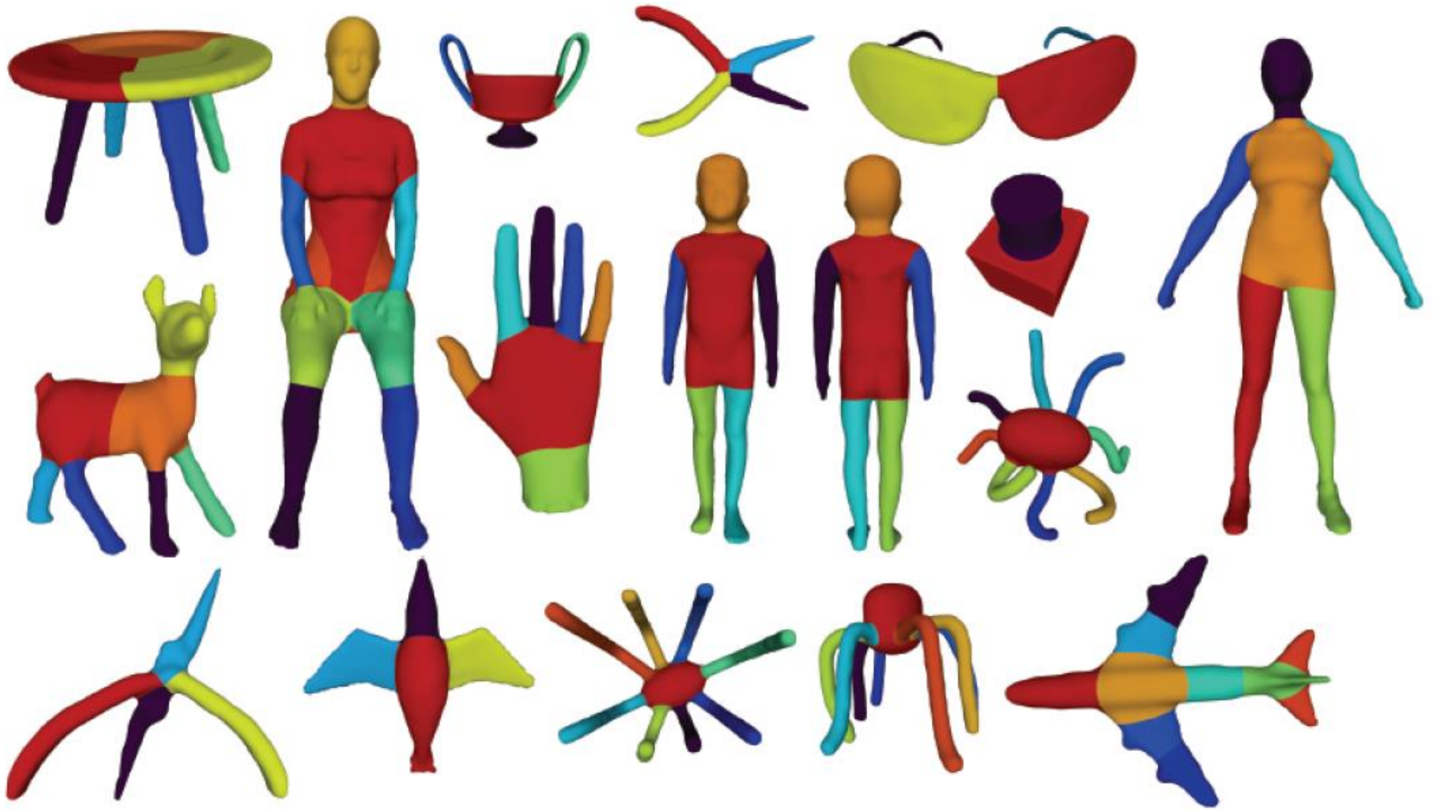


Shao et al. 2011

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
  - Segmentation
- Manipulation
- Animation



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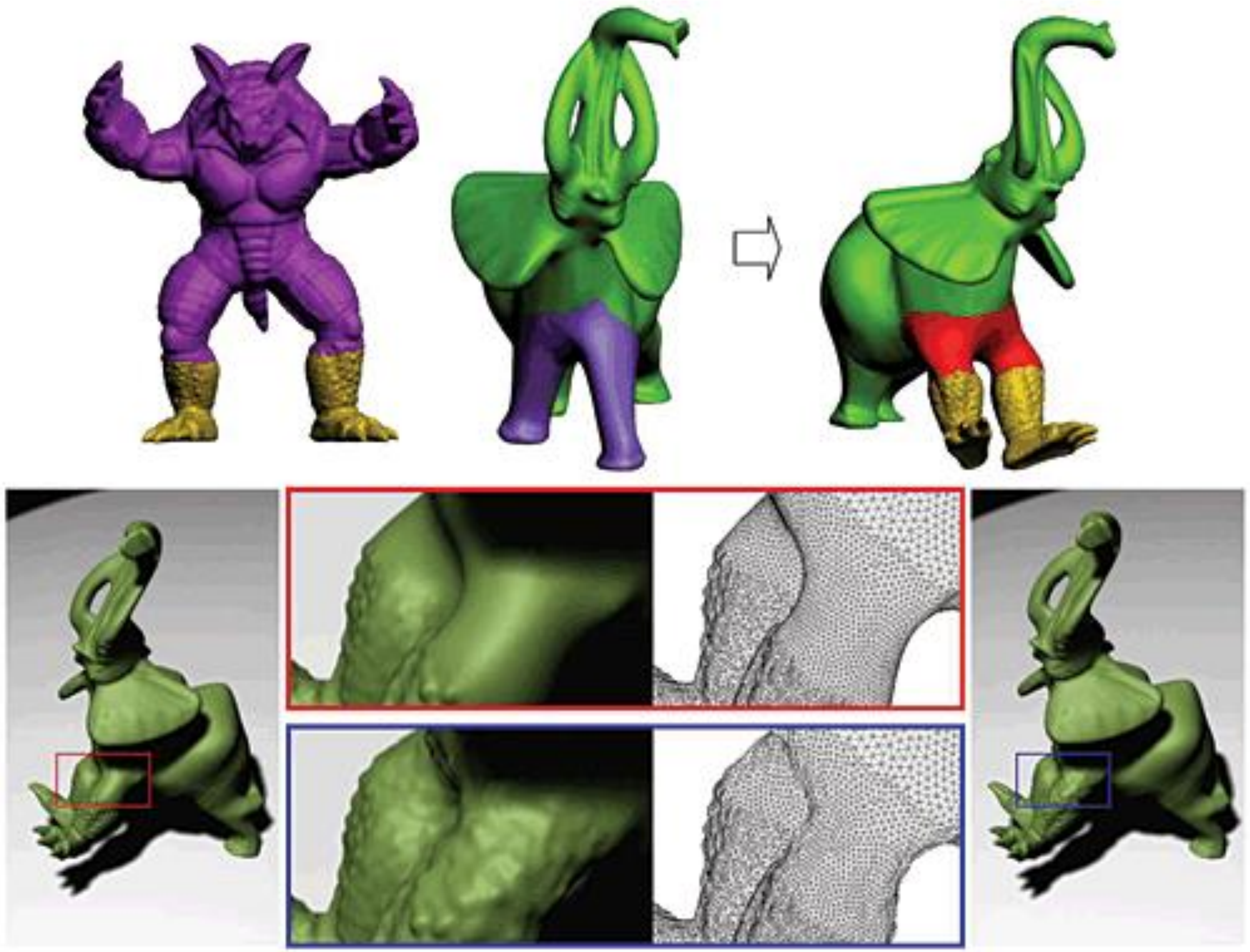
Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
    - Composition
- Manipulation
- Animation

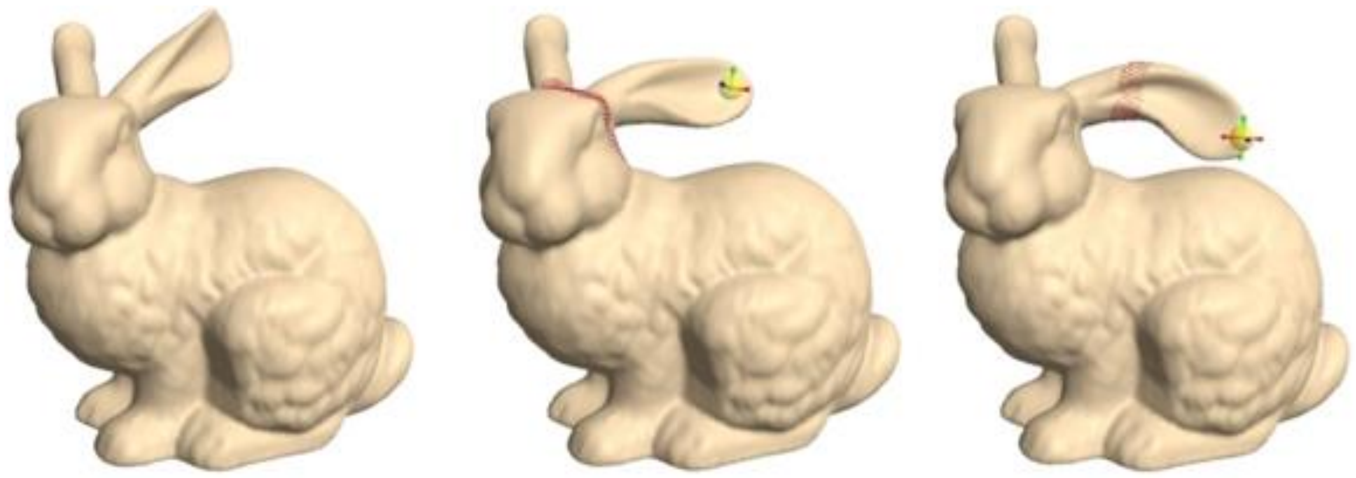


Lin et al. 2008

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
  - Deformation
- Animation

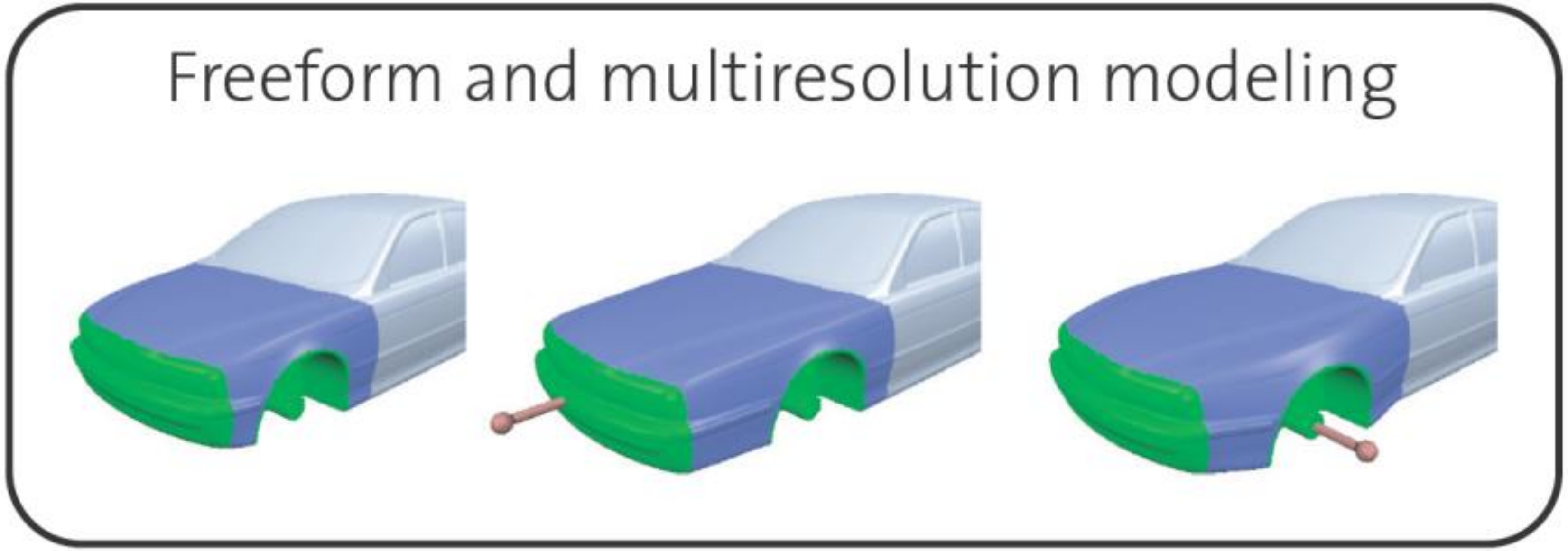


IGL

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
    - Deformation
- Animation



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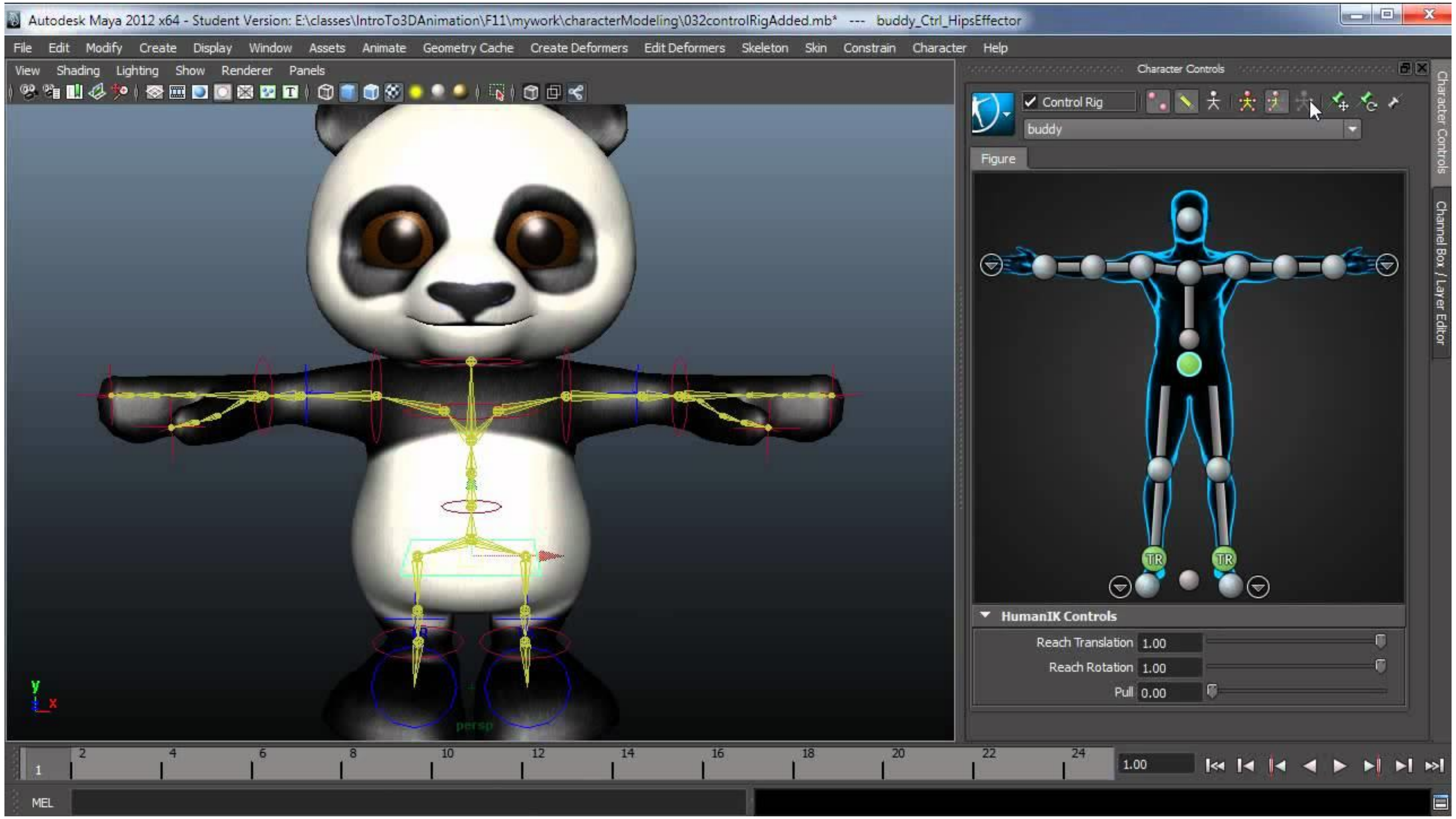
Why Different Representations?

Efficiency for different tasks

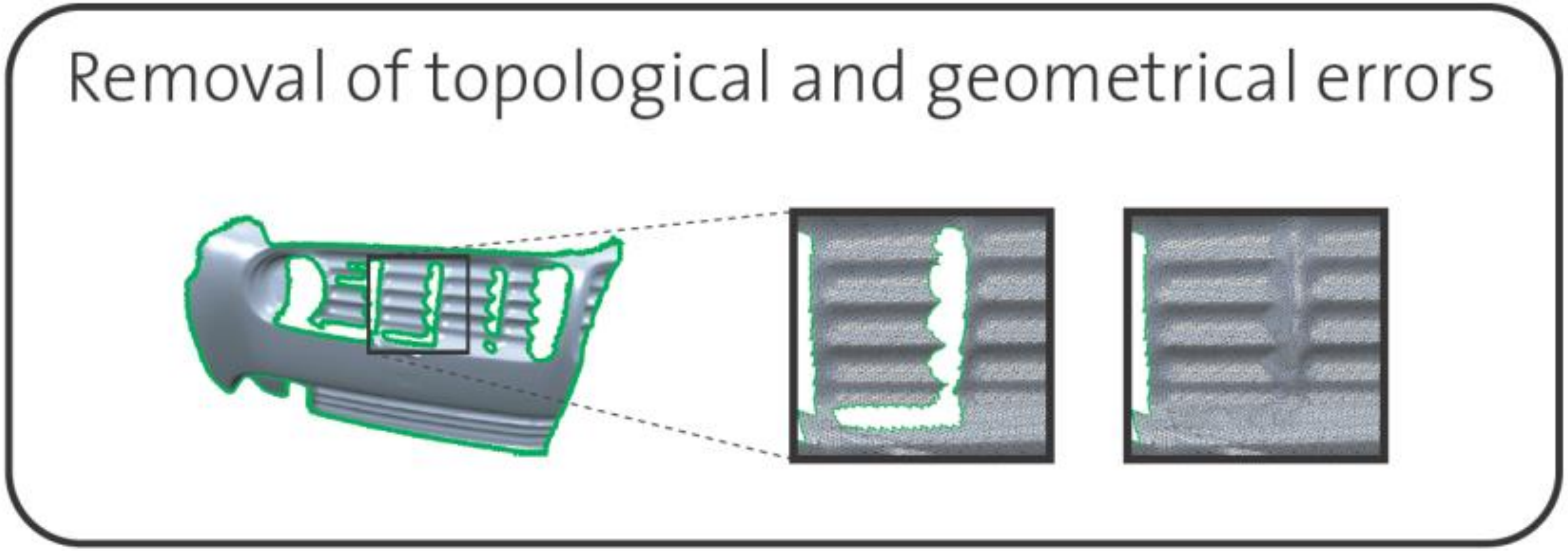
- Acquisition
- Rendering
- Analysis
- Manipulation
    - Control
- Animation



Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
    - Healing
- Animation

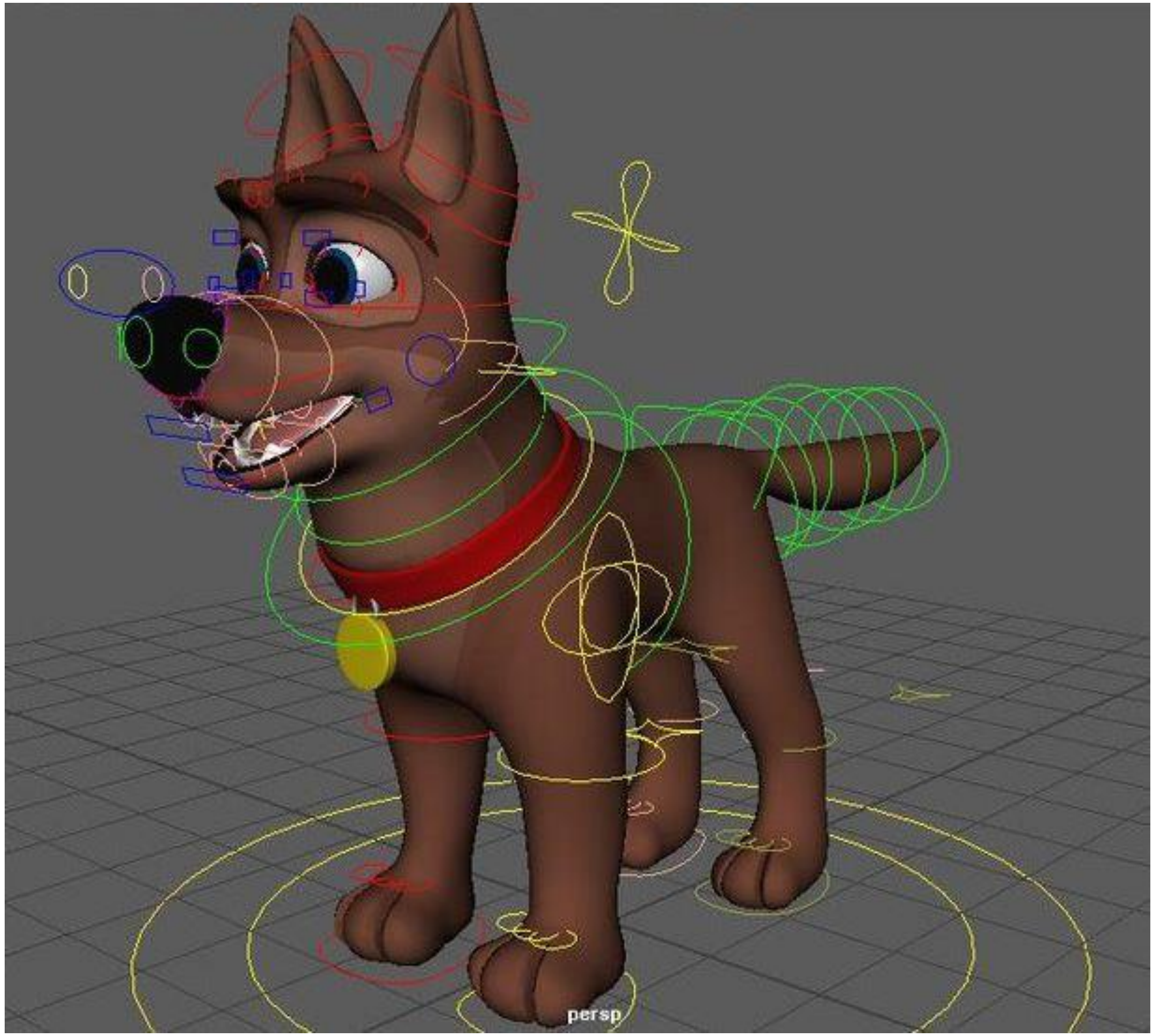


DGP course notes, Technion

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
  - Rigging

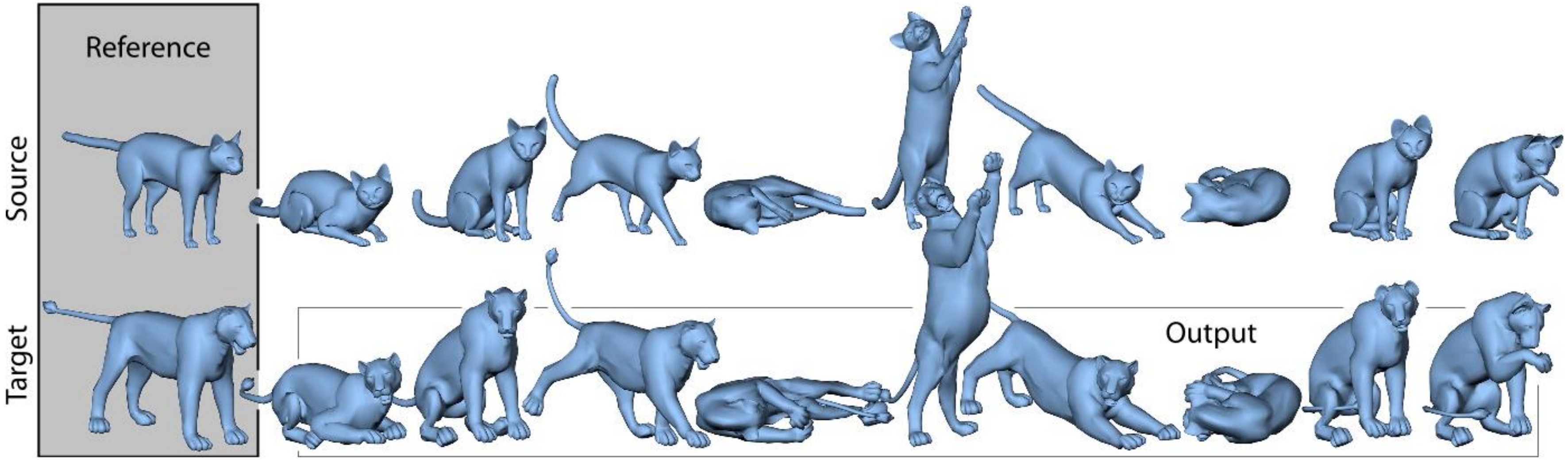


Animation Buffet

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
  - Deformation transfer

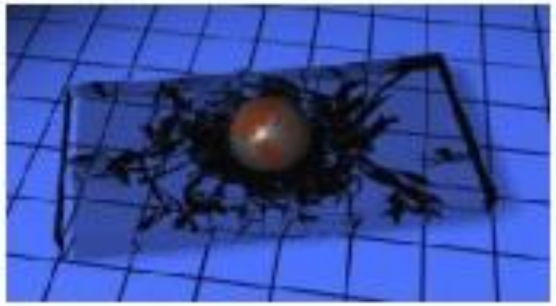


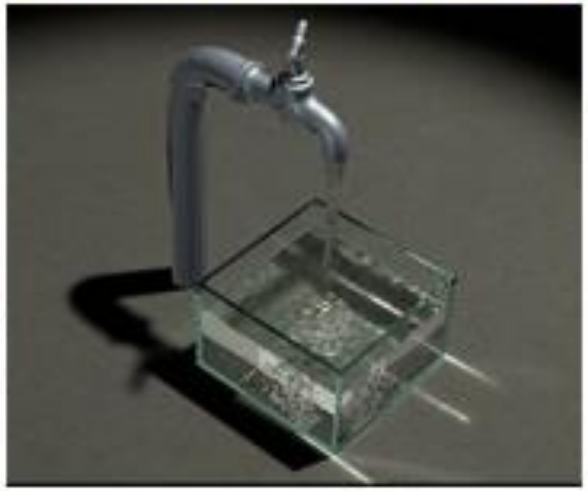
Sumner et al. 2004

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
    - Simulation



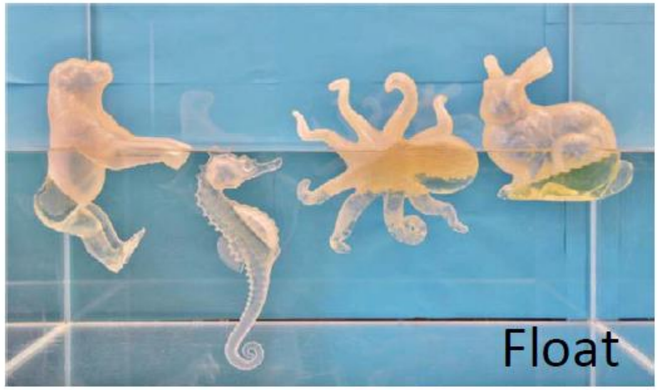


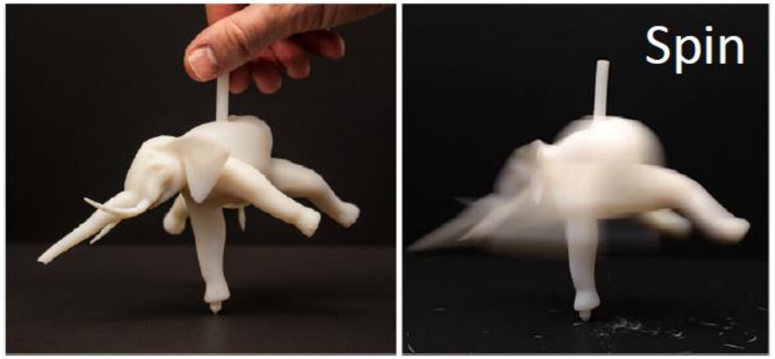
Physically Based Modelling course notes, USC

Why Different Representations?

Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
    - Fabrication





3D Object Representations

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

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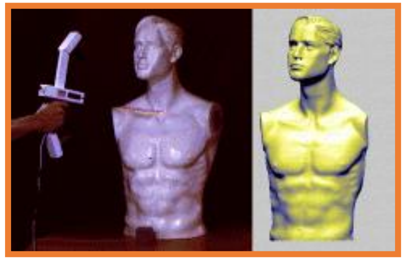
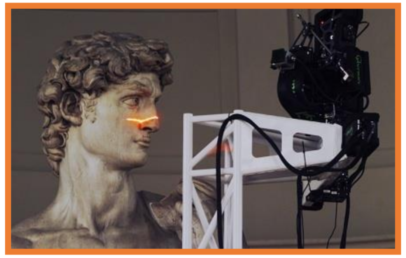
Range Image

Set of 3D points mapping to pixels of depth image

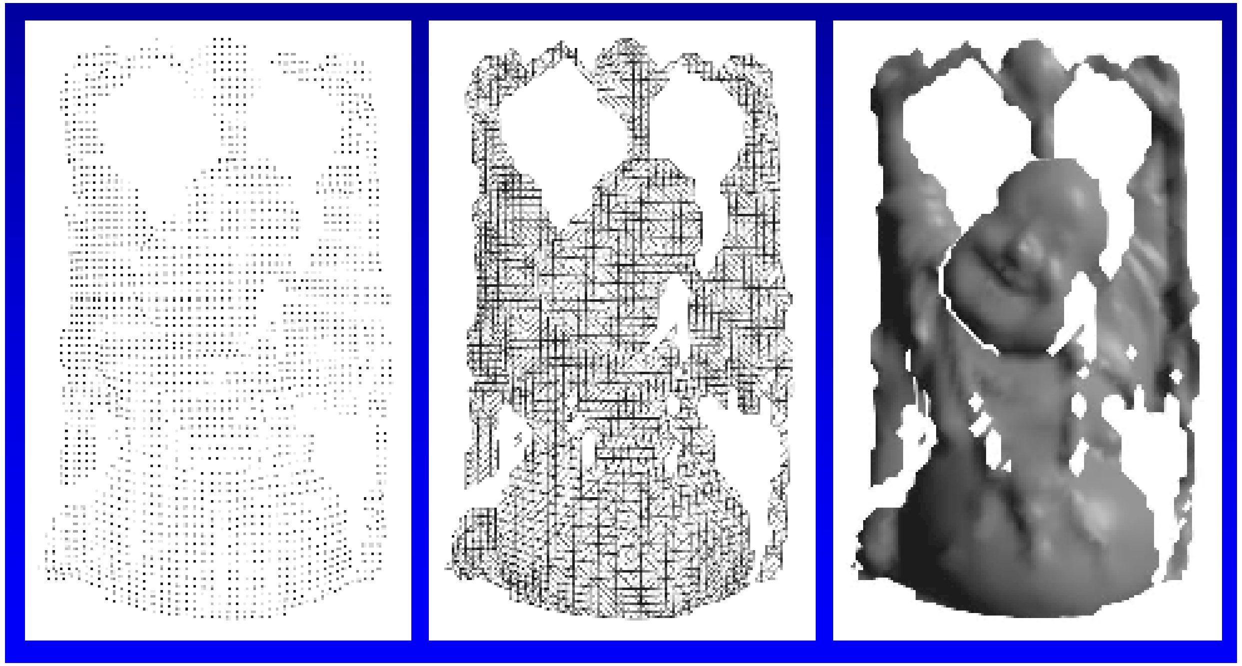
- Can be acquired from range scanner



Cyberware



Stanford



Range Image Tesselation Range Surface

Brian Curless
SIGGRAPH 99
Course #4 Notes

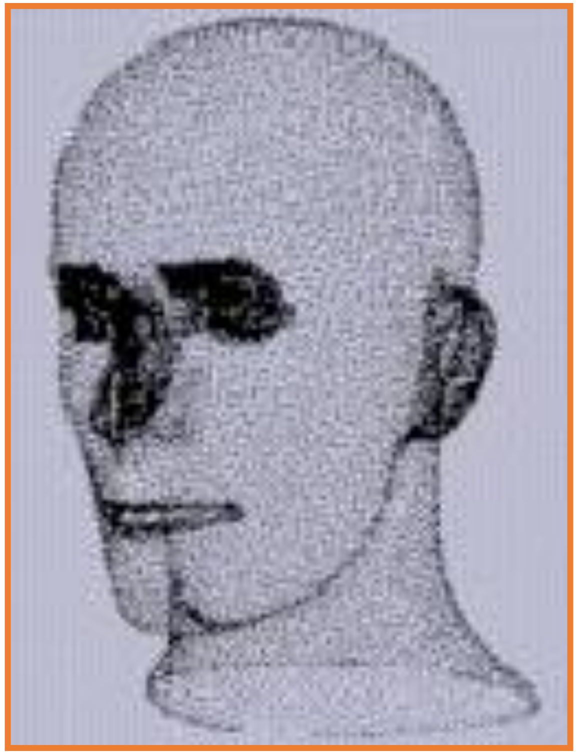
Point Cloud

Unstructured set of 3D point samples

- Acquired from range finder, computer vision, etc

Polhemus

Microscribe-3D



Hoppe

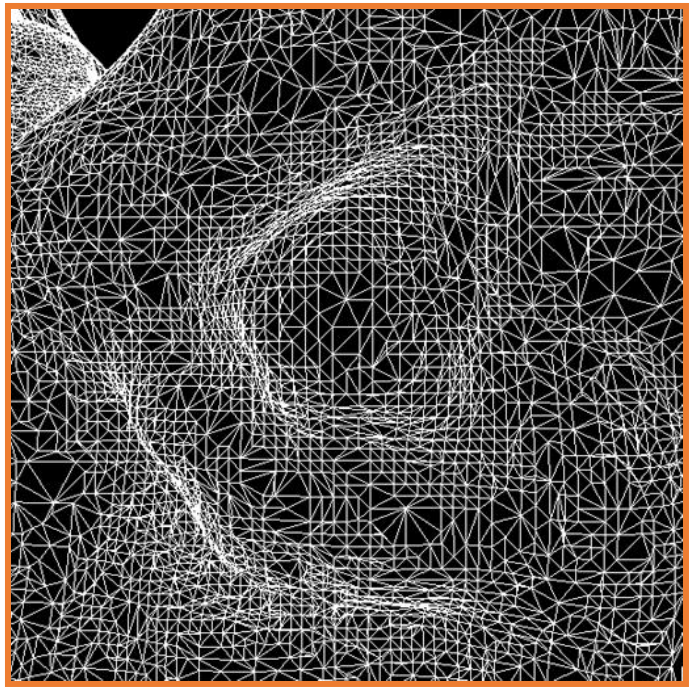
Hoppe

3D Object Representations

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

Polygonal Mesh

Connected set of polygons (often triangles)



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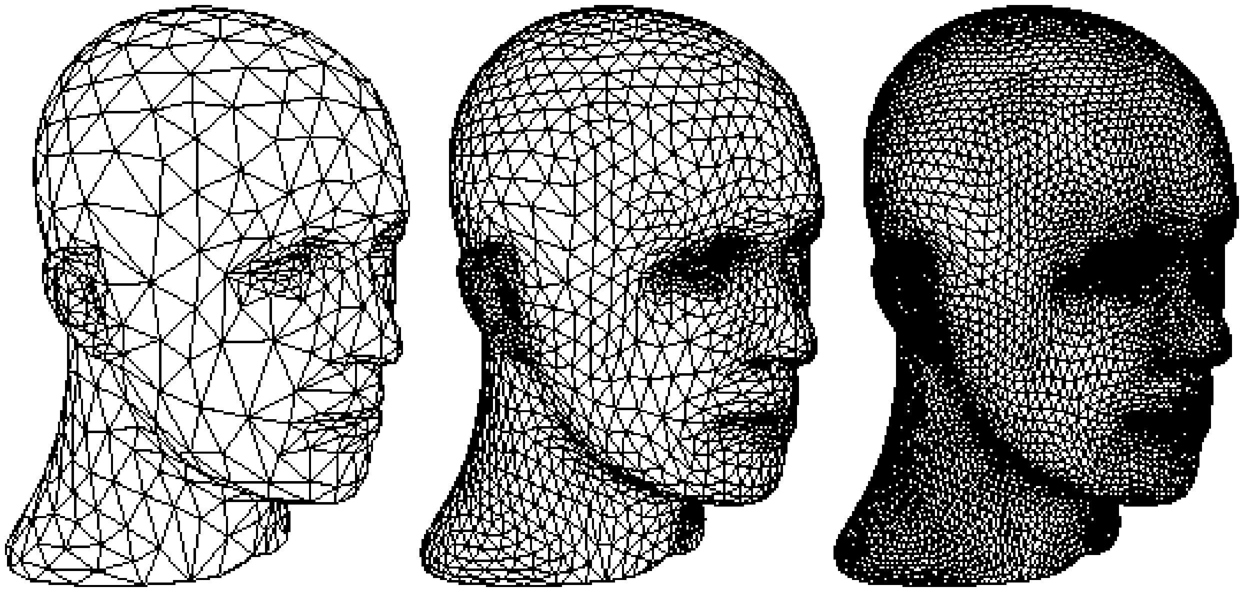
Subdivision Surface

Coarse mesh & subdivision rule

- Smooth surface is limit of sequence of refinements

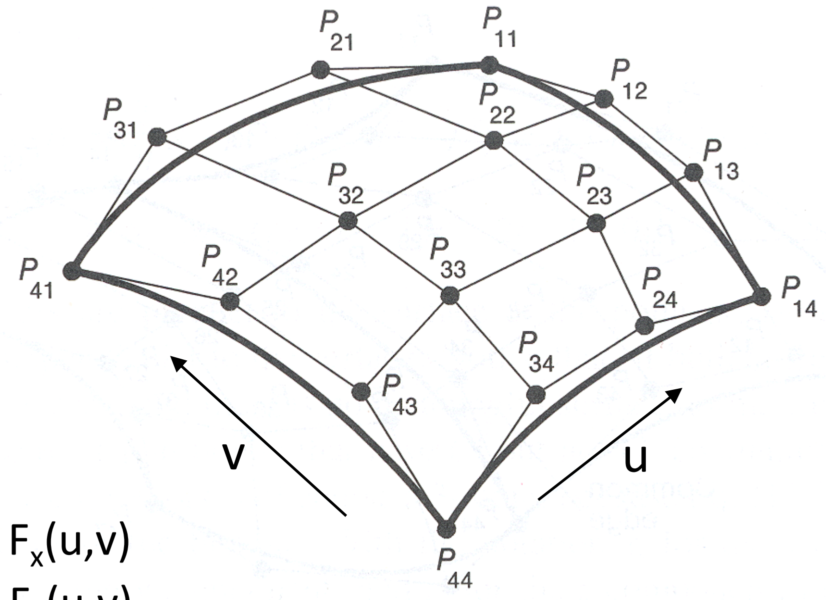


Zorin & Schroeder
SIGGRAPH 99
Course Notes

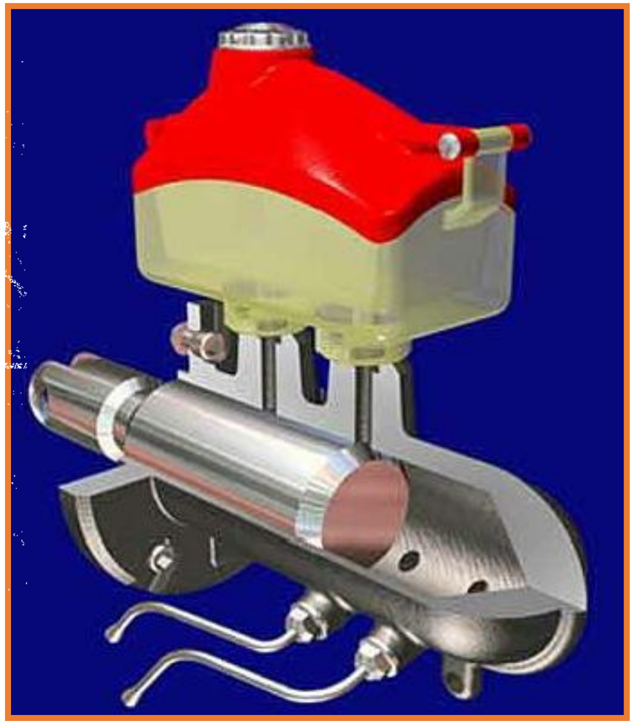
Parametric Surface

Tensor-product spline patches

- Each patch is parametric function
- Careful constraints to maintain continuity



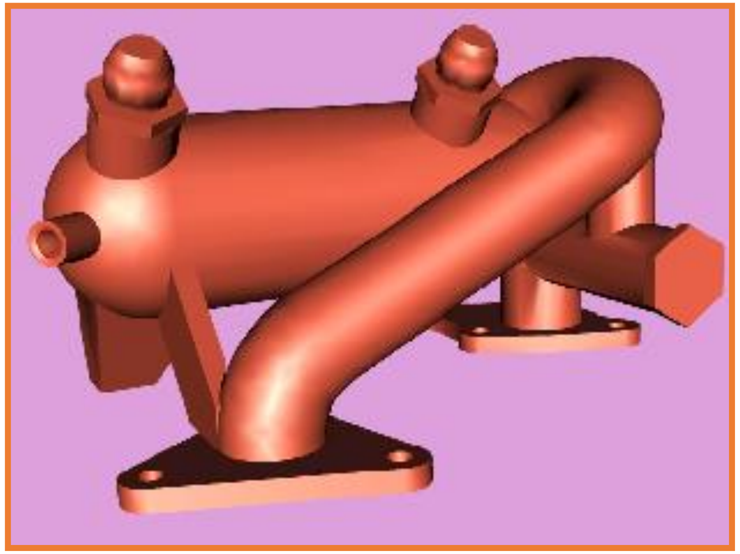
$x = F_x(u,v)$
$y = F_y(u,v)$
$z = F_z(u,v)$



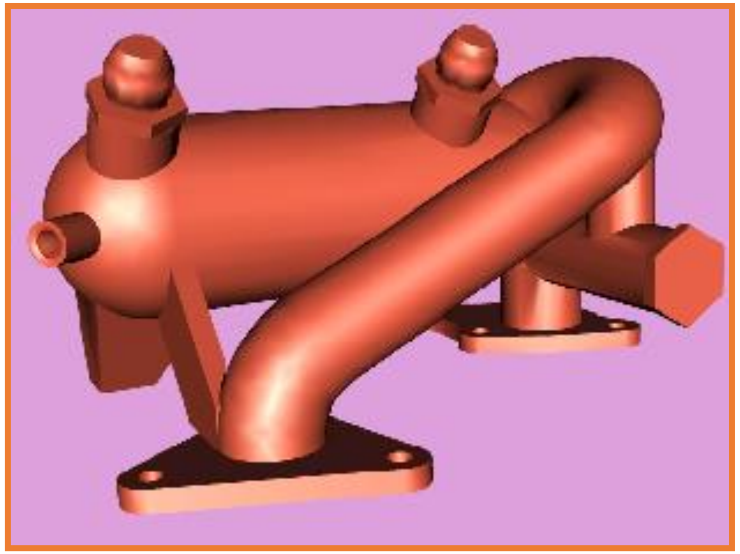
FvDFH Figure 11.44

Implicit Surface

Set of all points satisfying: F(x,y,z) = 0



Polygonal Model



Implicit Model

Bill Lorensen
SIGGRAPH 99
Course #4 Notes

3D Object Representations

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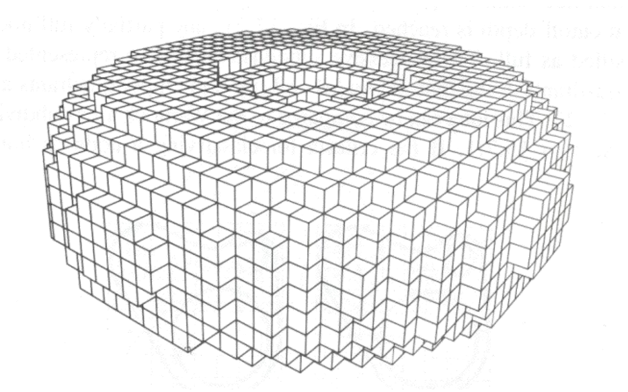
Voxel grid

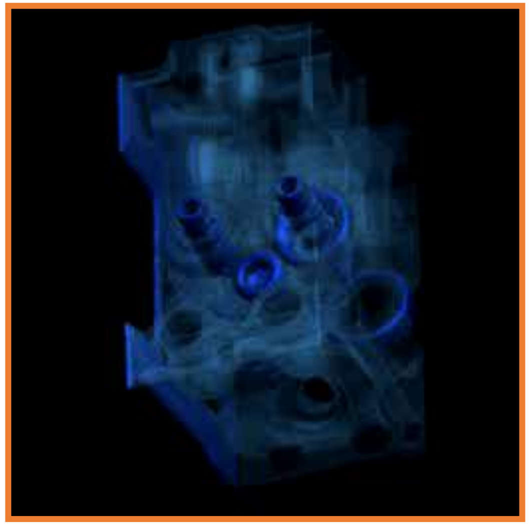
Uniform volumetric grid of samples:

- Occupancy
  (object vs. empty space)
- Density
- Color
- Other function
  (speed, temperature, etc.)

- Often acquired via
  simulation or from
  CAT, MRI, etc.



FvDFH Figure 12.20



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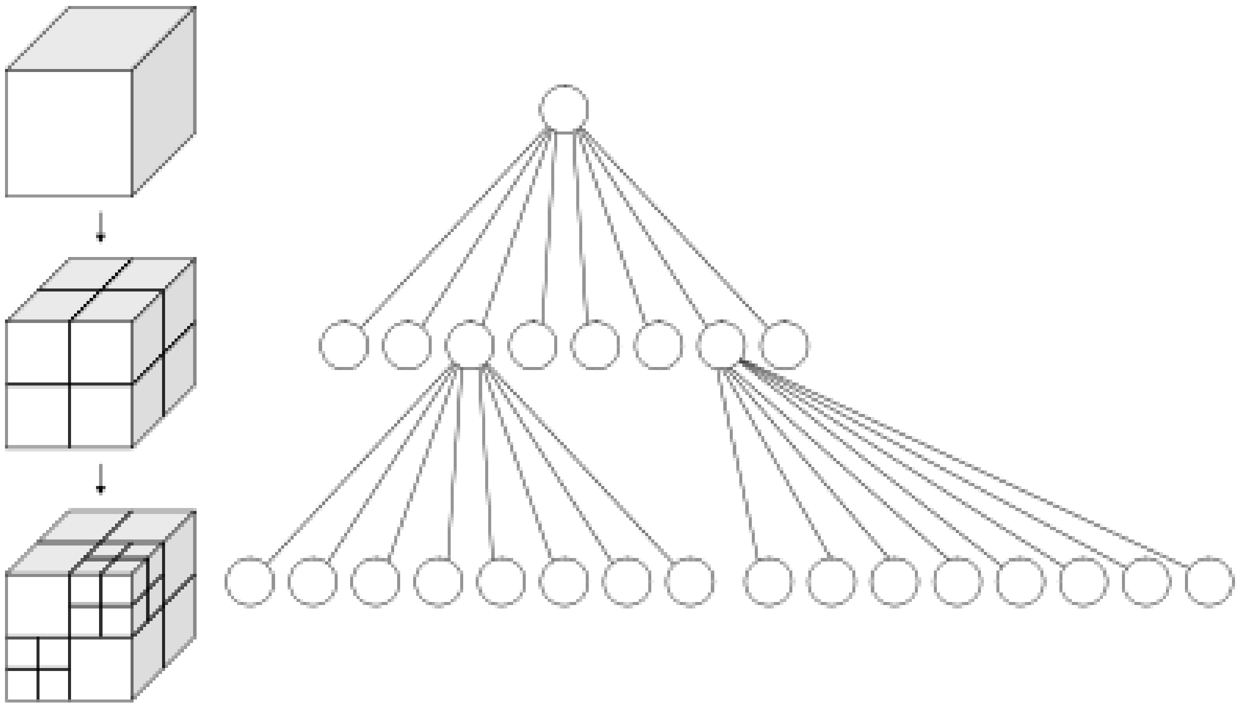
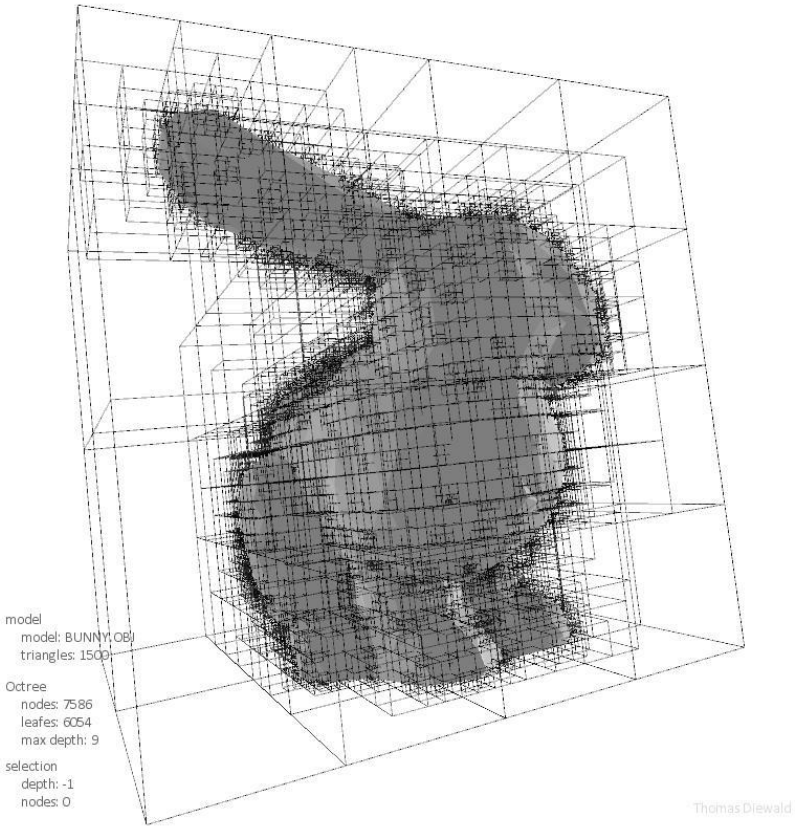
Octree

The adaptive version of the voxel grid

- Significantly more space efficient
- Makes operations more cumbersome

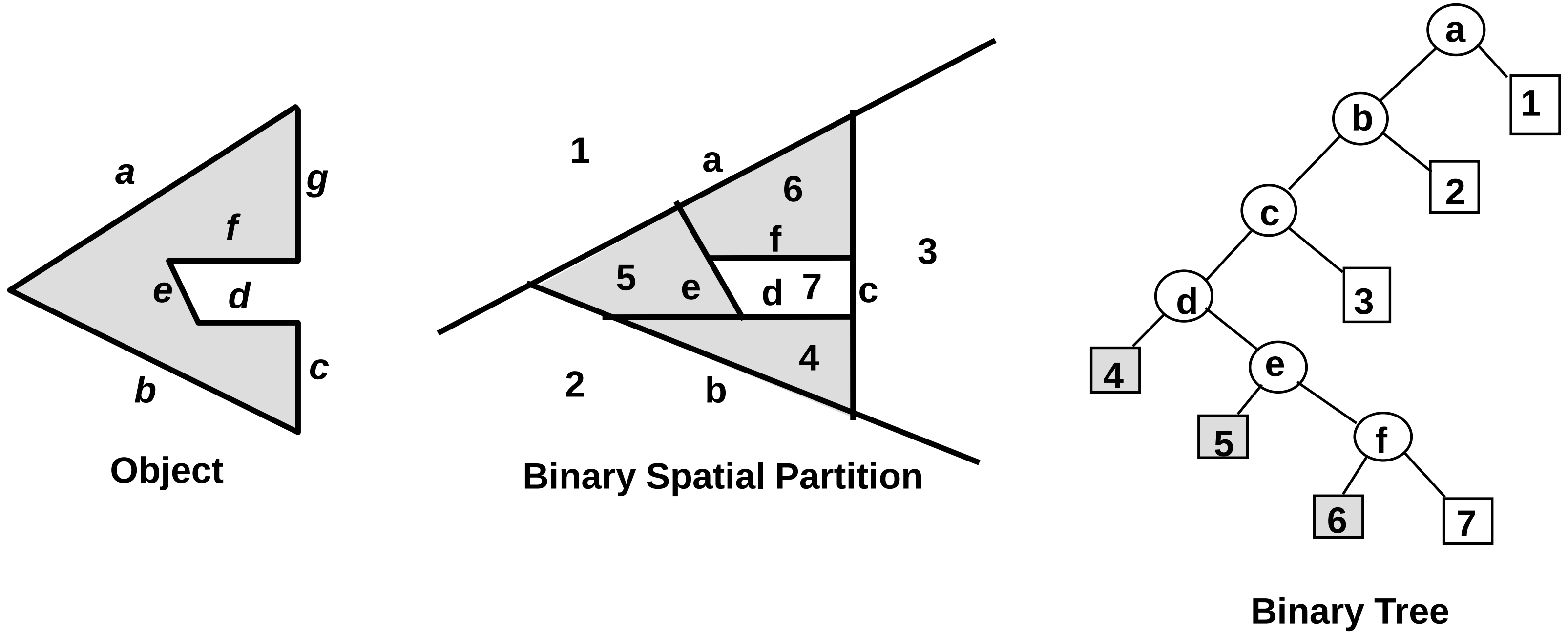




BSP Tree

Hierarchical **B**inary **S**pace **P**artition with solid/empty cells labeled

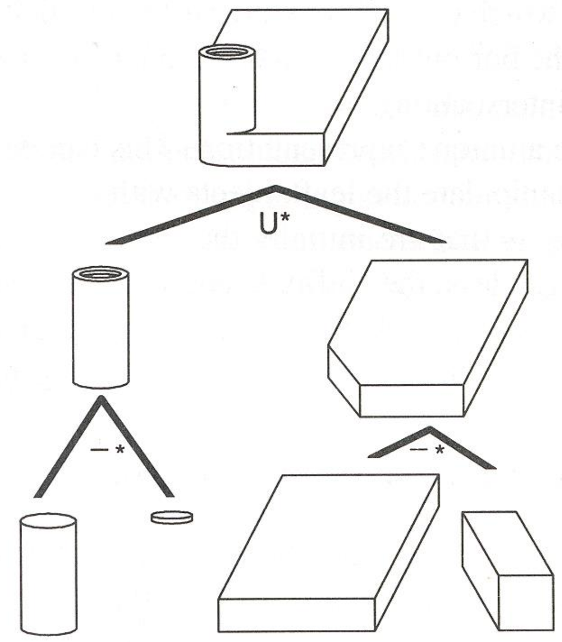
- Constructed from polygonal representations



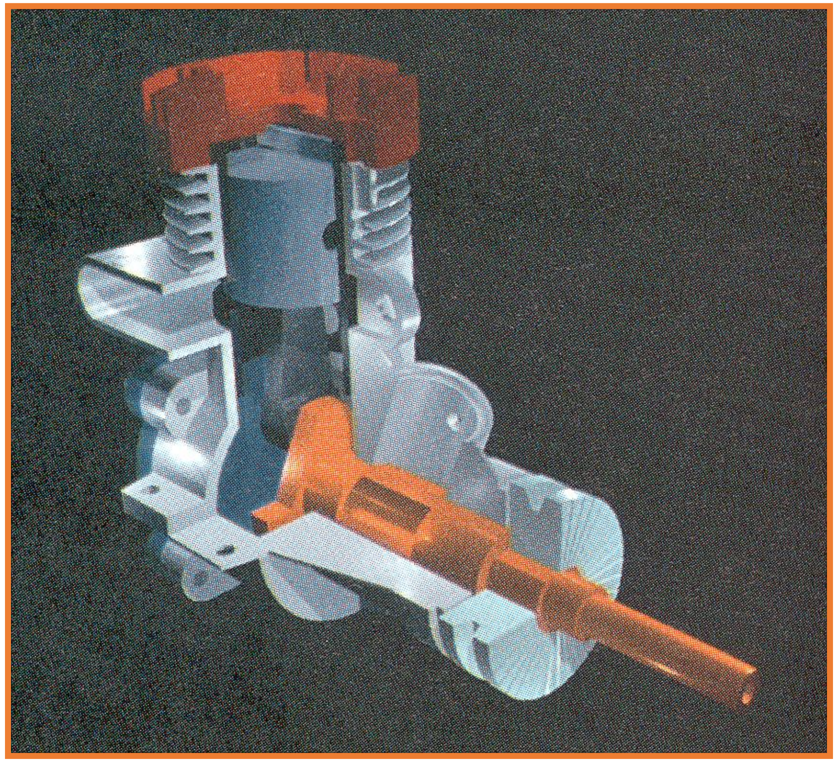
Naylor

CSG

Constructive **S**olid **G**eometry: set operations (union, difference, intersection) applied to simple shapes



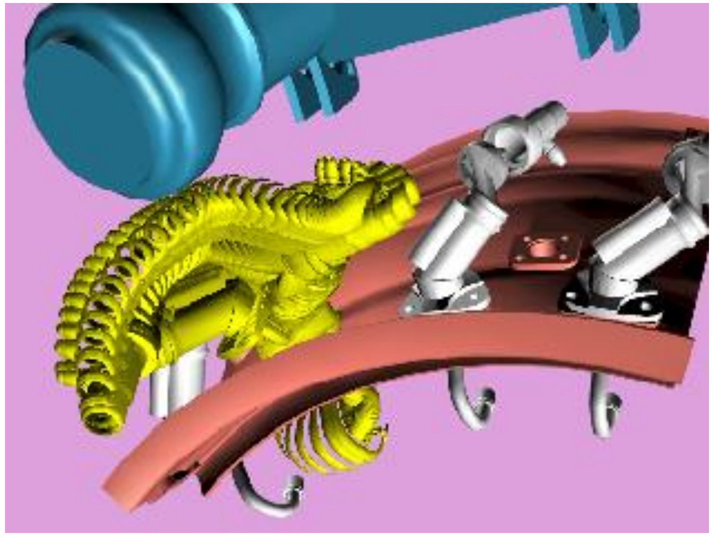
FvDFH Figure 12.27



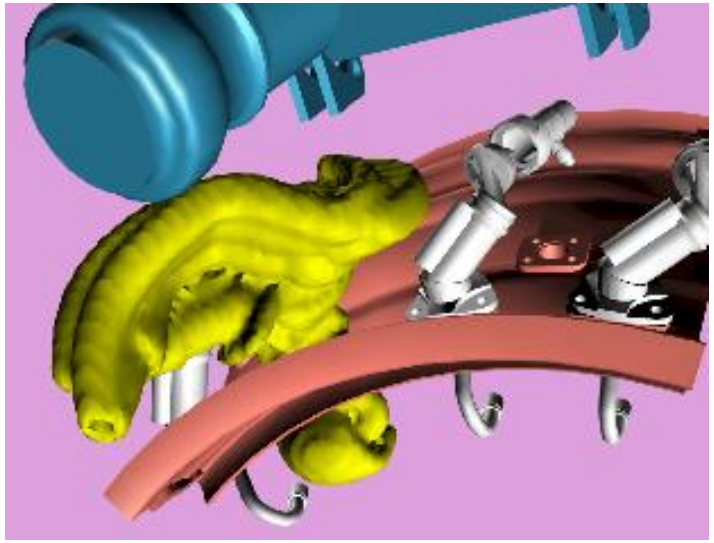
H&B Figure 9.9

Sweep

Solid swept by curve along trajectory



Removal Path



Sweep Model

Bill Lorensen
SIGGRAPH 99
Course #4 Notes

3D Object Representations

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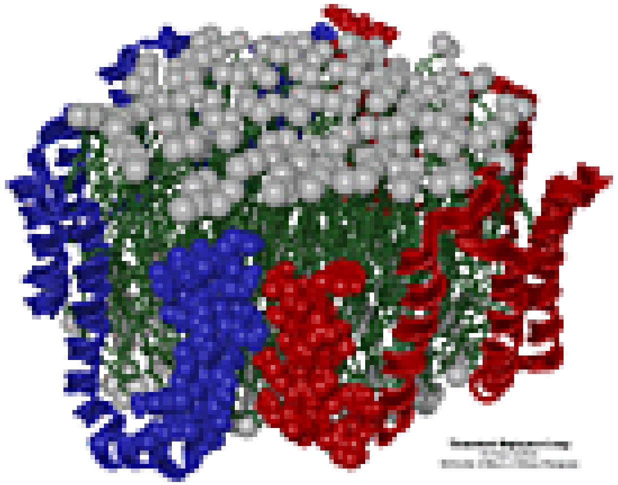
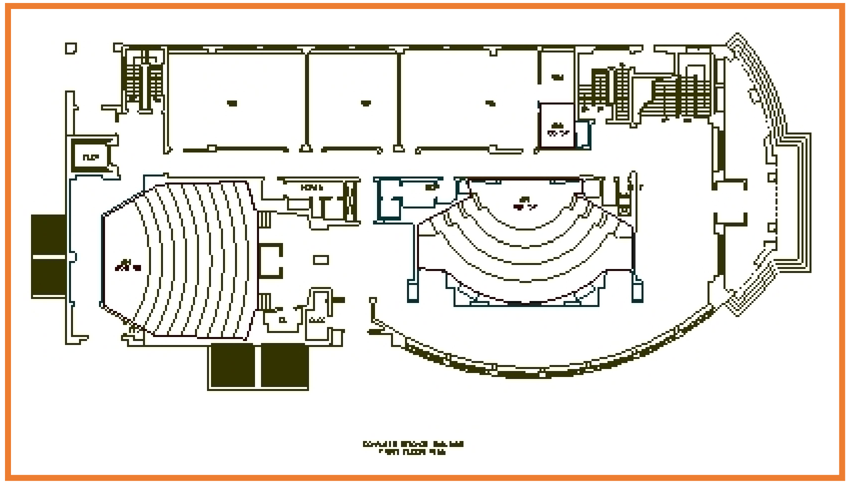
Application Specific



Apo A-1

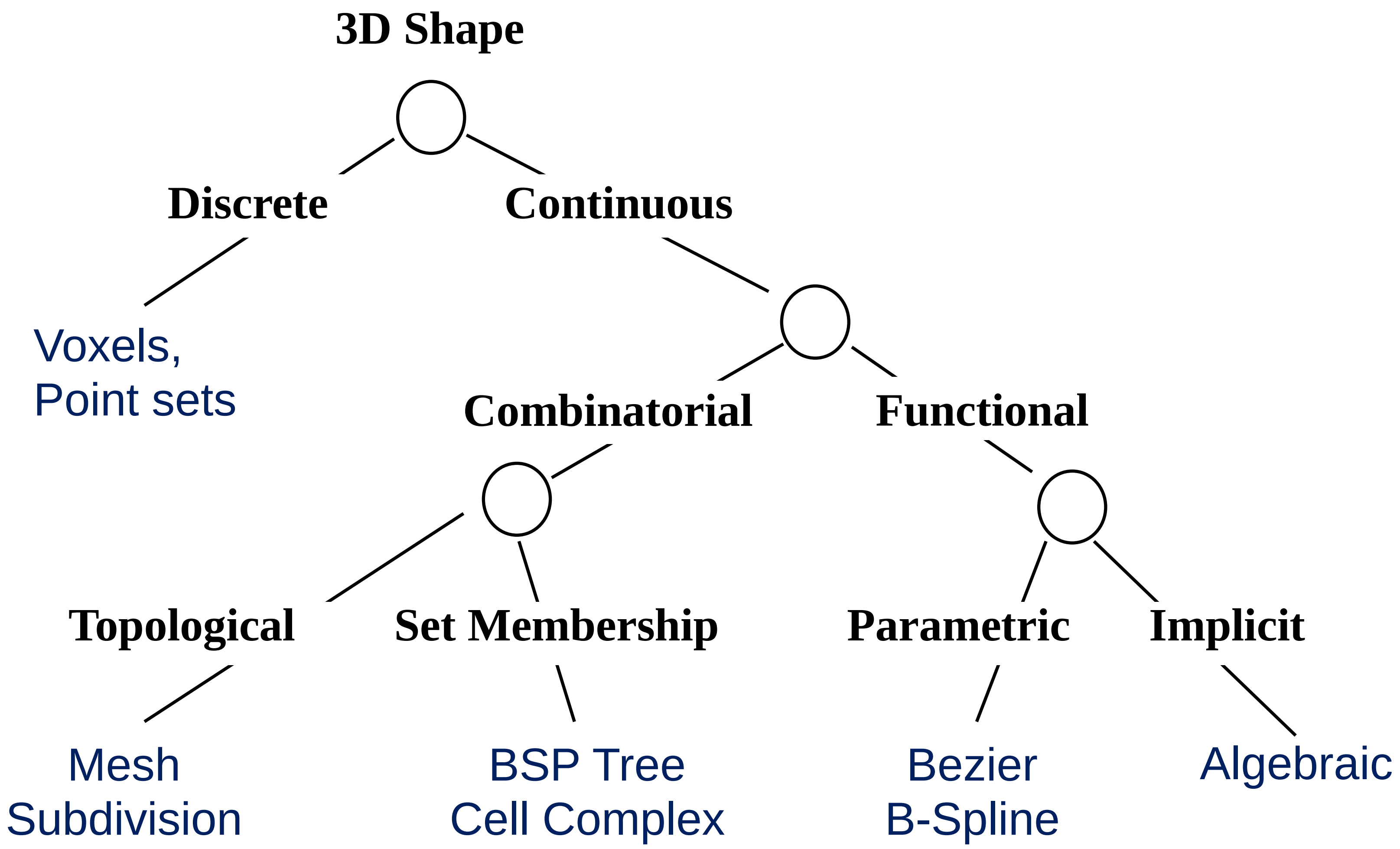
(Theoretical Biophysics Group,
University of Illinois at Urbana-Champaign)



Architectural Floorplan

(CS Building, Princeton University)

Taxonomy of 3D Representations



Naylor

Equivalence of Representations

- Thesis:
  - Each representation has enough expressive power to model the shape of any geometric object
  - It is possible to perform all geometric operations with any fundamental representation
- Analogous to Turing-equivalence
  - Computers and programming languages are Turing-equivalent, but each has its benefits…

Computational Differences

- Efficiency
  - Representational complexity (e.g. surface vs. volume)
  - Computational complexity (e.g. $O(n^2)$ vs $O(n^3)$)
  - Space/time trade-offs (e.g. tree data structures)
  - Numerical accuracy/stability (e.g. degree of polynomial)
- Simplicity
  - Ease of acquisition
  - Hardware acceleration
  - Software creation and maintenance
- Usability
  - Designer interface vs. computational engine

Upcoming Lectures

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
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