Image Processing

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Image Processing Operations

- Luminance
  - Brightness
  - Contrast
  - Gamma
  - Histogram equalization
- Color
  - Grayscale
  - Saturation
  - White balance
- Linear filtering
  - Blur & sharpen
  - Edge detect
  - Convolution
- Non-linear filtering
  - Median
  - Bilateral filter
- Dithering
  - Quantization
  - Ordered dither
  - Floyd-Steinberg

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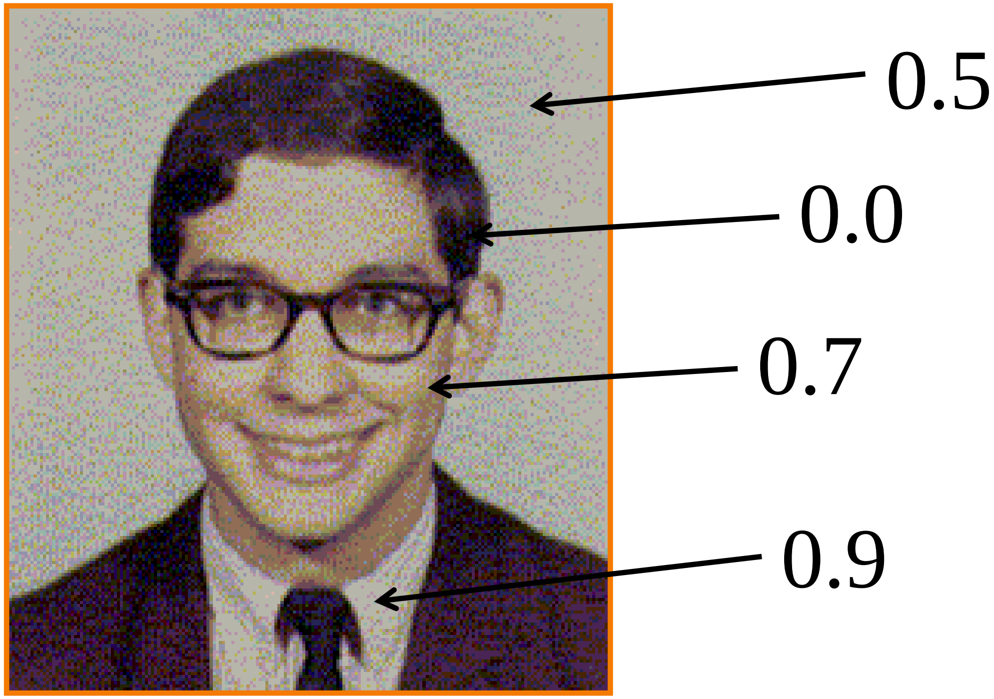
What is Luminance?

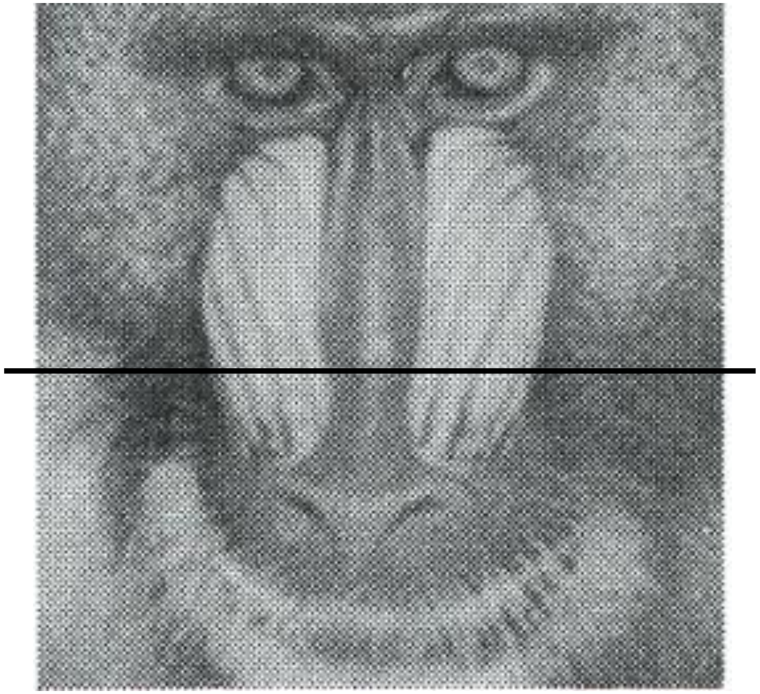
Measures perceived "gray-level" of pixel

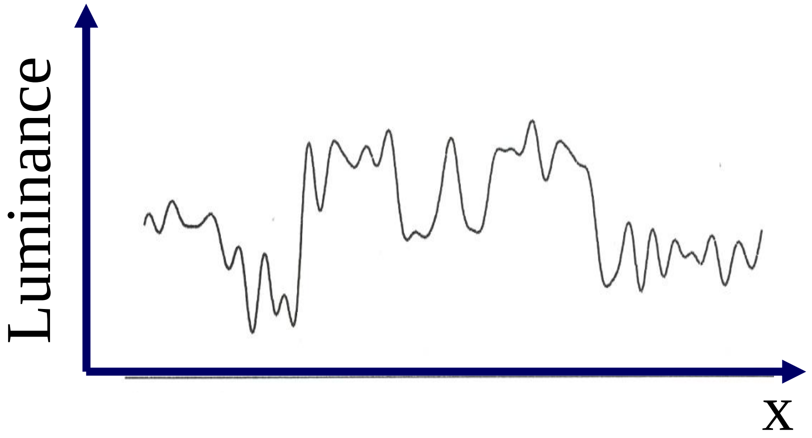
L = 0.30*red + 0.59*green + 0.11*blue



Luminance

Measures perceived "gray-level" of pixel



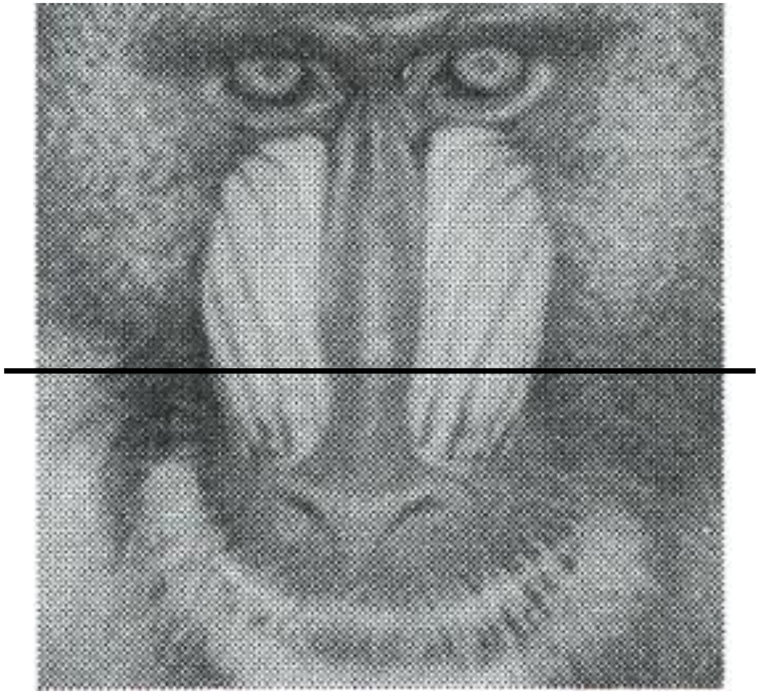


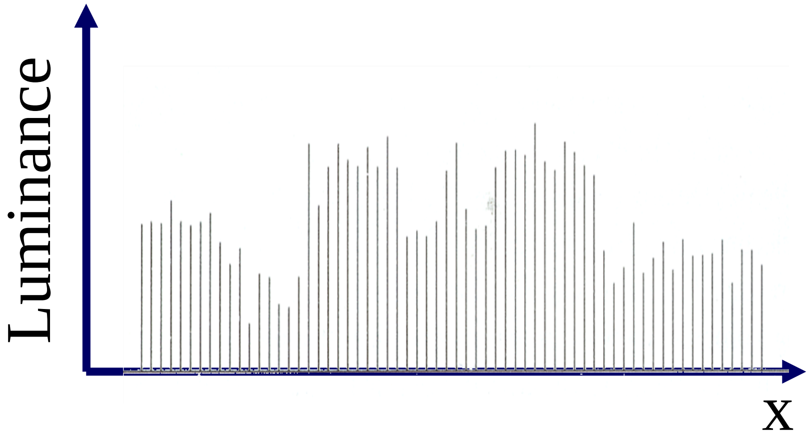
Samples of luminance for pixels on one horizontal row of pixels

Figure 19.9 FvDFH

Luminance

Measures perceived "gray-level" of pixel



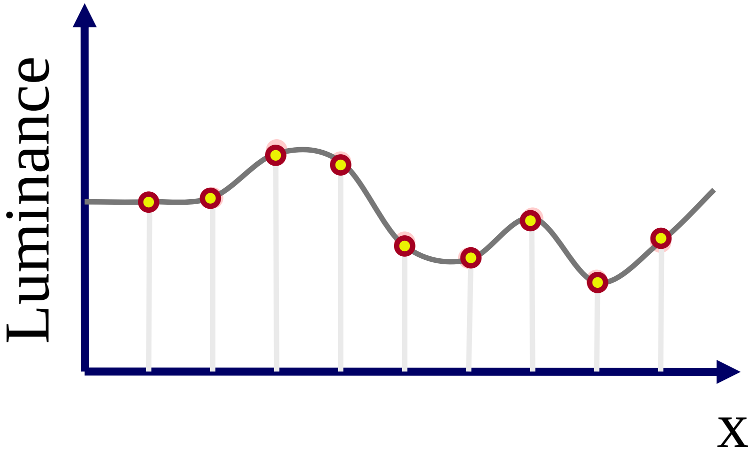


Samples of luminance for pixels on one horizontal row of pixels

Figure 19.9 FvDFH

Adjusting Brightness

- What must be done to the RGB values to make this image brighter?

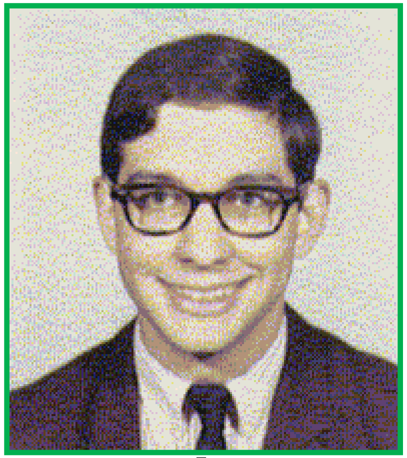


Adjusting Brightness

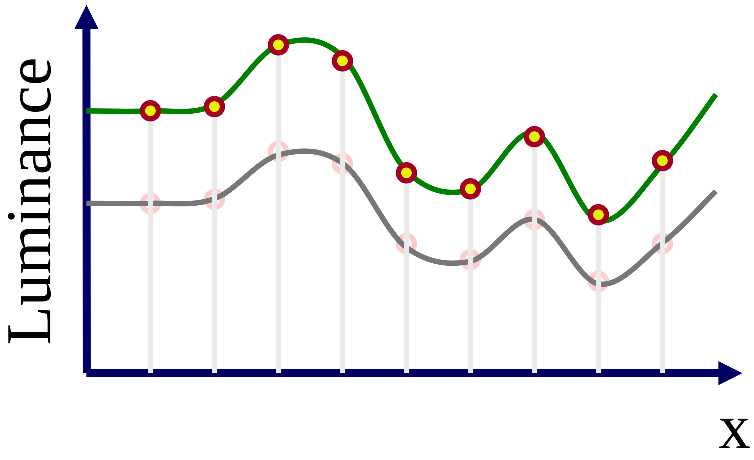
- Method 1: Convert to HSL, scale L, convert back (more on this shortly…)
- Method 2: Scale R, G, and B directly
  - o Multiply each of red, green, and blue by a factor
  - o Must clamp to [0..1] … always

    ([0..1] in floating point but often [0,255] for fixed point)

Original



Brighter

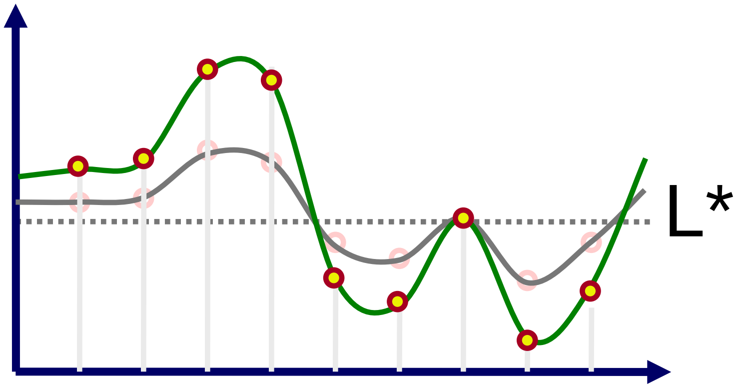


Adjusting Contrast

- Compute mean luminance L* over whole image
  Scale deviation from L* for each pixel

Original

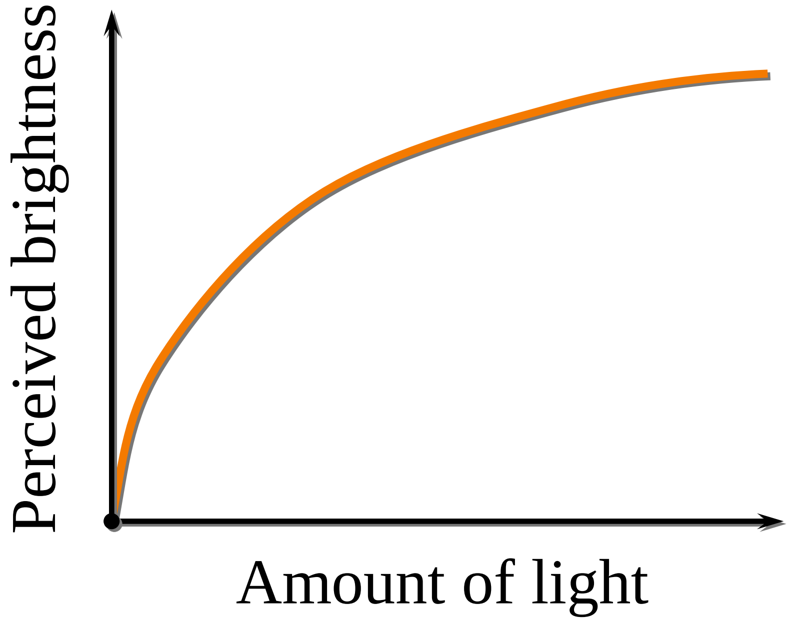
More Contrast



Adjusting Gamma

Apply non-linear function to account for difference between brightness and perceived brightness of display

$$I_{out} = I_{in}^{\gamma}$$

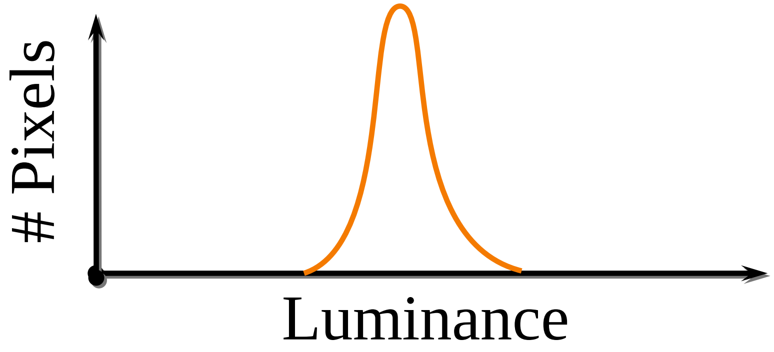


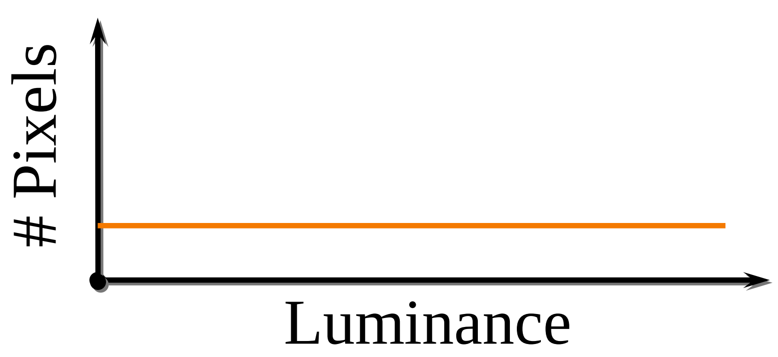
γ depends on camera and monitor

Histogram Equalization

Change distribution of luminance values to cover full range [0-1]

http://en.wikipedia.org/wiki/Histogram_equalization





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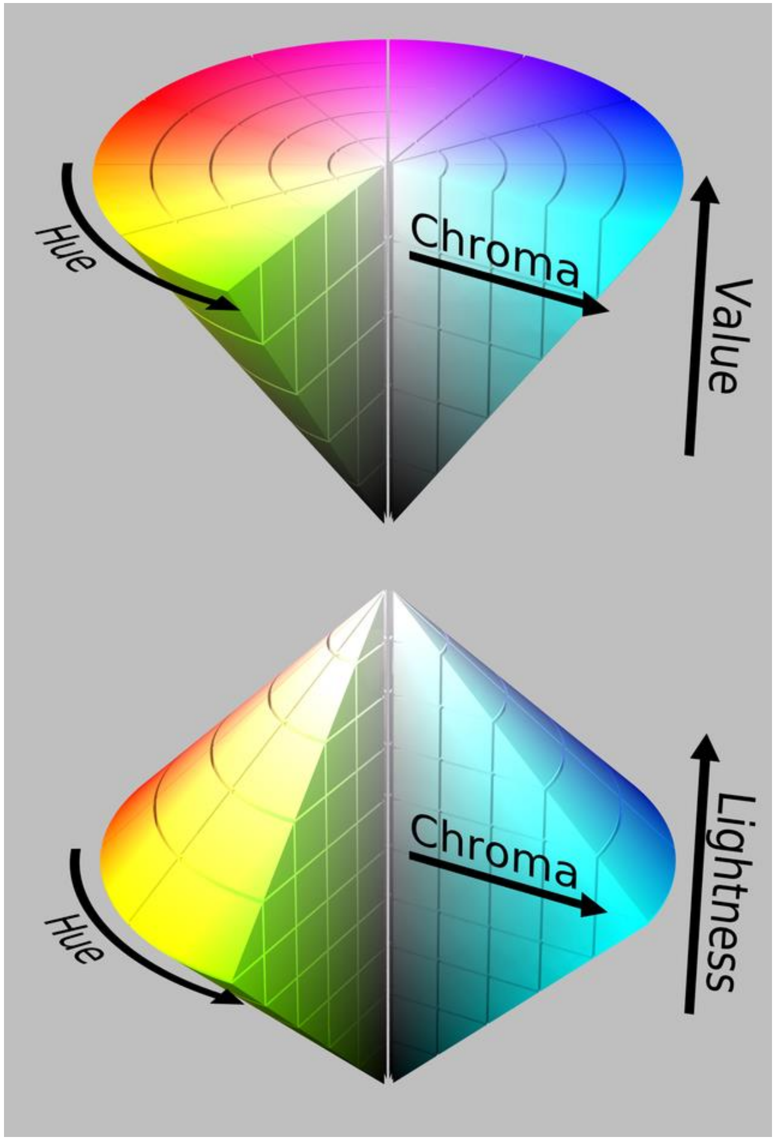
Color processing

- Color models (last lec.)
  - RGB
  - CMY
  - HSV
  - XYZ
  - La*b*
  - Etc.

HSV

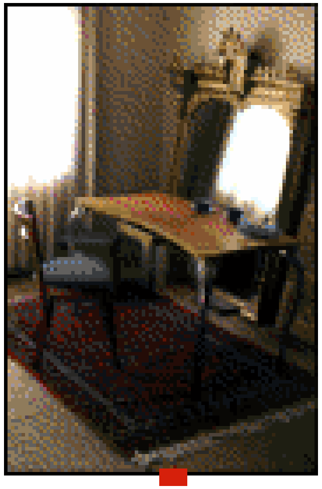
HSL



http://commons.wikimedia.org/wiki/File:HSV_color_solid_cone_chroma_gray.png

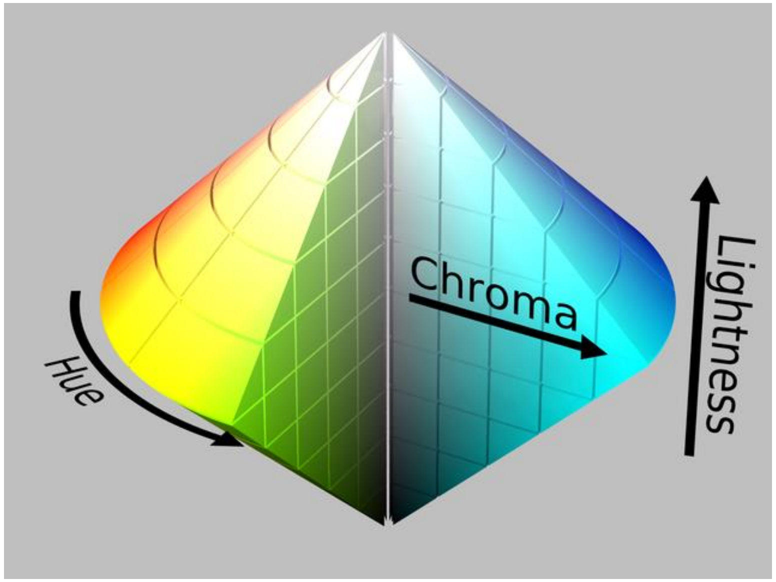
Grayscale

Convert from color to gray-levels



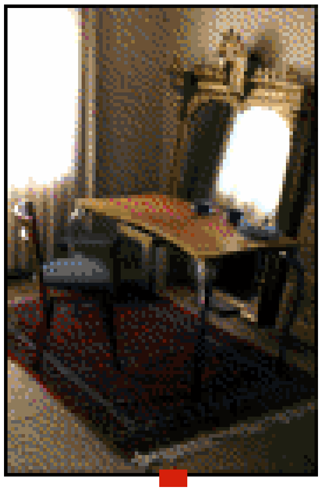
Original

Grayscale
(“black&white” photo)



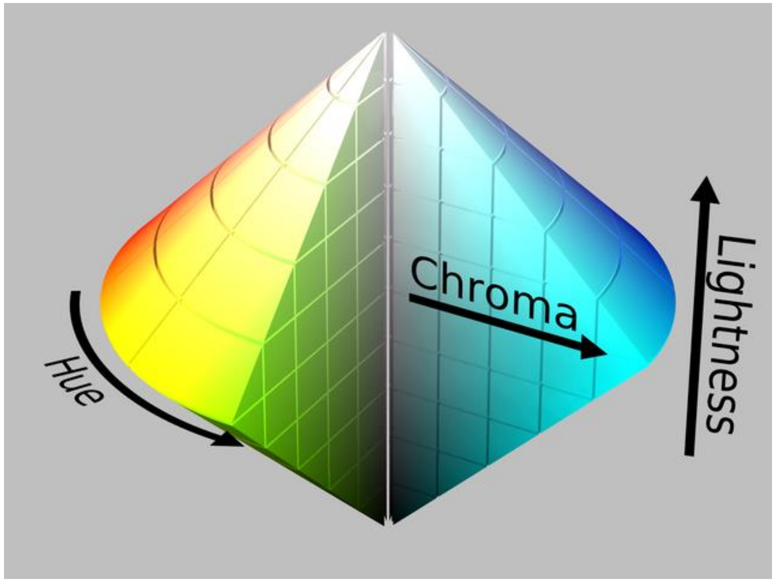
Grayscale

Convert from color to gray-levels



Original

Grayscale
(“black&white” photo)

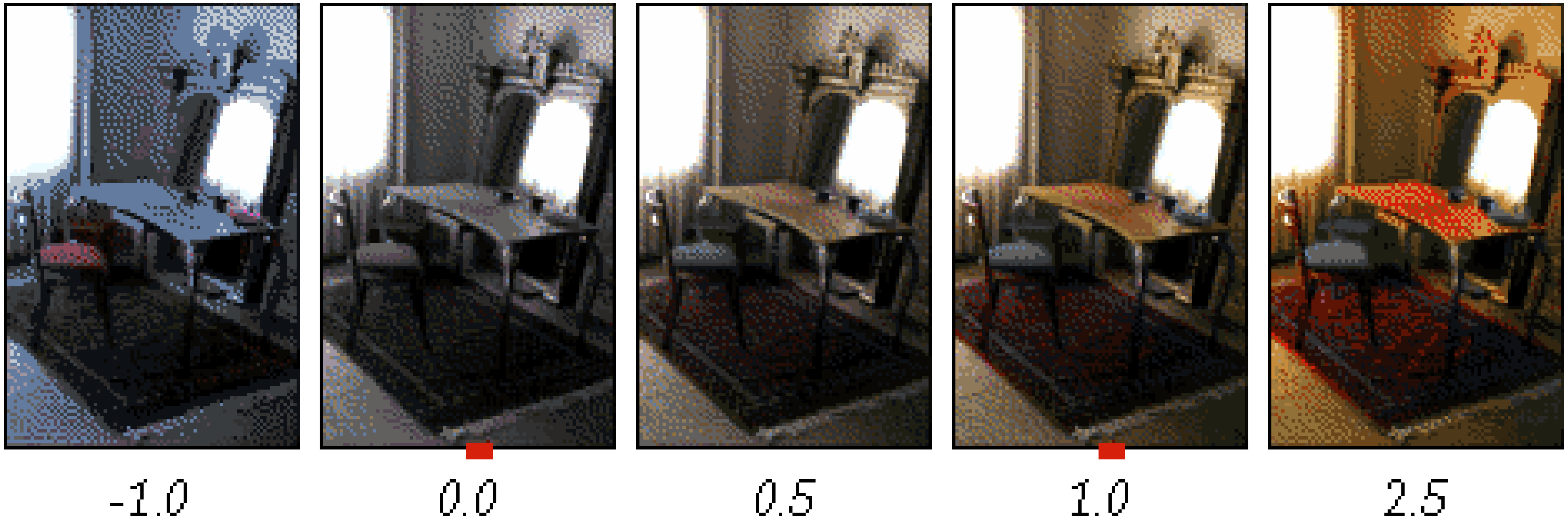


Method 1: Convert to HSL, set S=0, convert back to RGB

Method 2: Set RGB of every pixel to (L,L,L)

Adjusting Saturation

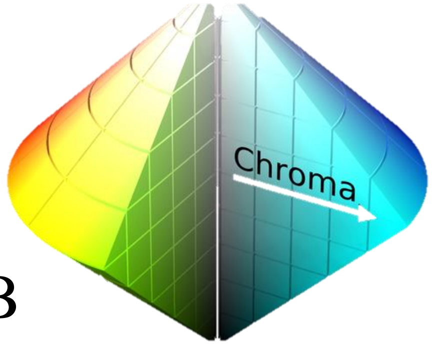
Increase/decrease color saturation of every pixel



Adjusting Saturation

Increase/decrease color saturation of every pixel

-1.0 0.0 0.5 1.0 2.5



Method 1: Convert to HSL, scale S, convert back

Method 2: R’ = L + scale * (R-L) … same for G&B

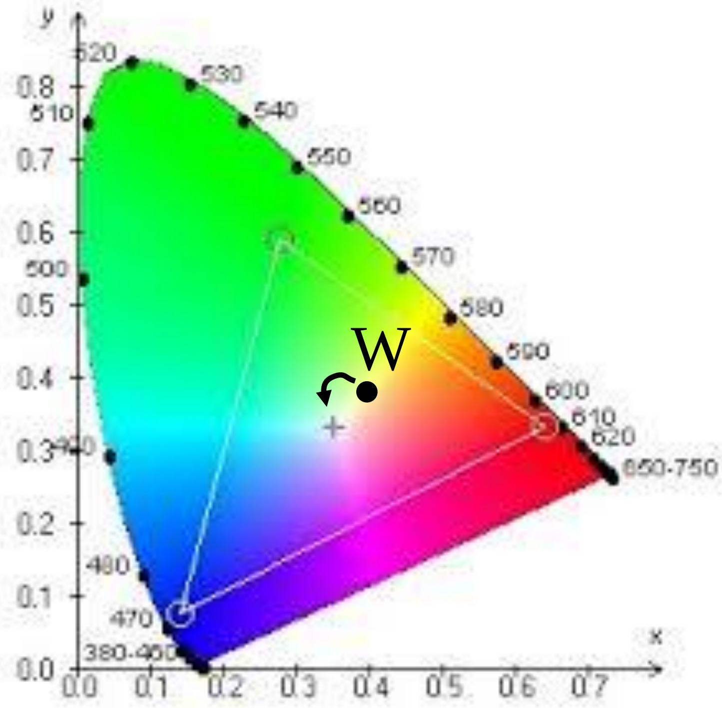
White Balance

Adjust colors so that a given RGB value is mapped to a neutral color

White Balance

Conceptually:

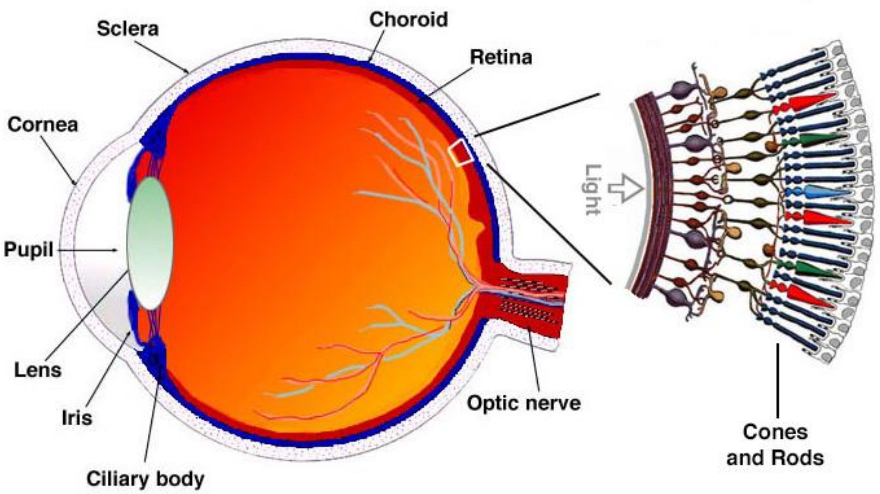
- Provide an RGB value W that should be mapped to white
- Perform transformation of color space



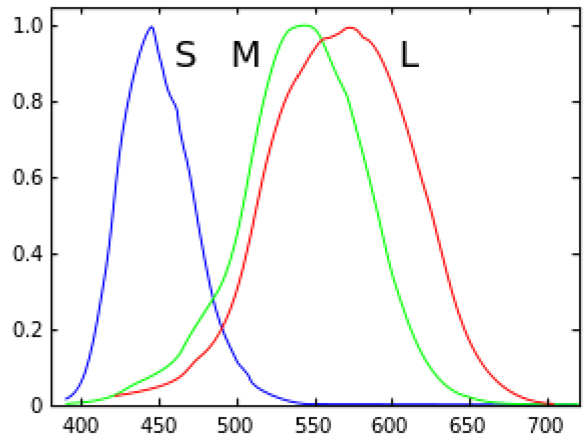
White Balance

Von Kries method: adjust colors in LMS color space

- LMS primaries represent the responses of the three different types of cones in our eyes



http://www.blueconemonochromacy.org



Wikipedia

White Balance

For each pixel RGB:

1) Convert to XYZ color space

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9502 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

2) Convert to LMS color space

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.40024 & 0.7076 & -0.08081 \\ -0.2263 & 1.16532 & 0.0457 \\ 0 & 0 & 0.91822 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3) Divide by $L_W M_W S_W$ the color of “white” in LMS

4) Convert back to RGB

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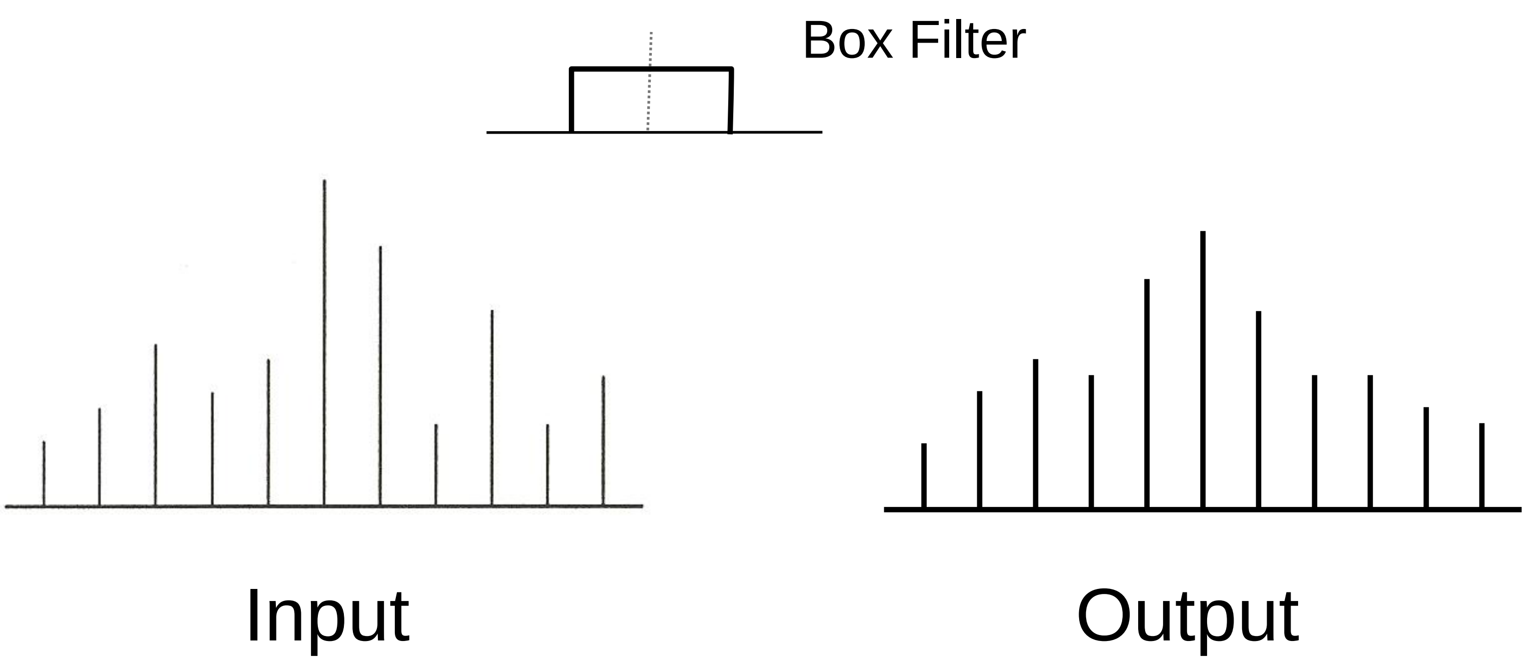
Blur

What is the basic operation for each pixel when blurring an image?

Basic Operation: Convolution

Output value is weighted sum of values in neighborhood of input image

- Pattern of weights is the “filter” or “kernel”



Basic Operation: Convolution

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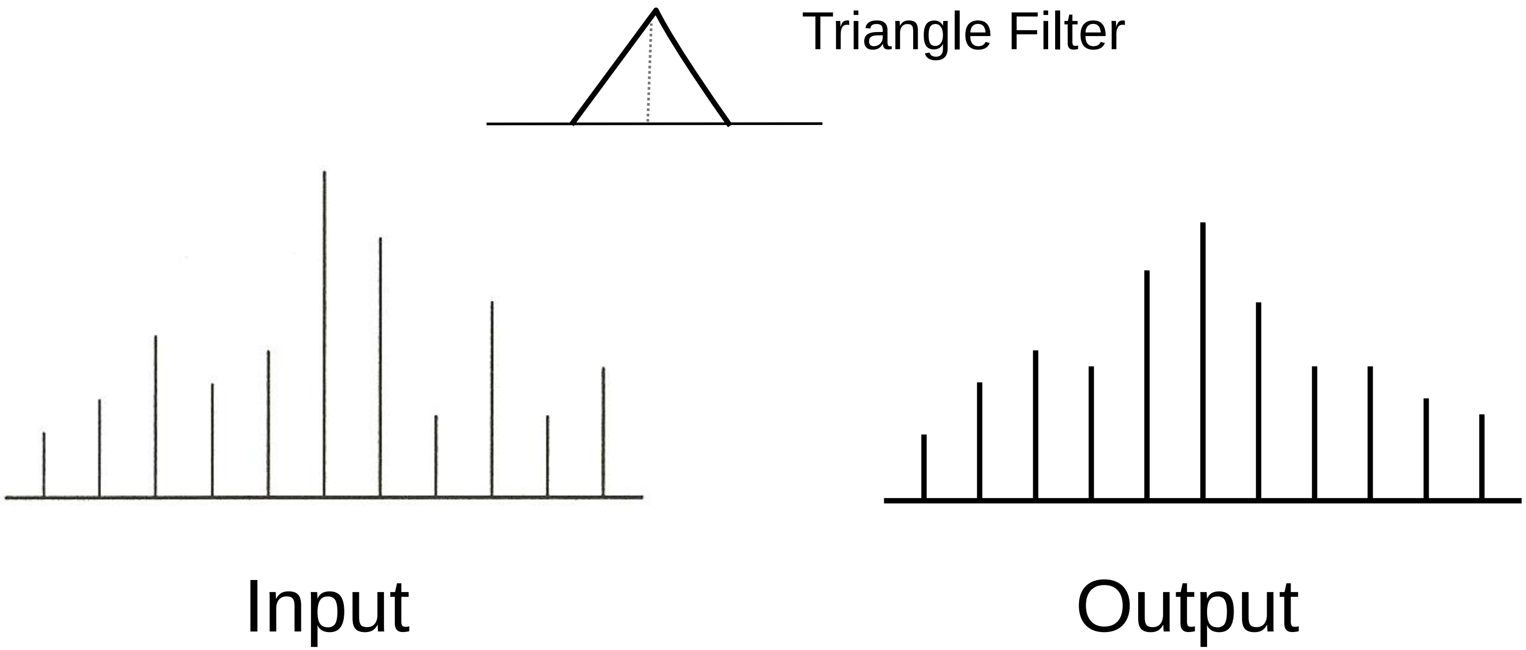

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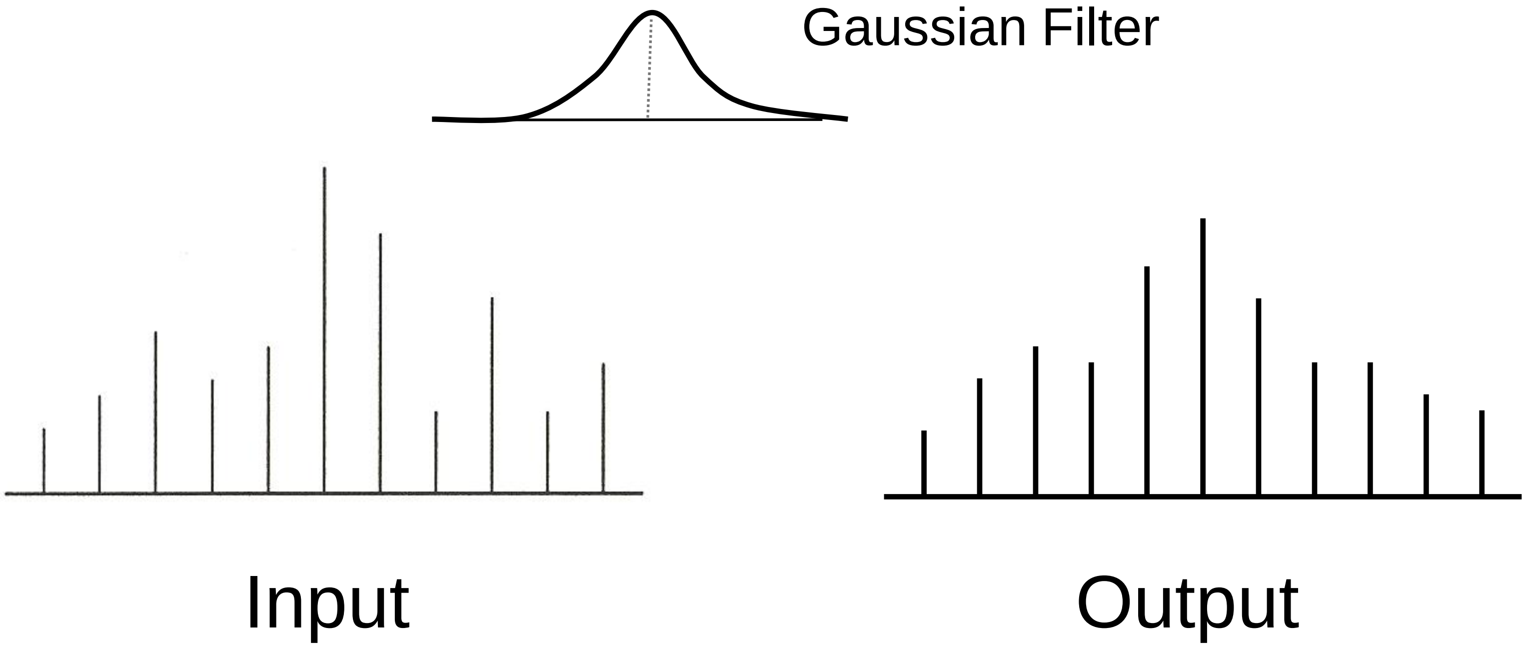
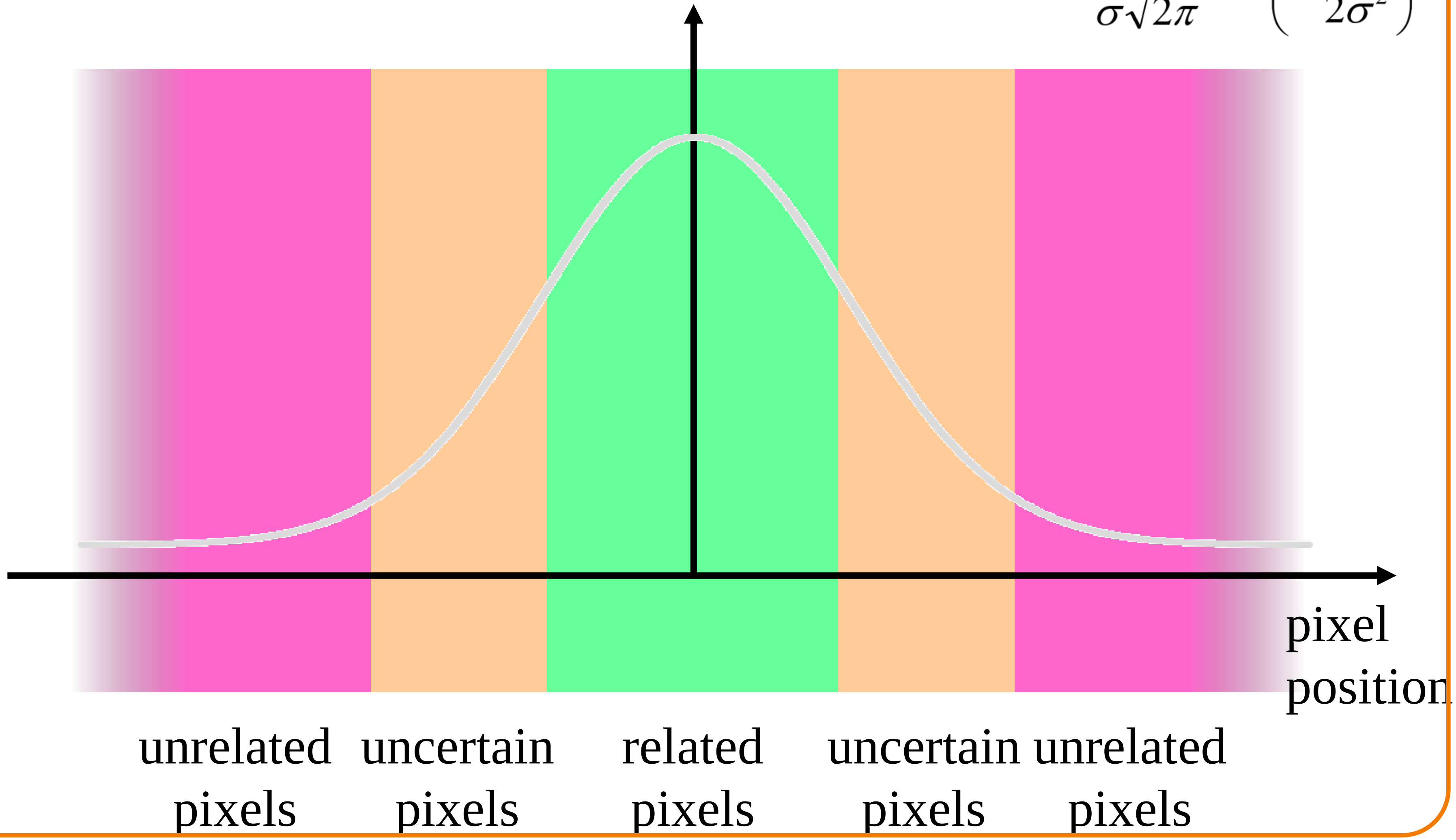


Convolution with a Gaussian Filter

$$G(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



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Output value is weighted sum of values in neighborhood of input image

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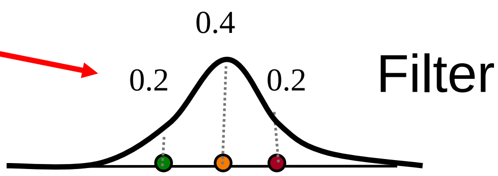
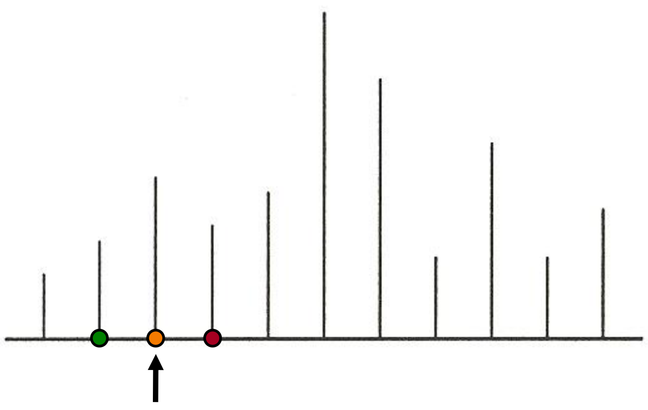
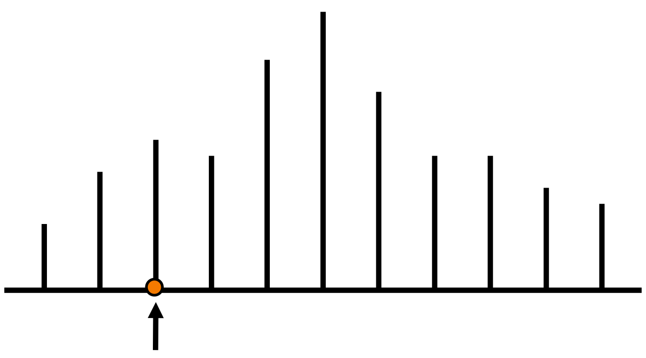
Problem: weights should sum to 1. Practical solution in next lecture: divide by sum of weights.





Input

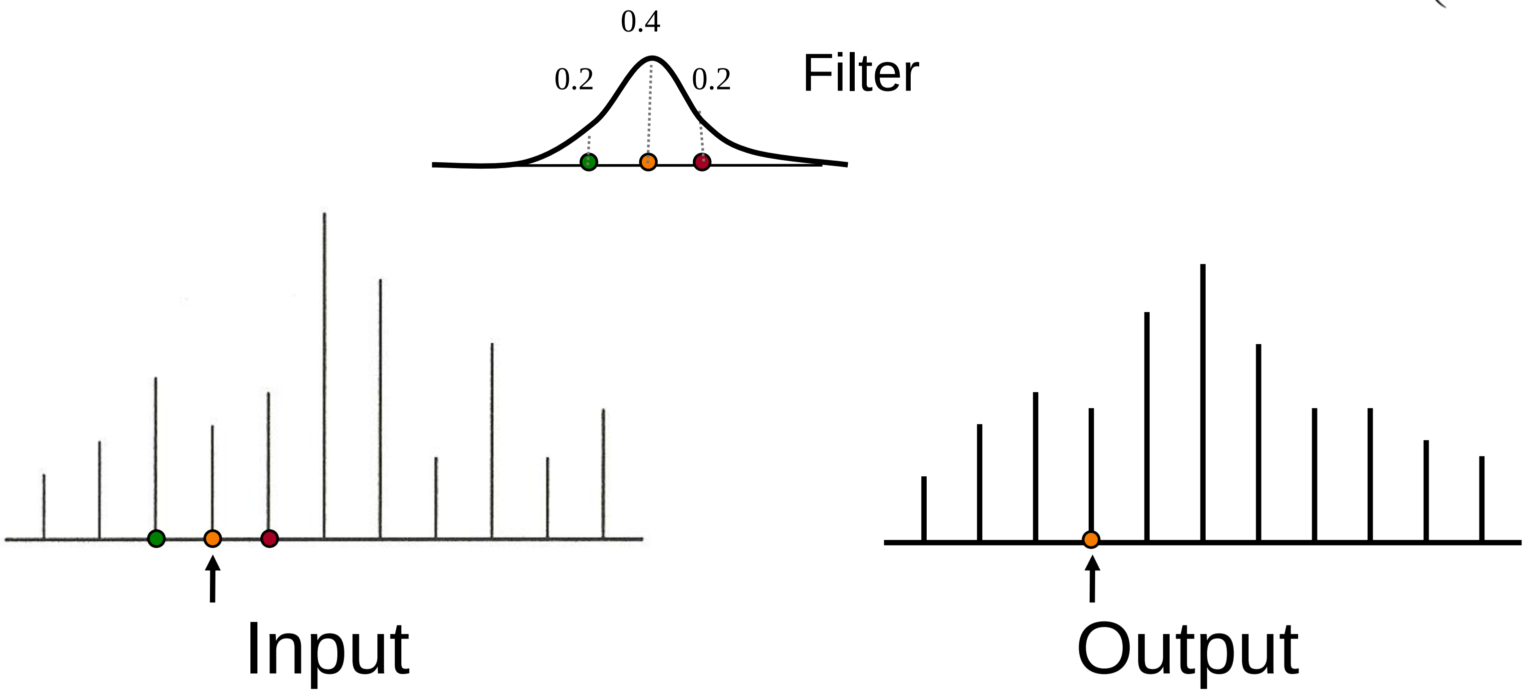


Output

Convolution with a Gaussian Filter

Output value is weighted sum of values in neighborhood of input image

$$G(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



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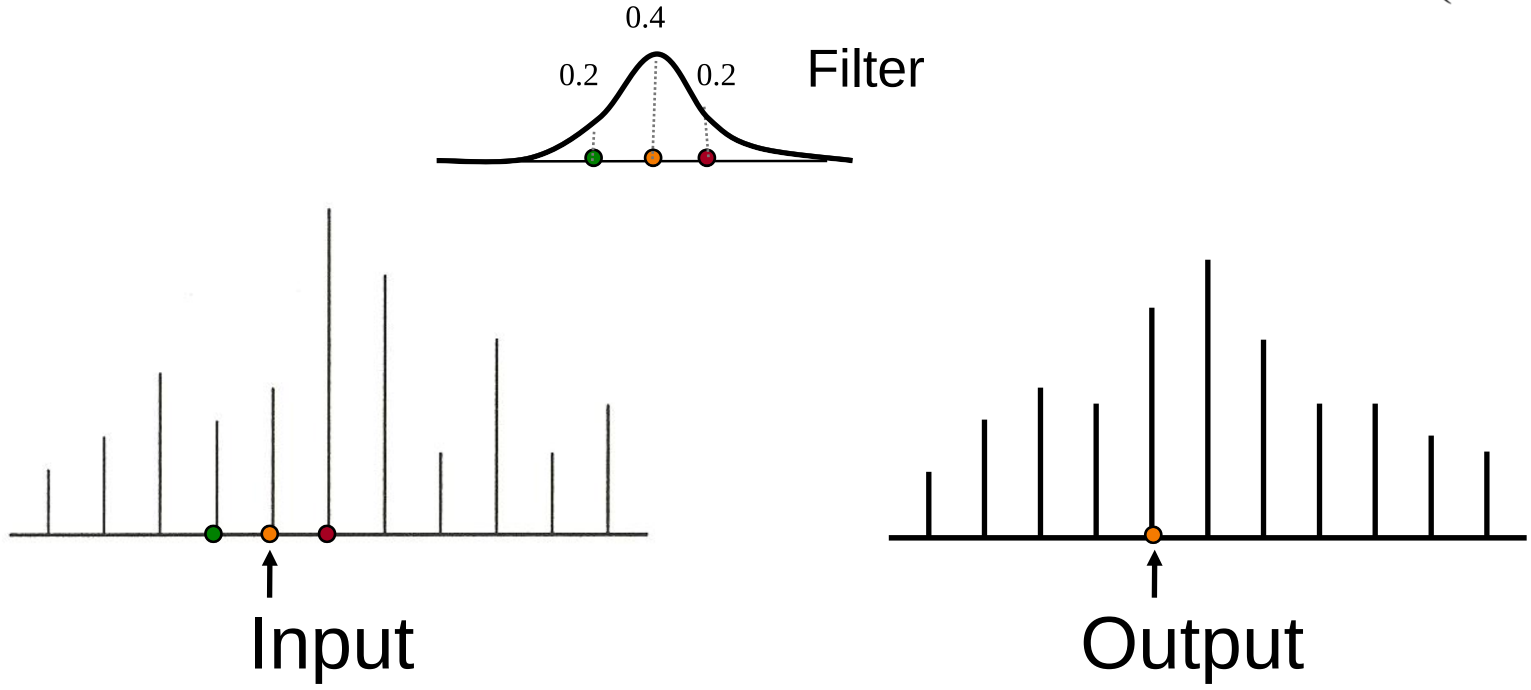

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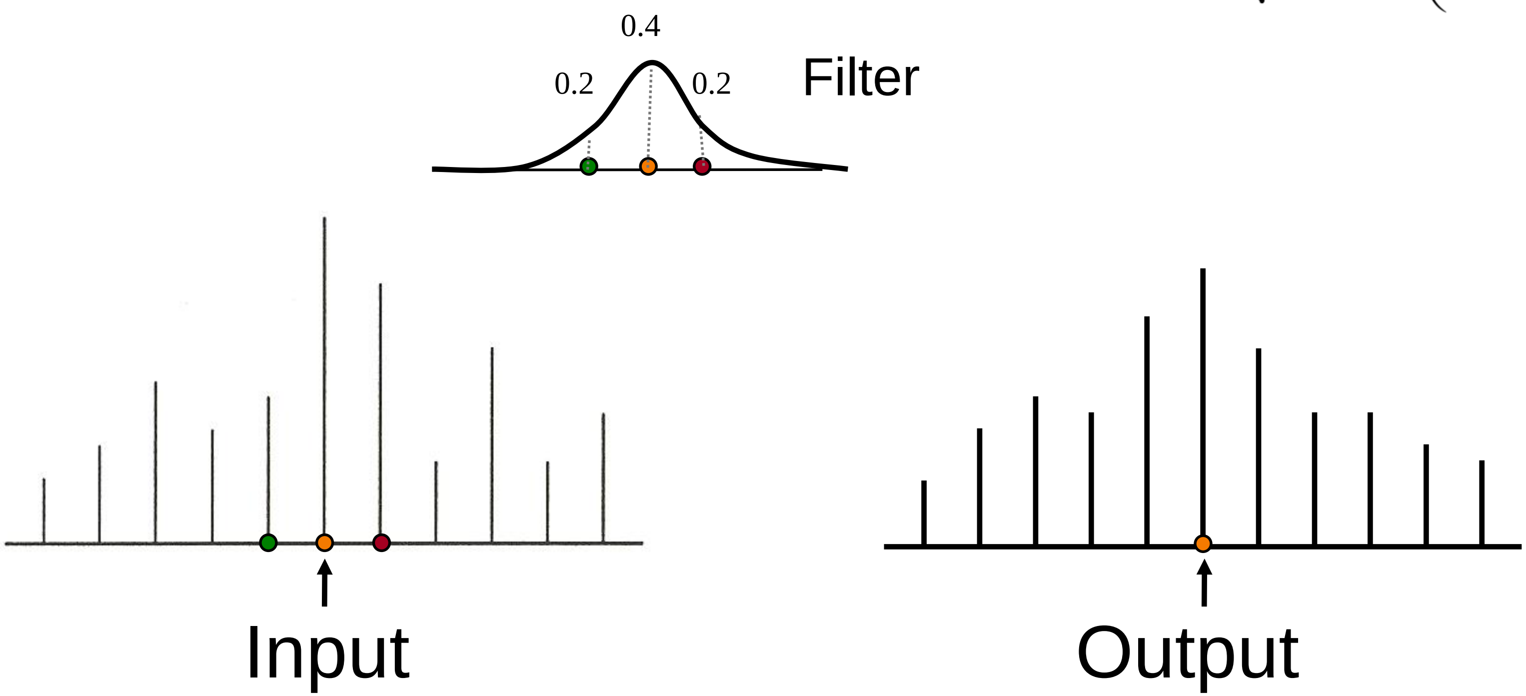

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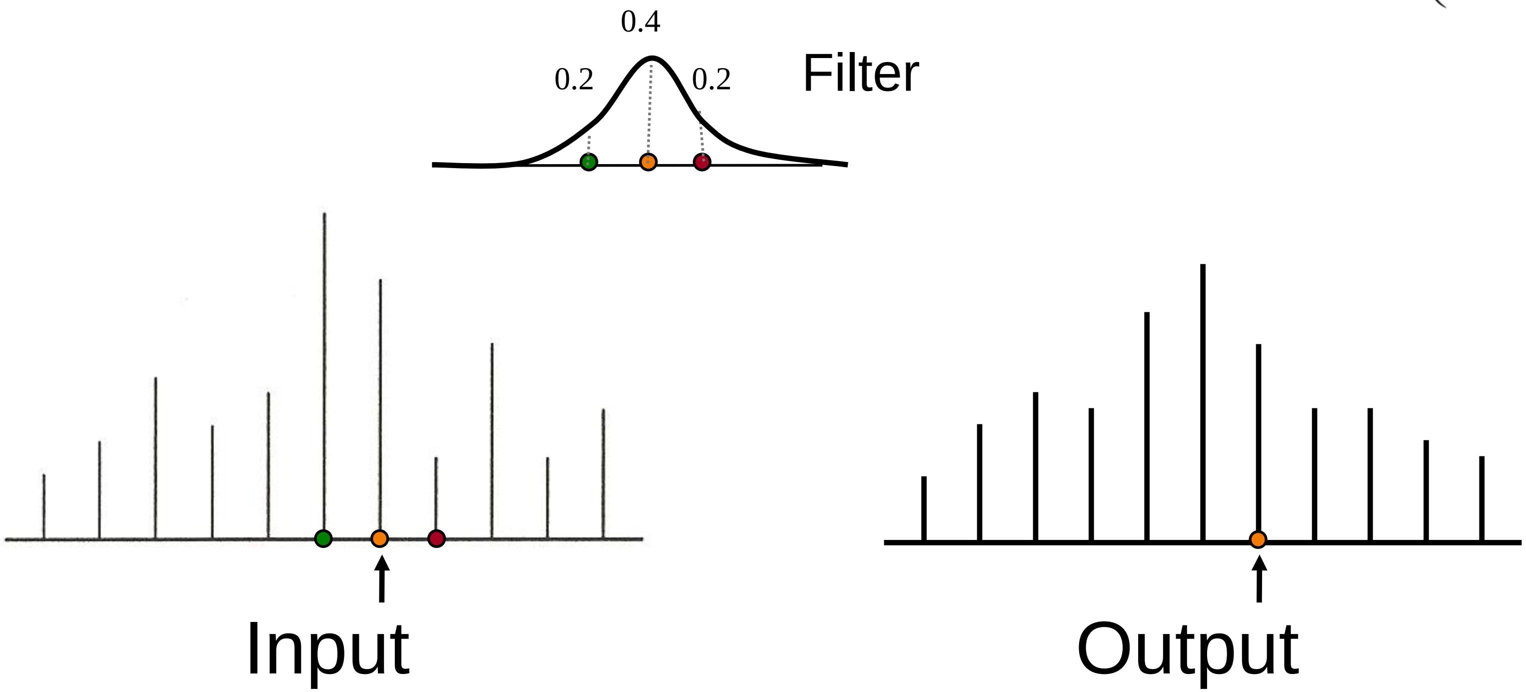
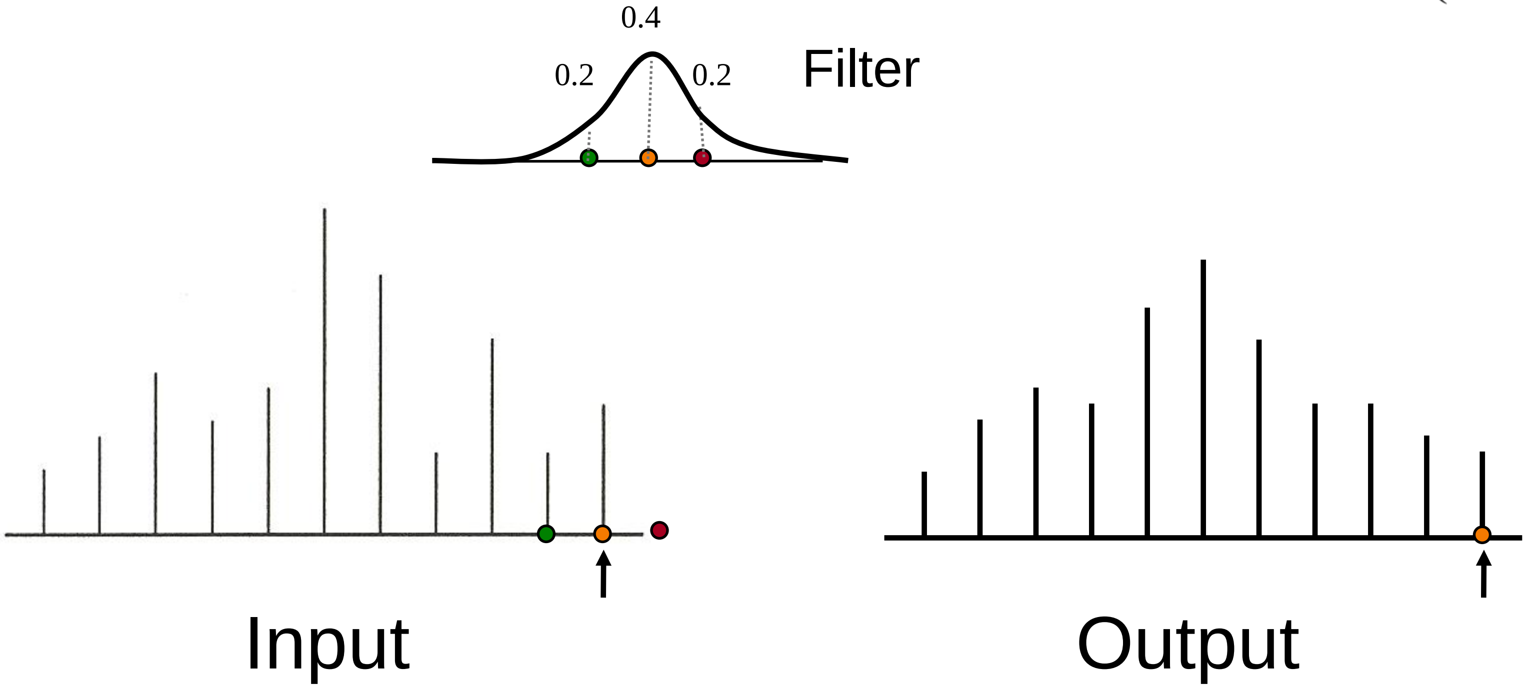


Convolution with a Gaussian Filter

What if filter extends beyond boundary?

$$G(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



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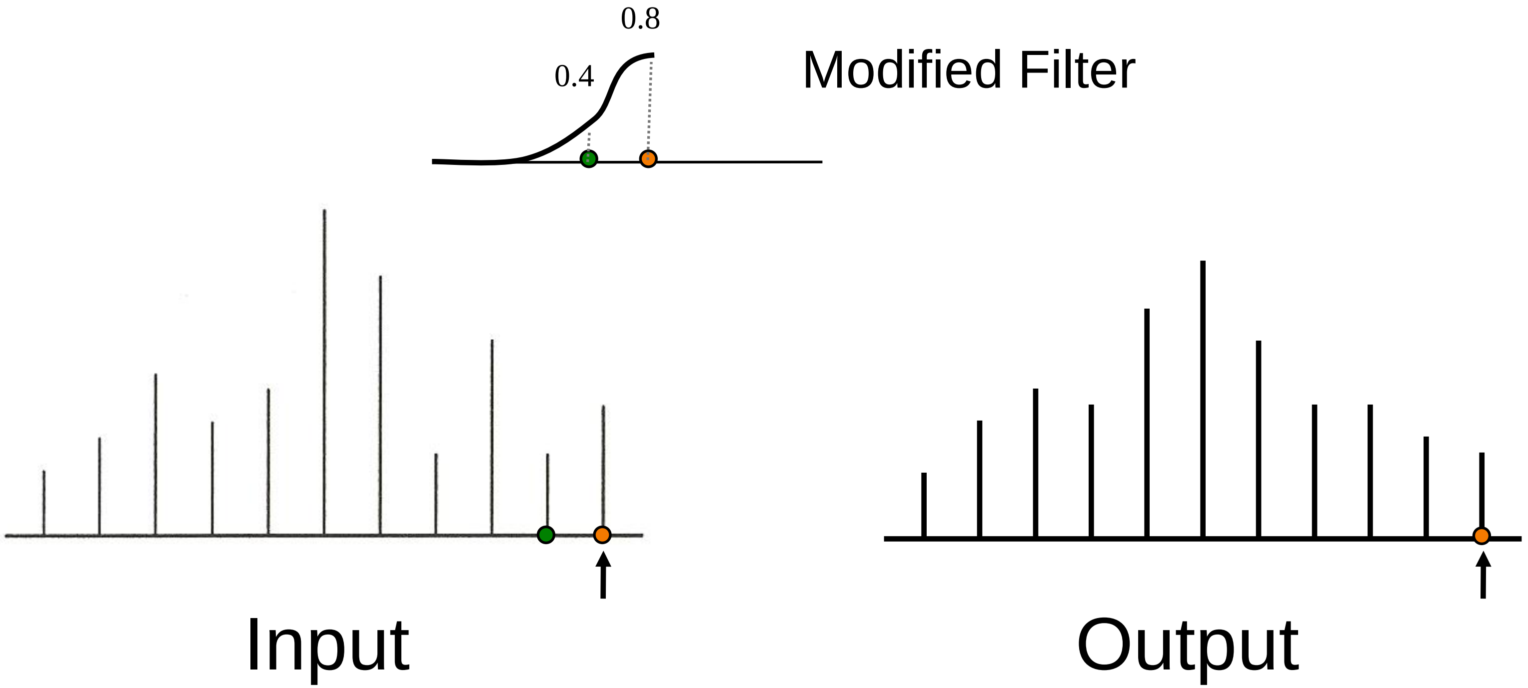


Convolution with a Gaussian Filter

Output contains samples from smoothed input

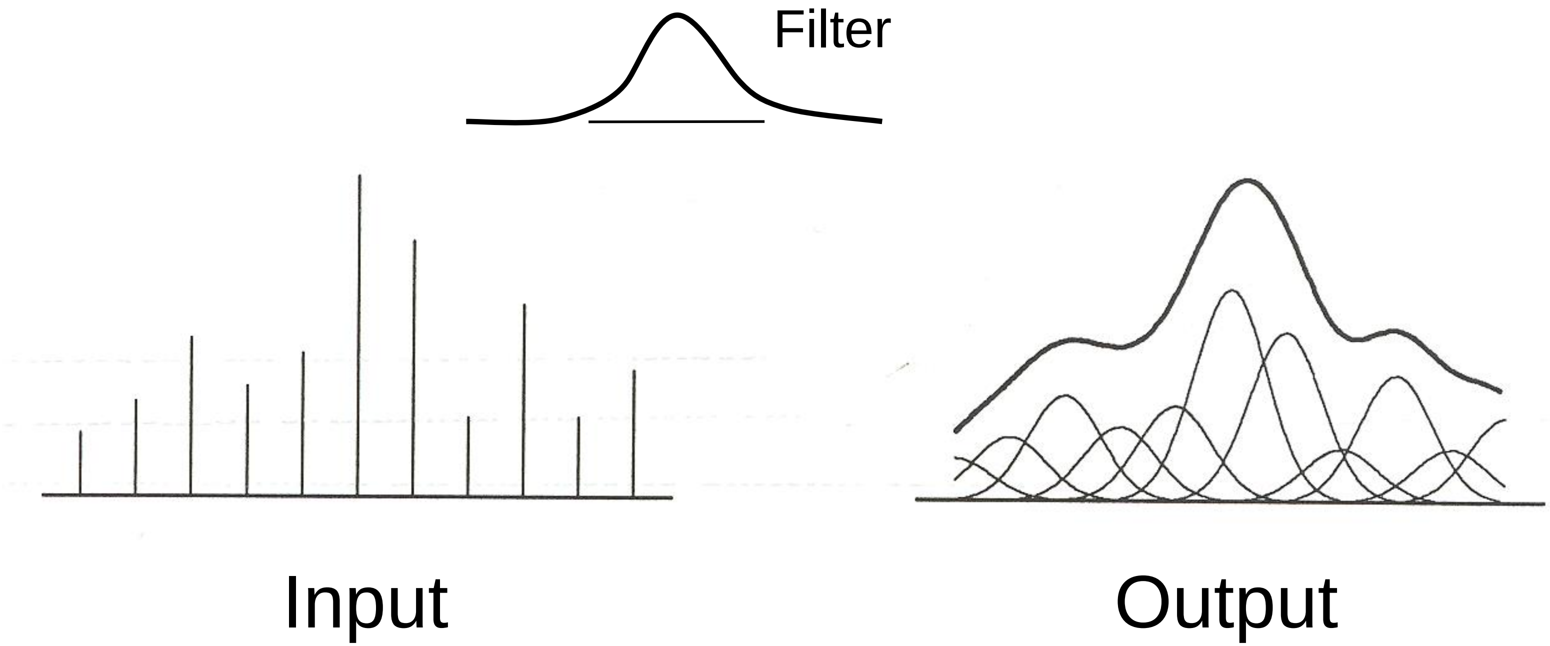
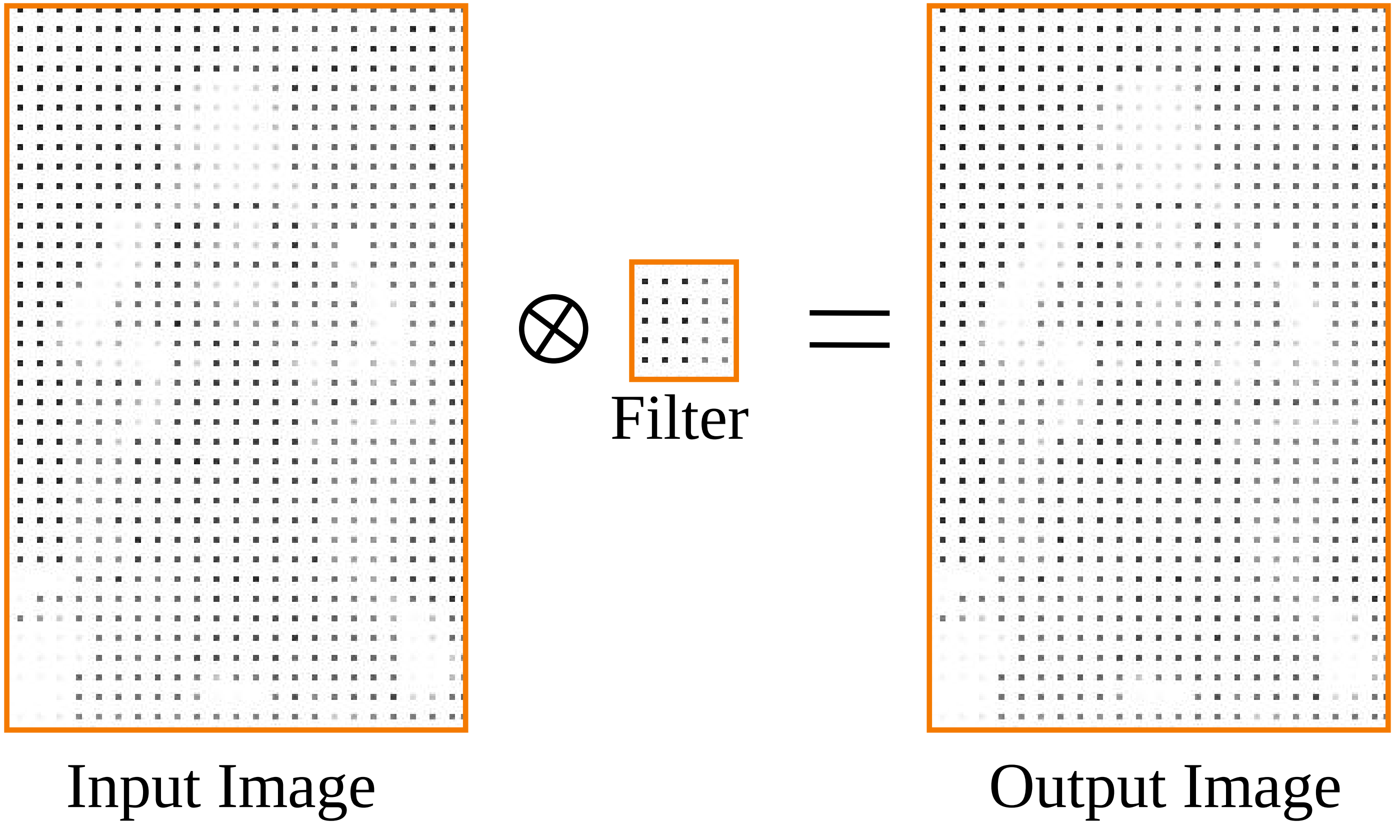


Figure 2.4 Wolberg

Linear Filtering

2D Convolution

- Each output pixel is a linear combination of input pixels in 2D neighborhood with weights prescribed by a filter



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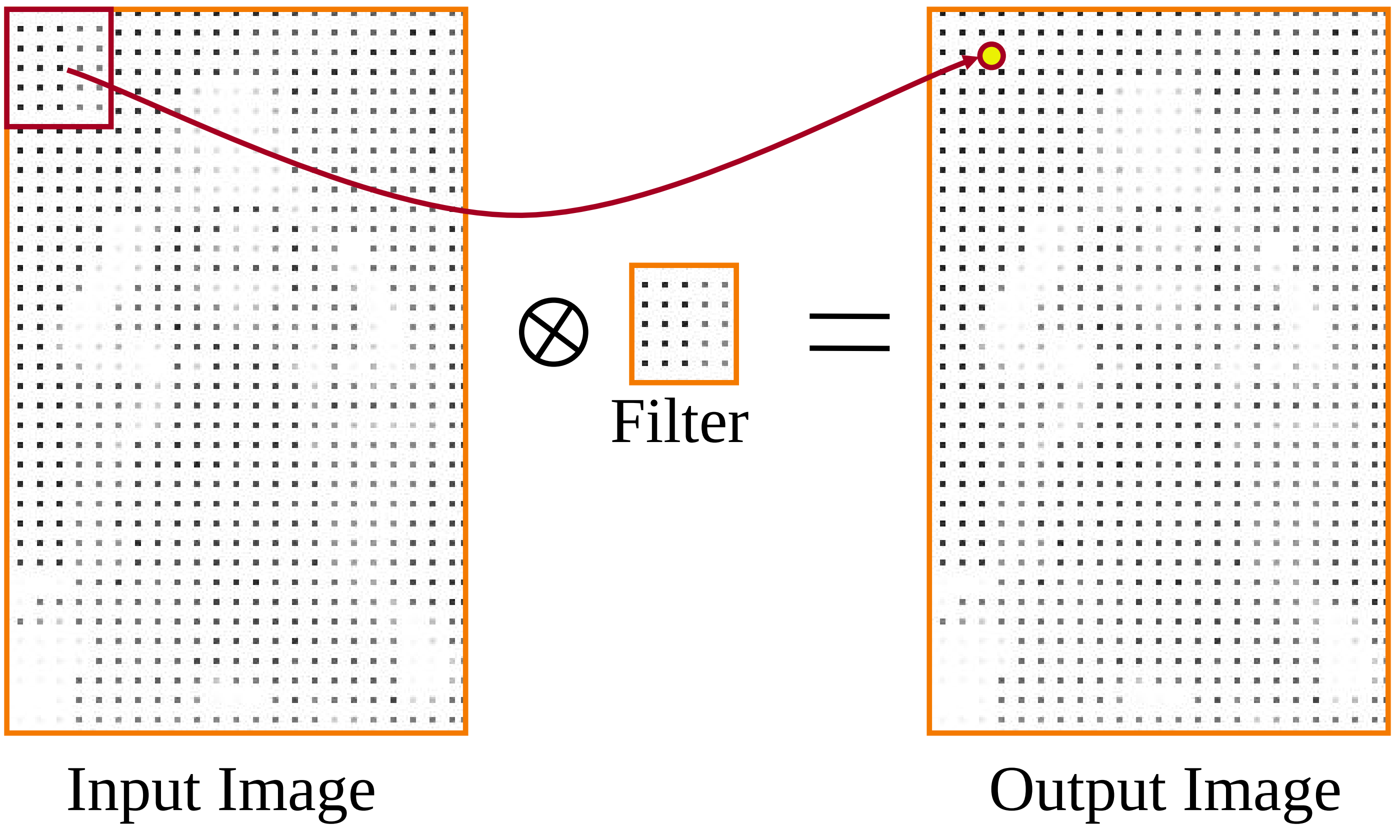

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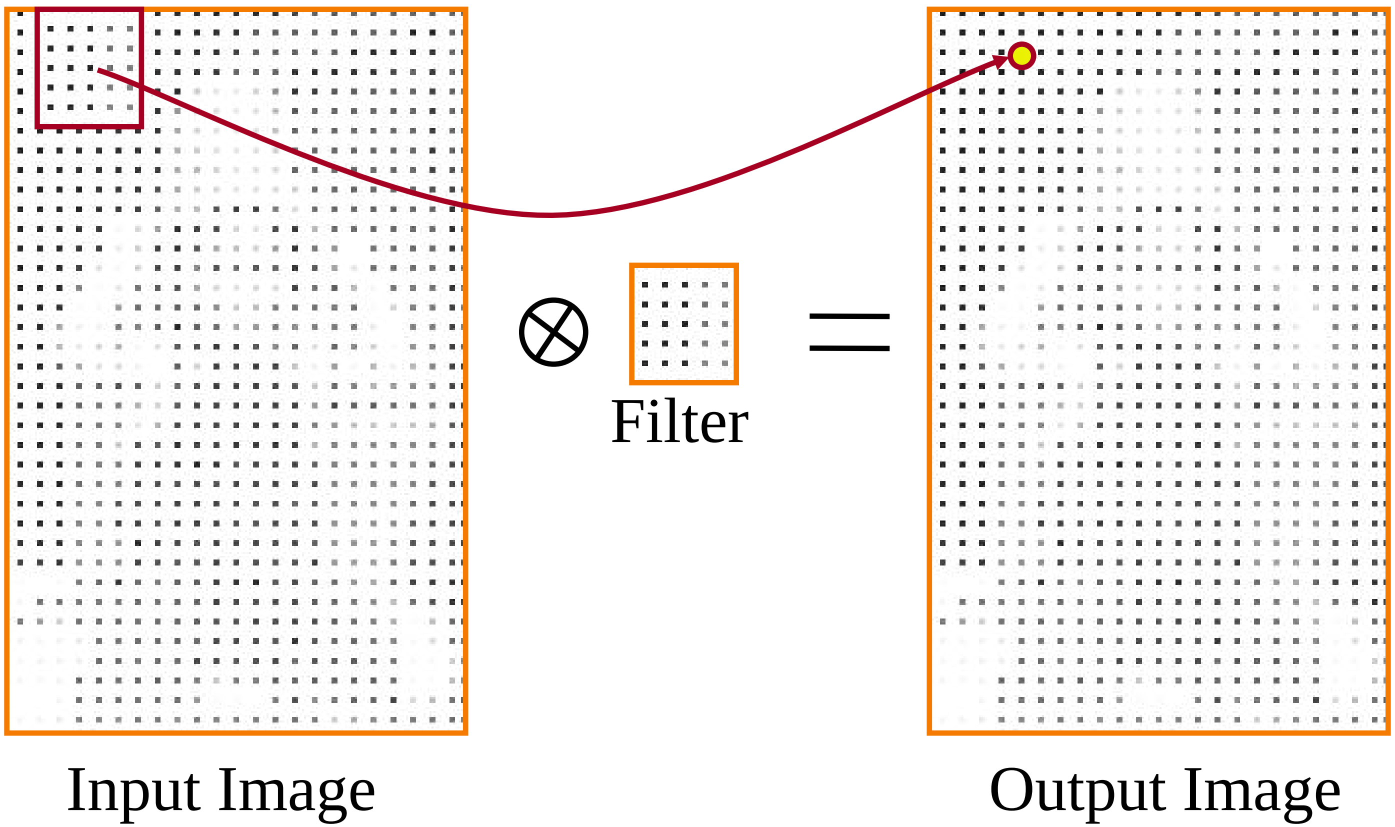

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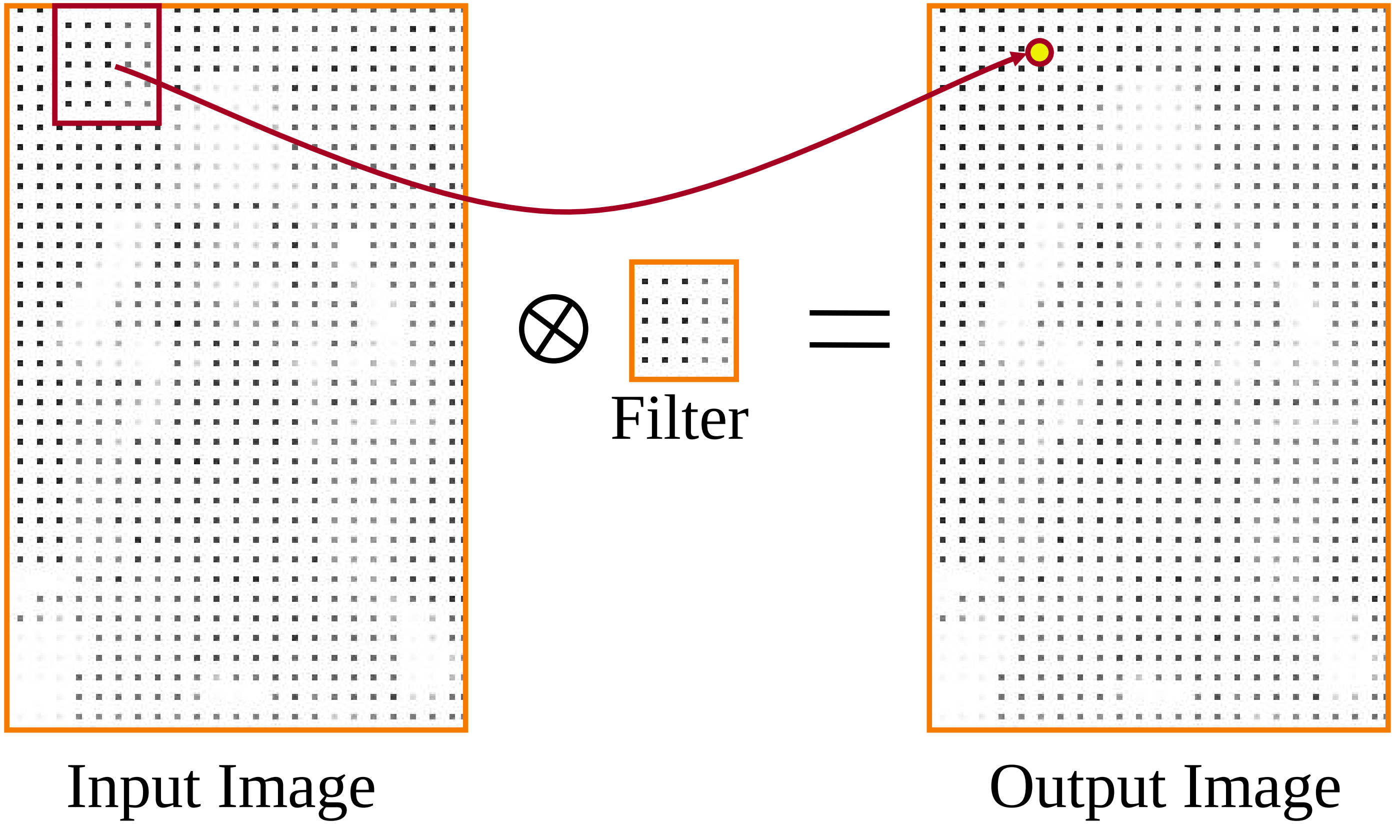

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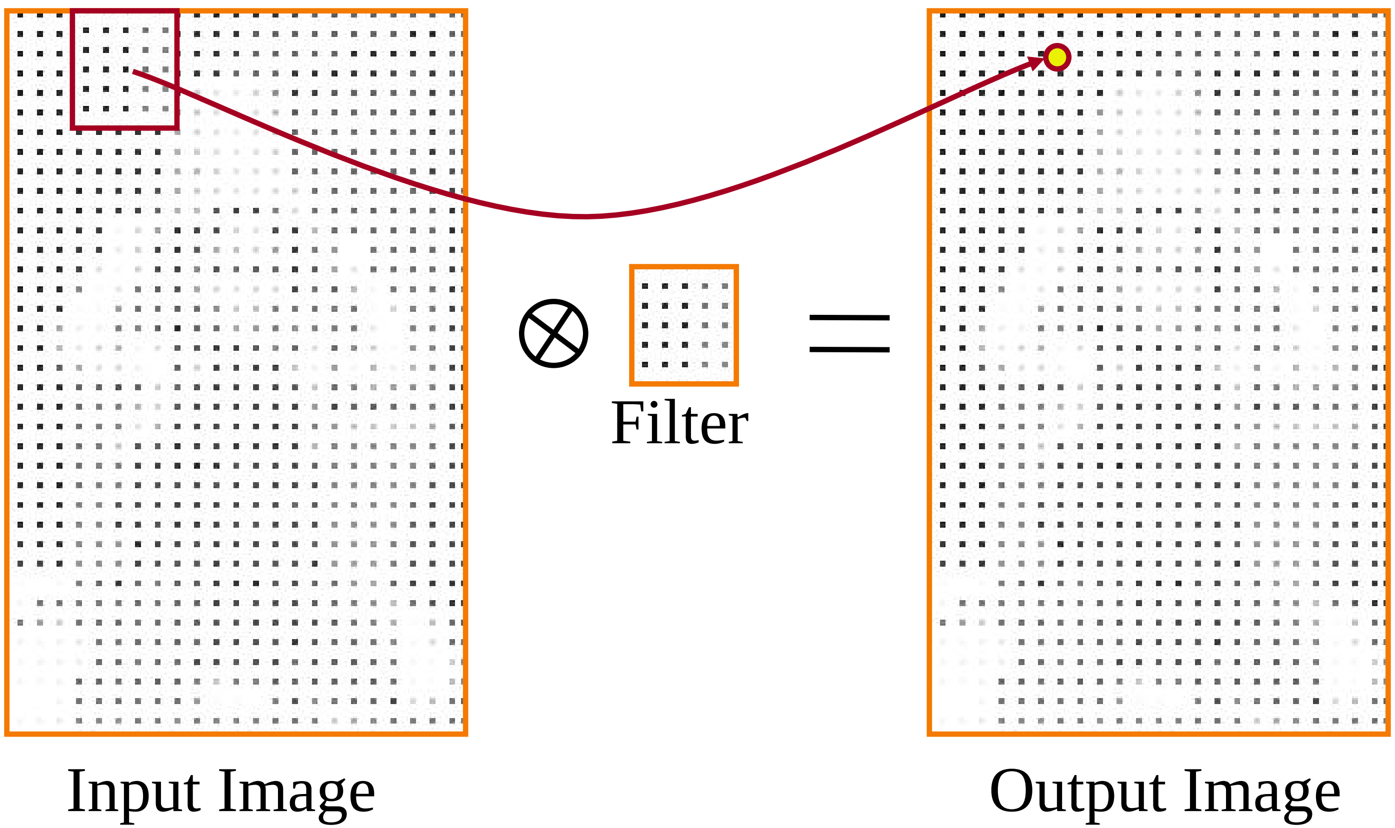
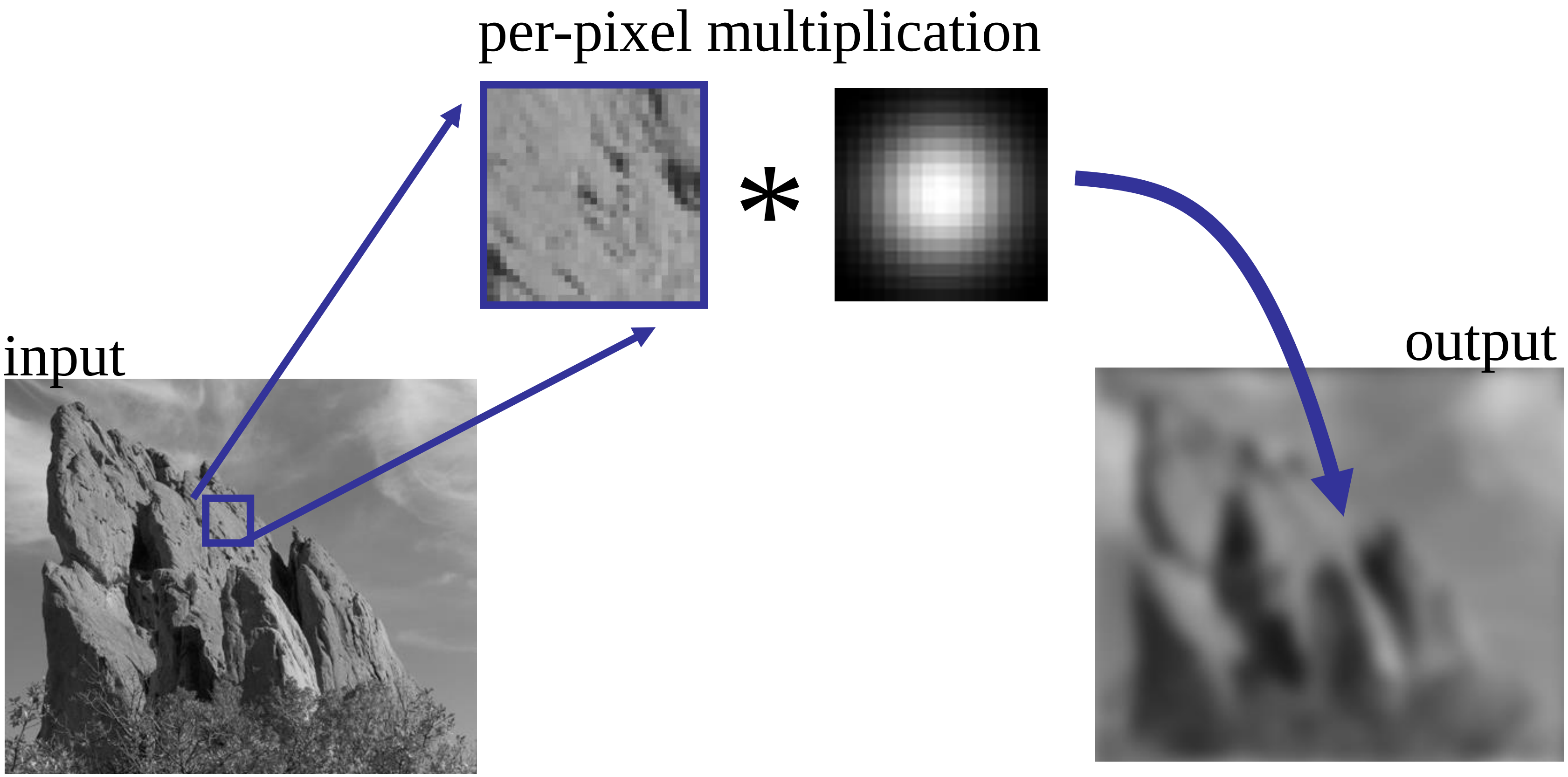


Gaussian Blur

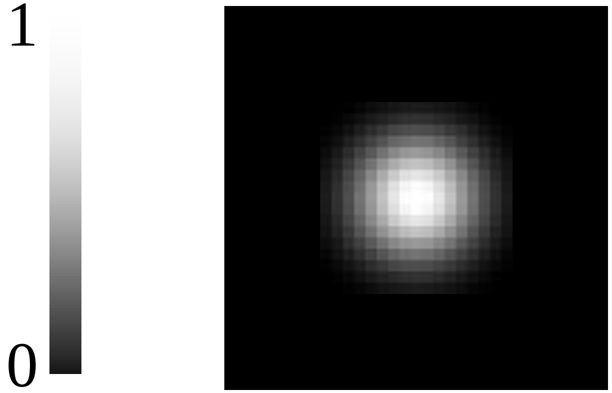


Gaussian Blur

Output value is weighted sum of values in neighborhood of input image

$$Blur(I_{\mathbf{p}}, \sigma) = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G(\|\mathbf{p} - \mathbf{q}\|, \sigma)\, I_{\mathbf{q}}$$

normalized
Gaussian function




input


Gaussian blur

Linear Filtering

- Many interesting linear filters
  - Blur
  - Edge detect
  - Sharpen
  - Emboss
  - etc.

Filter = ?

Blur

Convolve with a 2D Gaussian filter

Original

Blur

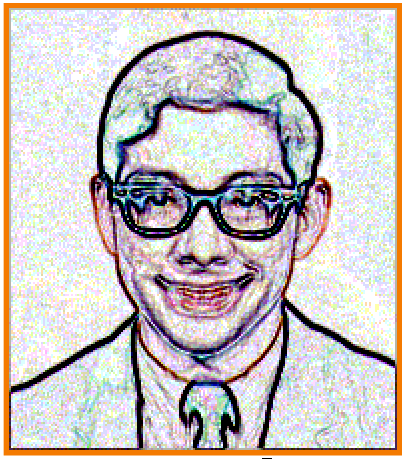
$$\text{Filter} = \begin{bmatrix} \tfrac{1}{16} & \tfrac{2}{16} & \tfrac{1}{16} \\ \tfrac{2}{16} & \tfrac{4}{16} & \tfrac{2}{16} \\ \tfrac{1}{16} & \tfrac{2}{16} & \tfrac{1}{16} \end{bmatrix}$$

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2} \right)$$

Edge Detection

Convolve with a 2D Laplacian filter that finds differences between neighbor pixels

Original



Detect edges

$$\text{Filter} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & +8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Sharpen

Sum detected edges with original image

Original

Sharpened

$$\text{Filter} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & +9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Emboss

Convolve with a filter that highlights gradients in particular directions

Original

Embossed

$$\text{Filter} = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Side Note: Separable Filters

Some filters are separable (e.g., Gaussian)

- First, apply 1-D convolution across every row
- Then, apply 1-D convolution across every column
- **HUGE** impact on performance (when kernel is big)

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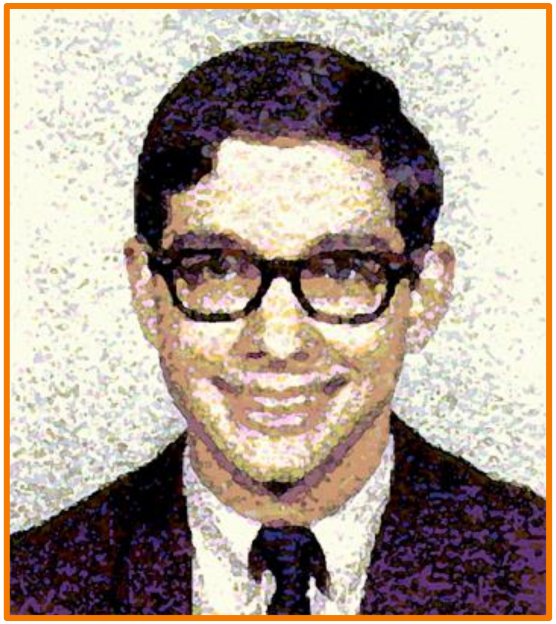
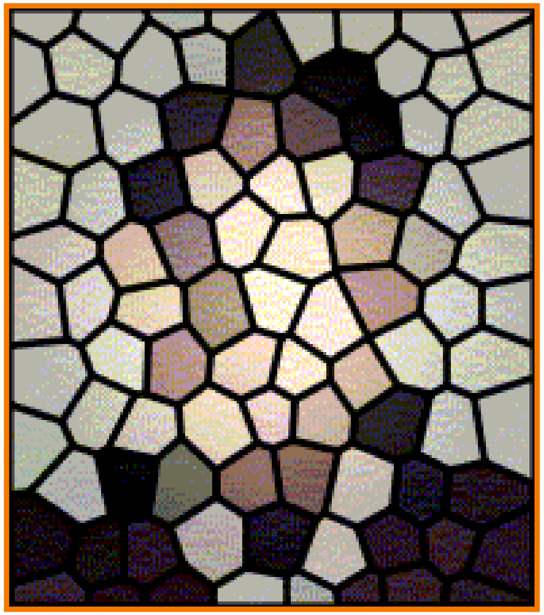
Non-Linear Filtering

Each output pixel is a non-linear function of input pixels in neighborhood (filter depends on input)

Original



Paint



Stained Glass

Median Filter

Each output pixel is median of input pixels in neighborhood

original image

1px median filter

3px median filter

10px median filter

Bilateral Filter

Gaussian blur uses same filter for all pixels

- Blurs across edges as much as other areas

Original

Gaussian Blur

Bilateral Filter

Gaussian blur uses same filter for all pixels

Prefer a filter that preserves edges (adapts to content)

Original

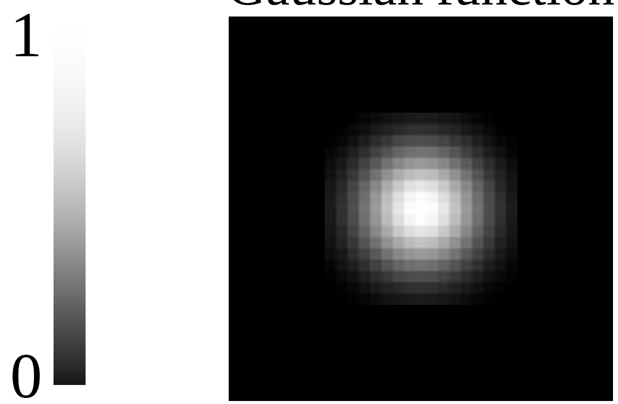
Bilateral Filter

Gaussian Blur

Output value is weighted sum of values in neighborhood of input image

$$Blur(I_{\mathbf{p}}, \sigma) = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G(\|\mathbf{p} - \mathbf{q}\|, \sigma)\, I_{\mathbf{q}}$$

normalized Gaussian function



Bilateral Filter

Combine Gaussian filtering in both spatial domain and color domain

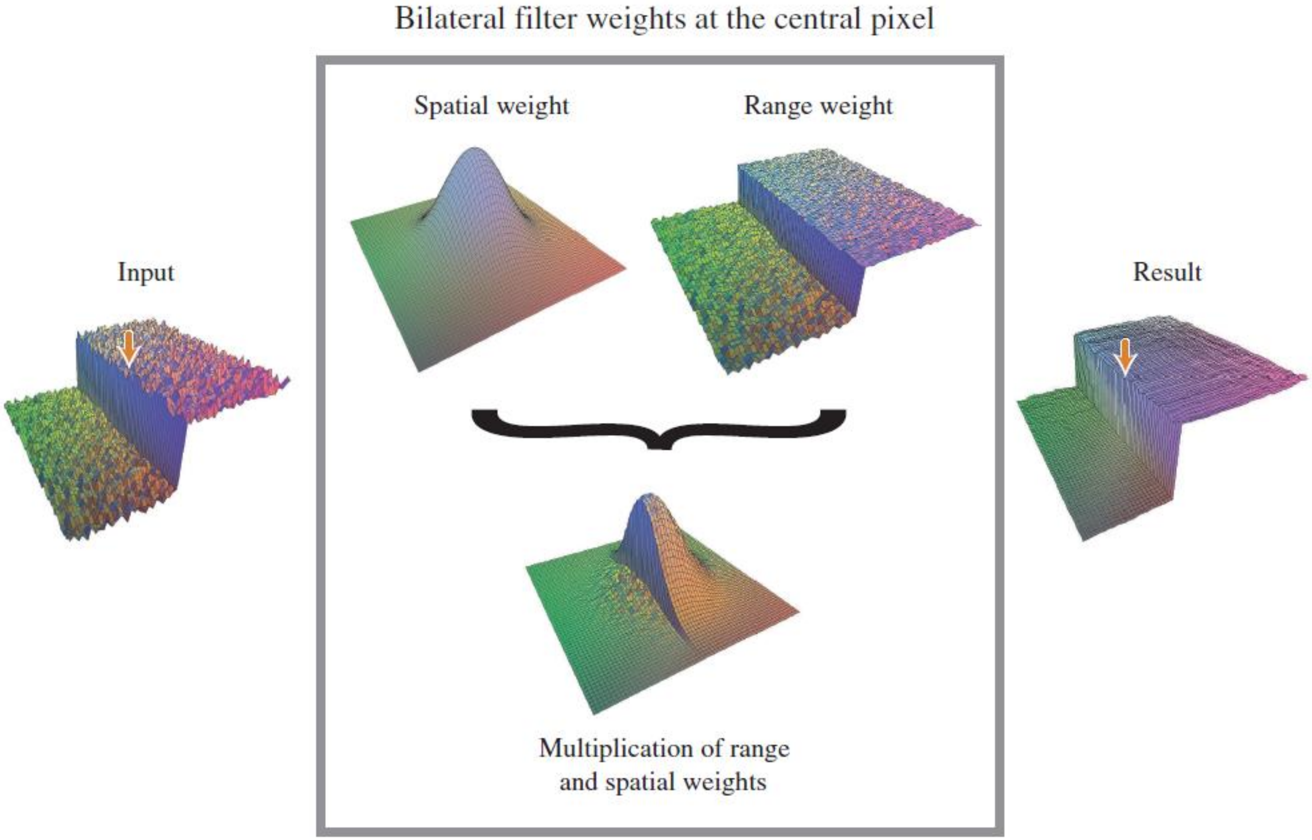
$$Bilateral[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \, G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) \, I_{\mathbf{q}}$$

↑ Spatial Proximity Weight

↑ Color Proximity Weight

Bilateral Filtering

Combine Gaussian filtering in both spatial domain and color domain



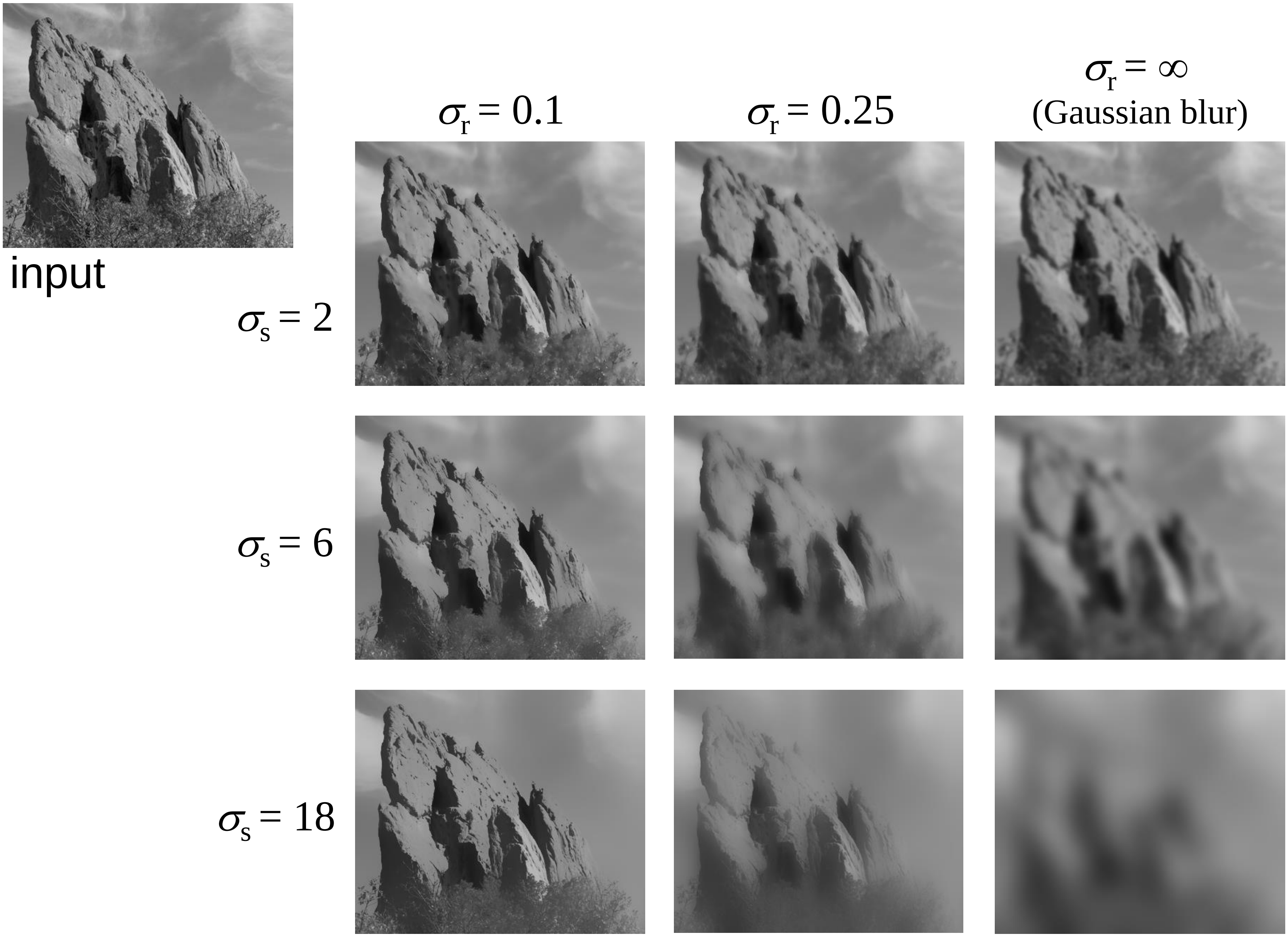

input
$\sigma_r = 0.1$
$\sigma_r = 0.25$
$\sigma_r = \infty$
(Gaussian blur)
$\sigma_s = 2$
$\sigma_s = 6$
$\sigma_s = 18$

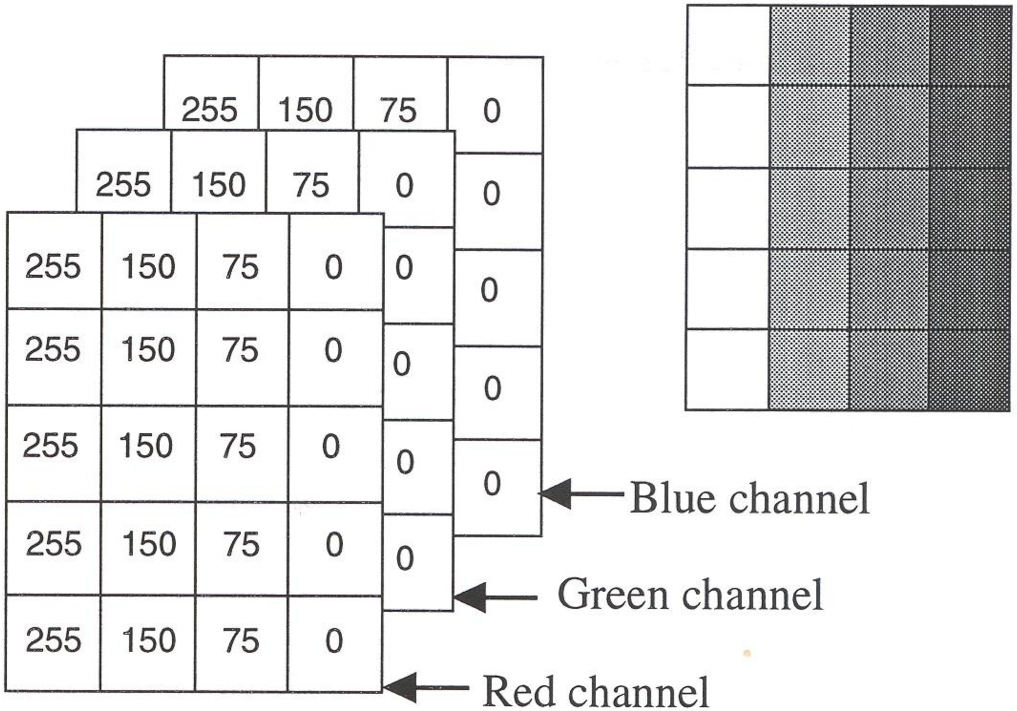
Image Processing Operations

- Luminance
  - Brightness
  - Contrast.
  - Gamma
  - Histogram equalization
- Color
  - Grayscale
  - Saturation
  - White balance
- Linear filtering
  - Blur & sharpen
  - Edge detect
  - Convolution
- Non-linear filtering
  - Median
  - Bilateral filter
- Dithering
  - Quantization
  - Ordered dither
  - Floyd-Steinberg

Quantization

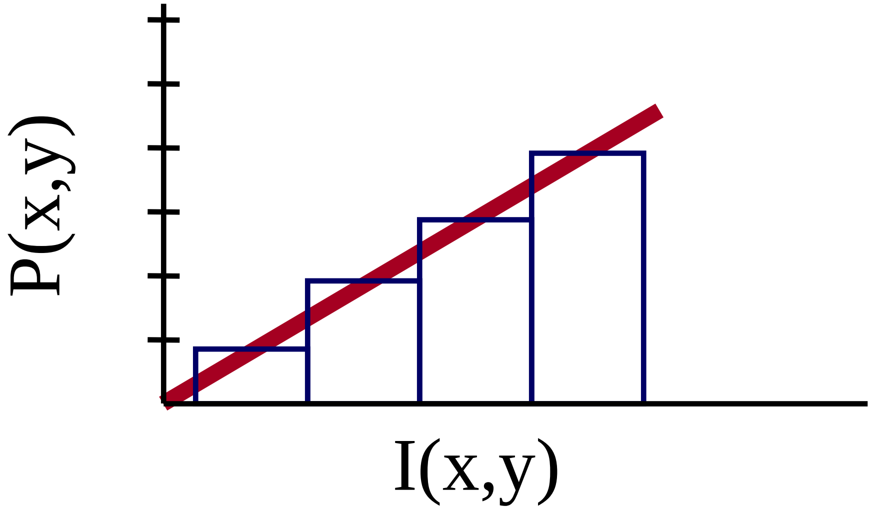
Reduce intensity resolution

- Frame buffers have limited number of bits per pixel
- Physical devices have limited dynamic range

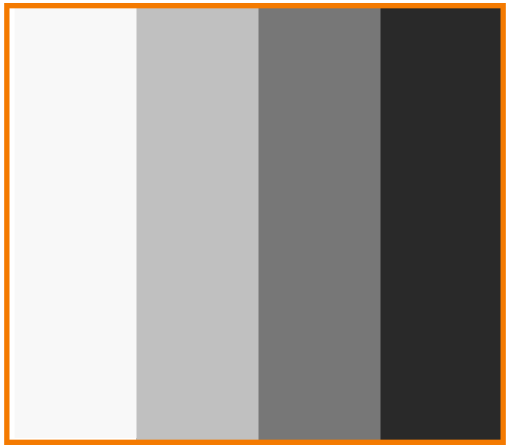


Uniform Quantization

P(x, y) = round(I(x, y))
where round() chooses nearest value that can be represented.



I(x,y)



P(x,y)
(2 bits per pixel)

Uniform Quantization

Images with decreasing bits per pixel:

8 bits 4 bits 2 bits 1 bit

Notice contouring.

Reducing Effects of Quantization

- Intensity resolution / spatial resolution tradeoff

- Dithering
  - Random dither
  - Ordered dither
  - Error diffusion dither
- Halftoning
  - Classical halftoning

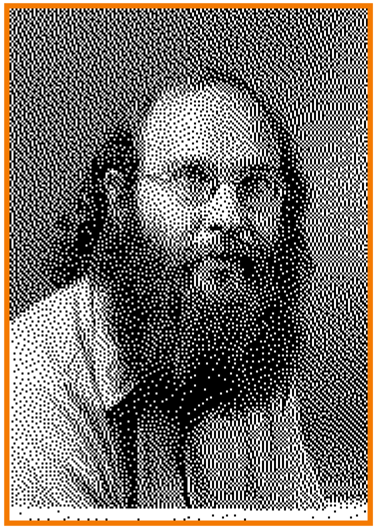
Dithering

Distribute errors among pixels

- o Exploit spatial integration in our eye
- o Display greater range of perceptible intensities

Original
(8 bits)

Uniform
Quantization
(1 bit)

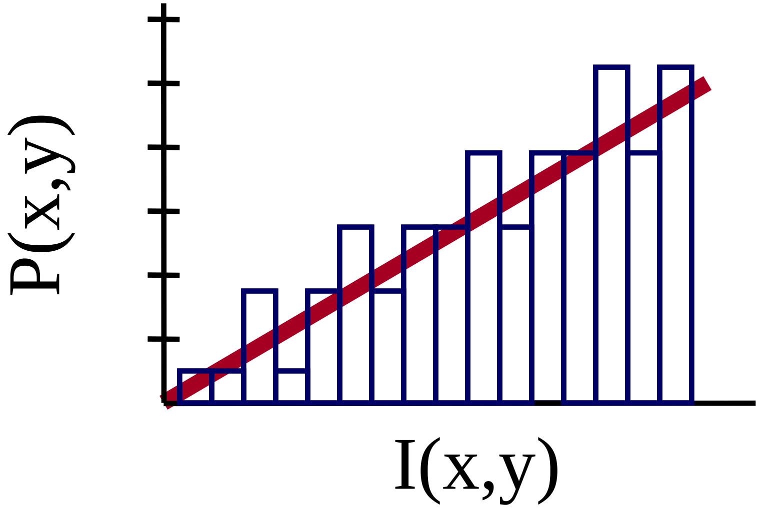


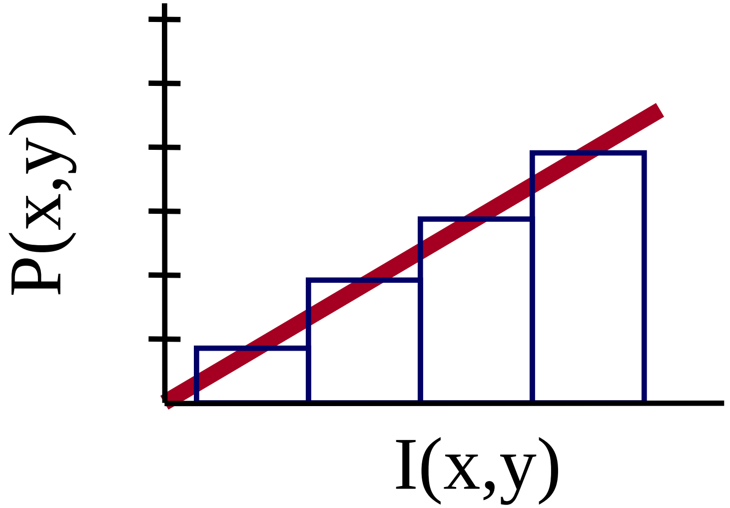
Floyd-Steinberg
Dither
(1 bit)

Random Dither

Randomize quantization errors

- Errors appear as noise





$$P(x, y) = \text{round}\big(I(x, y) + \text{noise}(x,y)\big)$$

Random Dither

Original
(8 bits)

Uniform
Quantization
(1 bit)

Random
Dither
(1 bit)

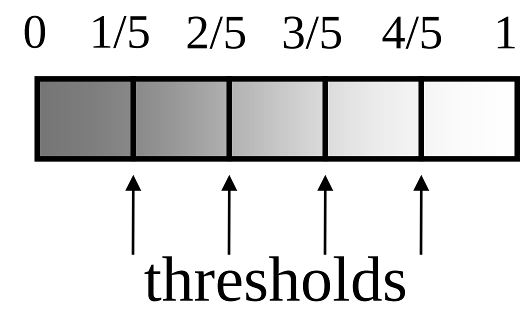
Ordered Dither

Pseudo-random quantization errors

- Matrix stores pattern of threshholds
- Pseudo-code for 1-bit output:

```
i = x mod n
j = y mod n
thresh = (D(i,j)+1)/(n^2+1)
if (I(x,y) > thresh)
       P(x,y) = 1
else
       P(x,y) = 0
```

$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

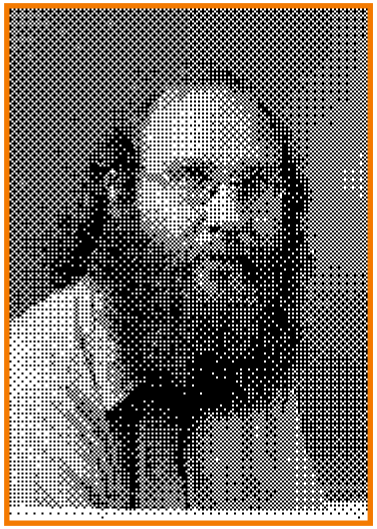


- Can be generalized to n-bit output, by comparing quantization error to threshhold.

Ordered Dither

Original
(8 bits)

Random
Dither
(1 bit)



Ordered
Dither
(1 bit)

Ordered Dither

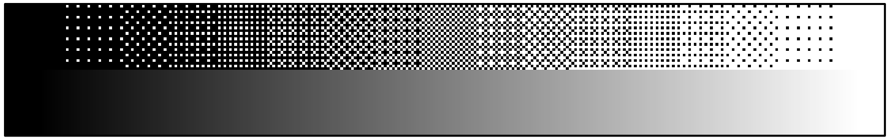
Recursion for Bayer's ordered dither matrices

$$D_n = \begin{bmatrix} 4D_{n/2} + D_2(1,1)U_{n/2} & 4D_{n/2} + D_2(1,2)U_{n/2} \\ 4D_{n/2} + D_2(2,1)U_{n/2} & 4D_{n/2} + D_2(2,2)U_{n/2} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \qquad D_4 = \begin{bmatrix} 15 & 7 & 13 & 5 \\ 3 & 11 & 1 & 9 \\ 12 & 4 & 14 & 6 \\ 0 & 8 & 2 & 10 \end{bmatrix}$$

4x4 matrix gives 17 gray levels:

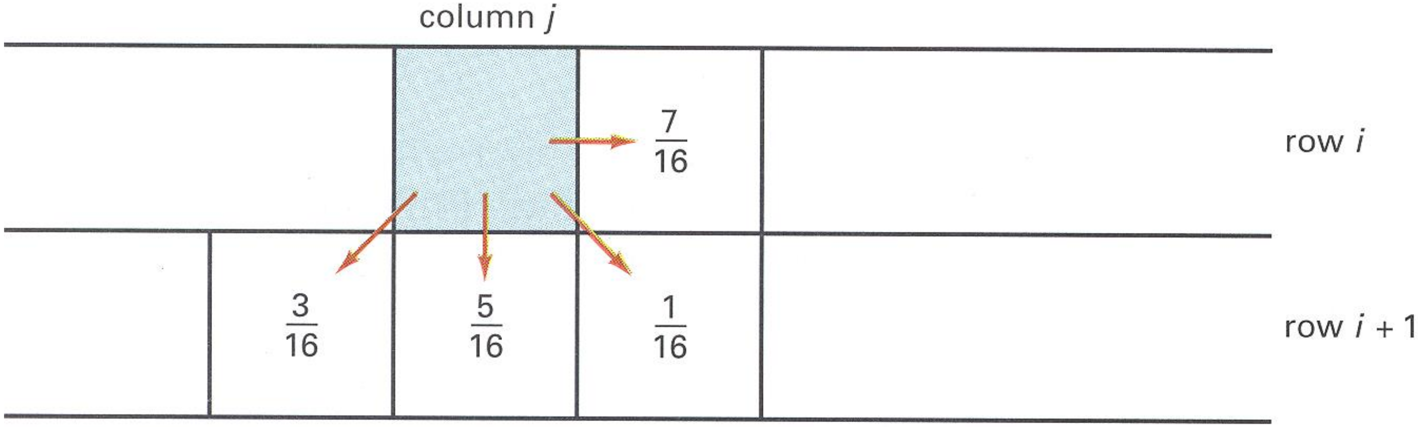
https://en.wikipedia.org/wiki/Ordered_dithering



Error Diffusion Dither

Spread quantization error over neighbor pixels

- Error dispersed to pixels right and below
- Floyd-Steinberg weights:



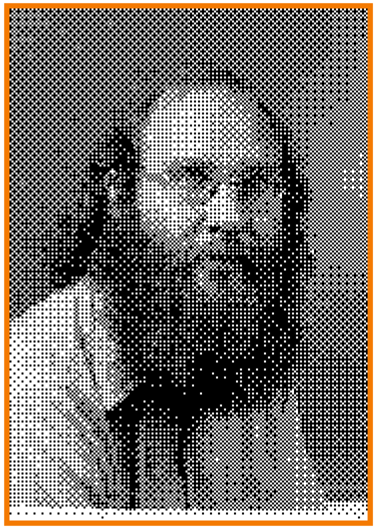
$$3/16 + 5/16 + 1/16 + 7/16 = 1.0$$

Figure 14.42 from H&B

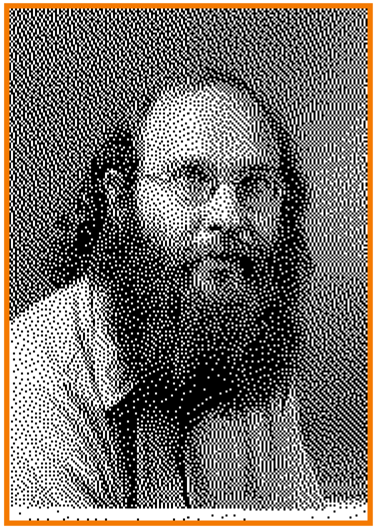
Error Diffusion Dither

Original
(8 bits)

Random
Dither
(1 bit)



Ordered
Dither
(1 bit)



Floyd-Steinberg
Dither
(1 bit)

Summary

- Color transformations
  - Different color spaces useful for different operations
- Filtering
  - Compute new values for image pixels based on function of old values in neighborhood
- Dithering
  - Reduce visual artifacts due to quantization
  - Distribute errors among pixels

    Exploit spatial integration in our eye