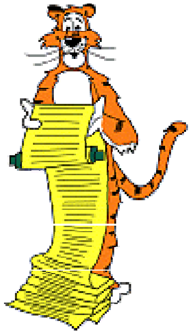
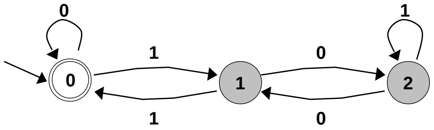
Lecture T4: Pattern Matching





Introduction to Theoretical CS

Two fundamental questions.
- What can a computer do?
- How fast can it do it?

General approach.
- Don't talk about specific machines or problems.
- Consider minimal abstract machines.
- Consider general classes of related problems.

Today.
- Simplest type of machine that is still interesting = FSA.
- Class of (pattern matching) problems that it can solve.

Future lectures.
- More complicated machines and problems.

Why Learn Theory

In theory . . .
- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice . . .
- Web search: theory of pattern matching.
- Sequential circuit: theory of finite state automata.
- Compilers: theory of context free grammar.
- Cryptography: theory of complexity.

Web Search Example

Standard Web search for `'+censorship +net'` might yield 1 million hits, ordered as follows:
- *www.epic.org/free_speech/action*
- *www.zepa.net/hypermail/asfar/1998/07/0466.html*
- *www.eserver.org/internet/censorship.html*
- *www.tiac.net/users/sojourn/censor0596.html*
- *www.anatomy.usyd.edu.au/danny/usenet/aus.net.news/*

Observation: not many useful pages.

Web Search Example

Standard Web search for `’+censorship +net’` might yield 1 million hits, ordered as follows:

Jon Kleinberg’s clever algorithm produces "authoritative" pages without human fine-tuning:

- *www.eff.org* *(Electronic Frontier Foundation)*
- *www.cdt.org* *(Center for Democracy and Technology)*
- *www.vtw.org* *(Voters Telecommunications Watch)*
- *www.aclu.org* *(American Civil Liberties Union)*

- Based on theoretical principles.
- Involves computing eigenvectors of Web matrix!
- Use *google.com*

Unix Tools

Unix.

- A large number of simple tools.

Some fundamental pattern matching tools.

- egrep, awk, sed, more, emacs, perl
- Useful for variety of applications including Web search.
- Not C programming, though tools are as powerful.
- Directly related to fundamentals tenets of computer science.

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egrep

General regular expressions pattern matching.

- Acts as filter.
- Sends lines from stdin to stdout that "match" argument string.

Elementary Examples

```
%egrep ’beth’ classlist
02/Condliffe/Elizabeth/3/condlife
03/Danaher/Elizabeth/8/edanaher
03/Smythe/Elizabeth/6/esmythe
03/Bethke/Kristen/3/kbethke

% egrep ’/3/’ classlist
03/Marin/Anthony/3/amarin
03/Parker/Andrew/3/aparker
. . .
03/Weiss/Jacob/3/weiss

%egrep ’zeuglodon’ mobydick.txt
 rechristened the monster zeuglodon and in his
```

Find all lines in file classlist with substring ’beth’

List all people in precept 3.

Crossword Puzzle or Scrabble Too Hard?

`/usr/dict/words` is a list of (25,143) words in dictionary.

More Examples

```
% egrep ’hh’ /usr/dict/words
    beachhead
    highhanded
    withheld
    withhold

% egrep ’u.u.u’ /usr/dict/words
    cumulus

% egrep ’..oo..oo’ /usr/dict/words
    bloodroot
    schoolbook
    schoolroom

% egrep -c ’..oo..oo’ /usr/dict/words
    3
```

Two consecutive h’s.

A dot matches any single character

Why not "cookbook"?

count number of matches

Excerpts From "man egrep"

Unix

```
% man egrep

Name:   egrep - search file using full regular expressions

Syntax:  egrep [option...] expression [file...]

Description:  Search the input files (standard input default) for lines
matching a pattern.  Normally, each line found is copied to the standard
output.

Take care when using special characters in the expression because they
are also meaningful to the Shell. It is safest to enclose the entire
expression argument in single quotes.

Options:
  -c  Produces count of matching lines only.
   -i  Considers upper and lowercase letter identical.
  -n  Precedes each matching line with its line number.
  -v  Displays all lines that do not match.

Restrictions:  Lines are limited to 256 chars; longer lines are truncated.
```

Grep Pattern Conventions

Conventions for `egrep`:

`c`	any non-special character matches itself
`.`	any single character
`r*`	zero or more occurrence of r
`(r)`	grouping
`r1|r2`	logical OR
`[...]`	any character in [a-z]
`[^ ...]`	any character not in [a-z]
`^`	beginning of line
`$`	end of line

Still More Examples

Unix

```
% egrep 'n(ie|ei)ther' /usr/dict/words
   neither

% egrep 'actg(atac)*gcta' human.data
   ggtactggctaggac

% egrep 'actg(atac)*gcta' student.data
   tatactgatacatacatacgctattac

% egrep '^y.(..)*y$' /usr/dict/words
   yesterday

% grep -v '[aeiou]' /usr/dict/words |
  grep '......'
   rhythm
   syzygy
```

Do spell checking by specifying what you know.

Starts and ends with y, odd number of characters.

Find all words with no vowels and 6 or more letters.

Pattern Matching Alternatives in Unix

Pattern matching.

- `grep, egrep`
- `more`

Substitution editing.

- `emacs, ex`
- `sed`: filter, line by line substitution

  `sed 's/apples/oranges/g' file.txt`

Pattern matching languages.

- `awk, perl`
- Matching, substitution, pattern manipulation, variables, numeric capabilities, control and logic.

Regular Expressions

Specifying "pattern" for grep can be complex.

`^[^aeiou]*a[^aeiou]*e[^aeiou]*i[^aeiou]*o[^aeiou]*u[^aeiou]*`

What kinds of patterns can be specified?

- Match all lines containing an even number?
- Match all lines containing a prime number?

Which aspects are essential?

- Unix `egrep` regular expressions are useful.
- But more complex than theoretical minimum.

Regular Expressions

Rules for creating regular expressions (RE's):

0 or 1	symbols
(a)	grouping
ab	concatenation
a + b	logical OR
a*	closure (0 or more replications)

use + instead of |

where a and b are regular expressions.

ε = empty string

Examples:

`(10)*`	`ε, 10, 1010, 101010, ...`
`0(0 + 1)*0`	`00, 000, 010, 0000, 0110, ...`
`(1*01*01*01*)*`	`ε, 000, 000000, 11101110101111, ...`

Formal Languages

An alphabet is a finite set of symbols.

- Binary alphabet = `{0, 1}`
- Lower-case alphabet = `{a, b, c, d, ..., y, z}`
- Genetic alphabet = `{a, c, t, g}`

A string is a finite sequence of symbols in the alphabet.

- `'0111011011'` is a string in the binary alphabet.
- `'tigers'` is a string in the lower-case alphabet.
- `'acctgaacta'` is a string in the genetic alphabet.

A formal language is an (unordered) set of strings in an alphabet.

- Can have infinitely many strings.
- Examples:
  `{0, 010, 0110, 01110, 011110, 0111110, ...}`
  `{11, 1111, 111111, 11111111, 1111111111, ...}`

Why Study Formal Languages?

Can cast any computation as a language recognition problem.

- Is x = 23,536,481,273 a prime number?

Regular Languages

Every RE describes a language (the set of all strings that match).

- `(1*01*01*)*` describes the language of all bit strings with an even number of 0's: `{1, 11, 1001, 00101110111, ...}`

Regular language.

- Any language that can be described by a RE.

Which languages are regular? (all but one of the following)

All bit strings that:	Example
Begin with 0 and end with 1.	00010110111
Have a multiple of 3 0's.	11000110100
Have more 1's than 0's.	01111001100
Have no consecutive 1's.	01001010010

Machines

Can cast any computation as a language recognition problem.

- Is x = 23,536,481,273 a prime number?
  - L = {2, 3, 5, 7, 11, 13, 17, . . . }
  - Is x in language L?

Start by trying to understand simple languages.

- Build a machine to recognize regular languages.

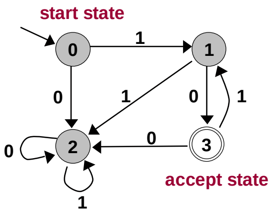
Finite State Automata

Simple machine with N states.

- Start in state 0.
- Read an input bit.
- Move to new state
  - depends on input bit and current state
- Stop when last bit read.
  - 'yes' if end in accept state(s)
  - 'no' otherwise

'Yes' also called *accepted* or *recognized* inputs from a language.

- FSA to right accepts all bit strings of the form 10(10)*
- Rejects all others.



C Code for 10(10)* FSA

fsa1.c

```
int main(void) {
  int c, state = START_STATE;
  while ((c = getchar()) != EOF) {
     if (state == 0 && c == ’0’) state = 2;
     if (state == 0 && c == ’1’) state = 1;
     if (state == 1 && c == ’0’) state = 3;
     if (state == 1 && c == ’1’) state = 2;
     if (state == 2 && c == ’0’) state = 2;
     if (state == 2 && c == ’1’) state = 2;
     if (state == 3 && c == ’0’) state = 2;
     if (state == 3 && c == ’1’) state = 1;
  }

  if (state == ACCEPT_STATE)
     printf("Accepted\n");
  else
     printf("Rejected\n");
  return 0;
```

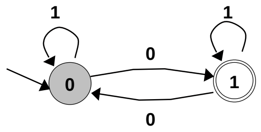
Better C Code for FSA

fsa.c

```
#include <stdio.h>
#define STATES          4
#define ALPHABET_SIZE   2
#define START_STATE     0
#define ACCEPT_STATE    3

int main(void) {
  int c, state = START_STATE
  int transition[STATES][ALPHABET_SIZE] =
      { {2, 1}, {3, 2}, {2, 2}, {2, 1} };

  while ((c = getchar()) != EOF)
     if (c >= ’0’ && c < ’0’ + ALPHABET_SIZE)
        state = transition[state][c - ’0’];

  if (state == ACCEPT_STATE) printf("Accepted\n");
  else printf("Rejected\n");
  return 0;
}
```

A Second Example

Consider the following two state FSA.



What bit strings does it accept?

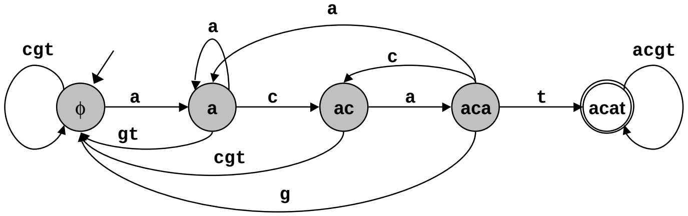
- Yes: `0, 11110, 00000, 100100111011`, all bit strings with an odd number of 0’s.
- No: `1, 1111, 00, 1011100111011`, all bit strings with an even number of 0’s.
- Recognizes same language as $\underbrace{\texttt{(1*01*01*)*}}_{\text{even \# of 0's}}\ \underbrace{\texttt{(1*01*)}}_{\text{exactly one 0}}$

A Third Example

Build an FSA that accepts all strings that contain ’acat’ as a substring.

- tgacatg
- acacatg

Finish building:



More Applications

Duality Between FSA's and RE's

Observation: every FSA we create generates a regular language.
(the inputs accepted can be specified by a regular expression)

Is this always the case?

What about the OTHER way around? Suppose I give you a regular expression. Can you always create a FSA that accepts precisely the same strings?

Stay tuned: see Lecture T2.

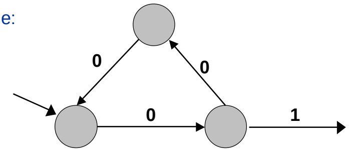
Limitations of FSA

FSA are simple machines.

- N states ⇒ can't remember more than N things.
- Some languages require remembering more than N things.

No FSA can recognize the language of all bit strings with an equal number of 0's and 1's.

A warmup exercise:



If `01xyz` accepted then so is `00001xyz`

Limitations of FSA

No FSA can recognize the language of all bit strings with an equal number of 0's and 1's.

- Suppose an N-state FSA can recognize this language.
- Consider following input: $\underbrace{00000000}_{\text{N+1 0's}}\underbrace{11111111}_{\text{N+1 1's}}$
- FSA must accept this string.
- Some state x is revisited during first N+1 0's since only N states.

```
0000000011111111
  x   x
```

- Machine would accept same string without intervening 0's.

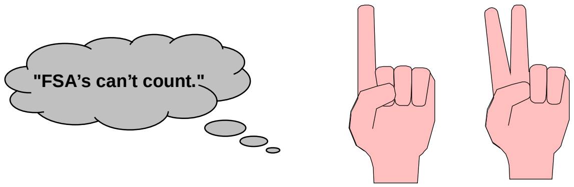
```
000011111111
```

- This string doesn't have an equal number of 0's and 1's.

Limitations of Regular Languages

Consequence: there are languages that are not regular.

- No FSA can recognize the language of all bit strings with an equal number of 0's and 1's.
- We claimed that FSA's are equivalent to regular expressions, i.e.,
  - for any regular language, there is a FSA that accepts precisely those strings
  - the language of all strings accepted by any specific FSA is regular
- Hence, the language above cannot be regular.



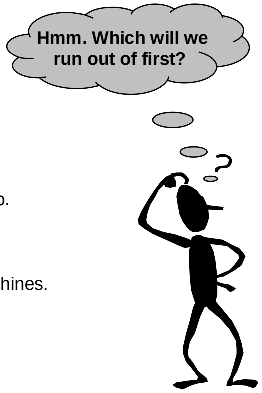
Looking Ahead

Today.

- Defined a simple abstract machine = FSA.
- Capable of pattern matching.
- Incapable of "counting."
- Need to consider more powerful machines.

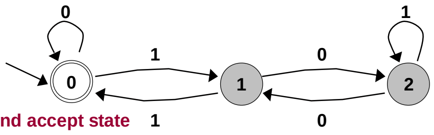
Future lectures.

- Define an abstract machine.
- Understand how it works and what it can do.
- Find things it can't do.
- Define a more powerful machine.
- Repeat until we run out of problems or machines.



A Fourth Example

FSA to decide if input (convert binary to decimal) is divisible by 3?



0 is start and accept state

What bit strings does it accept?

- Yes: 11 (3_{10}), 110 (6_{10}), 1001 (9_{10}), 1100 (12_{10}), 1111 (15_{10}), integers whose binary representation is divisible by 3.
- No: 1, 10, 100, 101, 111, integers not divisible by 3.

A Fourth Example

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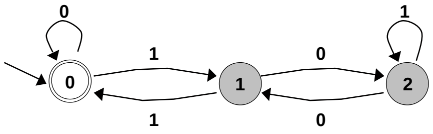


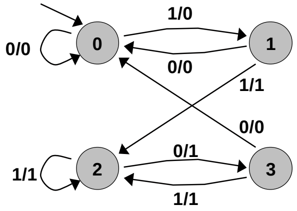
How does it work?

- State 0: input so far is divisible by 3.
- State 1: input has remainder 1 upon division by 3.
- State 2: input has remainder 2 upon division by 3.
- Transition example.
  - Input 1100 (12) ends in state 0.
  - If next bit is 0 then stay in state 0. Adding 0 to last bit is same as multiplying number by 2. Remains divisible by 3.

An Application: Bounce Filter

Bounce filter: remove isolated 0's and 1's in input.

- Input: `0 1 0 0 0 1 1 0 1 1 1 1`
- Output (one-bit delay): `- 0 0 0 0 0 1 1 1 1 1 1 1`
- `x/y`: if input is x, then change state and OUTPUT y



An Application: Bounce Filter

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- Output (one-bit delay): `- 0 0 0 0 0 1 1 1 1 1 1 1`
- `x/y`: if input is x, then change state and OUTPUT y

State interpretations.

- 0: at least two consecutive 0's
- 1: sequence of 0's followed by a 1
- 2: at least two consecutive 1's
- 3: sequence of 1's followed by a 0