



# 3D Modeling

COS 426, Spring 2020

Princeton University

Felix Heide

# Syllabus



I. Image processing

II. Modeling

III. Rendering

IV. Animation

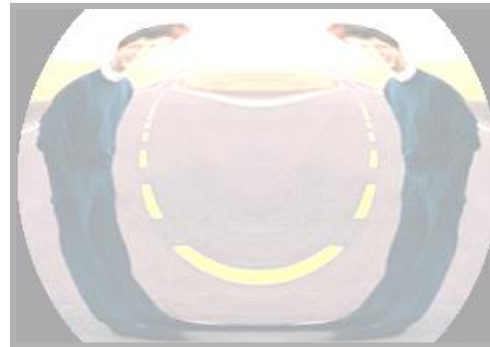
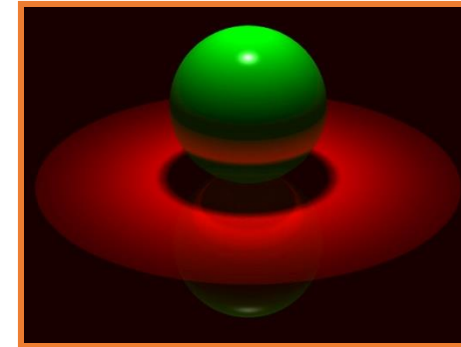
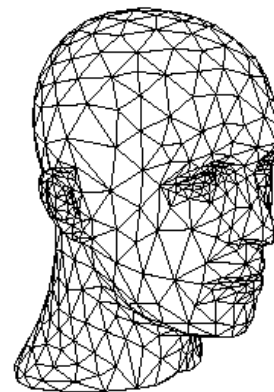


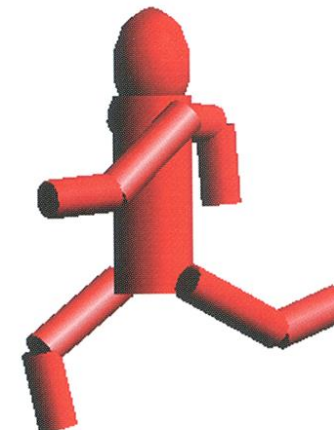
Image Processing  
*(Rusty Coleman, CS426, Fall99)*



Rendering  
*(Michael Bostock, CS426, Fall99)*



Modeling  
*(Denis Zorin, CalTech)*

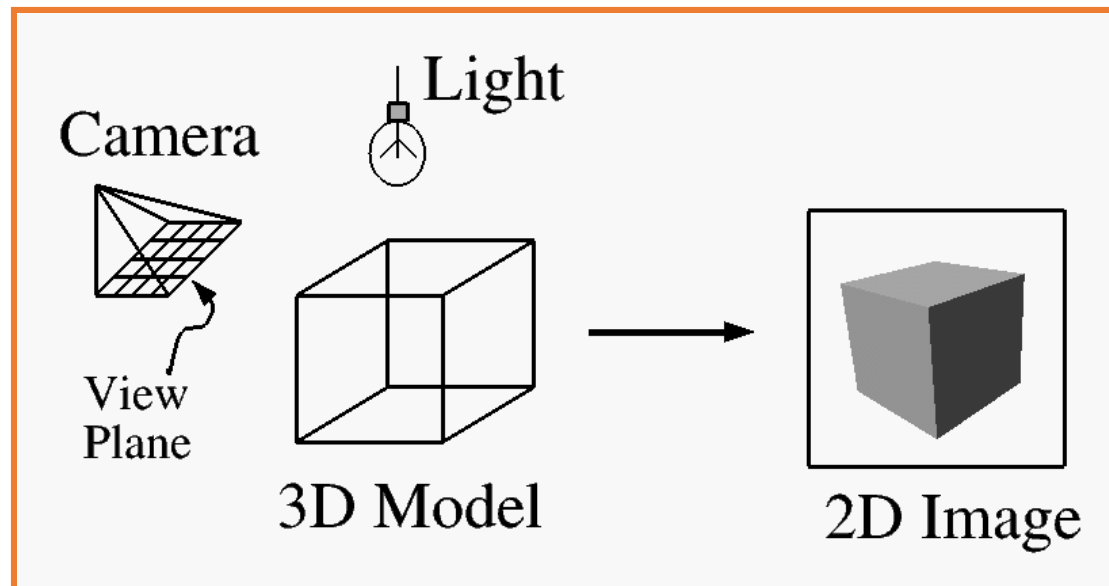


Animation  
*(Angel, Plate 1)*

# What is 3D Modeling?



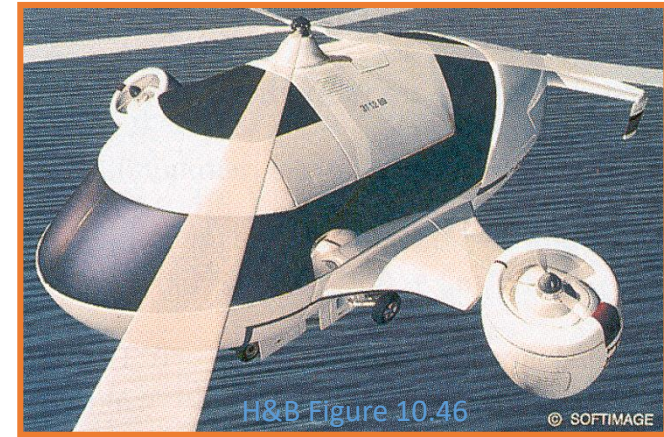
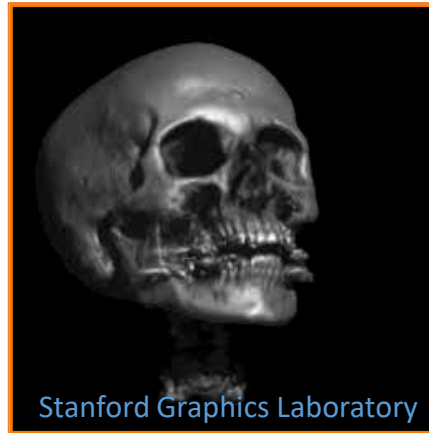
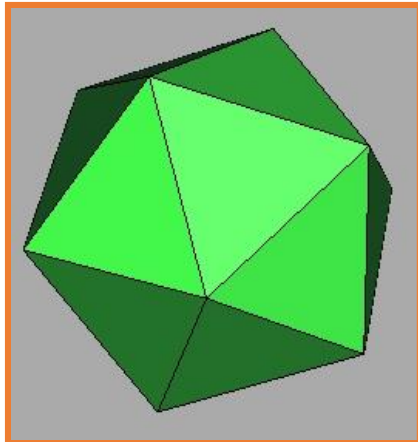
- Topics in computer graphics
  - Imaging = *representing 2D images*
  - Modeling = *representing 3D objects*
  - Rendering = *constructing 2D images from 3D models*
  - Animation = *simulating changes over time*



# Modeling



- How do we ...
  - Represent 3D objects in a computer?
  - Acquire computer representations of 3D objects?
  - Manipulate computer representations of 3D objects?



# Modeling Background



- Scene is usually approximated by 3D primitives
  - Point
  - Vector
  - Line segment
  - Ray
  - Line
  - Plane
  - Polygon

# 3D Point



- Specifies a location
  - Represented by three coordinates
  - Infinitely small

```
typedef struct {  
    Coordinate x;  
    Coordinate y;  
    Coordinate z;  
} Point;
```

• (x,y,z)

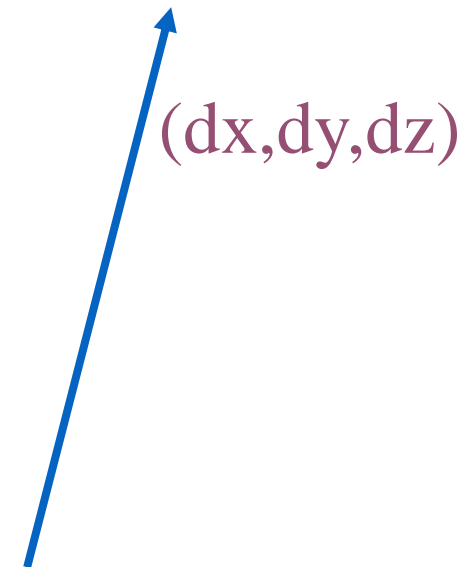


# 3D Vector



- Specifies a direction and a magnitude
  - Represented by three coordinates
  - Magnitude  $\|V\| = \sqrt{dx \ dx + dy \ dy + dz \ dz}$
  - Has no location

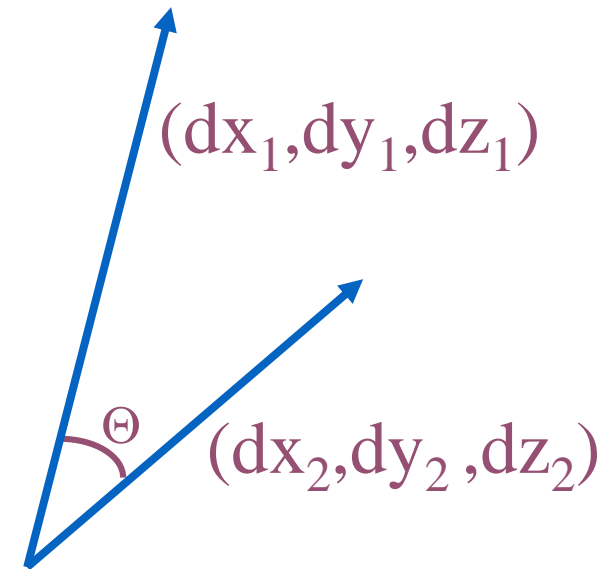
```
typedef struct {  
    Coordinate dx;  
    Coordinate dy;  
    Coordinate dz;  
} Vector;
```



# 3D Vector



- Dot product of two 3D vectors
  - $V_1 \cdot V_2 = \|V_1\| \|V_2\| \cos(\Theta)$

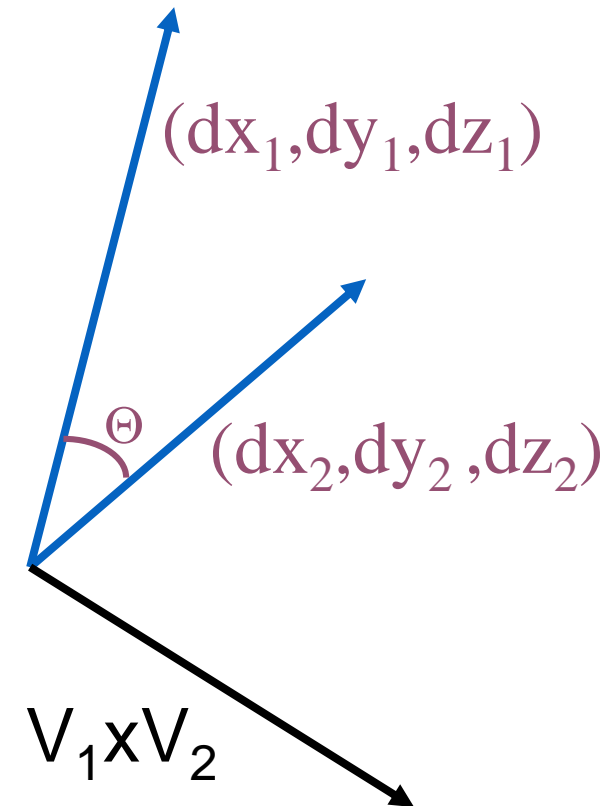




# 3D Vector



- Cross product of two 3D vectors
  - $V_1 \times V_2 =$  vector perpendicular to both  $V_1$  and  $V_2$
  - $\|V_1 \times V_2\| = \|V_1\| \|V_2\| \sin(\Theta)$

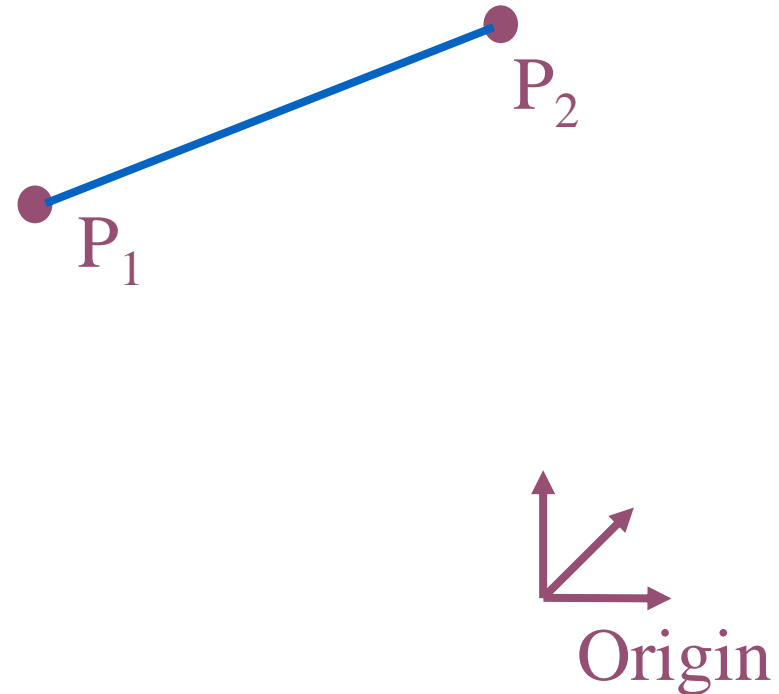


# 3D Line Segment



- Linear path between two points
  - Parametric representation:
    - $P = P_1 + t(P_2 - P_1), \quad (0 \leq t \leq 1)$

```
typedef struct {  
    Point P1;  
    Point P2;  
} Segment;
```

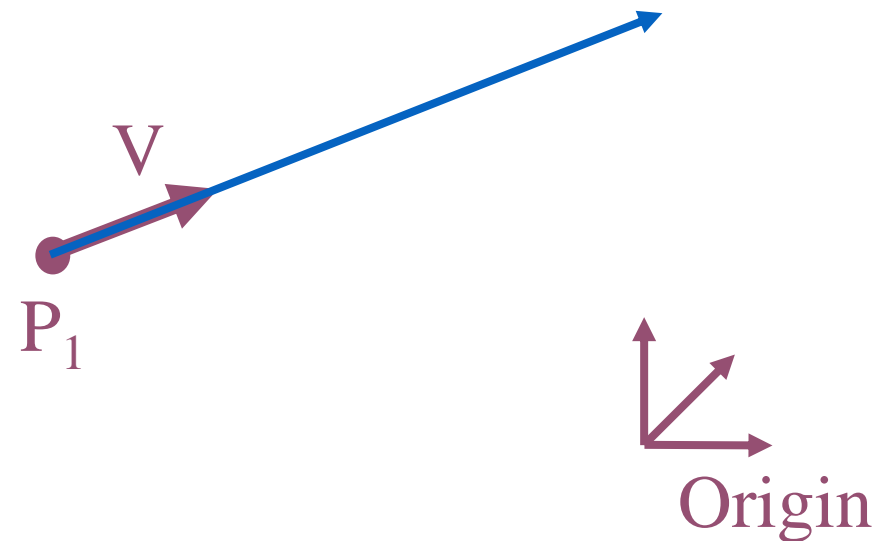


# 3D Ray



- Line segment with one endpoint at infinity
  - Parametric representation:
    - $P = P_1 + t V, \quad (0 \leq t < \infty)$

```
typedef struct {  
    Point P1;  
    Vector V;  
} Ray;
```

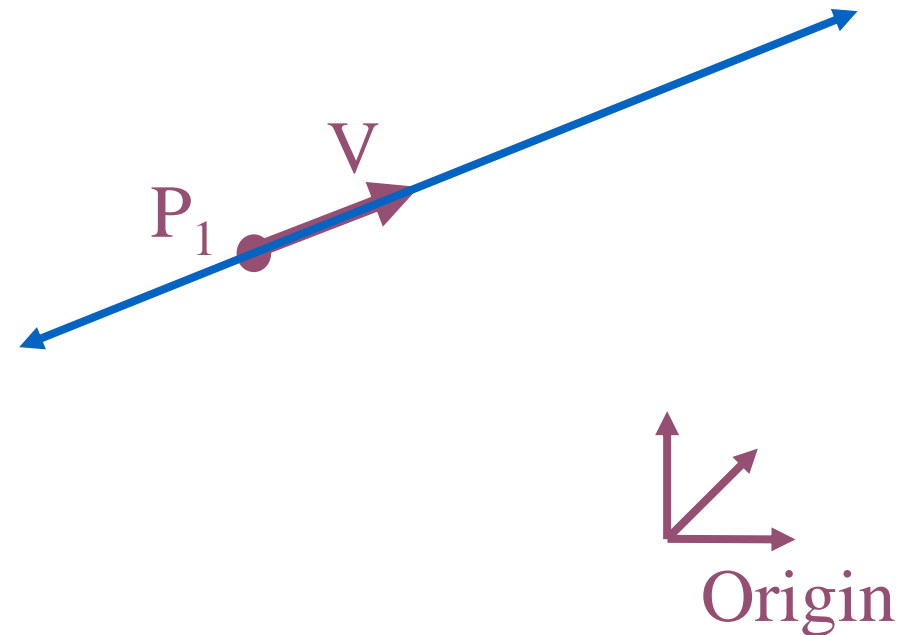


# 3D Line



- Line segment with both endpoints at infinity
  - Parametric representation:
    - $P = P_1 + t V, \quad (-\infty < t < \infty)$

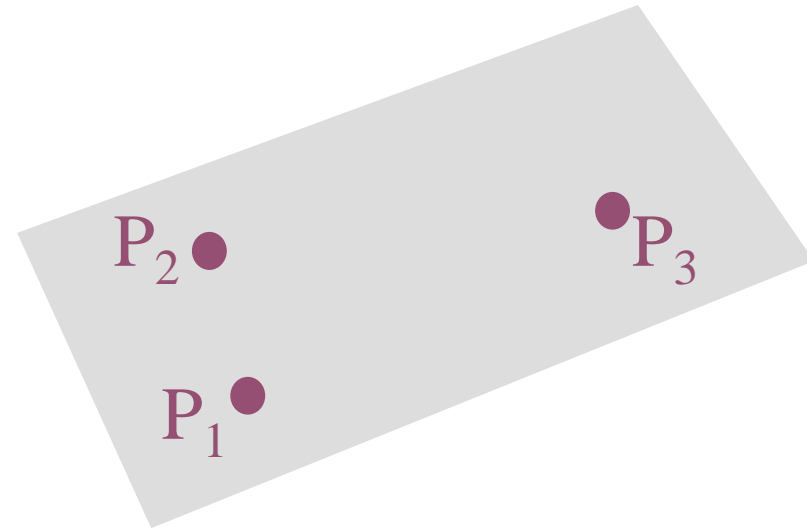
```
typedef struct {  
    Point P1;  
    Vector V;  
} Line;
```



# 3D Plane



- A linear combination of three points



# 3D Plane



- A linear combination of three points

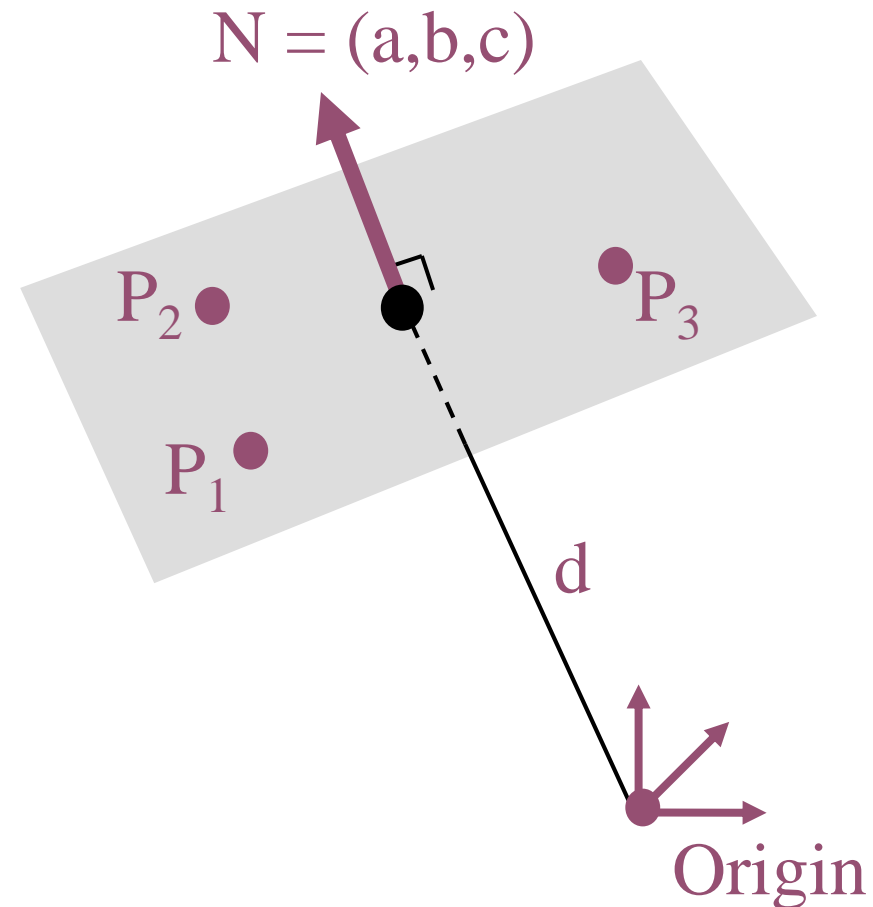
- Implicit representation:

- $P \cdot N - d = 0$ , or
- $ax + by + cz + d = 0$

```
typedef struct {  
    Vector N;  
    Distance d;  
} Plane;
```

- N is the plane “normal”

- Unit-length vector
- Perpendicular to plane

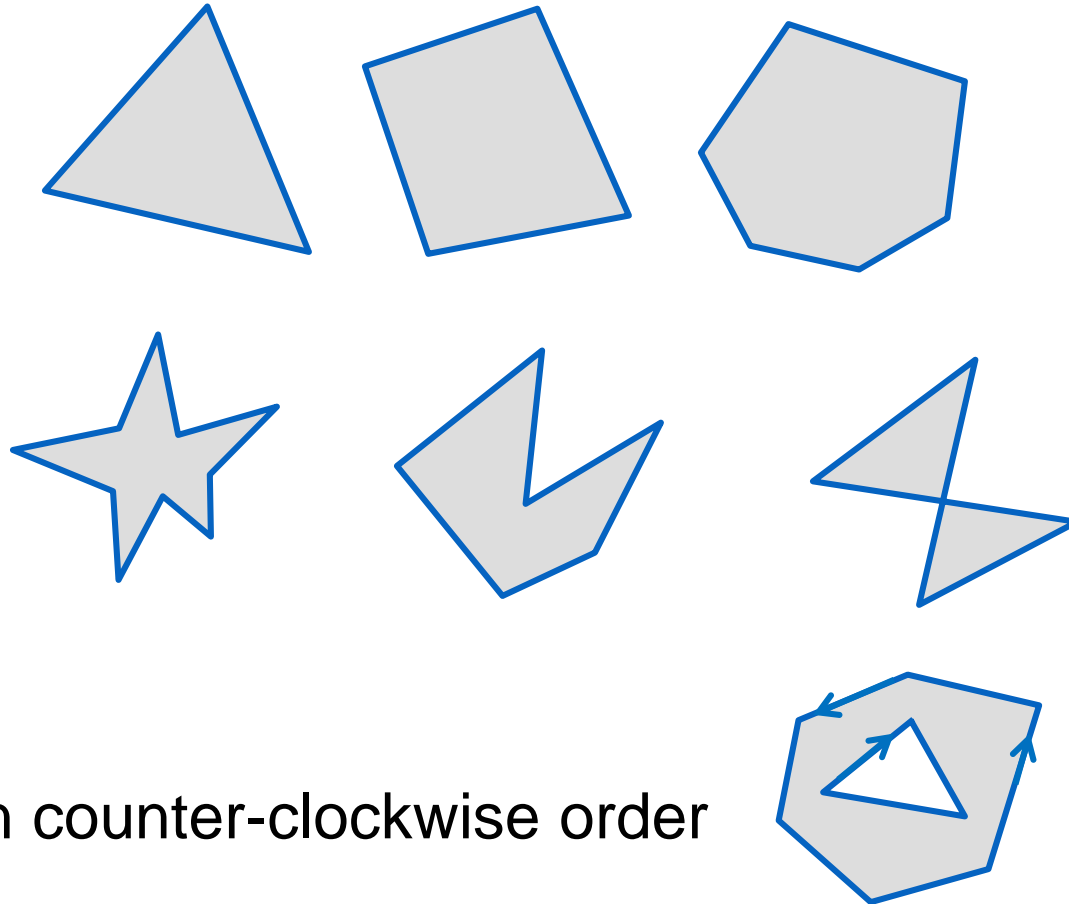


# 3D Polygon



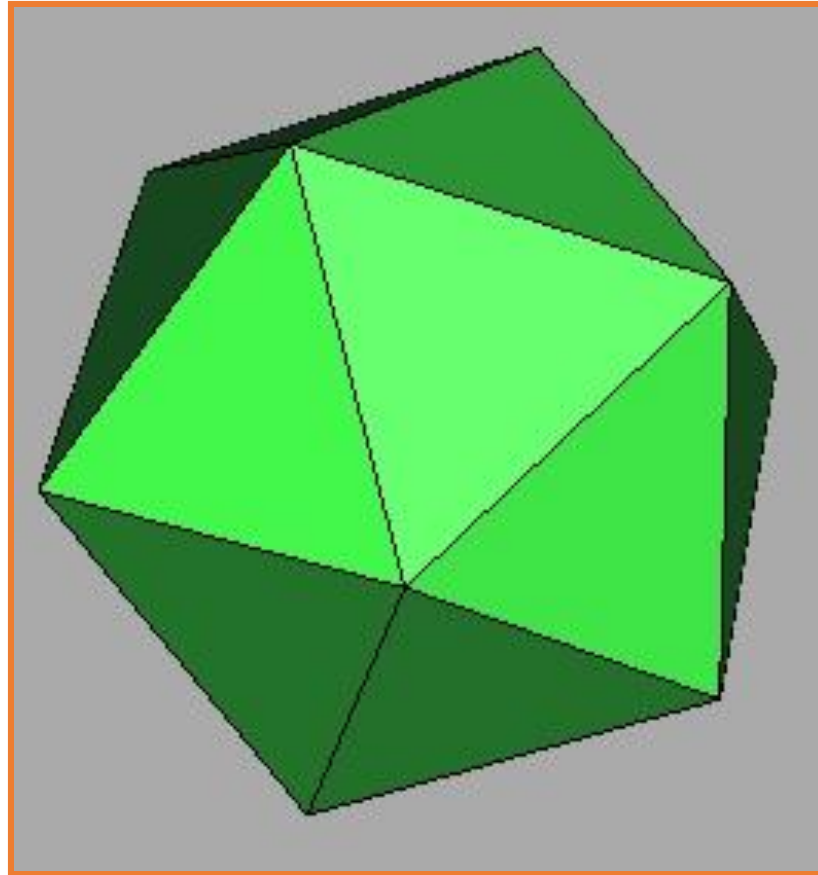
- Set of points “inside” a sequence of coplanar points

```
typedef struct {  
    Point *points;  
    int npoints;  
} Polygon;
```



Points are in counter-clockwise order

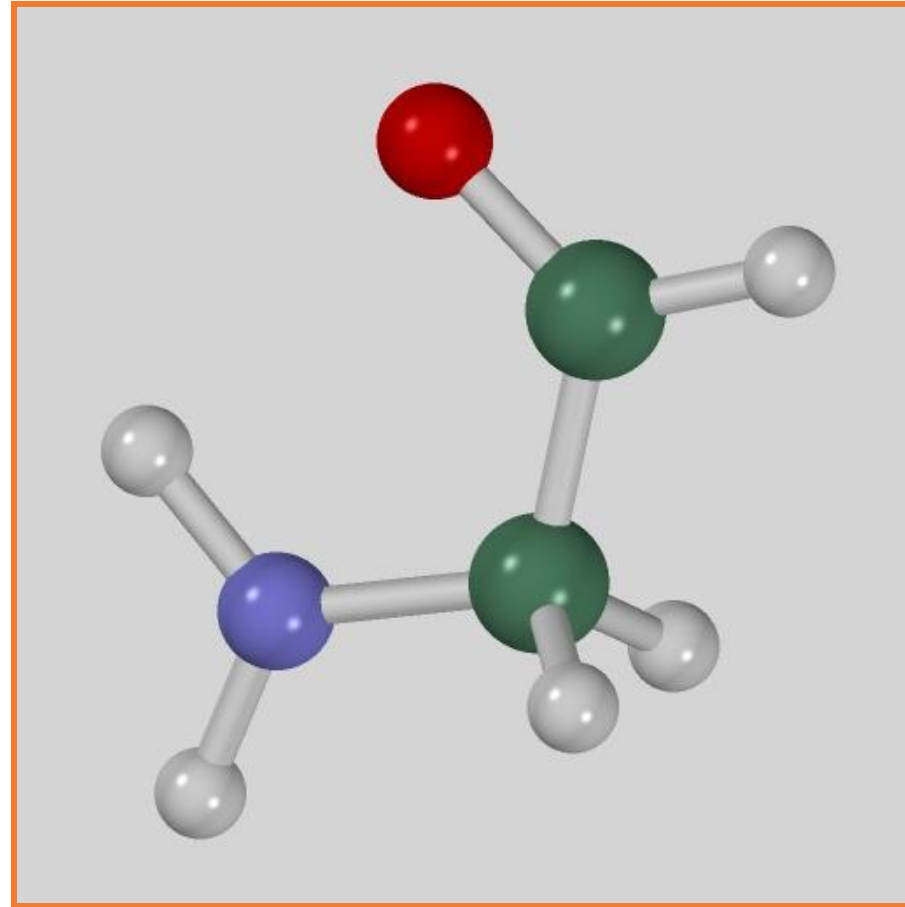
# 3D Object Representations



How can this object be represented in a computer?

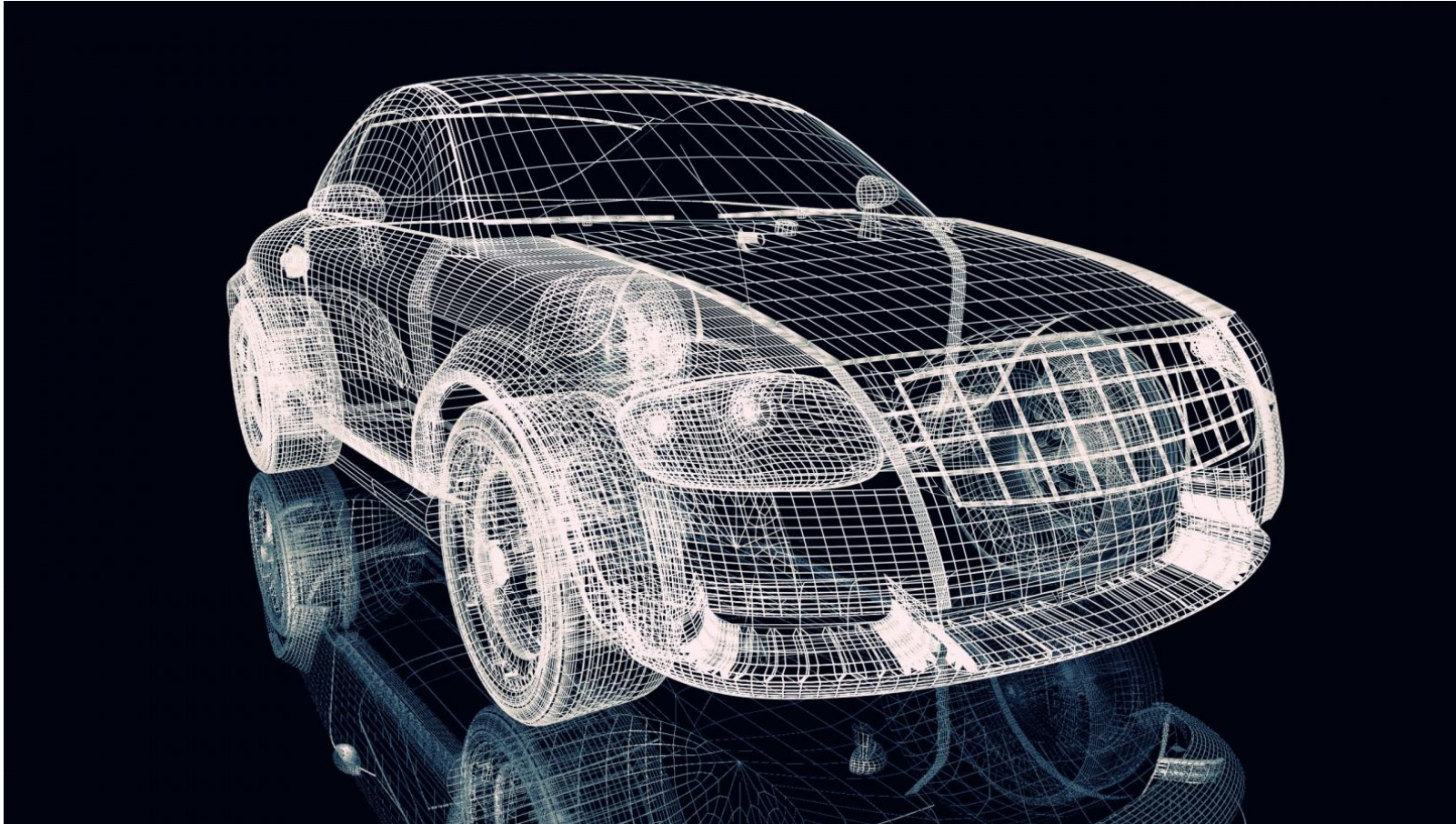


# 3D Object Representations



How about this one?

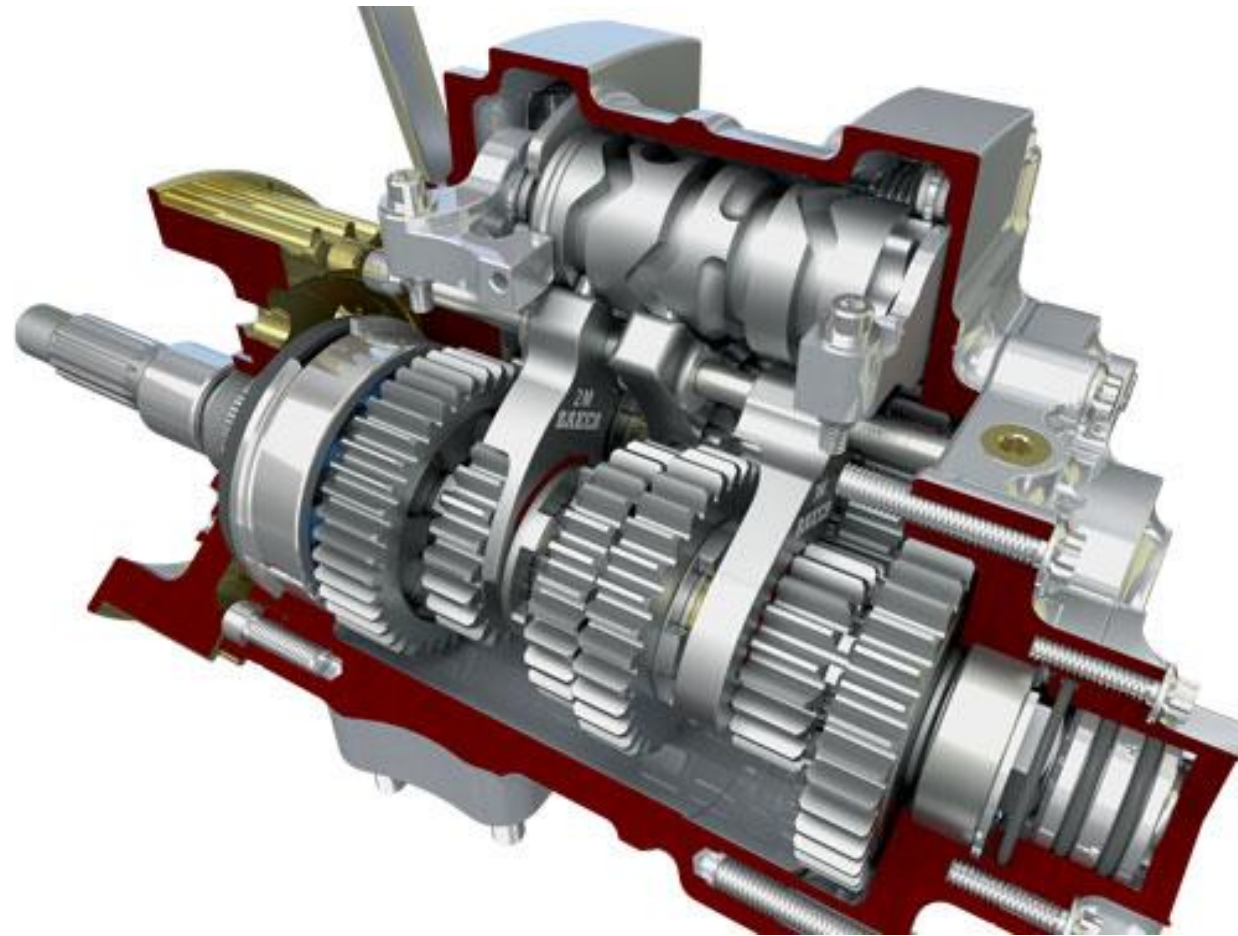
# 3D Object Representations



This one?

[Wallpaperonly.net](https://wallpaperonly.net)

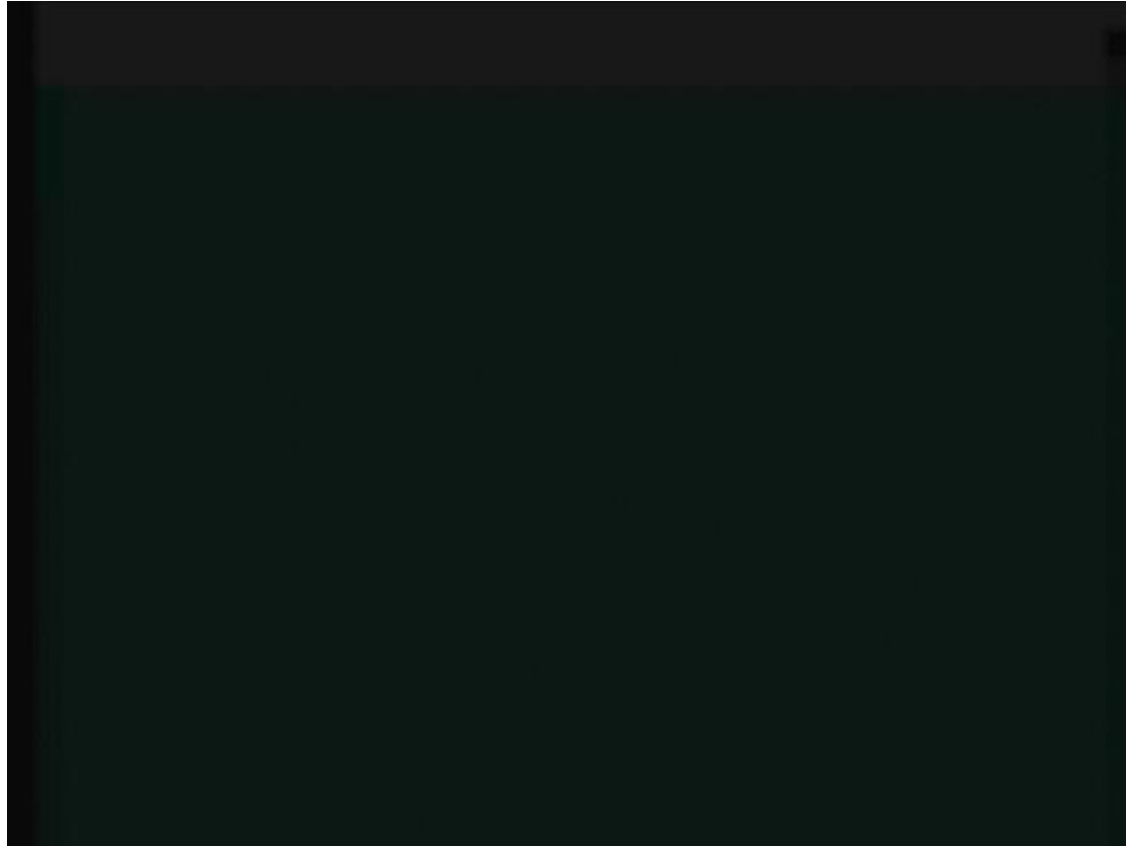
# 3D Object Representations



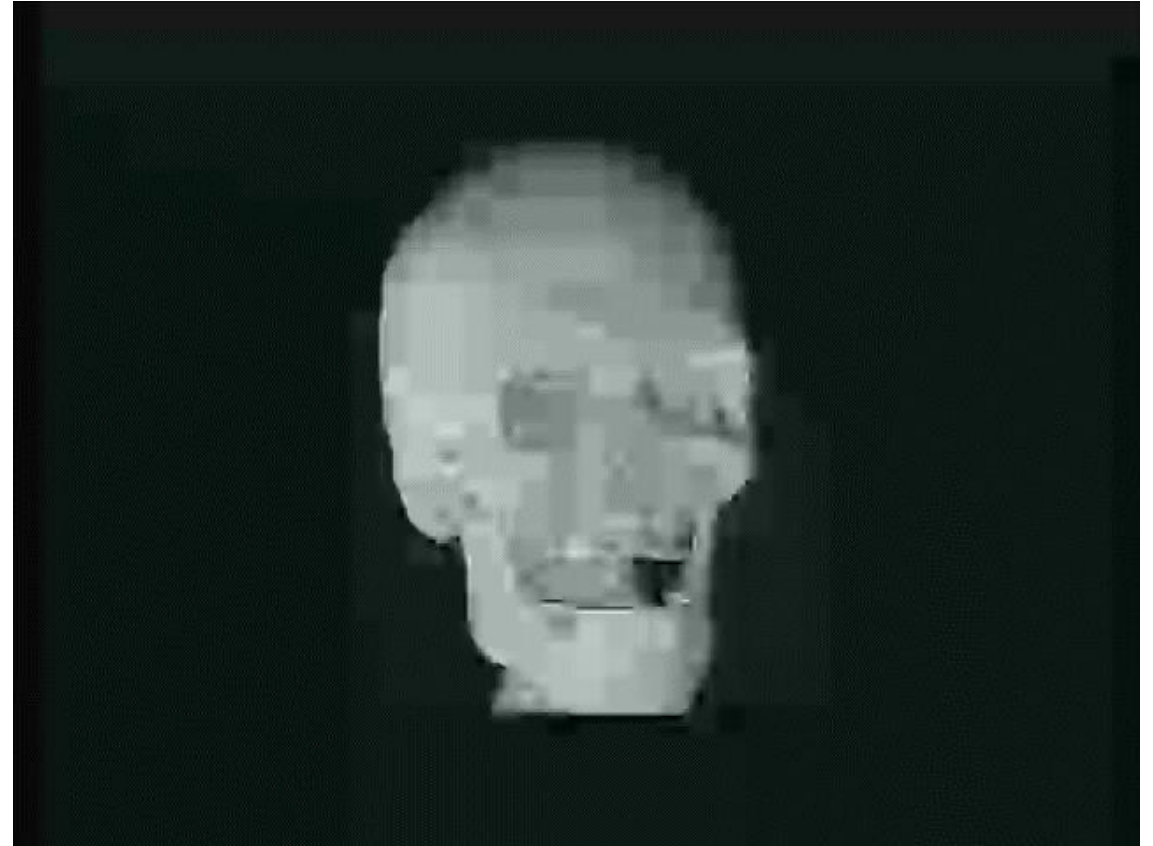
This one?

Solidworks

# 3D Object Representations

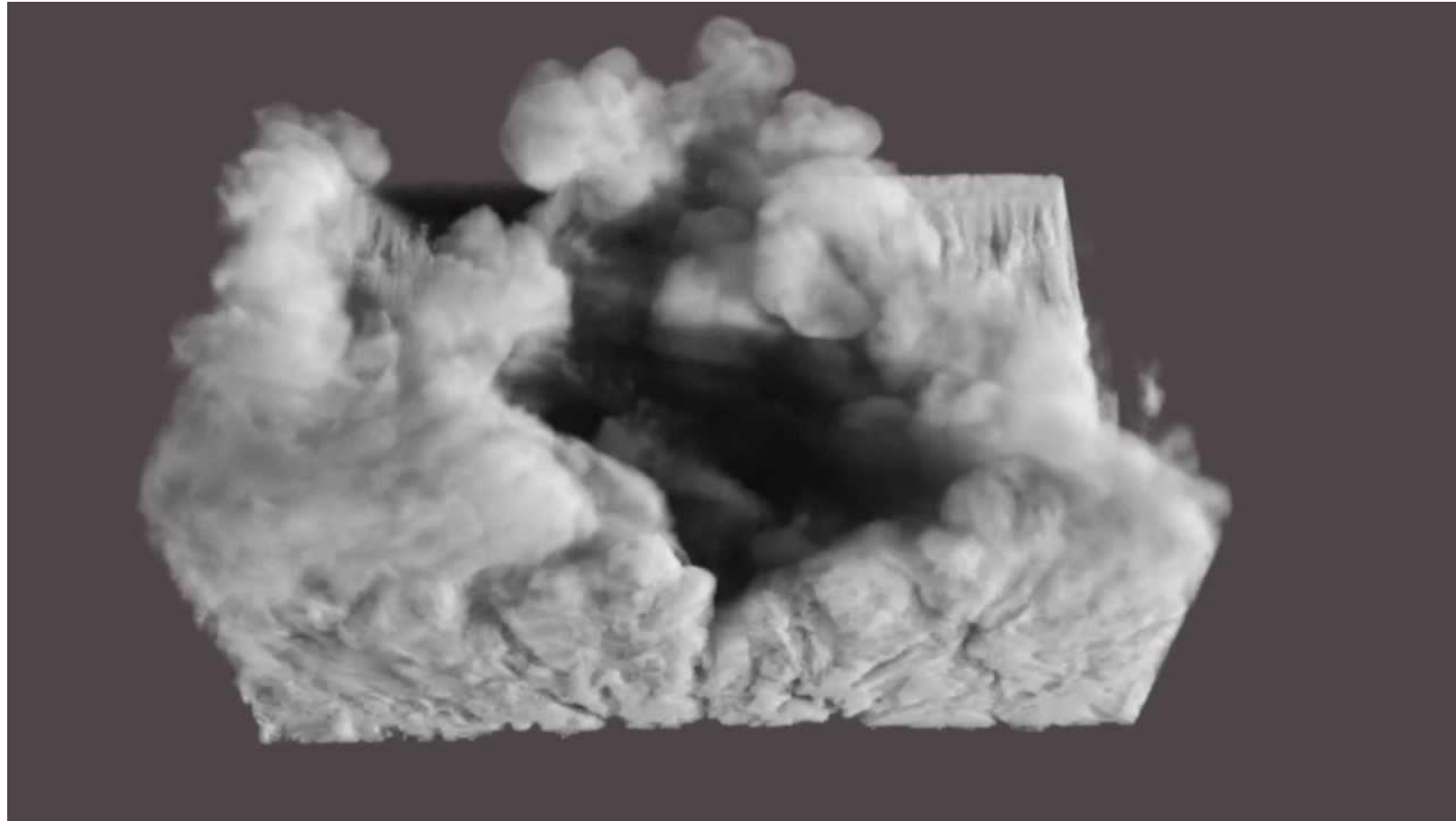


This one?



The visible human

# 3D Object Representations



This one?

FumeFx

# 3D Object Representations



- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# Equivalence of Representations



- Thesis:
  - Each representation has enough expressive power to model the shape of any geometric object
  - It is possible to perform all geometric operations with any fundamental representation
- Analogous to Turing-equivalence
  - Computers and programming languages are Turing-equivalent, but each has its benefits...

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation

Data structures determine algorithms

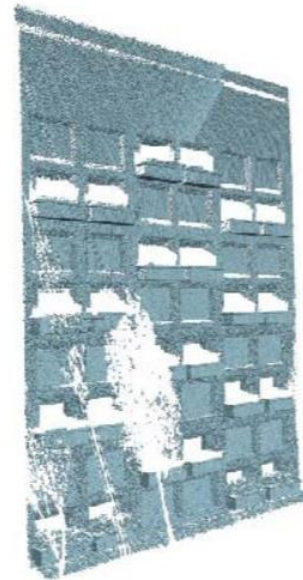


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
  - Range Scanning
- Rendering
- Analysis
- Manipulation
- Animation



# Why Different Representations?



## Efficiency for different tasks

- Acquisition
  - Computer Vision
- Rendering
- Analysis
- Manipulation
- Animation



Indiana University



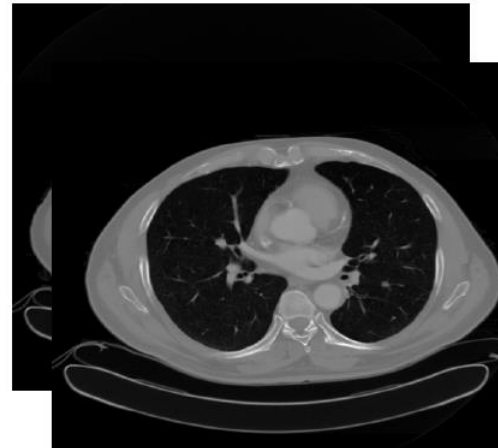
© 2012 CRS Interactive

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
  - Tomography
- Rendering
- Analysis
- Manipulation
- Animation



DGP course notes, Technion

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- **Rendering**
  - **Intersection**
- Analysis
- Manipulation
- Animation



# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Curvature, smoothness**
- Manipulation
- Animation

Analysis of surface quality



DGP course notes, Technion

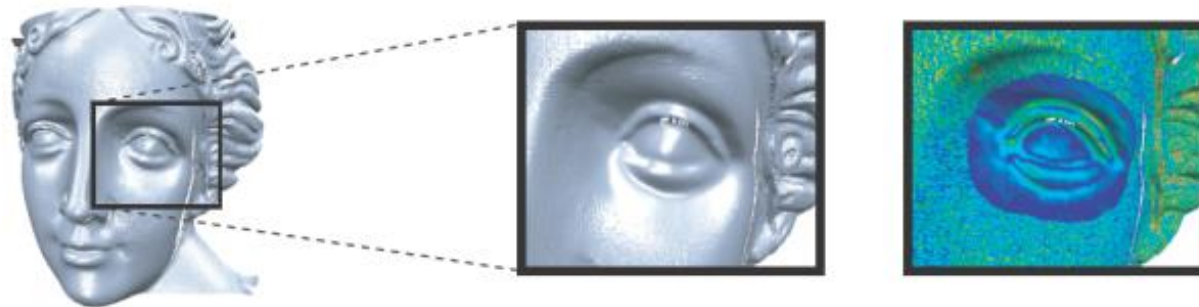
# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Fairing**
- Manipulation
- Animation

Surface smoothing for noise removal



DGP course notes, Technion

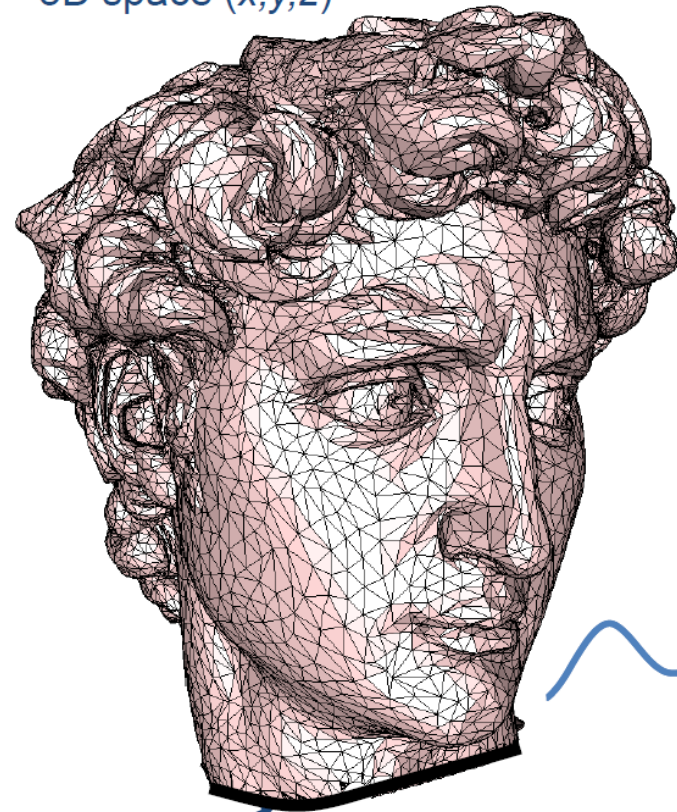
# Why Different Representations?



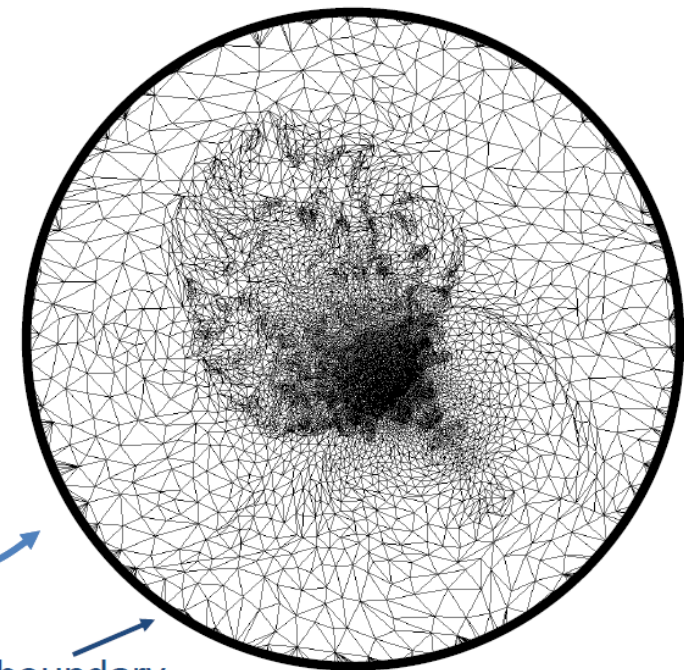
## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Parametrization**
- Manipulation
- Animation

3D space  $(x,y,z)$



2D parameter domain  $(u,v)$



boundary

boundary

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Texture mapping**
- Manipulation
- Animation



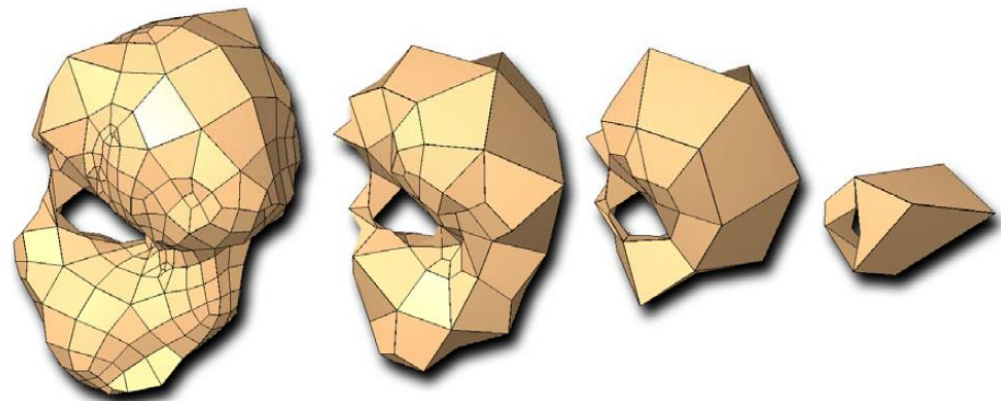
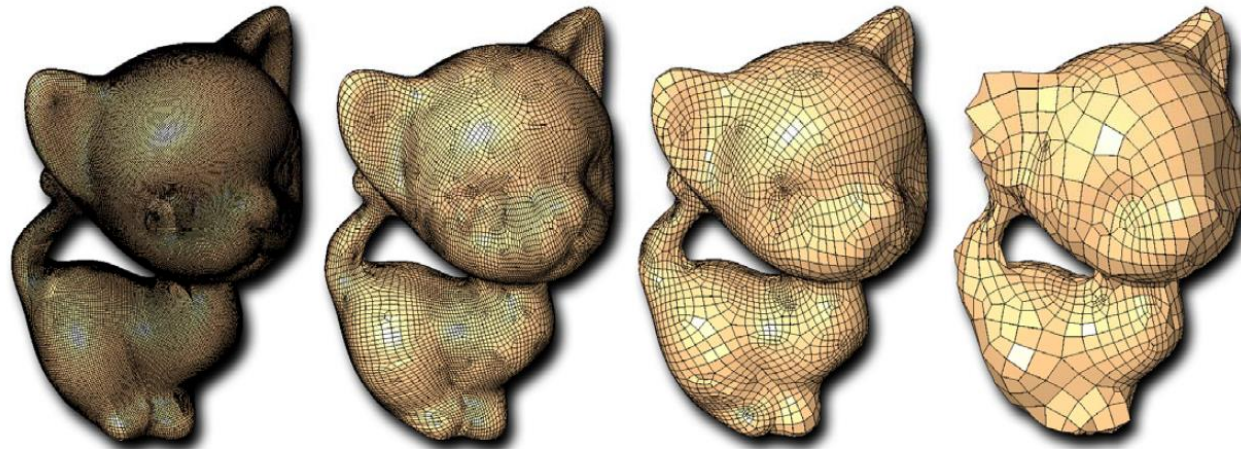


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Reduction**
- Manipulation
- Animation

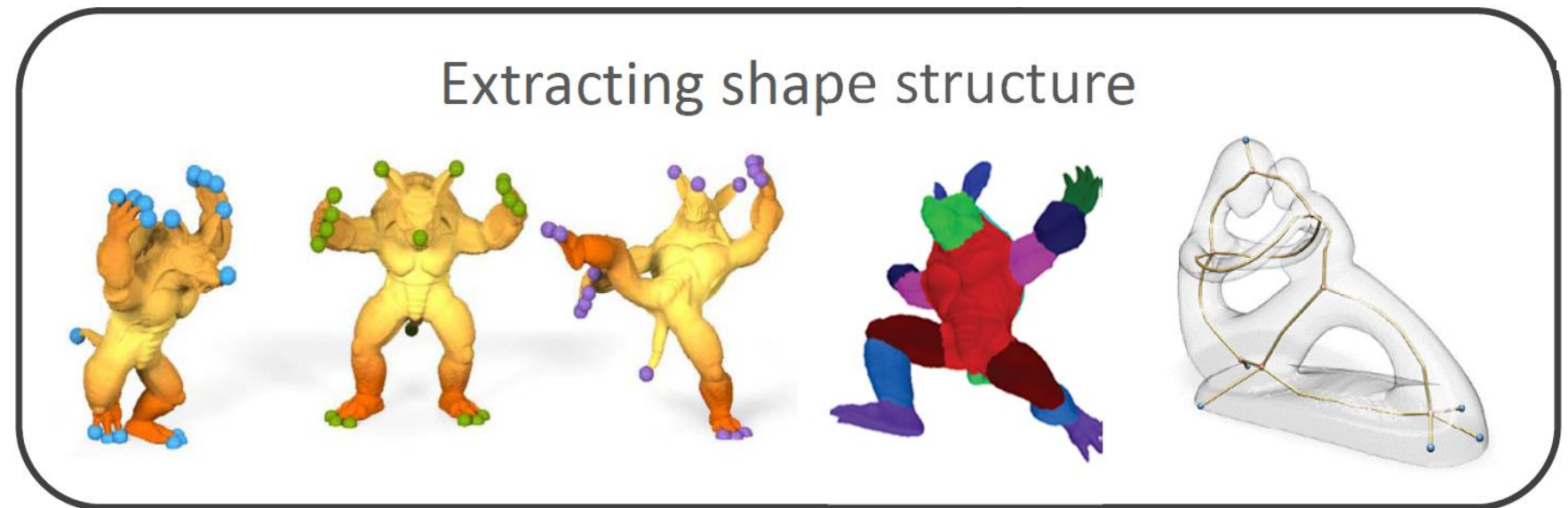


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Structure**
- Manipulation
- Animation



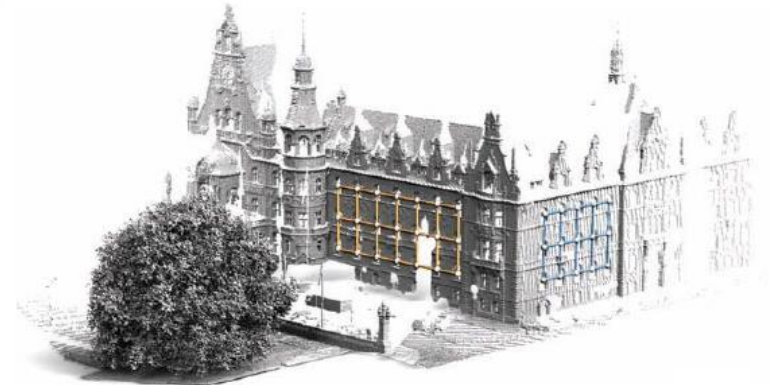
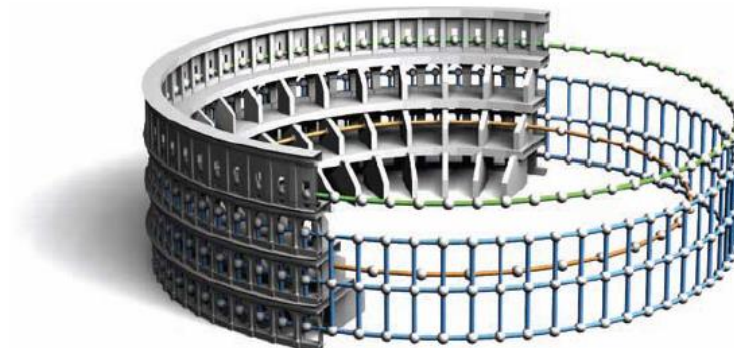
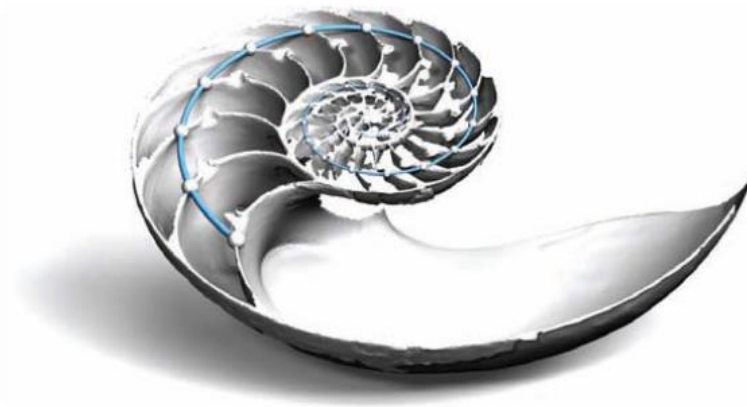
DGP course notes, Technion

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Symmetry detection**
- Manipulation
- Animation

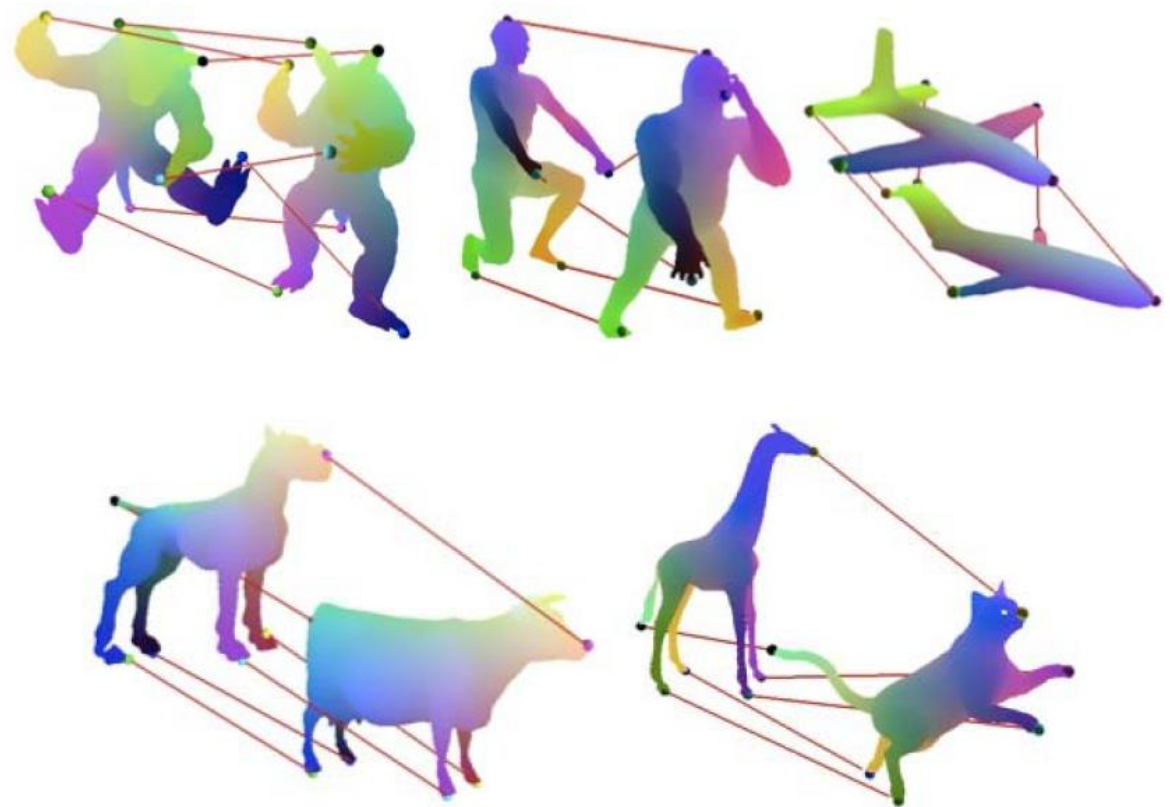


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Correspondence**
- Manipulation
- Animation

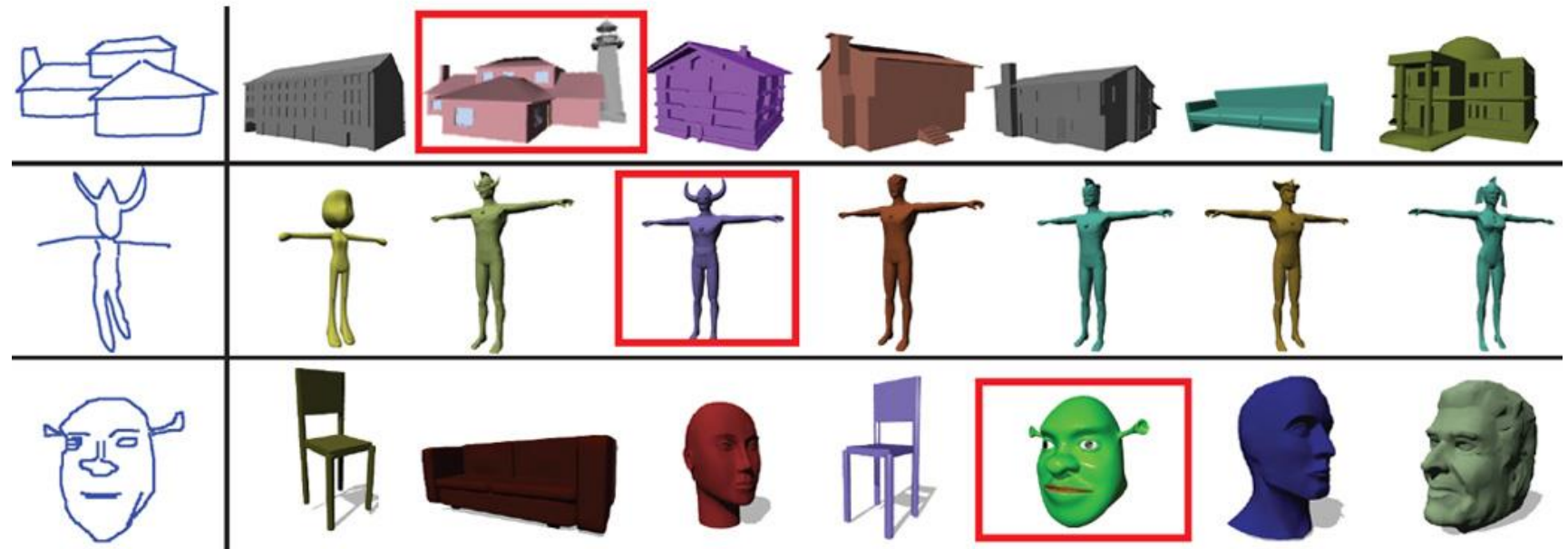


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Shape retrieval**
- Manipulation
- Animation



Shao et al. 2011

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Segmentation**
- Manipulation
- Animation

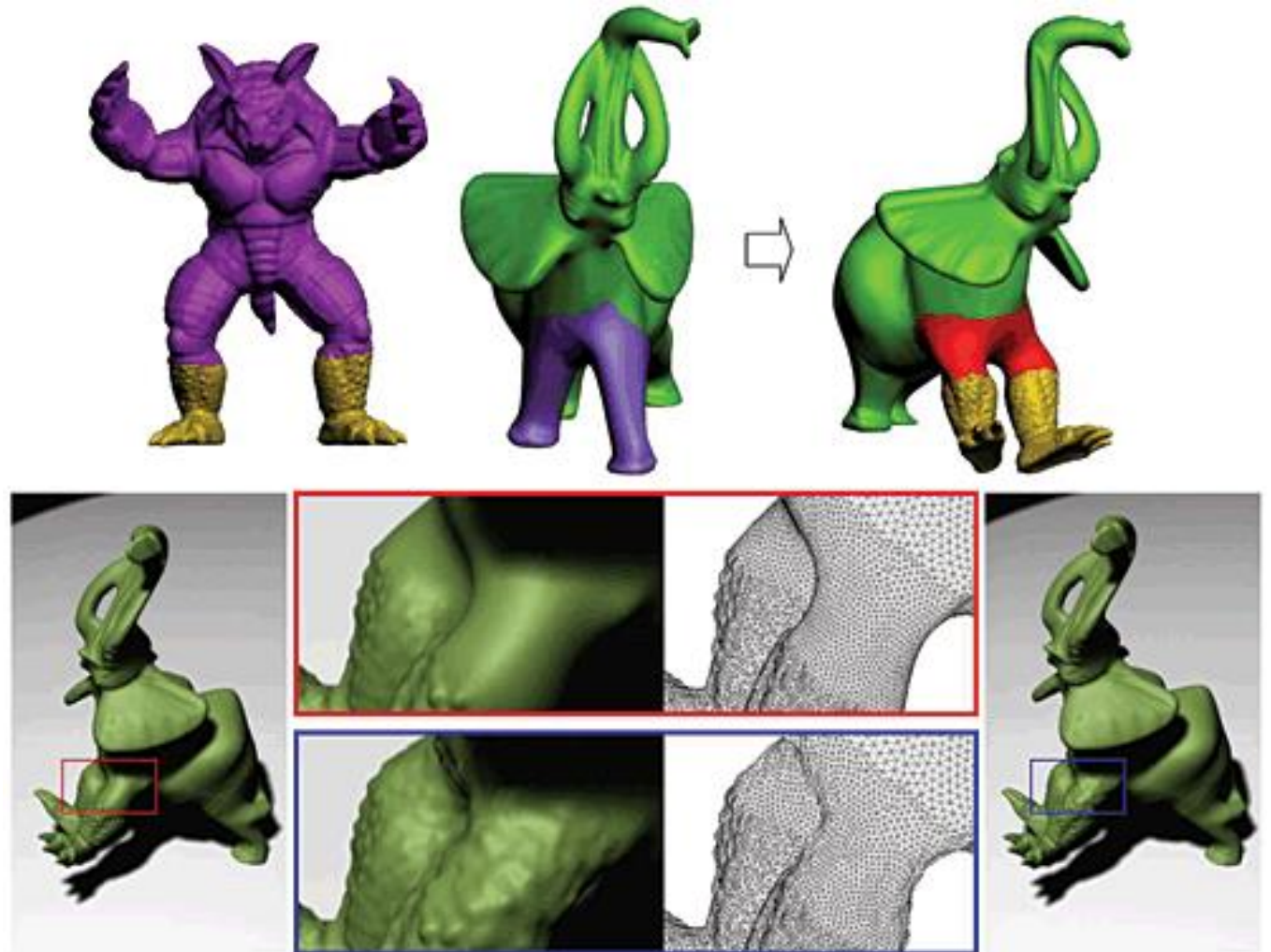


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- **Analysis**
  - **Composition**
- Manipulation
- Animation

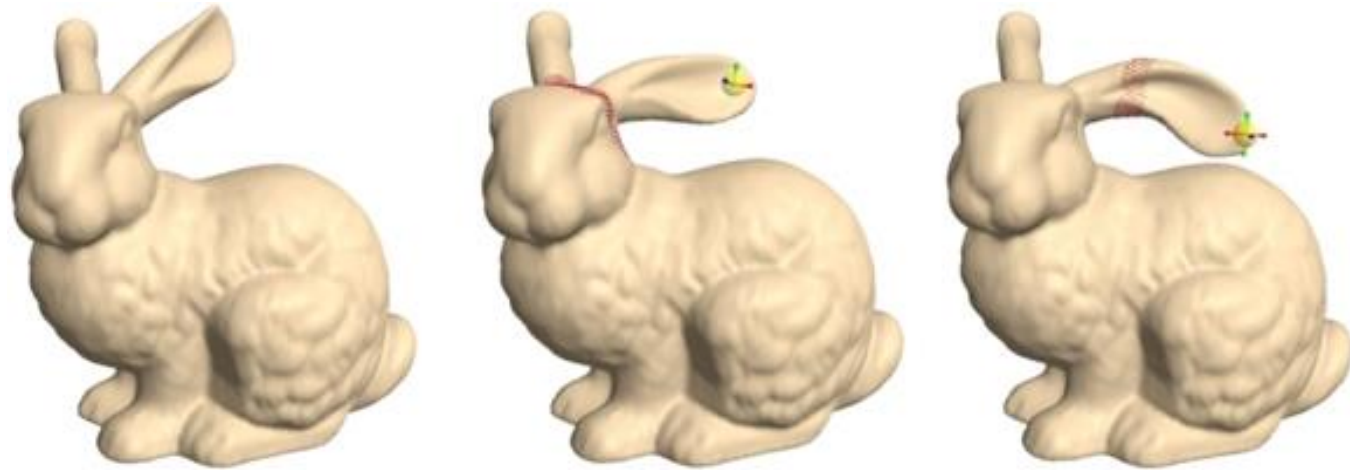


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- **Manipulation**
  - Deformation
- Animation



IGL



# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- **Manipulation**
  - Deformation
- Animation

Freeform and multiresolution modeling



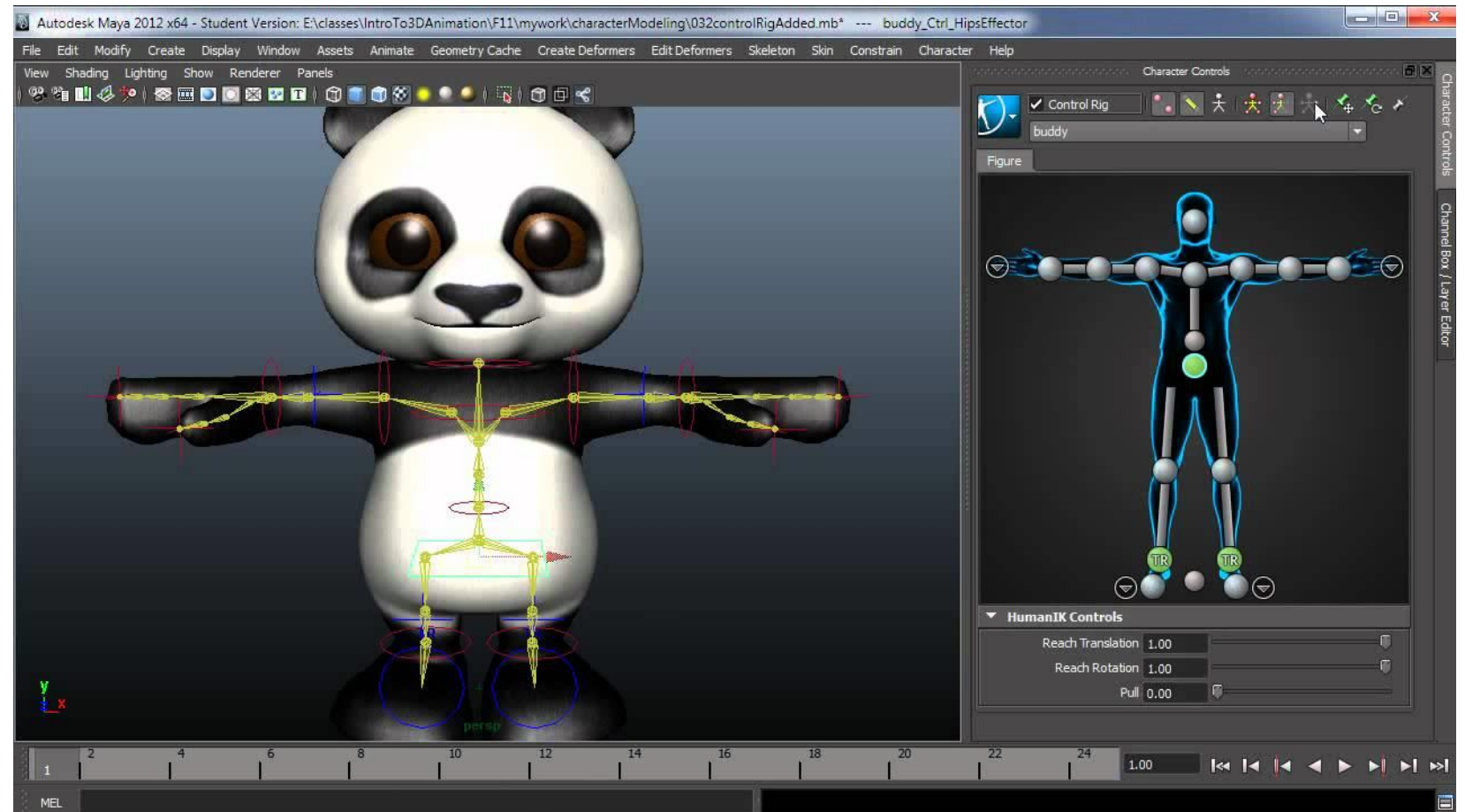
DGP course notes, Technion

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
  - Control
- Animation



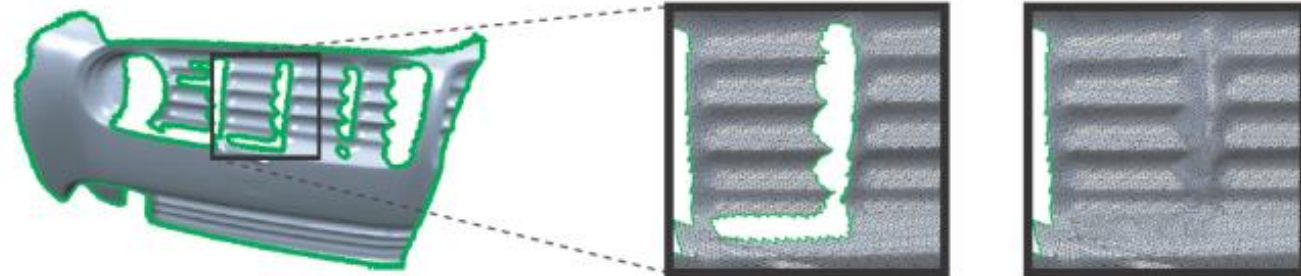
# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- **Manipulation**
  - **Healing**
- Animation

Removal of topological and geometrical errors



DGP course notes, Technion

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- **Animation**
  - **Rigging**

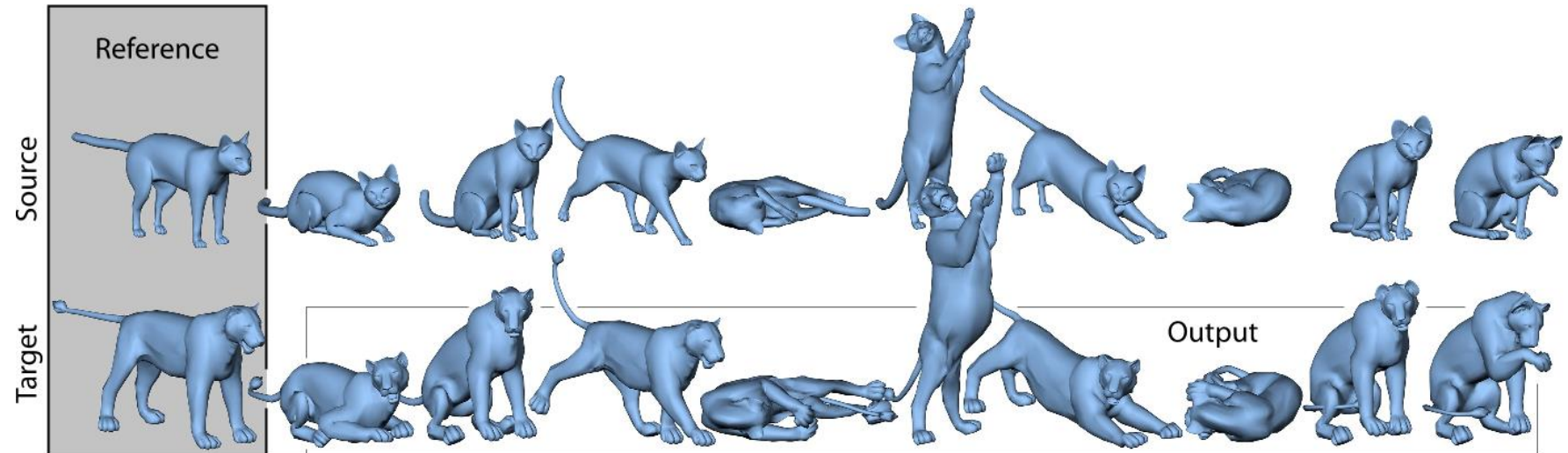


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- **Animation**
  - **Deformation transfer**



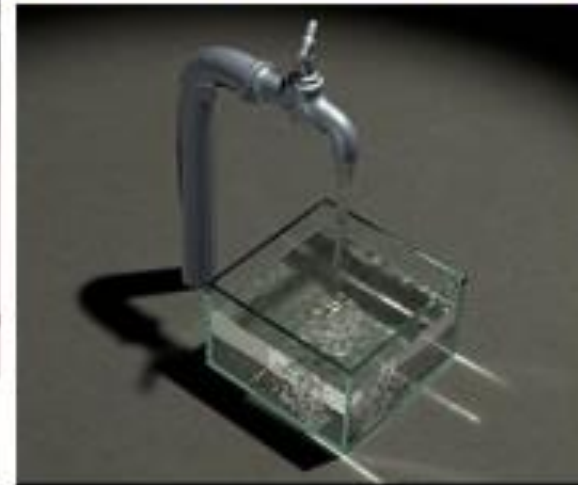
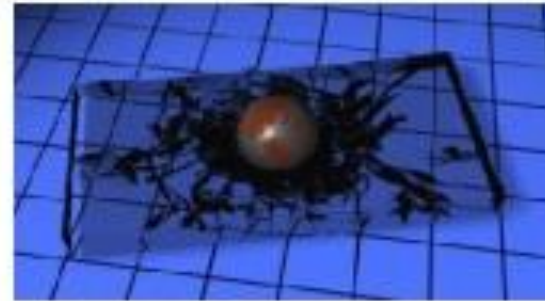
Sumner et al. 2004

# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- **Animation**
  - **Simulation**

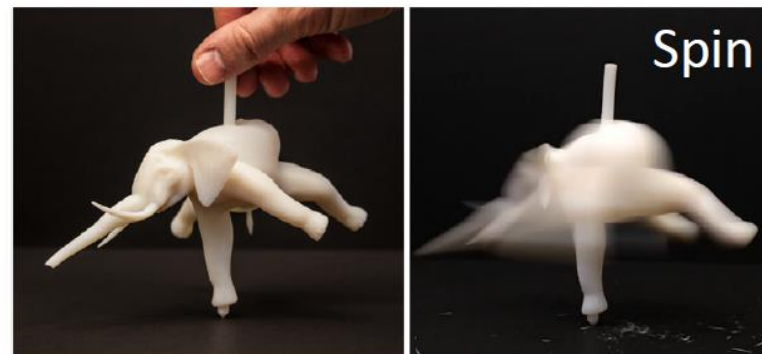


# Why Different Representations?



## Efficiency for different tasks

- Acquisition
- Rendering
- Analysis
- Manipulation
- Animation
  - Fabrication



# 3D Object Representations



- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific



# 3D Object Representations



- Points

- Range image
- Point cloud

- Surfaces

- Polygonal mesh
- Subdivision
- Parametric
- Implicit

- Solids

- Voxels
- BSP tree
- CSG
- Sweep

- High-level structures

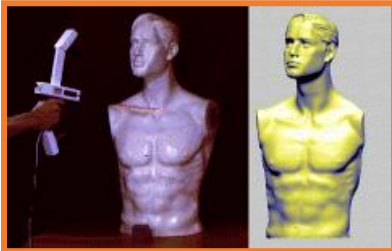
- Scene graph
- Application specific

# Range Image



Set of 3D points mapping to pixels of depth image

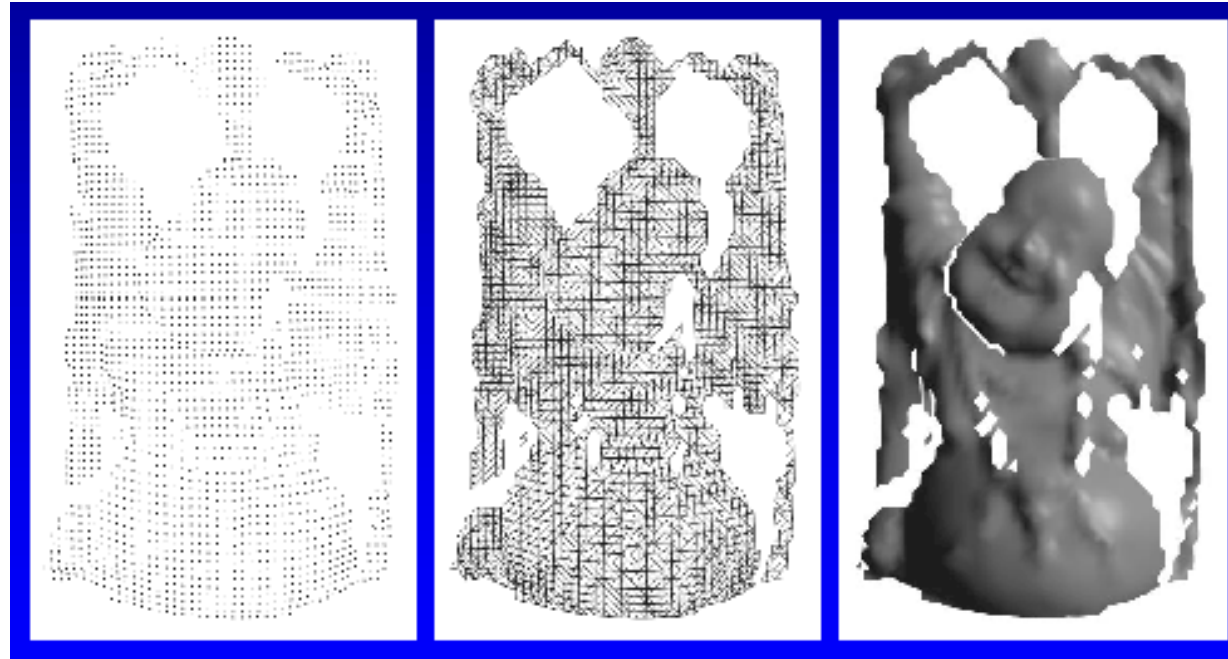
- Can be acquired from range scanner



Cyberware



Stanford



Range Image

Tessellation

Range Surface

# Point Cloud



Unstructured set of 3D point samples

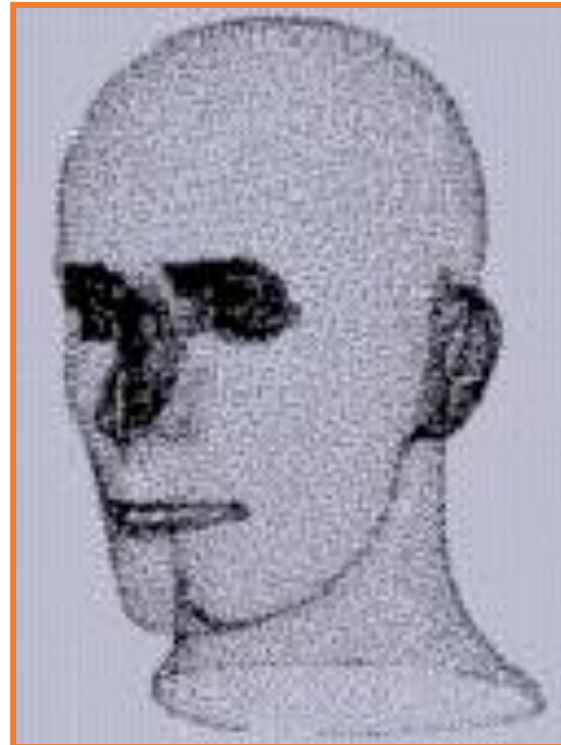
- Acquired from range finder, computer vision, etc



Polhemus



Microscribe-3D



Hoppe



Hoppe

# 3D Object Representations

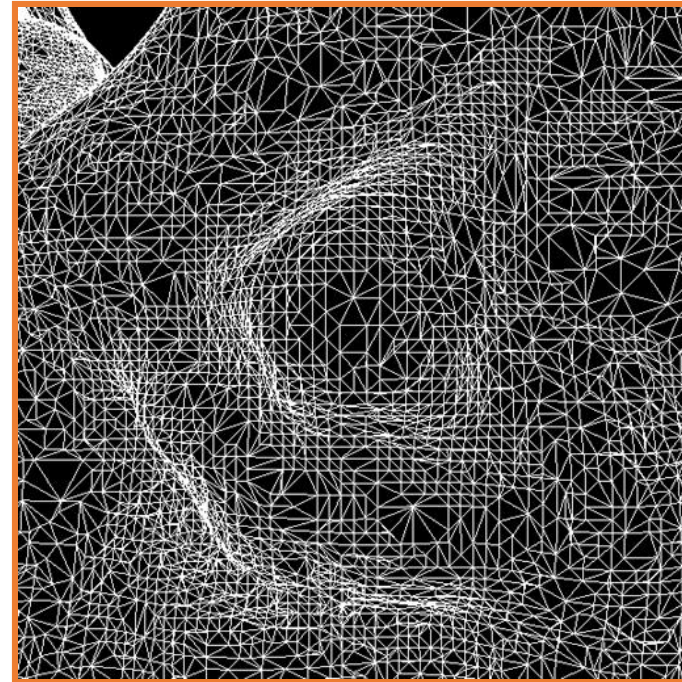


- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# Polygonal Mesh



Connected set of polygons (often triangles)

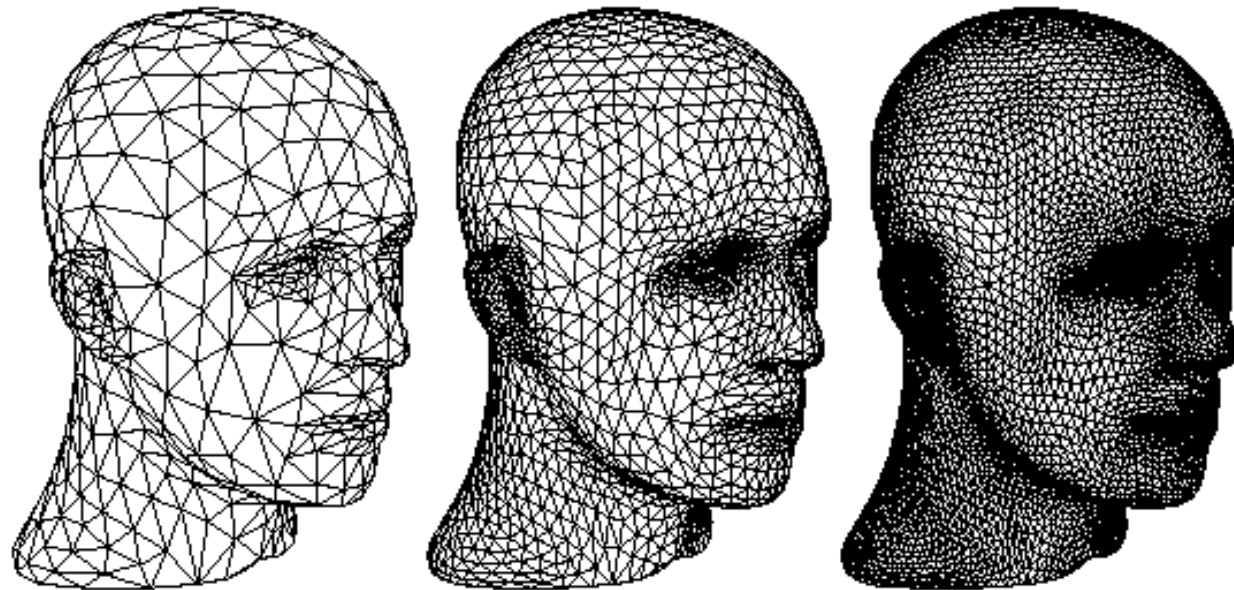


# Subdivision Surface



## Coarse mesh & subdivision rule

- Smooth surface is **limit** of sequence of refinements



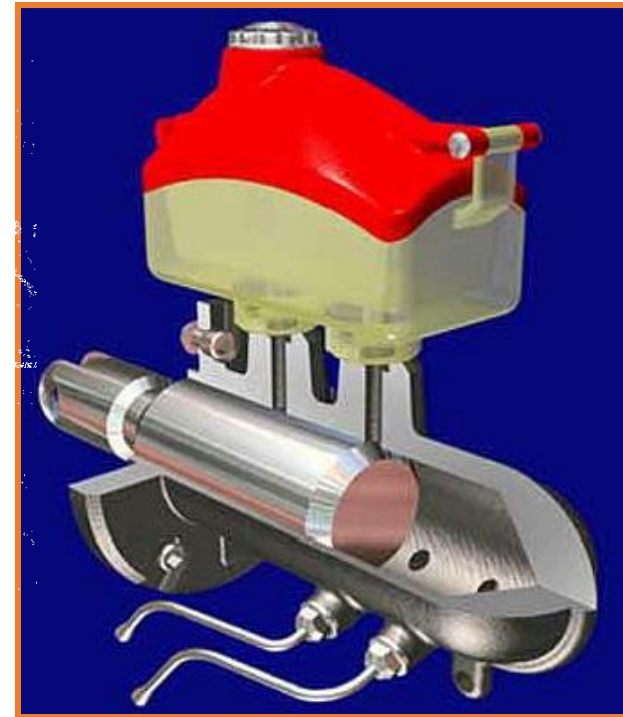
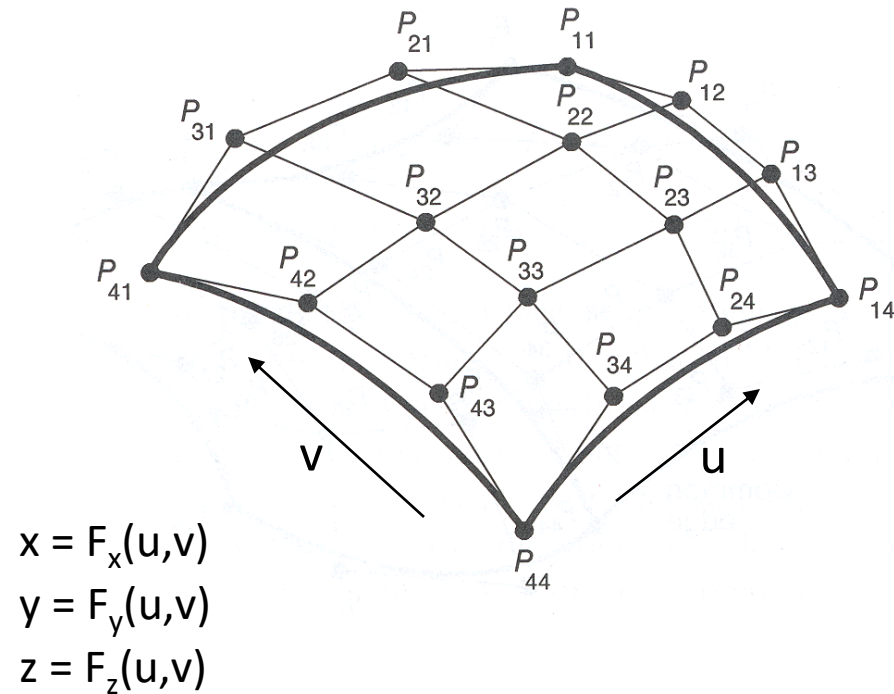
Zorin & Schroeder  
SIGGRAPH 99  
Course Notes

# Parametric Surface



## Tensor-product spline patches

- Each patch is parametric function
- Careful constraints to maintain continuity

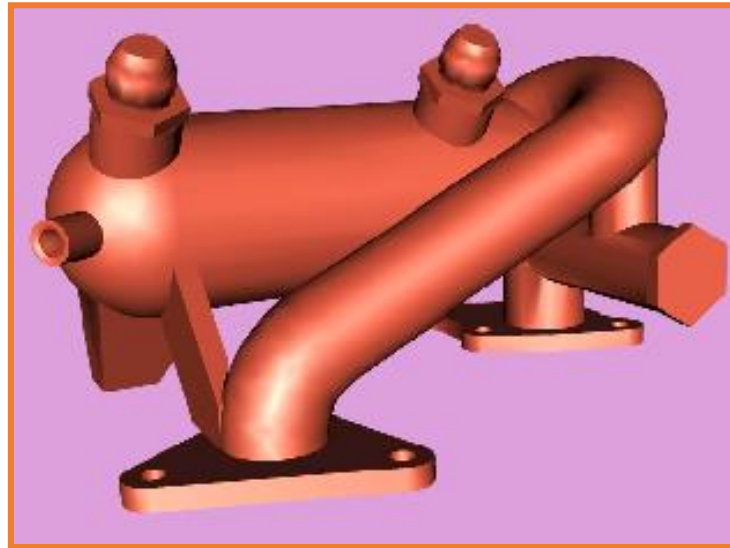


FvDFH Figure 11.44

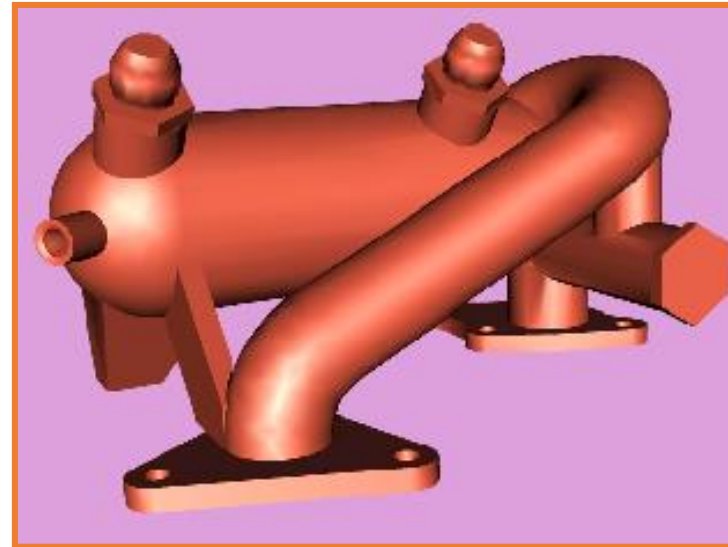
# Implicit Surface



Set of all points satisfying:  $F(x,y,z) = 0$



Polygonal Model



Implicit Model

Bill Lorensen  
SIGGRAPH 99  
Course #4 Notes



# 3D Object Representations



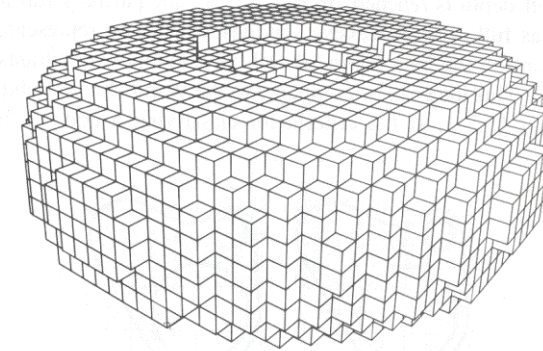
- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# Voxel grid

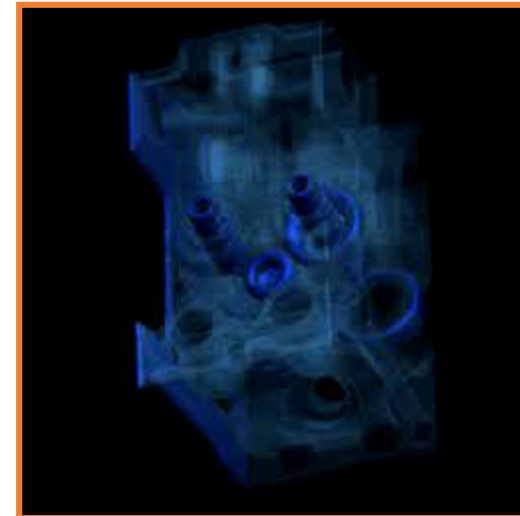


Uniform volumetric grid of samples:

- Occupancy  
(object vs. empty space)
- Density
- Color
- Other function  
(speed, temperature, etc.)
  
- Often acquired via  
simulation or from  
CAT, MRI, etc.



FvDFH Figure 12.20

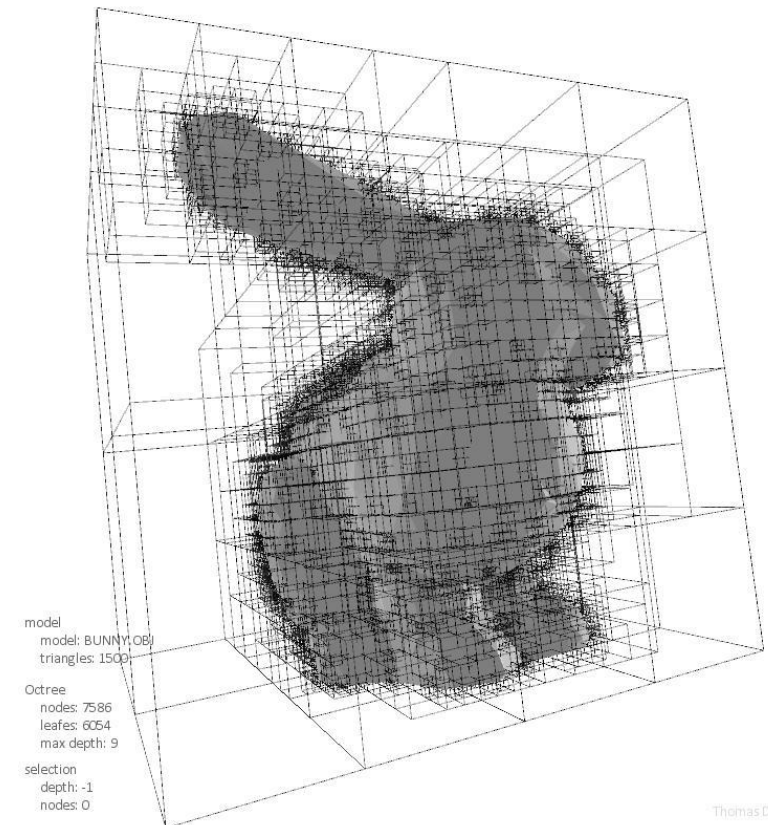
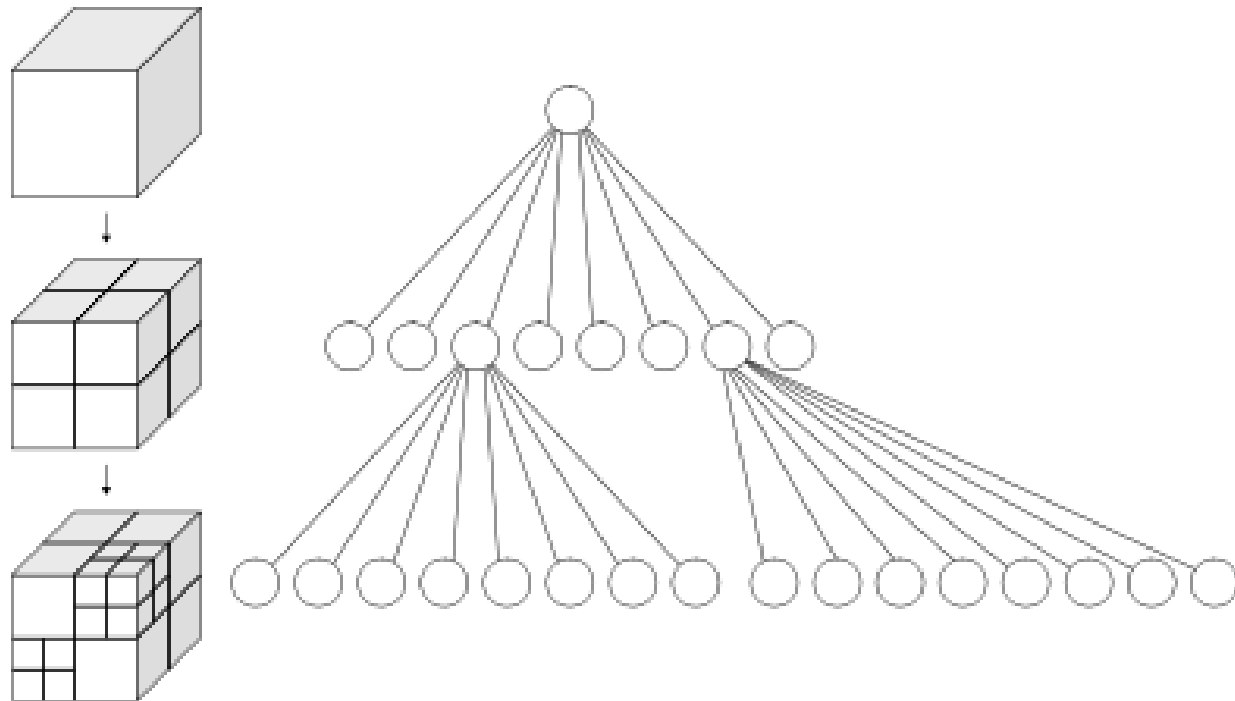


# Octree



The adaptive version of the voxel grid

- Significantly more space efficient
- Makes operations more cumbersome



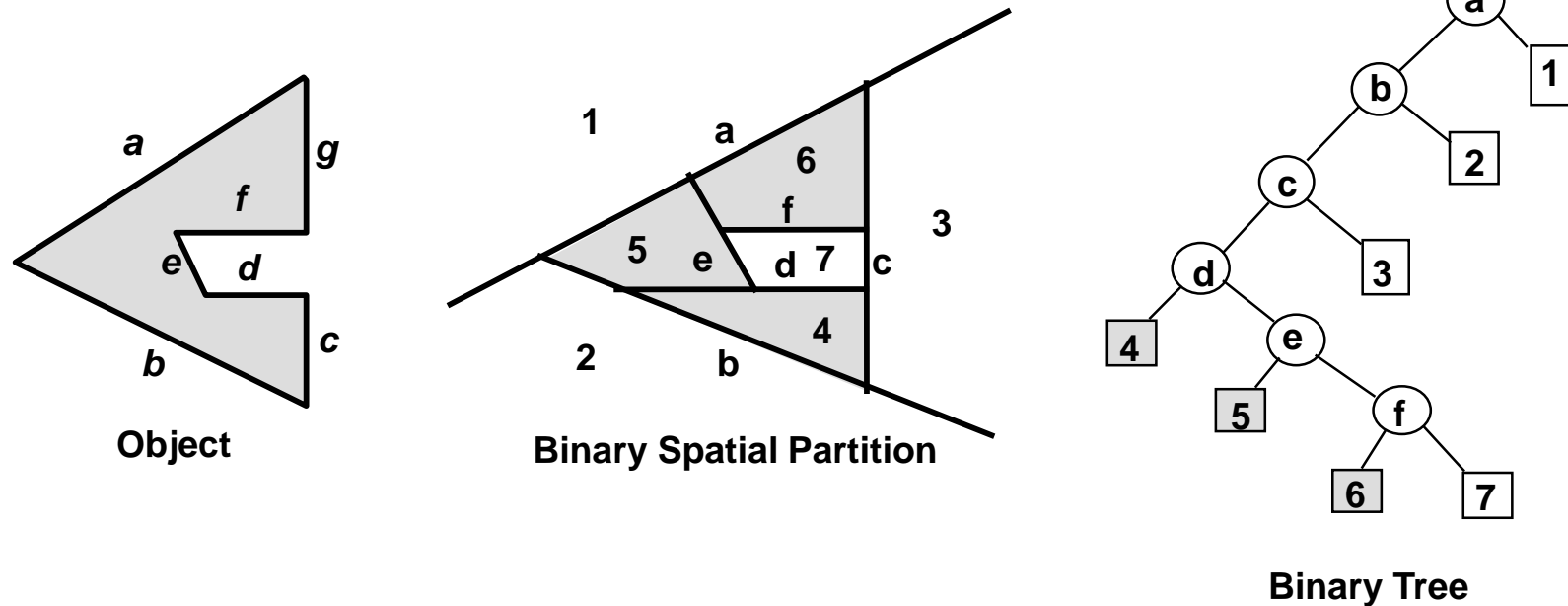
Thomas Diewald

# BSP Tree



Hierarchical **B**inary **S**pace **P**artition with solid/empty cells labeled

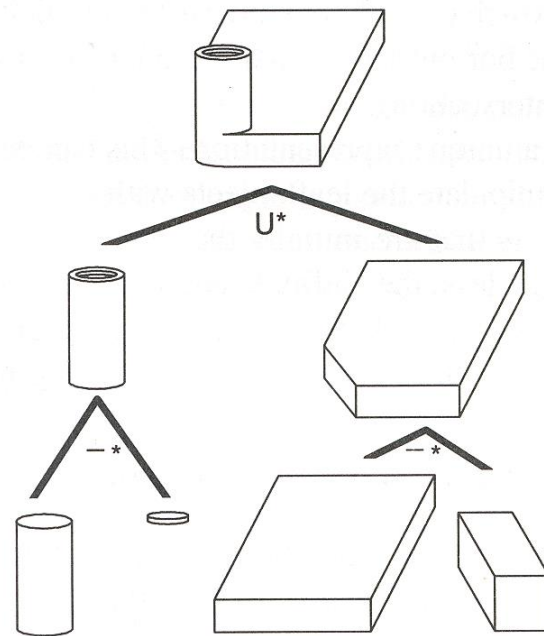
- Constructed from polygonal representations



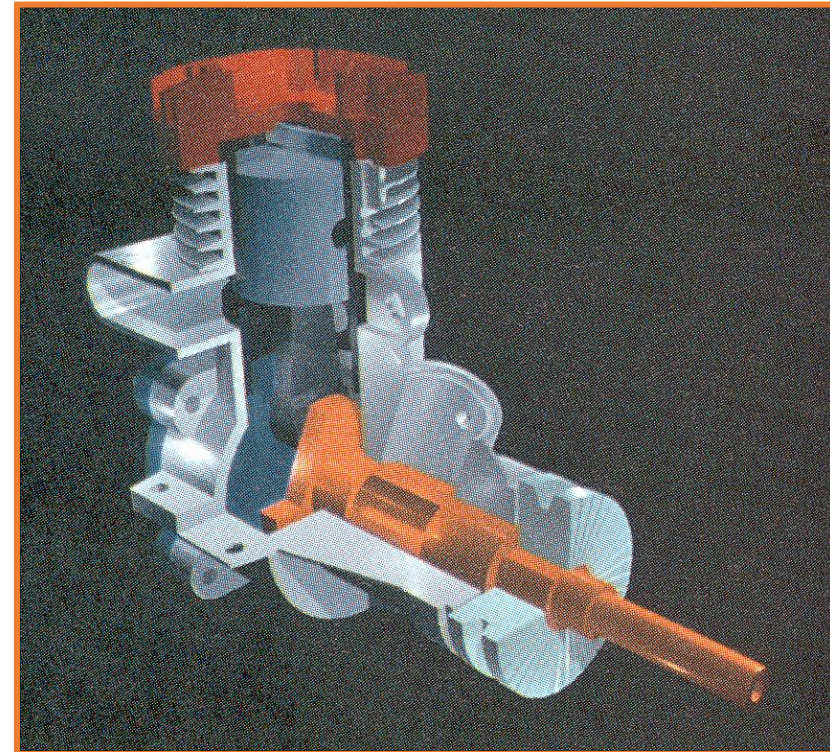
# CSG



**C**onstructive **S**olid **G**eometry: set operations (union, difference, intersection) applied to simple shapes



FvDFH Figure 12.27

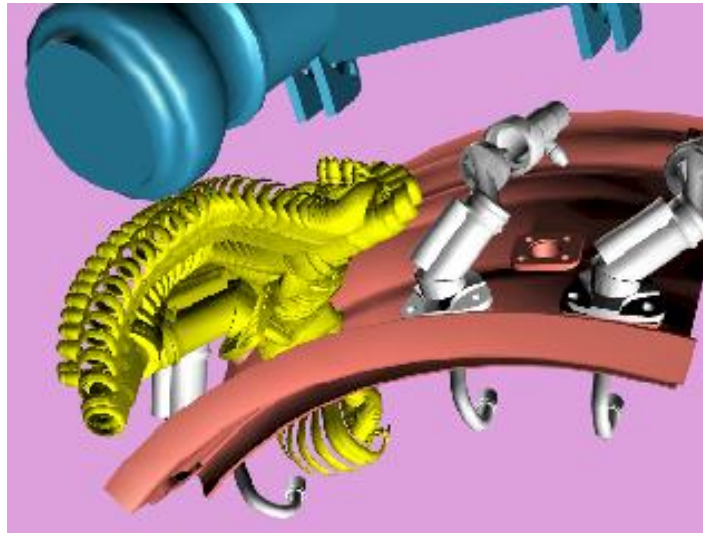


H&B Figure 9.9

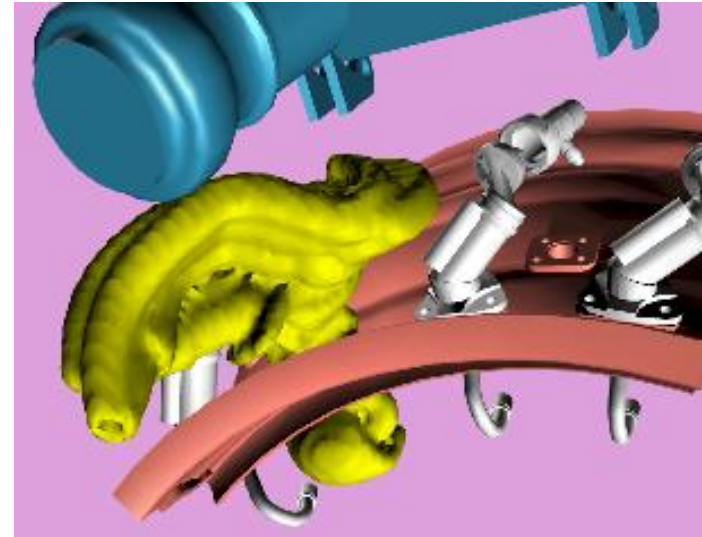
# Sweep



Solid swept by curve along trajectory



Removal Path



Sweep Model

Bill Lorensen  
SIGGRAPH 99  
Course #4 Notes

# 3D Object Representations



- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

# Scene Graph



Union of objects at leaf nodes



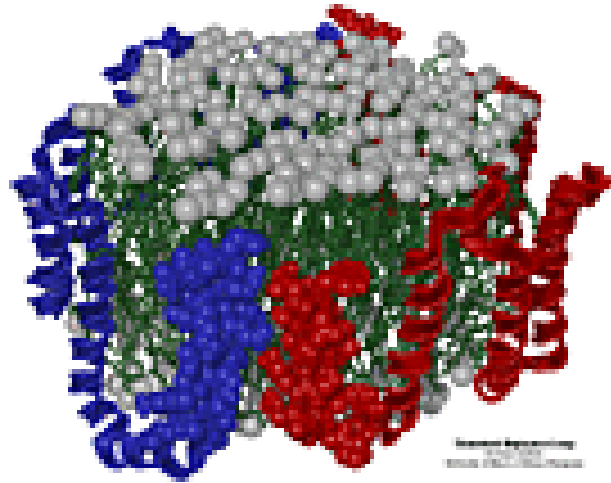
Bell Laboratories



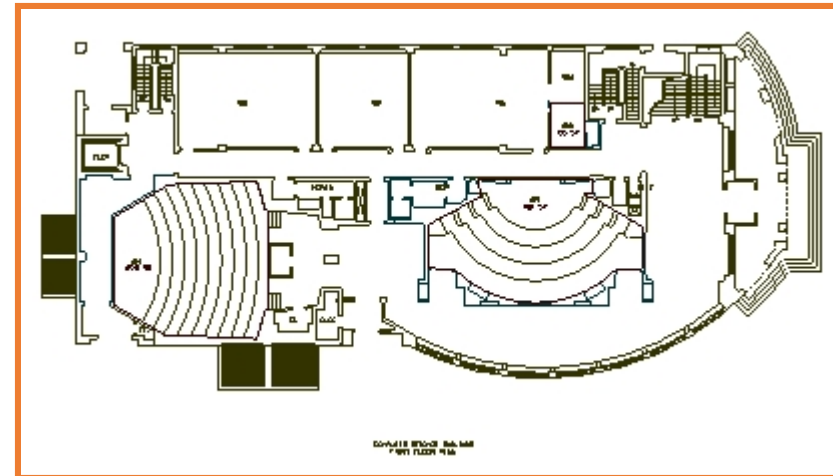
[avalon.viewpoint.com](http://avalon.viewpoint.com)



# Application Specific

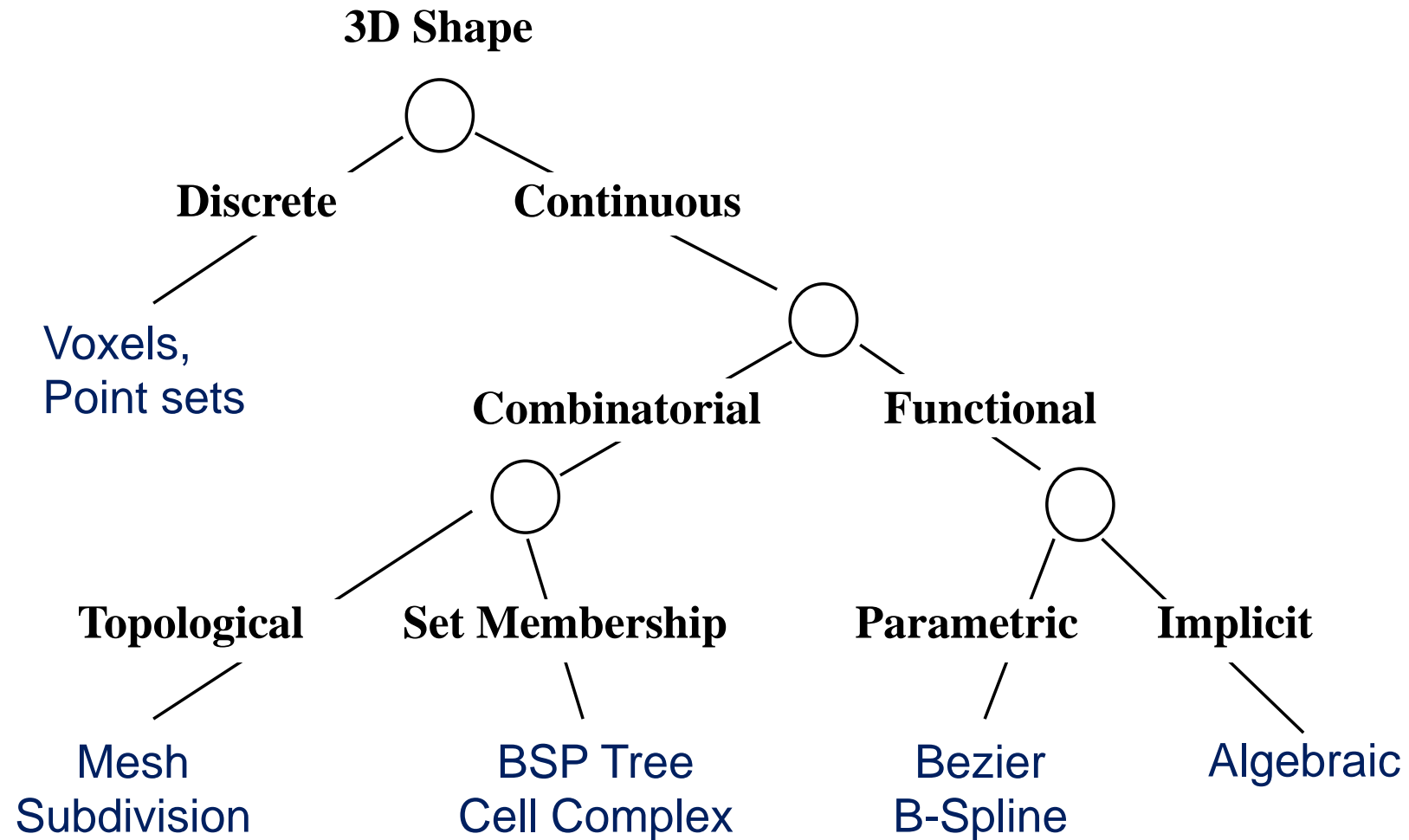


**Apo A-1**  
*(Theoretical Biophysics Group,  
University of Illinois at Urbana-Champaign)*



**Architectural Floorplan**  
*(CS Building, Princeton University)*

# Taxonomy of 3D Representations



# Equivalence of Representations



- Thesis:
  - Each representation has enough expressive power to model the shape of any geometric object
  - It is possible to perform all geometric operations with any fundamental representation
- Analogous to Turing-equivalence
  - Computers and programming languages are Turing-equivalent, but each has its benefits...

# Computational Differences



- Efficiency
  - Representational complexity (e.g. surface vs. volume)
  - Computational complexity (e.g.  $O(n^2)$  vs  $O(n^3)$  )
  - Space/time trade-offs (e.g. tree data structures)
  - Numerical accuracy/stability (e.g. degree of polynomial)
- Simplicity
  - Ease of acquisition
  - Hardware acceleration
  - Software creation and maintenance
- Usability
  - Designer interface vs. computational engine

# Upcoming Lectures



- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific