3D Rendering Intro & Ray Casting

COS 426, Spring 2020

Felix Heide

Princeton University

Ed Catmull and Pat Hanrahan win ACM Turing award

Syllabus

I. Image processing

II. Modeling

III. Rendering

IV. Animation

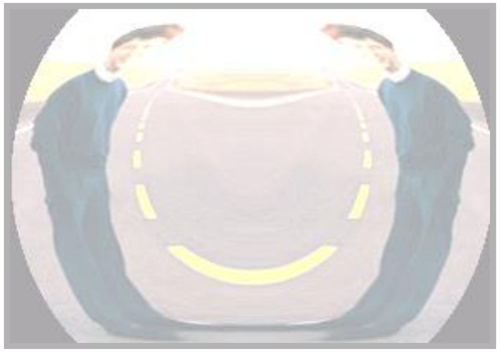
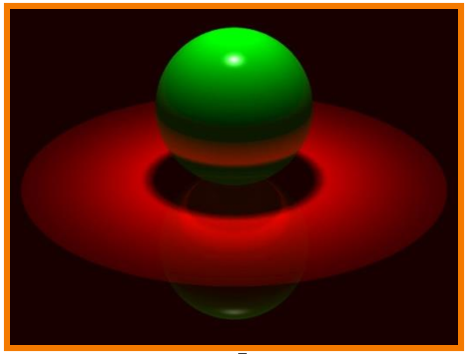
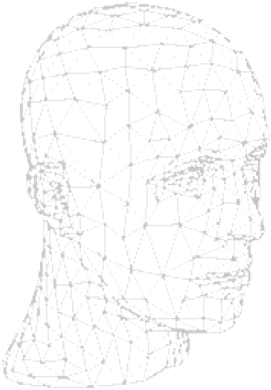


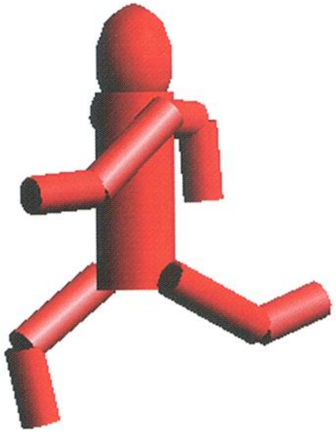
Image Processing
(Rusty Coleman, CS426, Fall99)



Rendering
(Michael Bostock, CS426, Fall99)



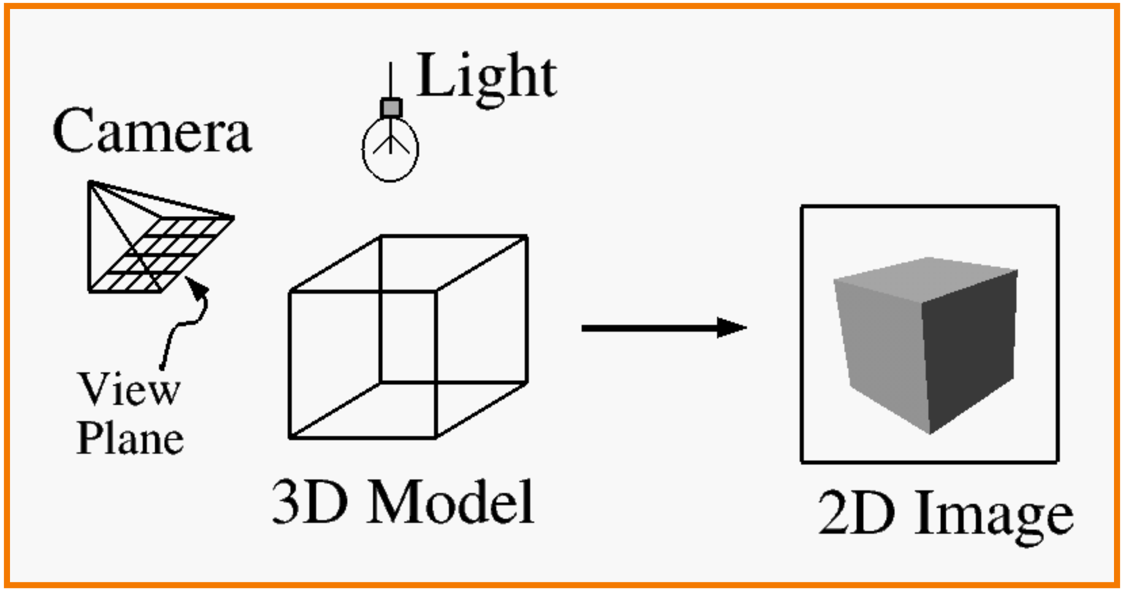
Modeling
(Dennis Zorin, CalTech)



Animation
(Angel, Plate 1)

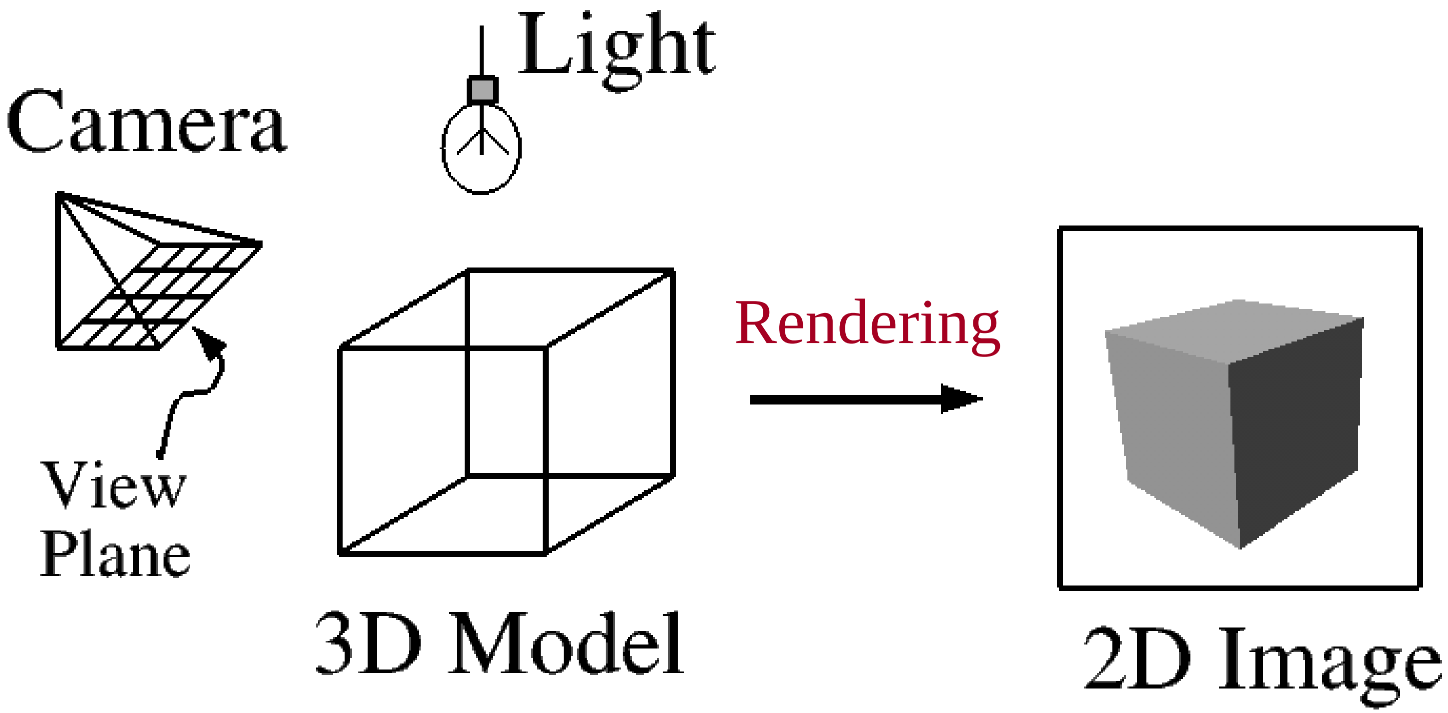
What is 3D Rendering?

- Topics in computer graphics
  - Imaging = *representing 2D images*
  - Modeling = *representing 3D objects*
  - Rendering = *constructing 2D images from 3D models*
  - Animation = *simulating changes over time*



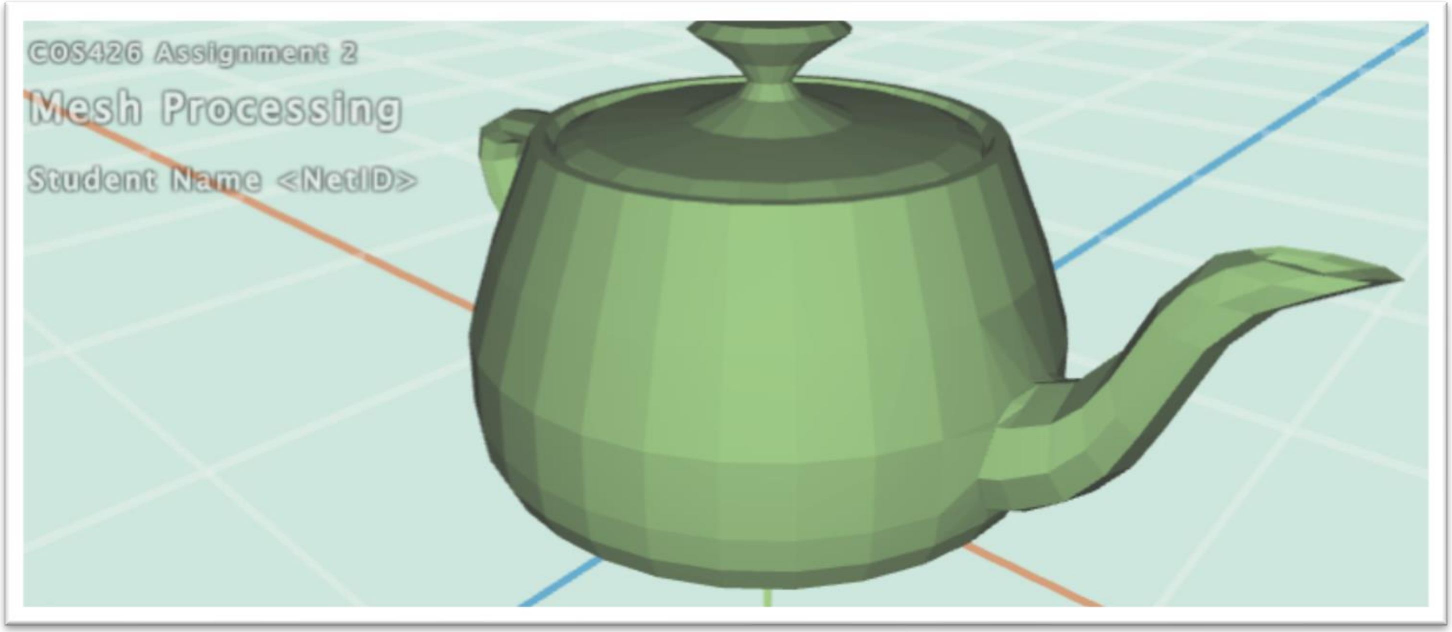
What is 3D Rendering?

- Construct image from 3D model



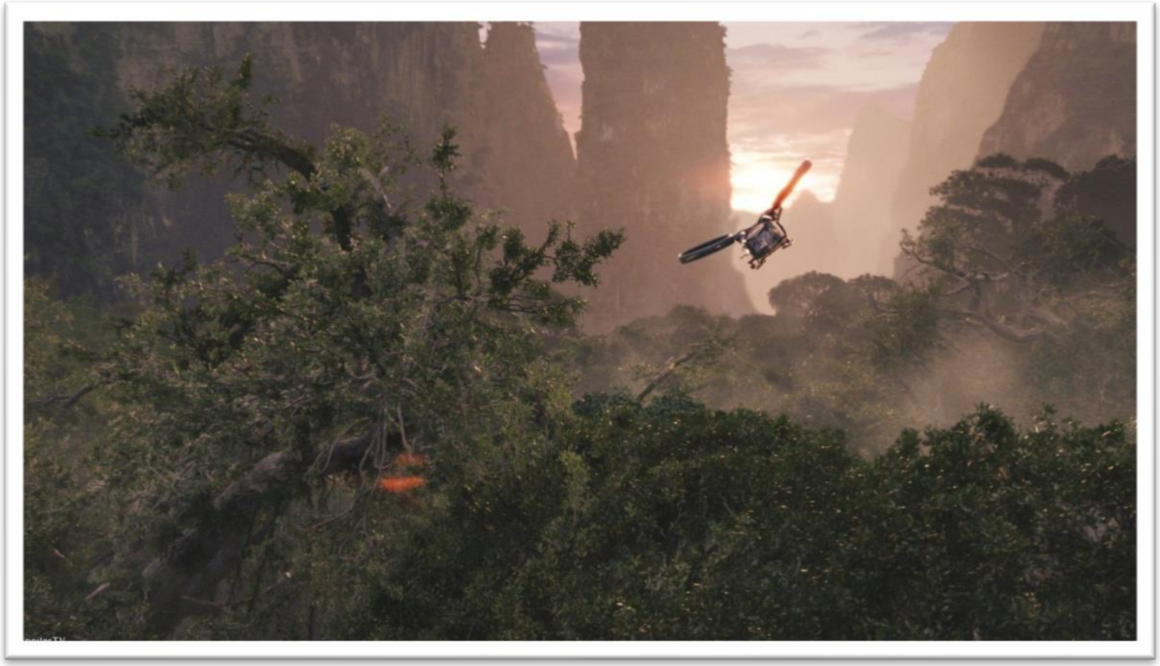
Interactive 3D Rendering

- Images generated in fraction of a second (e.g., 1/30) as user controls rendering parameters (e.g., camera)
  - » Achieve highest quality possible in given time
  - » Useful for visualization, games, etc.



Offline 3D Rendering

- One image generated with as much quality as possible for a particular set of rendering parameters
  - » Take as much time as is needed (minutes, hours…)
  - » Photorealism: movies, cut scenes, etc.



Avatar

3D Rendering Issues

- What issues must be addressed by a 3D rendering system?

Pixar

3D Rendering Issues

- What issues must be addressed by a 3D rendering system?
  - Camera
  - Visible surface determinaton
  - Lights
  - Reflectance
  - Shadows
  - Indirect illumination
  - Sampling
  - etc.

3D Rendering Issues

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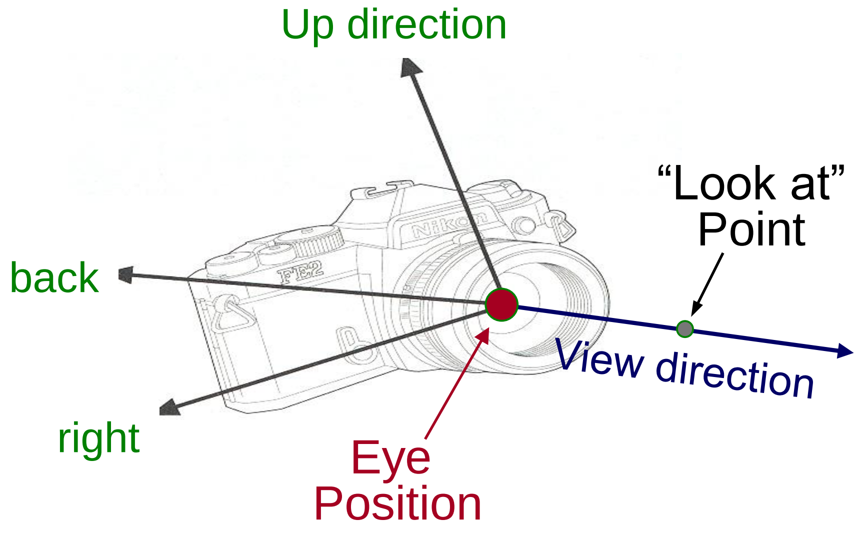
  - etc.

Pinhole Camera Parameters

- Position
  - Eye position (p_x, p_y, p_z)
- Orientation
  - View direction (d_x, d_y, d_z) or "look at" point
  - Up direction (u_x, u_y, u_z)
- Coverage
  - Field of view (fov_x, fov_y)
- Resolution
  - x and y



3D Rendering Issues

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  - Shadows
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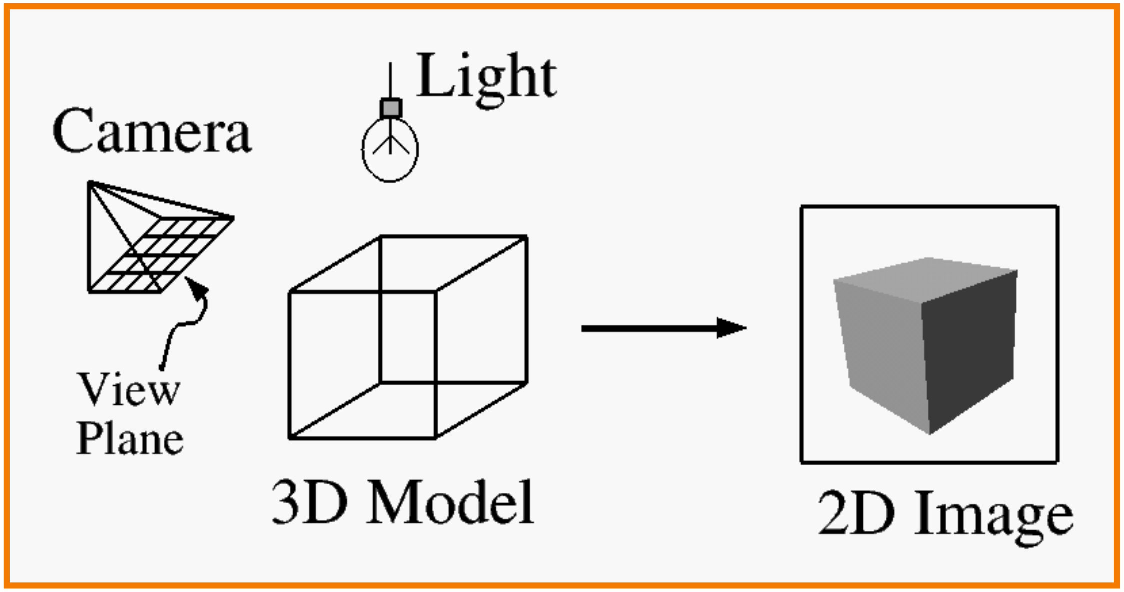
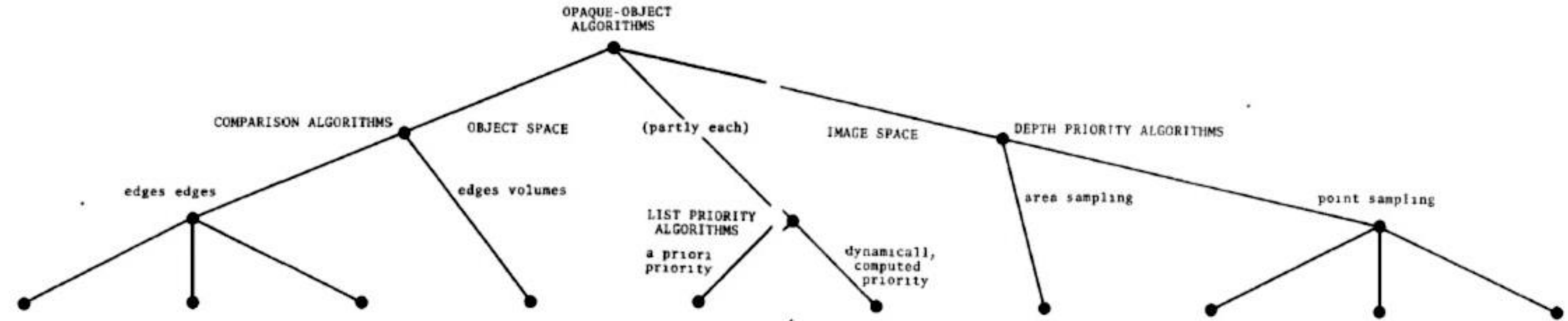
  - Sampling
  - etc.

Visible Surface Determination

- The color of each pixel on the view plane depends on the radiance ("amount of light") emanating from visible surfaces

How find visible surfaces?





RESTRICTIONS

COHERENCE

SORTING

(1) What, what property

(2) Method

(3) Type

(4) Result structure

(5) Number per frame, number of objects

(merge) Number of new entries per frame, length of list

(search) Number of searches, length of list

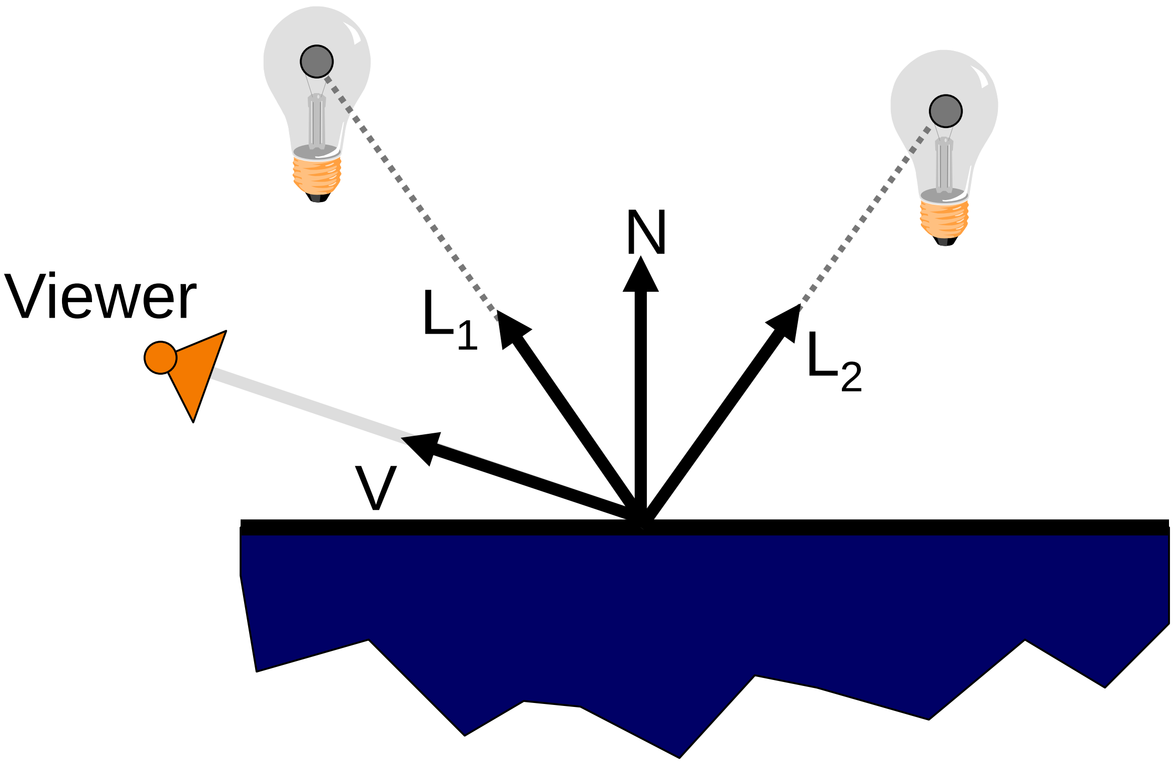
	APPEL 1967	GALIMBERTI, *et al* 1969	LOUTREL 1967	ROBERTS 1963	SCHUMACKER, *et al* 1969	NEWELL, *et al* 1972	WARNOCK 1968	WATKINS 1970	ROMNEY, *et al* 1967	BOUKNIGHT 1969
RESTRICTIONS	TP,NP	TP,NP	TP,NP	TP, CC, CF, NP	CF, NP, LS (TP)	None	(TR) None	None	TR,CF,NP	
COHERENCE	Promote visibility of a vertex to all edges at vertex	Promote visibility of a vertex to all edges at vertex	Promote visibility of a vertex to all edges at vertex		Frame coherence in depth No X coherence used	None used	Area coherence	Scanline X coherence	Scanline Depth Coherence	Scanline X Coherence
SORTING	<u>Back Edge Cull</u> 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	<u>Back Edge Cull</u> 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	<u>Back Edge Cull</u> 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	<u>Back Edge Cull</u> 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	<u>Intra-Cluster Priority</u> 1) Faces - visibility 2) Dot product with normals 3) Exhaustive search 4) Ordered table 5) 0, (off-line)	<u>Z Sort</u> 1) Faces, max Z 2) Comparison of max points 3) n log m 4) Ordered table 5) 1, F_r	<u>Z Sort (Opt.)</u> 1) Faces, max Z 2) Comparison of max points 3) n log m 4) Ordered table 5) 1, F_r	<u>Y Sort</u> 1) Edges, min Y 2) Comparison 3) Bucket 4) Table of Lists 5) 1, E_r	<u>Y Sort</u> 1) Polygons, Y endpoints 2) Comparison 3) 2 bucket 4) Table of lists 5) 1, F_r	<u>Y Sort</u> 1) Edges, Min Y 2) Comparison 3) Bucket 4) Table of lists 5) 1, E_r
	<u>Contour Edge Cull</u> 1) Edges separating front & back faces 2) Dot product with normals & topology 3) Cull 4) List, E_c 5) 1, E_t	(Omitted)	(Omitted)	<u>Clipping Cull</u> 1) Intersect edge with visible volume 2) 3) Cull 4) E_s 5) 1, E_s	<u>Inter-Cluster Priority</u> 1) Clusters 2) Dot product with separating planes 3) Prefix scan binary tree 4) ordered table 5) 1, C_t	<u>Newell Special</u> 1) Faces, pairwise visibility 2) Depth, bounding boxes, separation 3) Bubble, splitting 4) Ordered table 5) 1, F_r+split faces	<u>Warnock Special</u> 1) Faces with window 2) Depth, mini-max in X and Y, sum of angles 3) Radix 4 subdivision with overlap 4) Stacks of unordered tables 5) L_v, F_r/factor 1	<u>X Merge</u> 1) Edges, X value 2) Comparison 3) Merge (ordred) 4) 2-way linked list 5) E_r, S_ℓ	<u>X Sort</u> 1) Edges, X value 2) Comparison 3) 2 bucket 4) Table of lists 5) n, S_ℓ	<u>X Merge</u> 1) Edges, X value 2) Comparison 3) Merge (ordered) 4) Linked list 5) E_r, $2S_\ell$ (edges)
	<u>Initial Visibility</u> 1) Ray to vertex against all faces 2) Depth, Surroundedness 3) Exhaustive search 4) Quantitative visibility of vertex 5) # Objects, F_r	<u>Initial Visibility</u> 1) Ray to vertex against all faces 2) Depth, surroundedness 3) Exhaustive search 4) Quantitative visibility of vertex 5) #objects, F_r	<u>Initial Visibility</u> 1) Ray to vertex against all faces 2) Betweenness, surroundedness 3) Exhaustive search 4) Quantitative visibility of vertex 5) #objects, F_r	<u>Edge/Volume Test</u> 1) Edges, visibility relative to volumes 2) Linear Programming 3) Mini-max sort 4) Answer 5) E_s* split edges, #objects	<u>Back-Face Cull</u> 1) Faces 2) Dot product with face normal 3) Cull 4) Smaller ordered table 5) 1, F_t	<u>Y Sort</u> 1) Face segment by Y range 2) Y intercept 3) Bucket 4) None 5) F_r + split faces H_f^r	<u>Depth Search</u> 1) Surrounder faces 2) 4-corner compare 3) Exhaustive 4) Answer/failure 5) L_v, F_r/factor 2	<u>X Sort</u> 1) Segments, λ left 2) Comparison 3) Bubble 4) 2-way linked list 5) n, S_ℓ	<u>X Priority Search</u> 1) Edges, X value 2) Comparison 3) Priority search 4) Active segment list 5) n, m	<u>X Sort</u> 1) Edges, X value 2) Comparison 3) Bubble 4) 1-way linked list 5) N, $2S_\ell$ (edges)
	<u>Edge Intersection</u> 1) Intersect one E_s with all E_c 2) Penetration with sweep triangle 3) Cull (unordered) 4) Intersection list 5) E_s, E_c	<u>Edge Intersection</u> 1) Intersect one E_s with all E_c 2) Intersect in picture plane, depth 3) Cull (unordered) 4) Intersection list 5) E_s, $E_s - 1$	<u>Edge Intersection</u> 1) Intersect one E_s with all E_c 2) Intersect in picture plane, depth 3) Cull (unordered) 4) Intersection list 5) E_s, $E_s - 1$		<u>Y Cull</u> 1) Faces by Y extent 2) Mini-max on X intercepts 3) Cull (unordered) 4) X intercepts of relevant segments 5) n, E_s	<u>X Merge</u> 1) Segments, X intercept 2) Comparison 3) Ordered merge 4) Ordered list 5) S_r, $S_v/2$	<u>TV Sort (Opt.)</u> Sort windows into scan-line order if needed	<u>Span Cull</u> 1) Segments, overlap with sample span 2) Double comparison 3) Cull ordered list 4) Active list 5) $n{*}S_v * f\ (>1)$, S_ℓ	<u>Z Search</u> 1) Segments, depth 2) Linear equations and comparison 3) Search (unordered) 4) Visible segment 5) $n{*}2S_\ell$, D_c (Omitted if X priorities same as last time)	<u>Z Search</u> 1) Segments, depth 2) Linear equations and comparison 3) Search of unordered active list 4) Visible segment 5) $n{*}2S_\ell$, D_c
	<u>Sort Along Edge</u> 1) Intersections on edge, ordering 2) Comparison 3) Bubble 4) Answer 5) E_s, X_v/E_s (Omit if well hidden)	<u>Sort Along Edge</u> 1) Intersections on edge, ordering 2) 3) 4) Answer 5) E_s, X_v/E_s (must be done)	<u>Sort Along Edge</u> 1) Intersections on edge, ordering 2) 3) 4) Answer 5) E_s, X_v/E_s (Omit if well hidden)		<u>X Sort</u> 1) Segments 2) Counters 3) Hardware 4) Segments at this X 5) nm, S_ℓ			<u>Z Search</u> 1) Segments, Z 2) Depth by logarithmic search 3) Search (unordered) 4) Visible segment 5) $n{*}S_v{*}f(>1)$, D_c		
					<u>Priority Search</u> 1) Segments, priority 2) Logic network 3) Logic network 4) Visible segment 5) nm, S_ℓ					

Figure 29. Characterization of ten opaque-object algorithms b. Comparison of the algorithms.

3D Rendering Issues

- What issues must be addressed by a 3D rendering system?
  - Camera
  - Visible surface determination
  - Lights
  - Reflectance
  - Shadows
  - Indirect illumination
  - Sampling
  - etc.

Lighting Simulation



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Shadows

- Occlusions from light sources
  - Soft shadows with area light source

Moller

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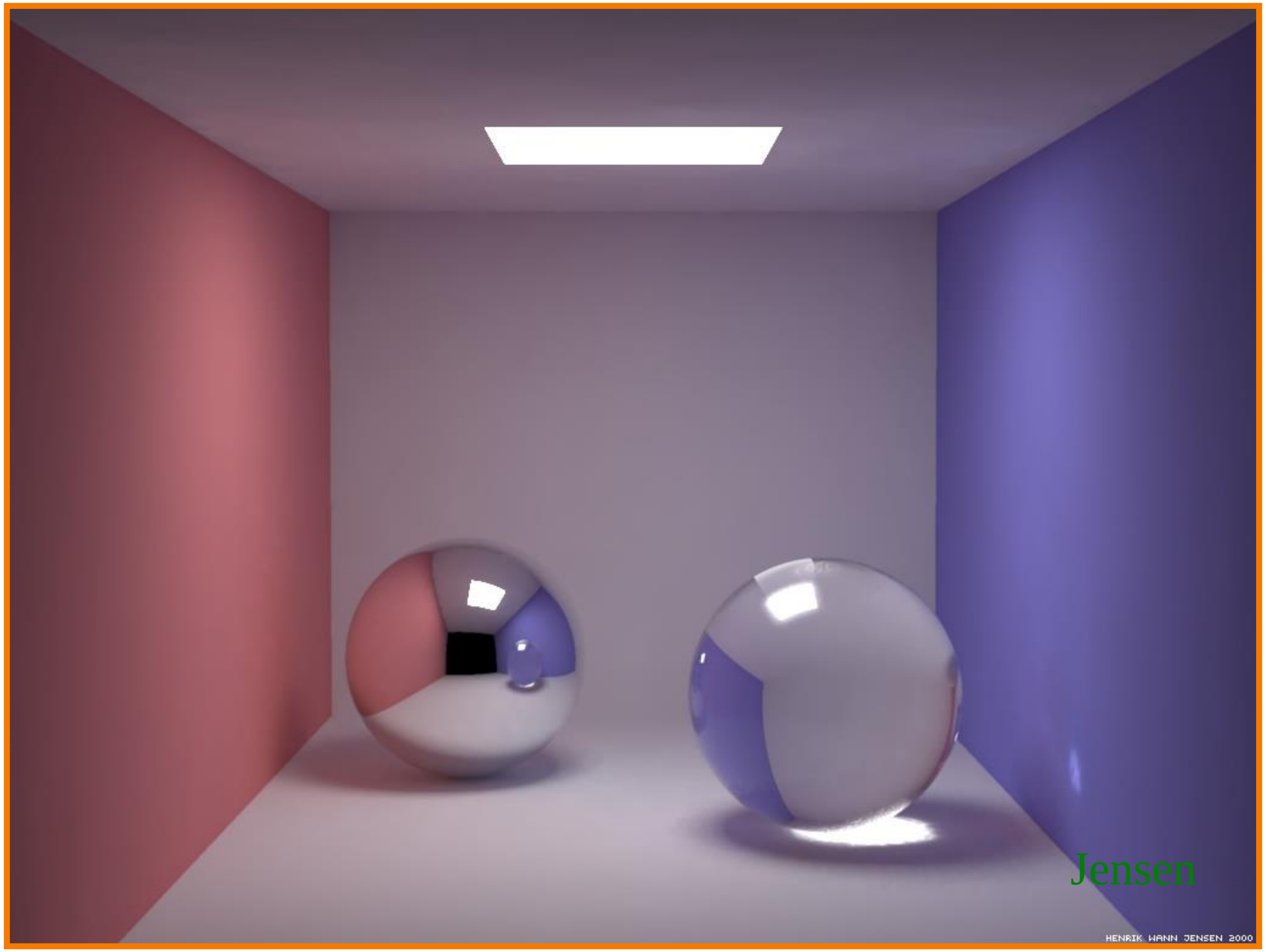
Indirect Illumination



Henrik Wann Jensen

3D Rendering Issues

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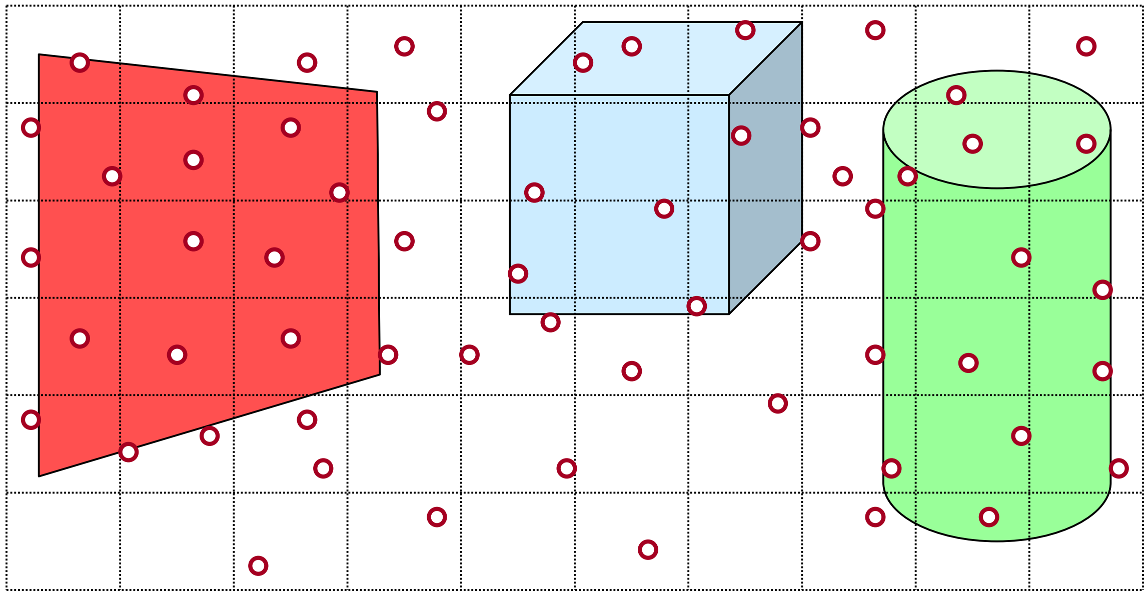
  - etc.

Sampling

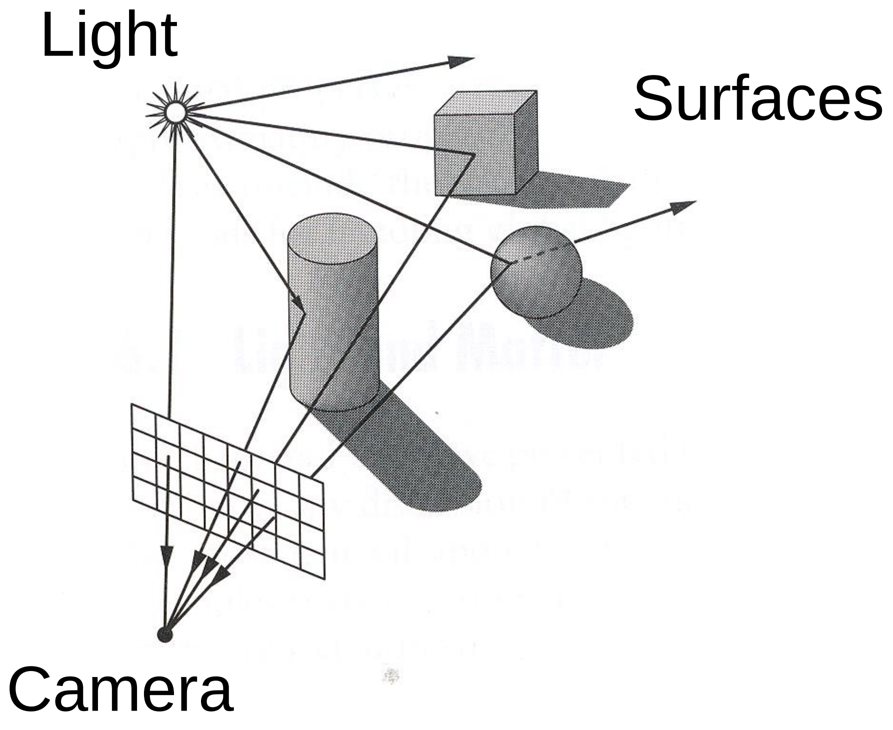
- Scene can be sampled with any ray
  - Rendering is a problem in sampling and reconstruction



Rendering Method I:
Ray Casting

Ray Casting

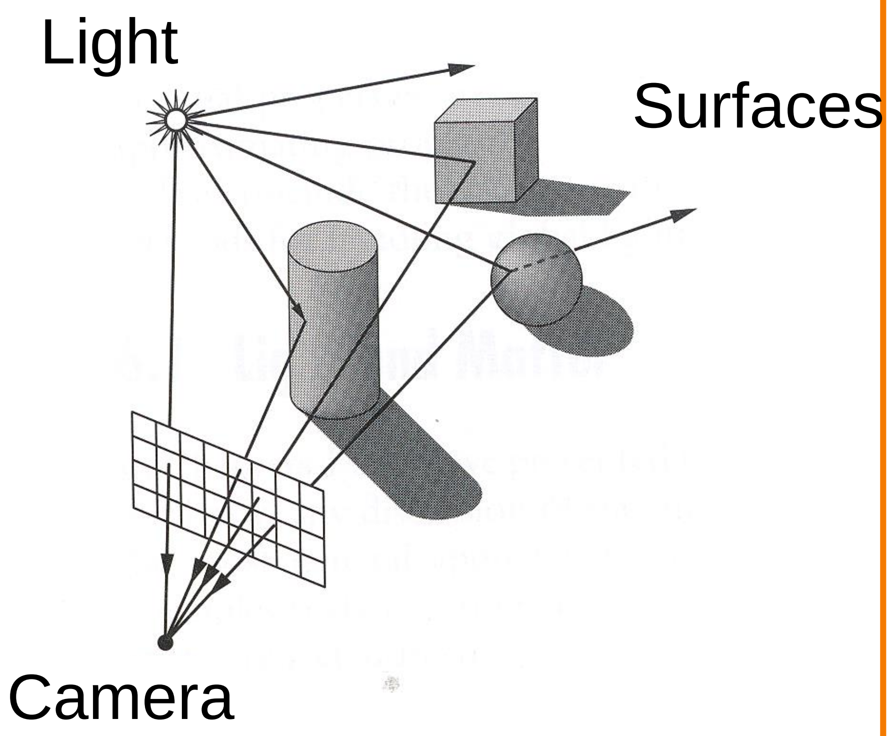
- The color of each pixel on the view plane depends on the radiance emanating along rays from visible surfaces in scene



Scene

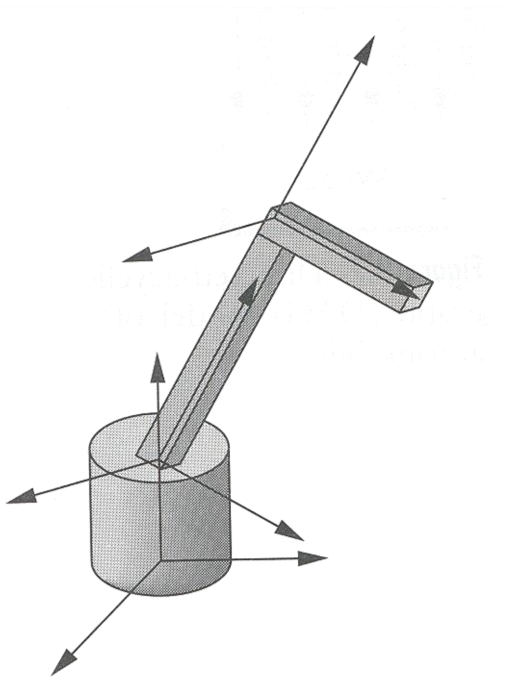
- Scene has:
    - Scene graph with surface primitives
    - Set of lights
    - Camera

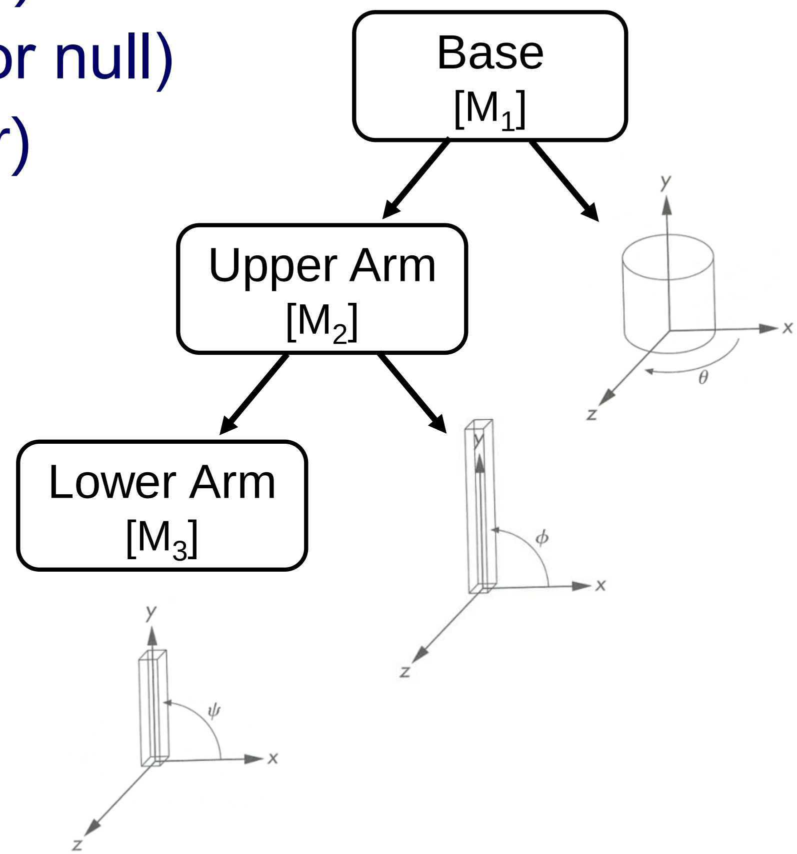
```
struct R3Scene {
      R3Node *root;
      vector<R3Light *> lights;
      R3Camera camera;
      R3Box bbox;
      R3Rgb background;
      R3Rgb ambient;
};
```



Scene Graph

- Scene graph is hierarchy of nodes, each with:
  - Bounding box (in node's coordinate system)
  - Transformation (4x4 matrix)
  - Shape (mesh, sphere, … or null)
  - Material (more on this later)





Scene Graph

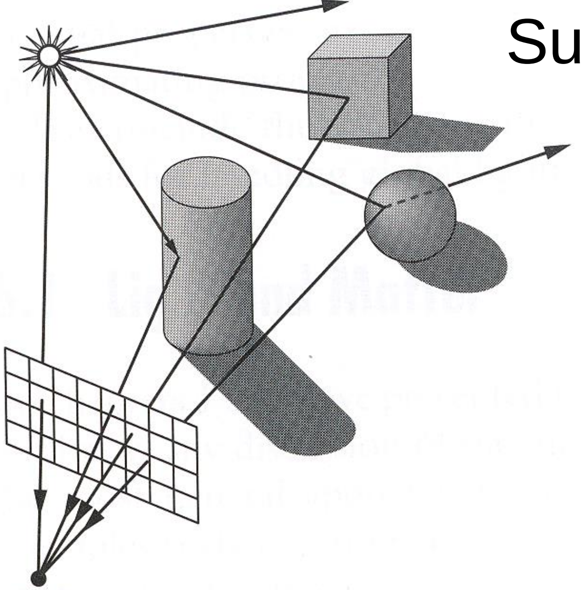
- Simple scene graph implementation:

```
struct R3Node {
    struct R3Node *parent;
    vector<struct R3Node *> children;
    R3Shape *shape;
    R3Matrix transformation;
    R3Material *material;
    R3Box bbox;
};
```

```
struct R3Shape {
    R3ShapeType type;
    R3Box *box;
    R3Sphere *sphere;
    R3Cylinder *cylinder;
    R3Cone *cone;
    R3Mesh *mesh;
};
```

Ray Casting

- For each sample (pixel) …
  - Construct ray from eye position through view plane
  - Compute radiance leaving first point of intersection between ray and scene



Ray Casting

- Simple implementation:

```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene,  &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

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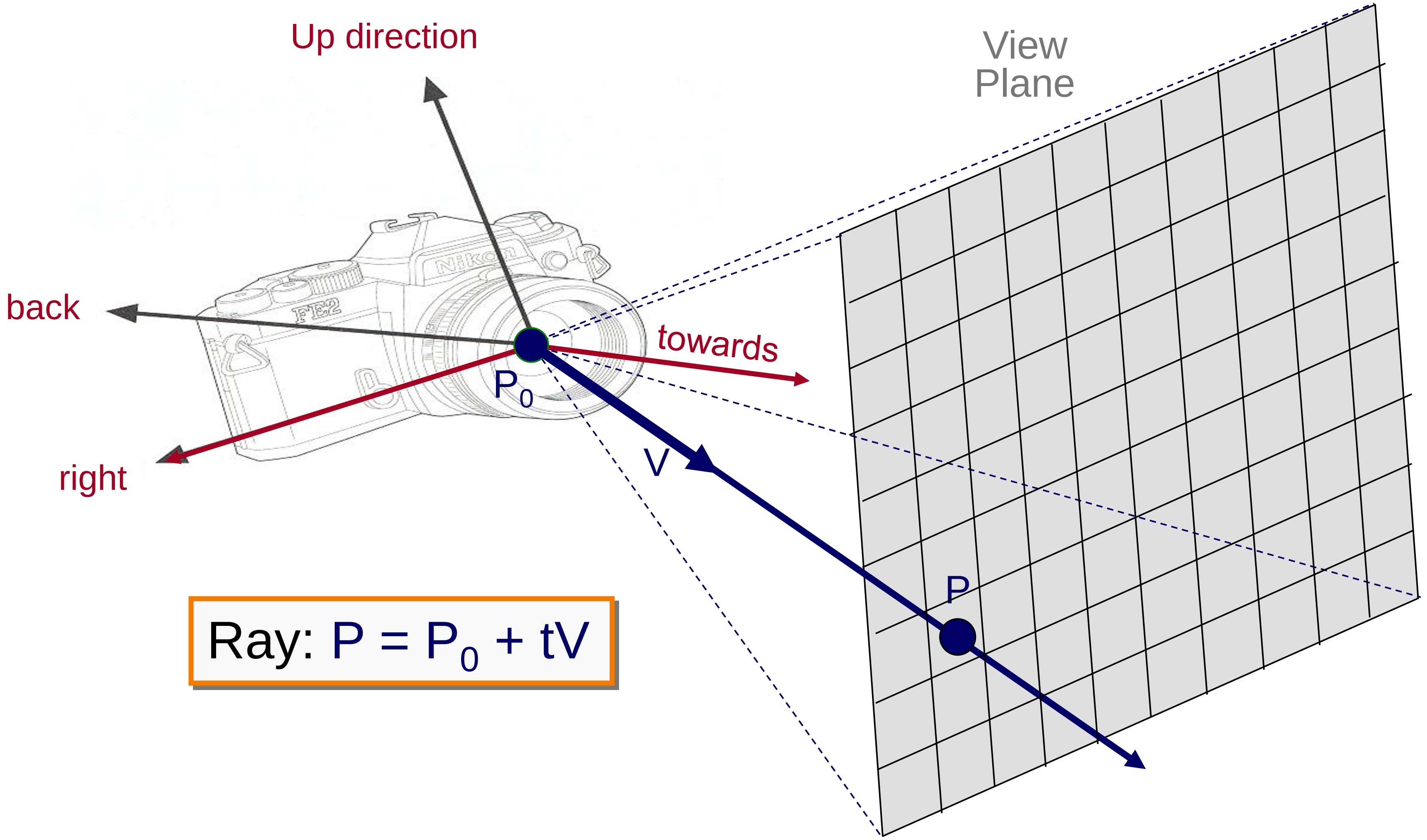
Constructing Ray Through a Pixel



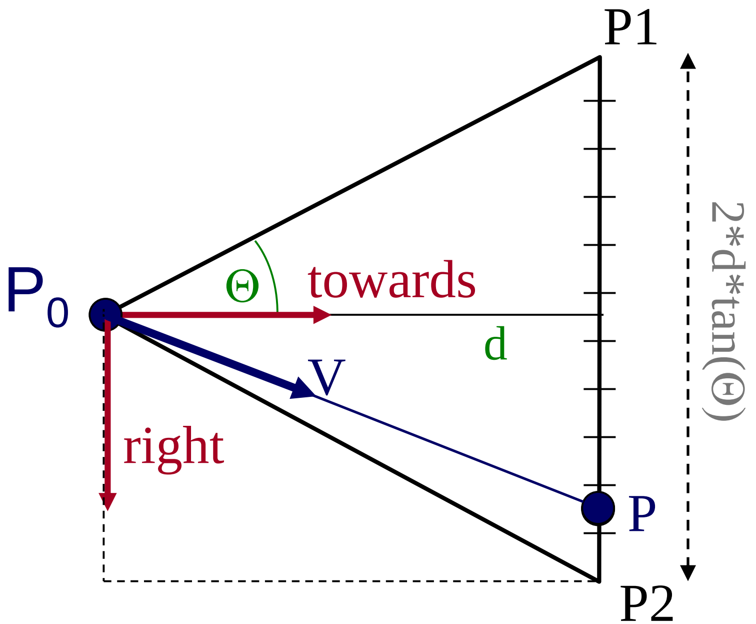
Ray: $P = P_0 + tV$

Constructing Ray Through a Pixel

- 2D Example

Θ = frustum **half**-angle
d = distance to view plane

right = towards × up



$P1 = P_0 + d*towards - d*\tan(\Theta)*right$
$P2 = P_0 + d*towards + d*\tan(\Theta)*right$

$P = P1 + ((i + 0.5) / width) * (P2 - P1)$
$V = (P - P_0) / \|P - P_0\|$
(d cancels out…)

Ray: $P = P_0 + tV$

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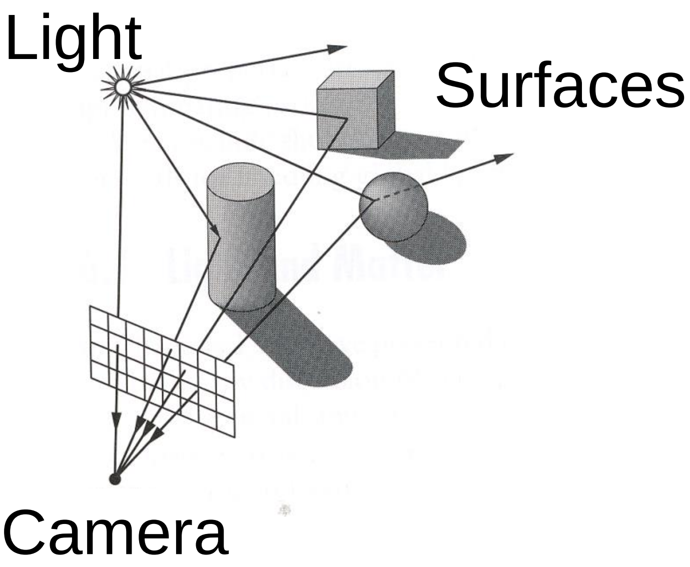
        }
    }
    return image;
}
```

Ray Casting

- Simple implementation:

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);
    return ComputeRadiance(scene, ray, intersection);
}
```

```
struct R3Intersection {
    bool hit;
    R3Node *node;
    R3Point position;
    R3Vector normal;
    double t;
};
```



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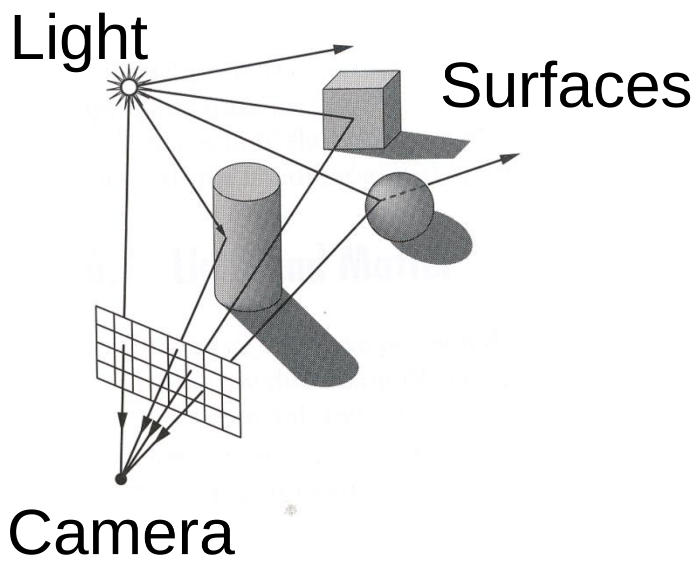

Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene
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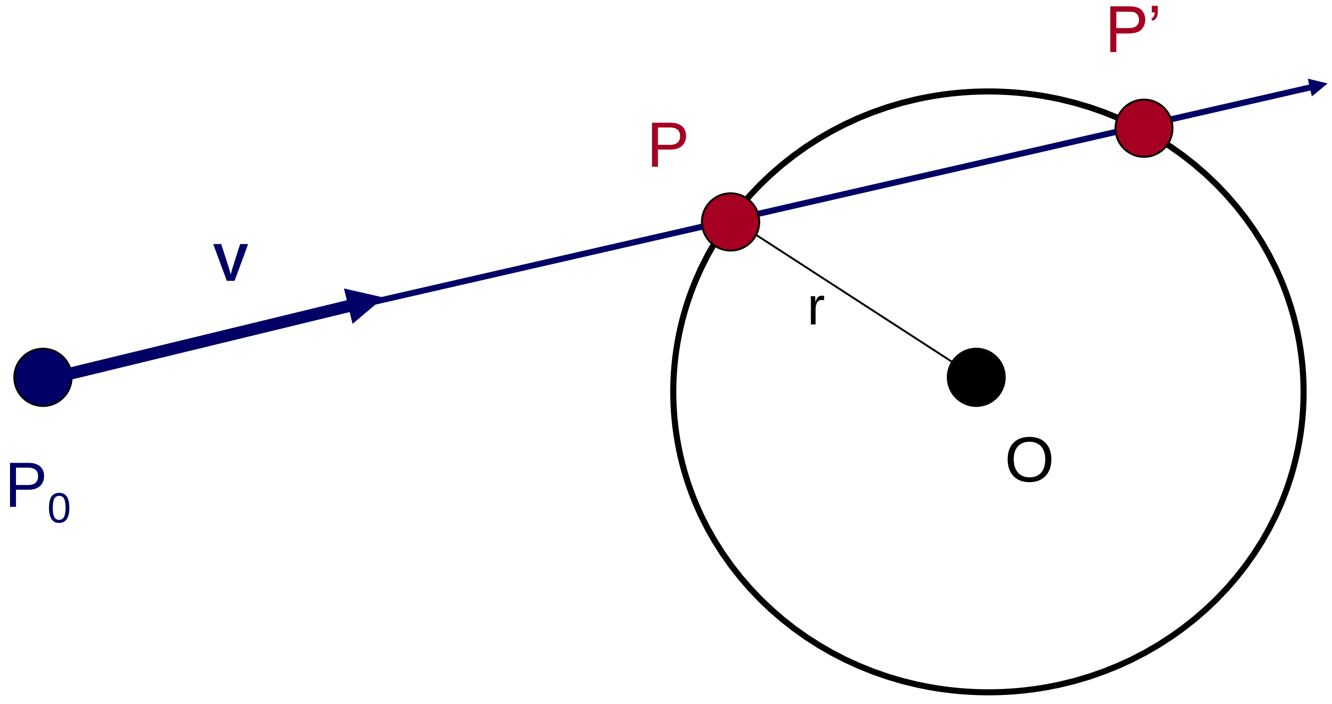
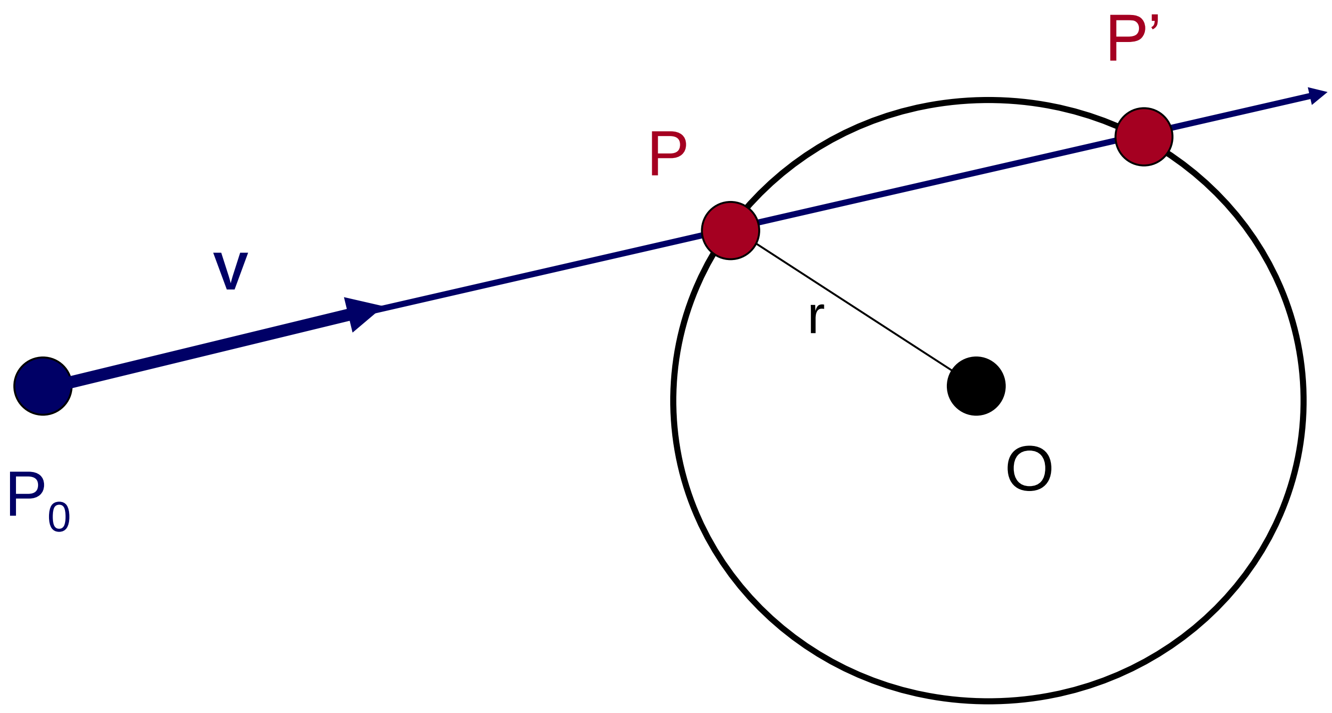
Ray-Sphere Intersection



Ray-Sphere Intersection

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$



Ray-Sphere Intersection I

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0} + \mathbf{tV} - O|^2 - r^2 = 0$$

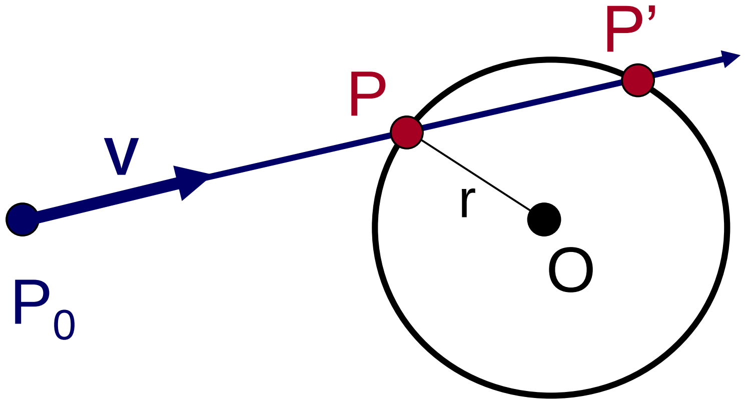
Solve quadratic equation:

$$at^2 + bt + c = 0$$

where:

$$a = V^2$$
$$b = 2\, V \bullet (P_0 - O)$$
$$c = |P_0 - C|^2 - r^2 = 0$$

$P = P_0 + tV$



Ray-Sphere Intersection II

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

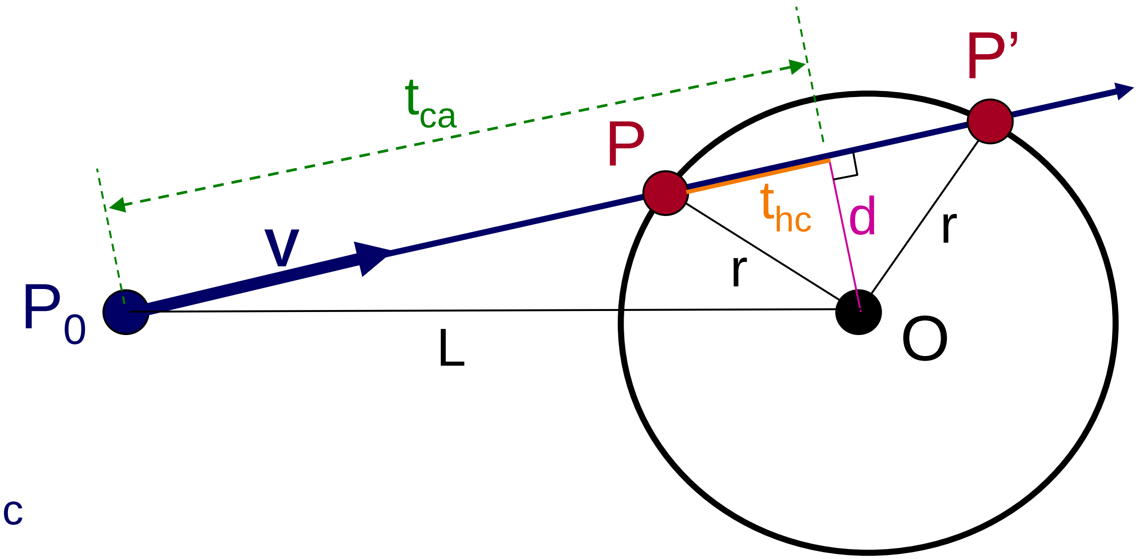
$L = O - P_0$

$t_{ca} = L \cdot V$
if ($t_{ca} < 0$) return INF

$d^2 = L \cdot L - t_{ca}^2$
if ($d^2 > r^2$) return INF

$t_{hc} = \mathrm{sqrt}(r^2 - d^2)$
$t = t_{ca} - t_{hc}$ and $t_{ca} + t_{hc}$

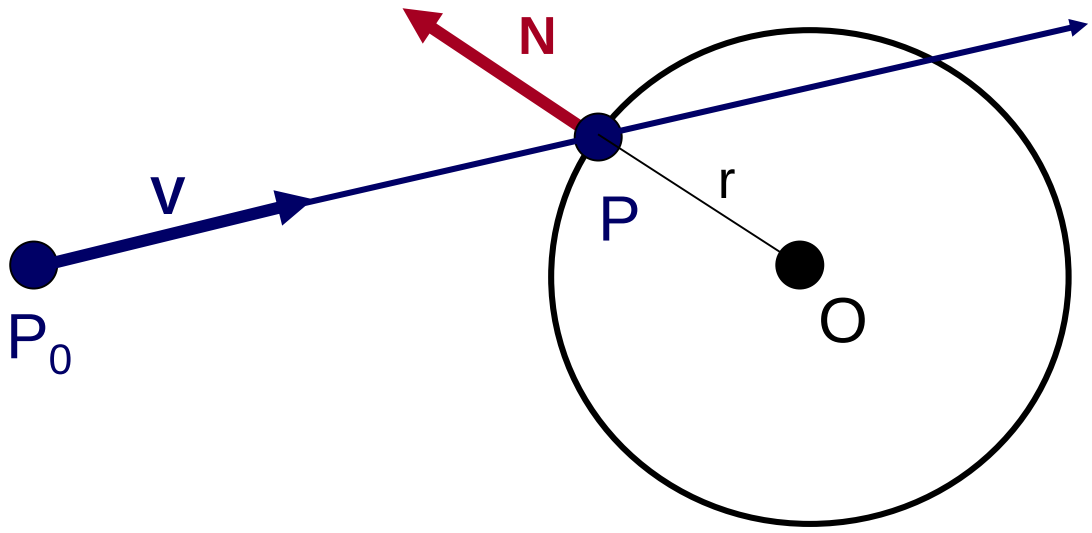
$P = P_0 + tV$



Ray-Sphere Intersection

- Need normal vector at intersection for lighting calculations (next lecture)

$$N = (P - O) / ||P - O||$$

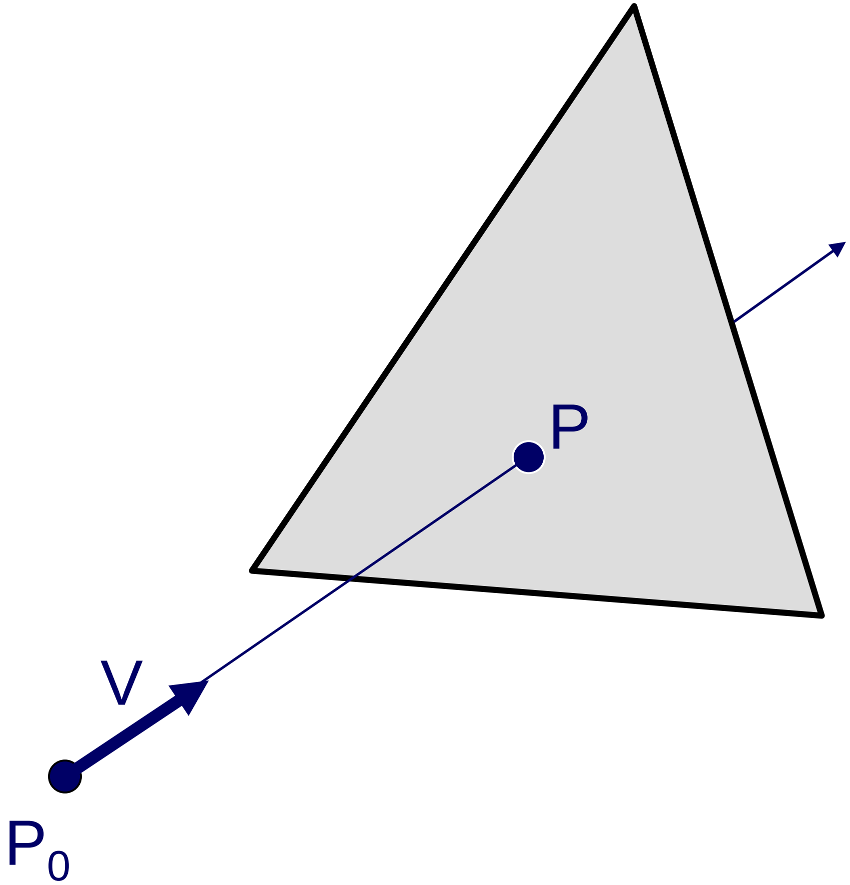


Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

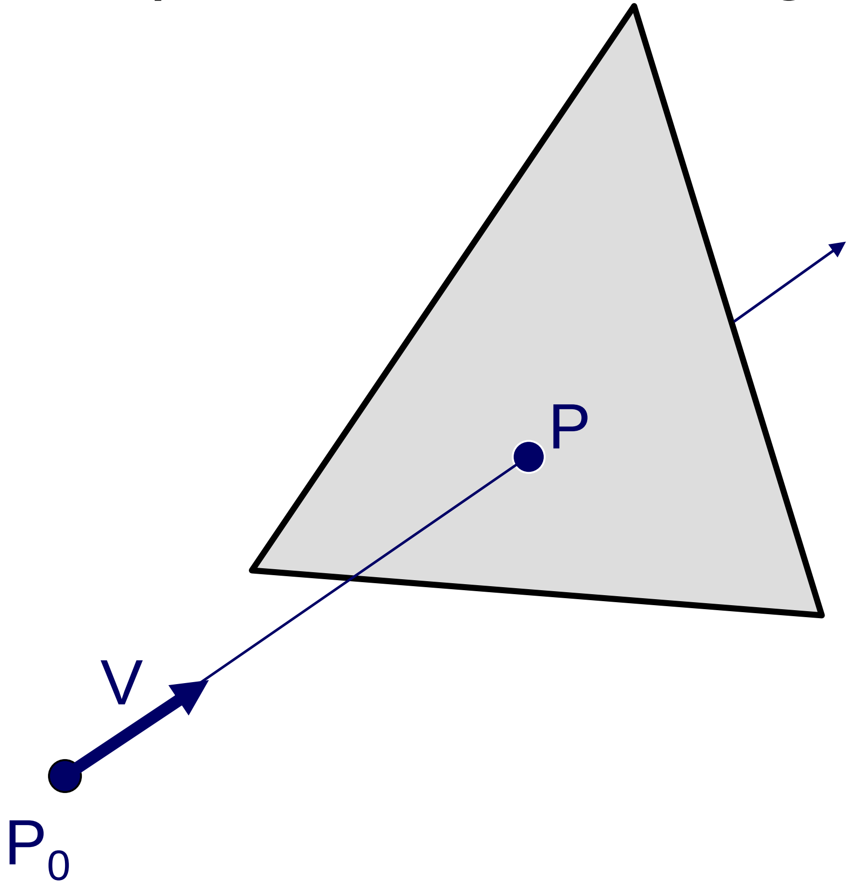
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

Ray-Triangle Intersection



Ray-Triangle Intersection

- First, intersect ray with plane
- Then, check if intersection point is inside triangle



Ray-Plane Intersection

Ray: $P = P_0 + tV$
Plane: $P \cdot N + d = 0$

Algebraic Method

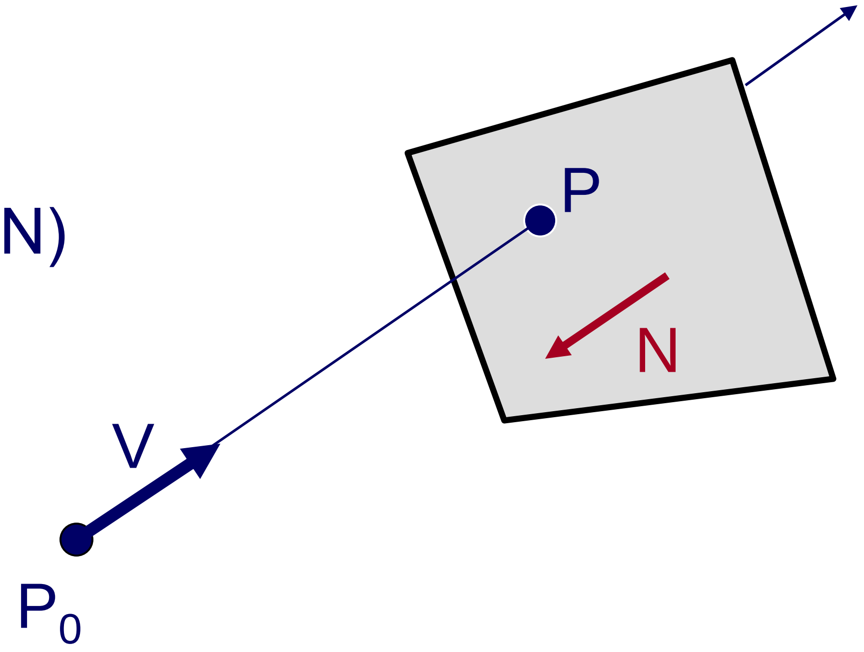
Substituting for P, we get:

$$(\mathbf{P_0 + tV}) \cdot N + d = 0$$

Solution:

$$t = -(P_0 \cdot N + d) / (V \cdot N)$$

$$P = P_0 + tV$$



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

For each side of triangle

$V_1 = T_1 - P_0$

$V_2 = T_2 - P_0$

$N_1 = V_2 \times V_1$

Normalize N_1

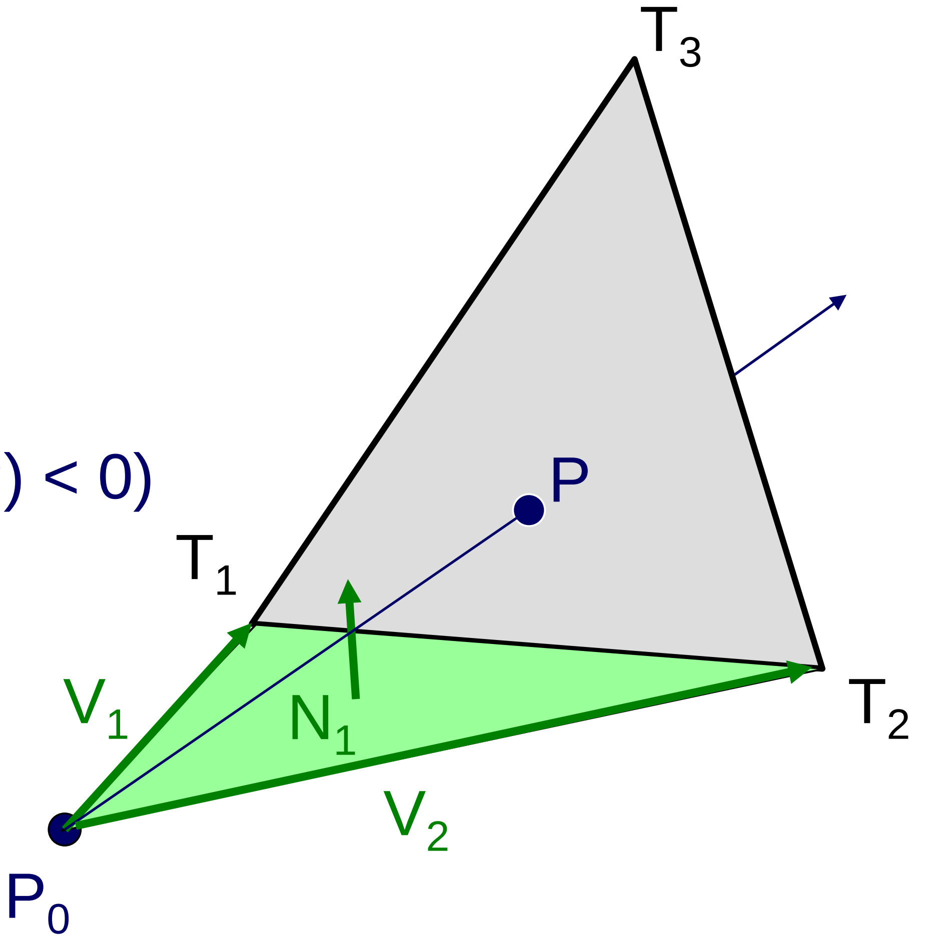
Plane $p(P_0, N_1)$

if (SignedDistance(p, P) < 0)

return FALSE

end

return TRUE



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

For each side of triangle

$V_1 = T_1 – P_0$

$V_2 = T_2 – P_0$

$N_1 = V_2 \times V_1$

Normalize N_1

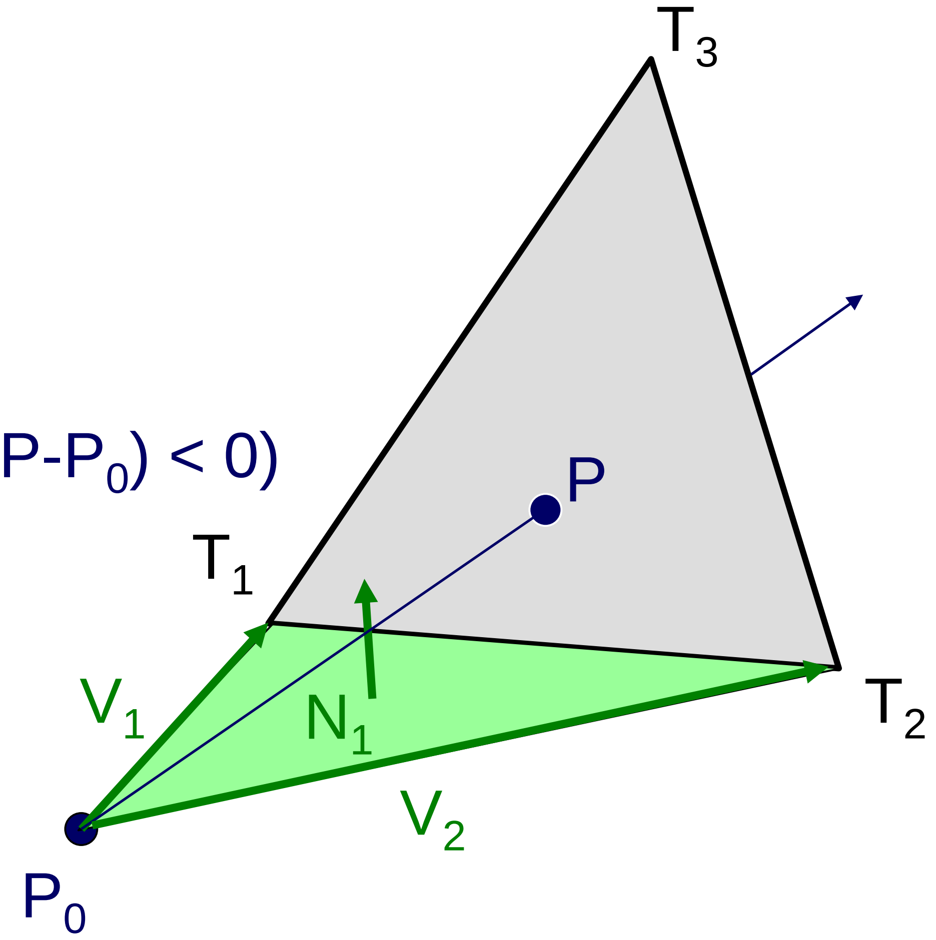
Plane $p(P_0, N_1)$

if (SignedDistance(p, P-P_0) < 0)

return FALSE

end

return TRUE



Ray-Triangle Intersection II

- Check if point is inside triangle algebraically

For each side of triangle

$V_1 = T_1 - P$

$V_2 = T_2 - P$

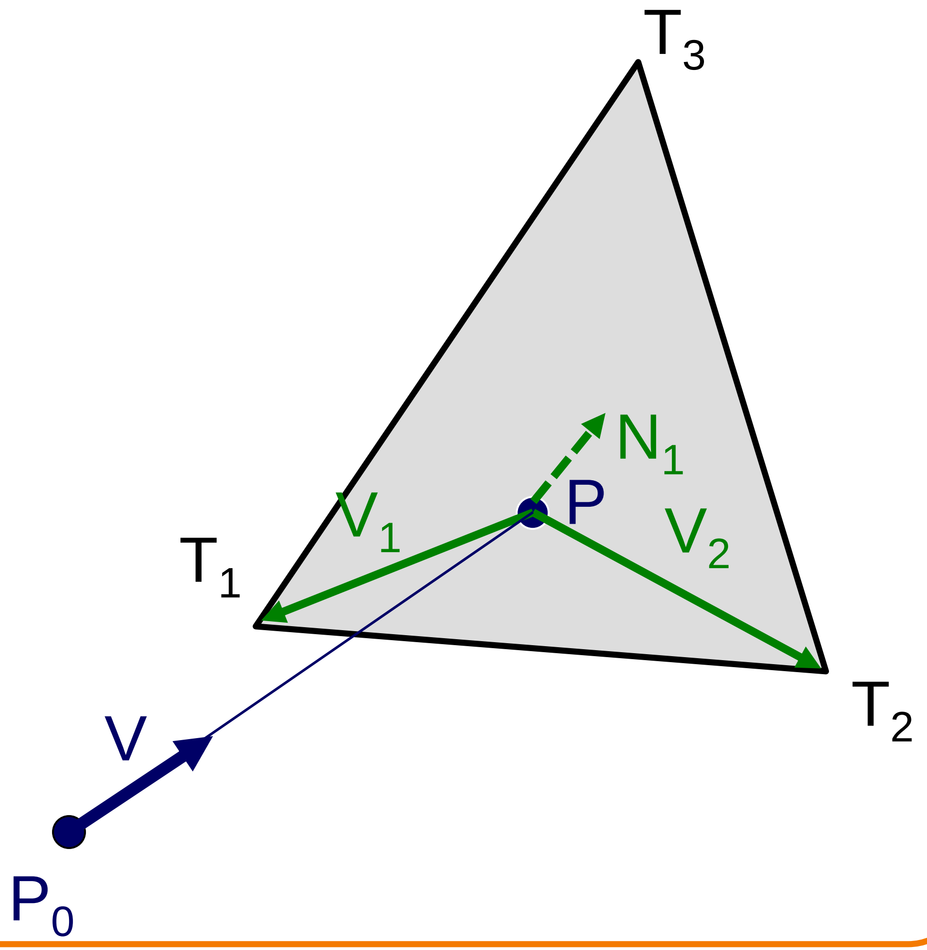
$N_1 = V_2 \times V_1$

if ($V \cdot N_1$ < 0)

return FALSE

end

return TRUE



Ray-Triangle Intersection II

- Check if point is inside triangle algebraically

For each side of triangle

$V_1 = T_1 - P$

$V_2 = T_2 - P$

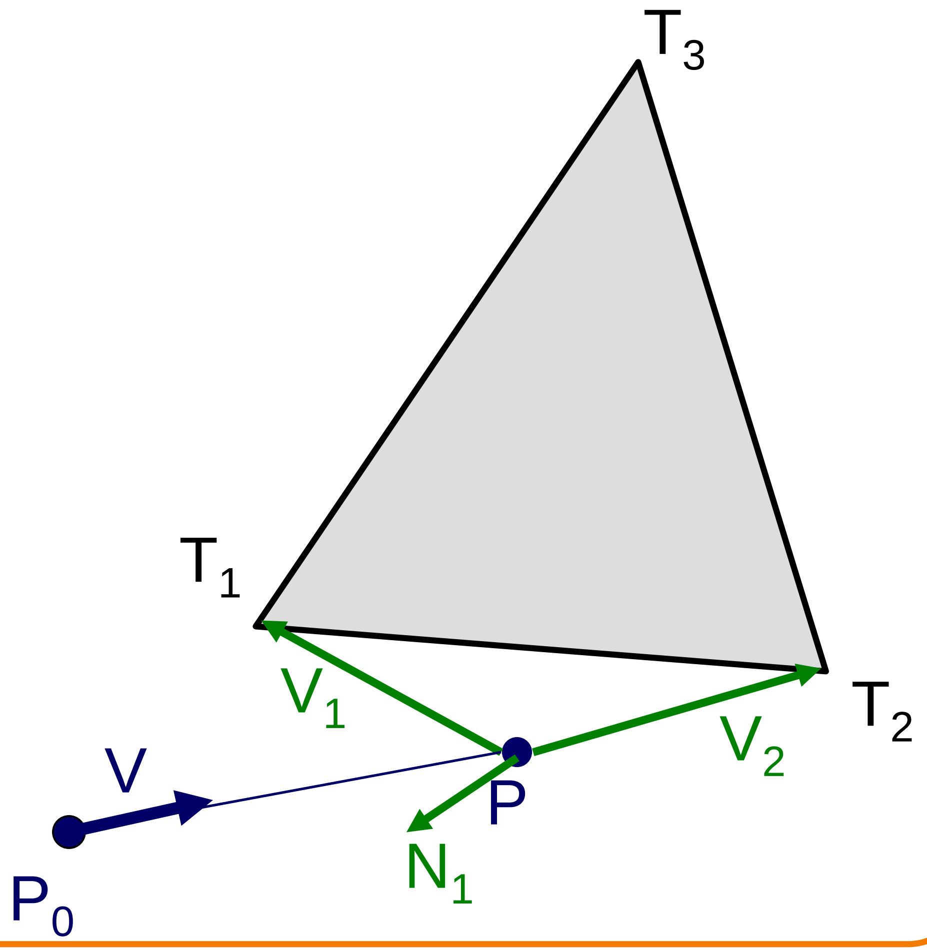
$N_1 = V_2 \times V_1$

if $(V \bullet N_1 < 0)$

return FALSE

end

return TRUE



Ray-Triangle Intersection III

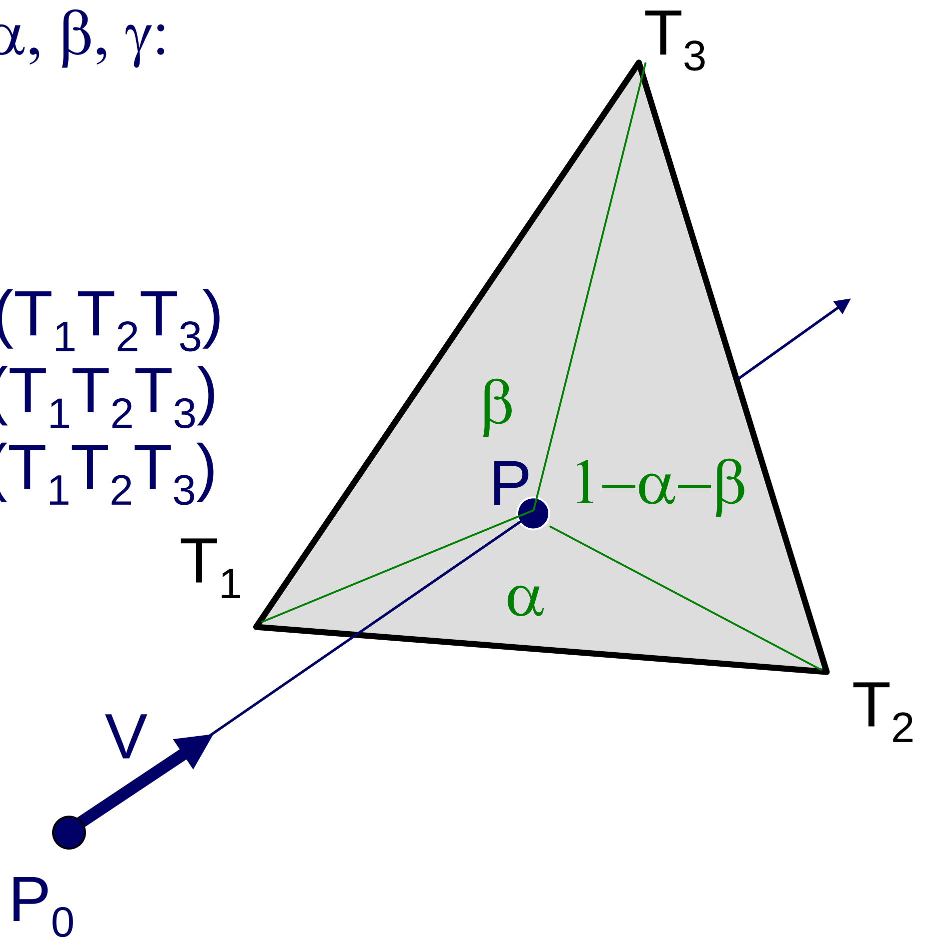
- Check if point is inside triangle parametrically

"Barycentric coordinates" α, β, γ:

$$P = \alpha T_3 + \beta T_2 + \gamma T_1$$

where $\alpha + \beta + \gamma = 1$

$$\alpha = \text{Area}(T_1 T_2 P) / \text{Area}(T_1 T_2 T_3)$$
$$\beta = \text{Area}(T_1 P T_3) / \text{Area}(T_1 T_2 T_3)$$
$$\gamma = \text{Area}(P T_2 T_3) / \text{Area}(T_1 T_2 T_3)$$
$$= 1 - \alpha - \beta$$



Ray-Triangle Intersection III

- Check if point is inside triangle parametrically

Compute "barycentric coordinates" α, β:

α = Area(T_1T_2P) / Area($T_1T_2T_3$)

β = Area(T_1PT_3) / Area($T_1T_2T_3$)

Area($T_1T_2T_3$) = ½ || (T2-T1) x (T3-T1) ||

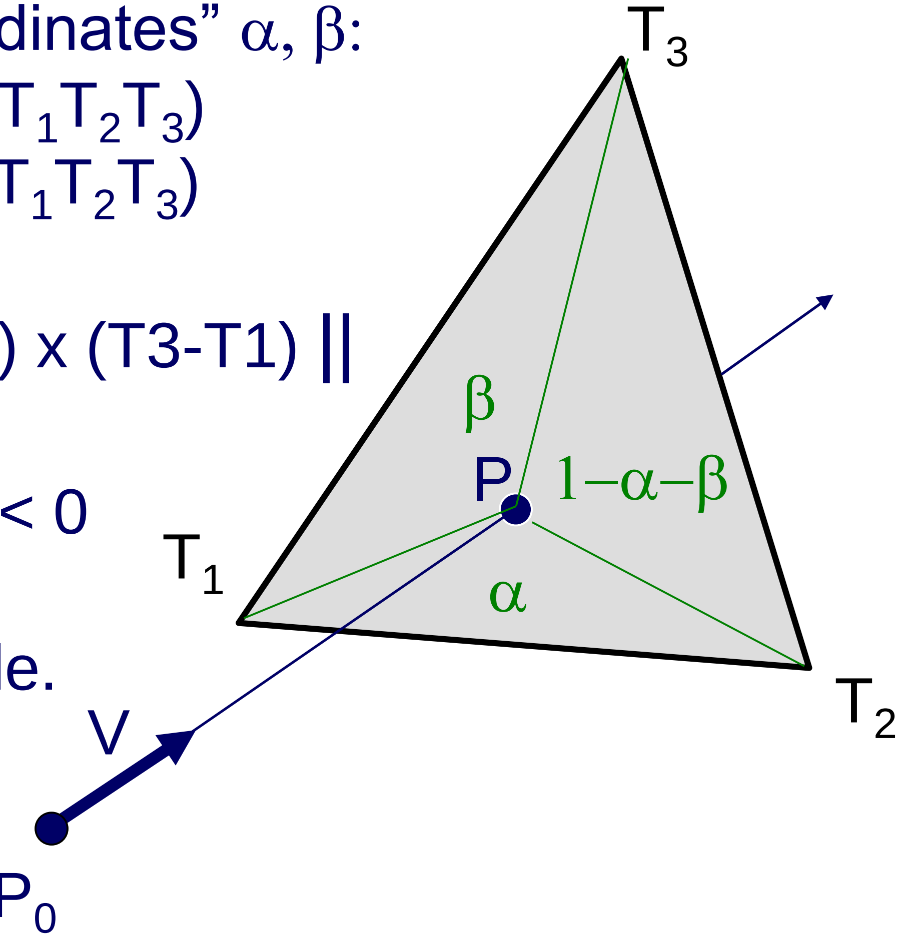
check if backfacing:

$((T2-T1) \times (T3-T1)) \cdot N < 0$

Check if point inside triangle.

$0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$

and $\alpha + \beta \leq 1$

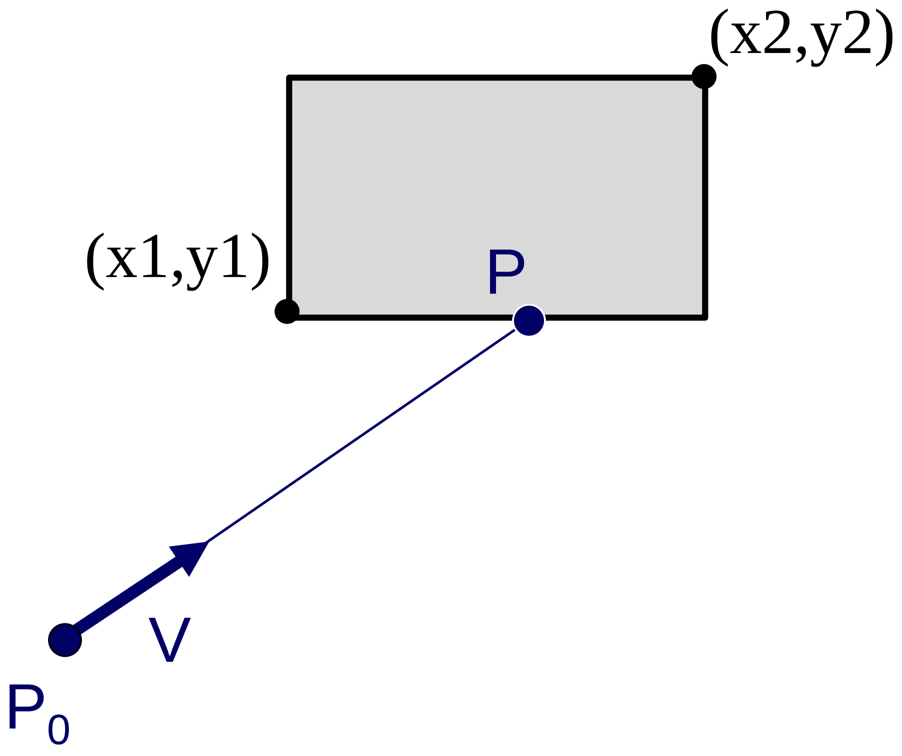


Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

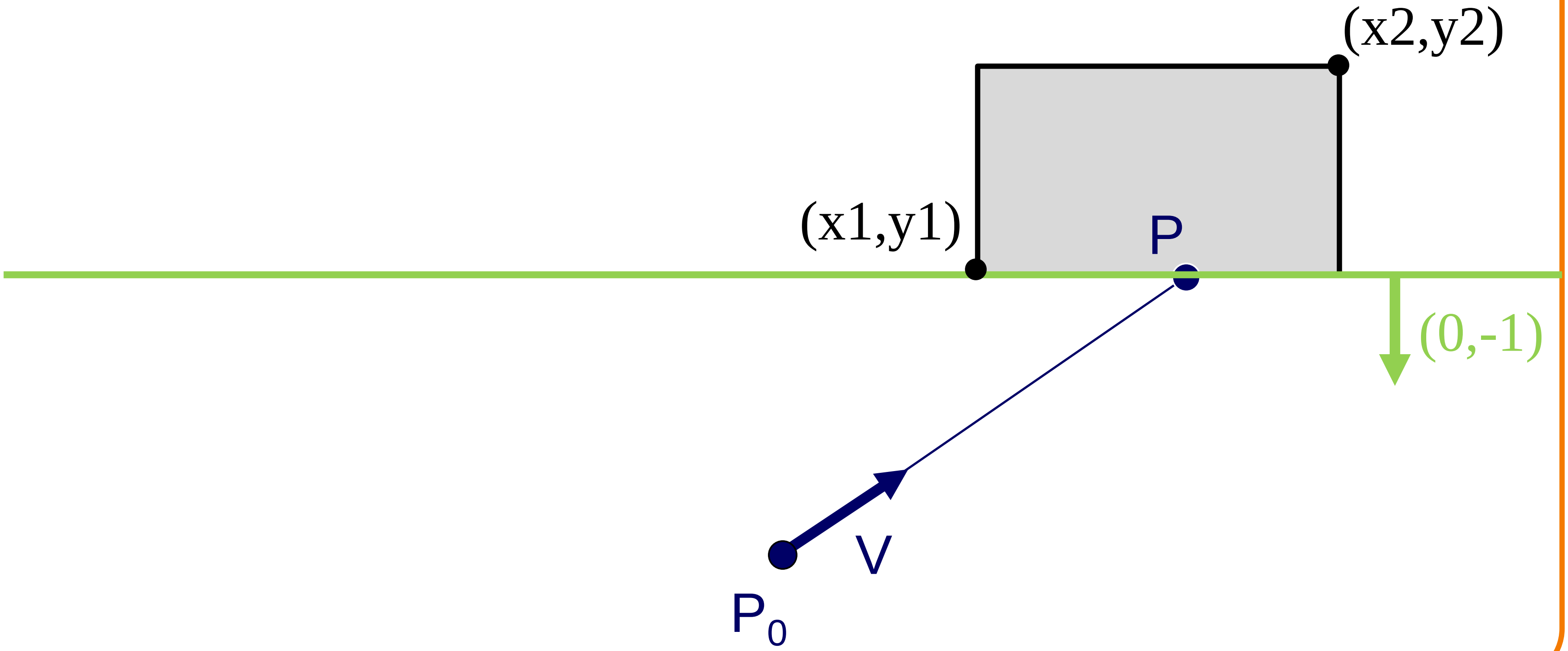
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least t)



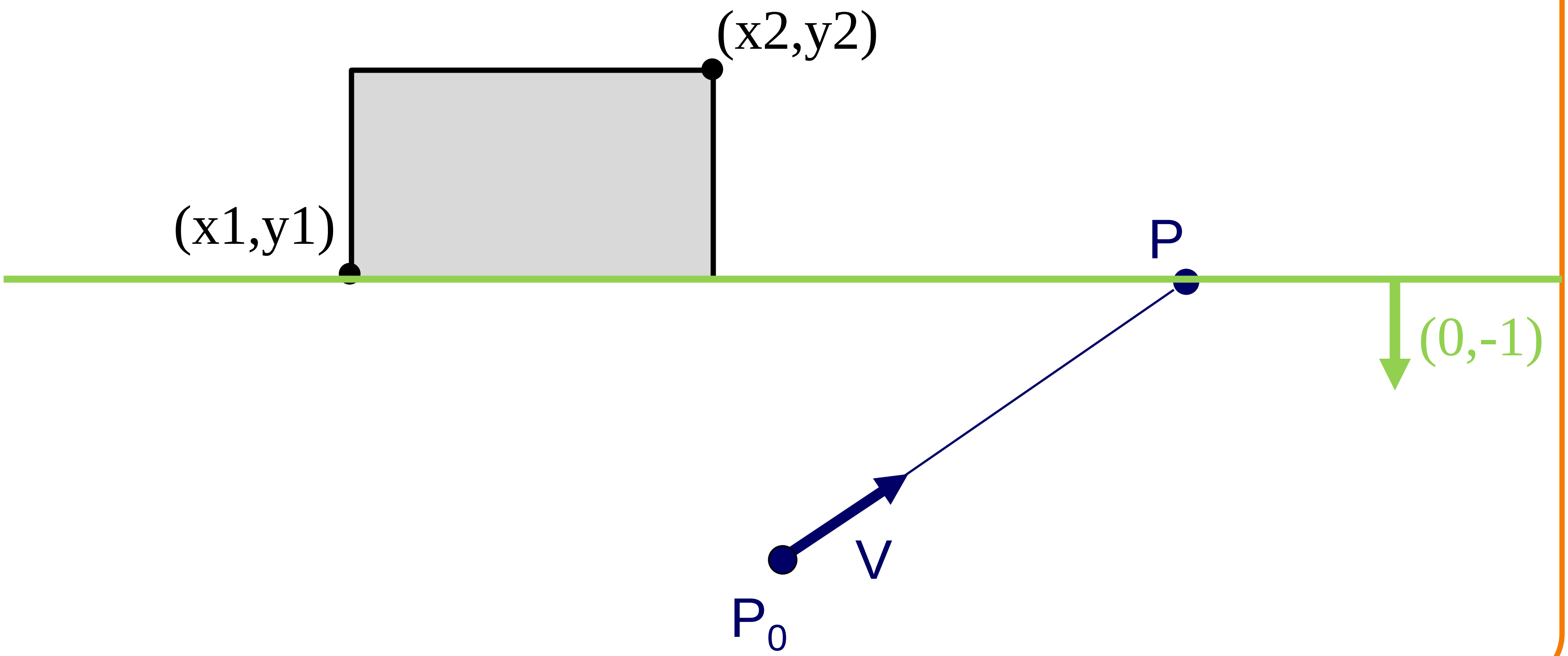
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least t)
  - Find intersection with plane
  - Check if point is inside rectangle



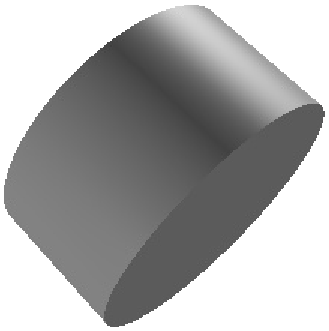
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least t)
  - Find intersection with plane
  - Check if point is inside rectangle



Other Ray-Primitive Intersections

- Cone, cylinder:
  - Similar to sphere
  - Must also check end caps



- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
  - Or, decompose into triangles, and check all of them
- Mesh
  - Compute intersection for all polygons
  - Return closest intersection (least t)

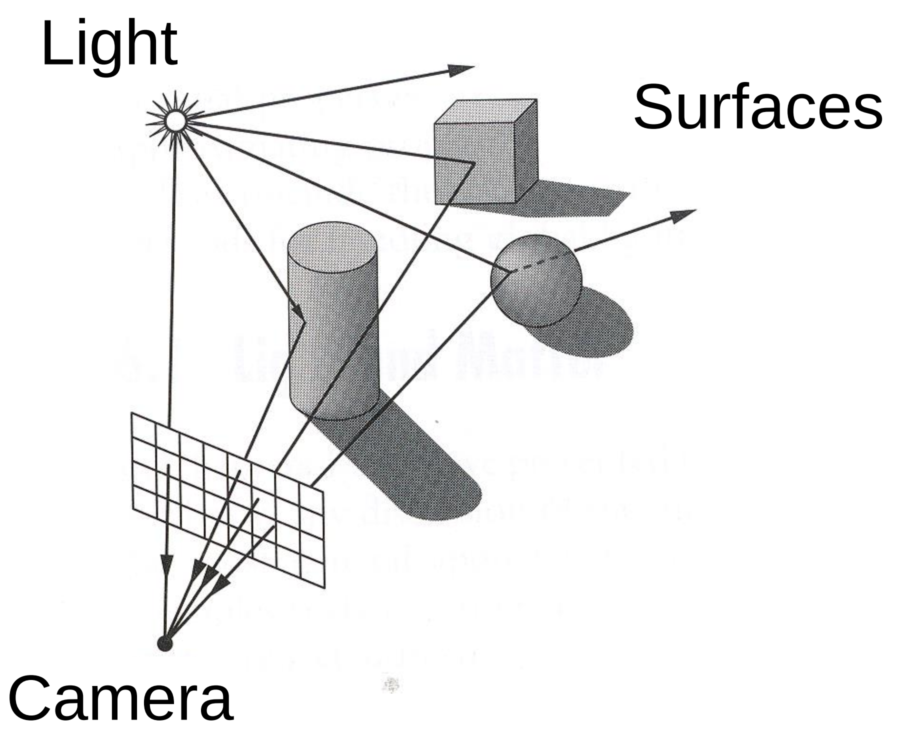
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

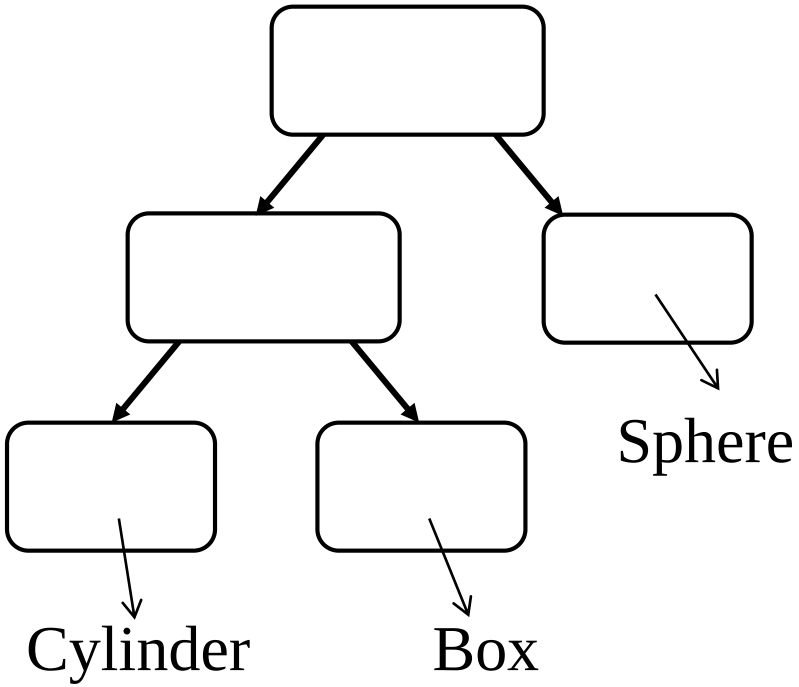
Ray-Scene Intersection

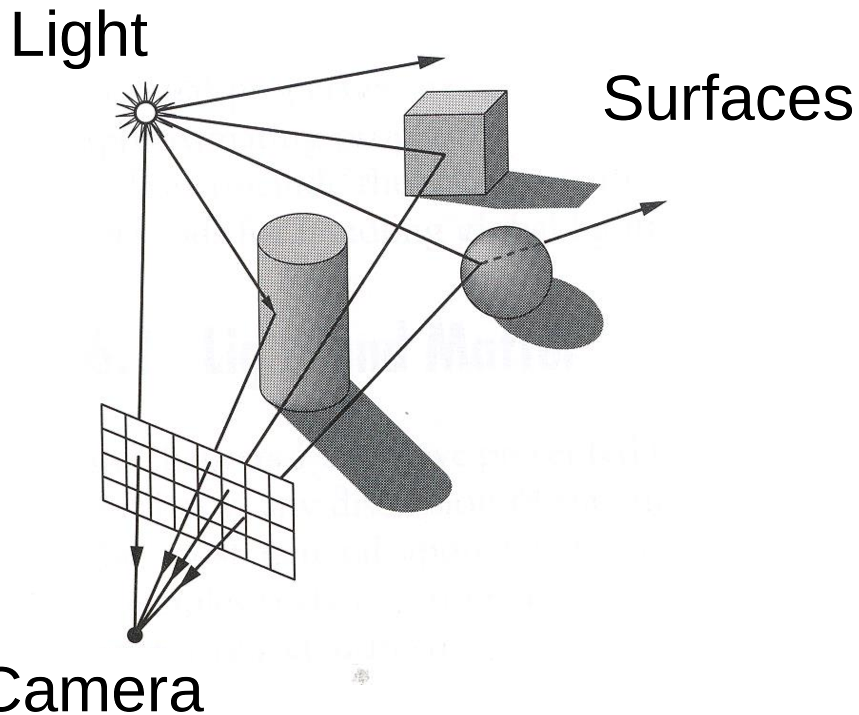
- Intuitive method
  - Compute intersection for all nodes of scene graph
  - Return closest intersection (least t)



Ray-Scene Intersection

- Scene graph is a DAG
  - Traverse with recursion





Ray-Scene Intersection I

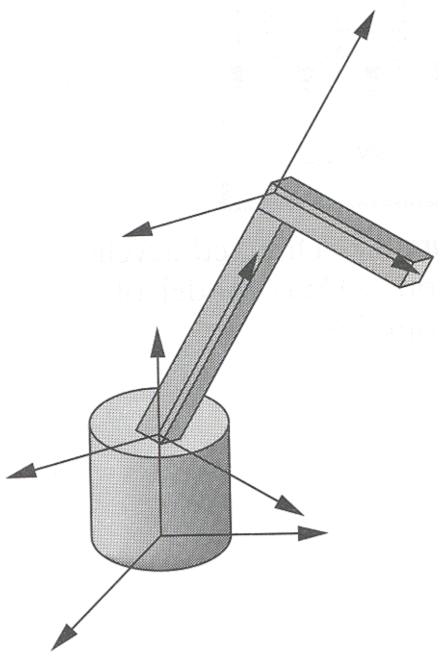
```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Check for intersection with shape
    shape_intersection = Intersect node's shape with ray
    if (shape_intersection is a hit) closest_intersection = shape_intersection
    else closest_intersection = infinitely far miss

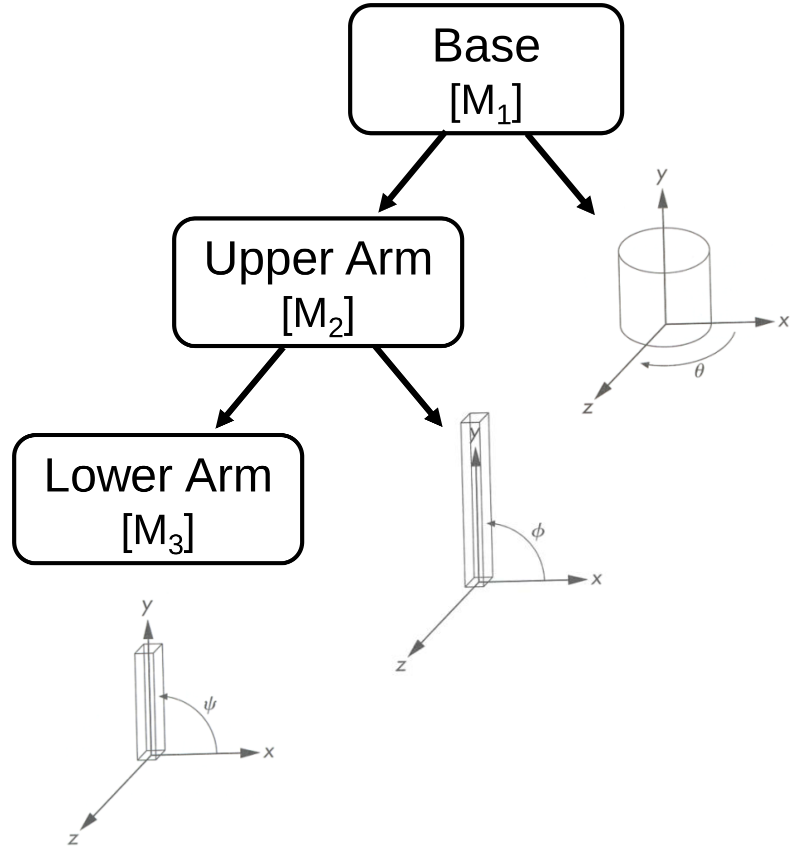
    // Check for intersection with children nodes
    for each child node
        // Check for intersection with child contents
        child_intersection = ComputeIntersection(scene, child, ray);
        if (child_intersection is a hit and is closer than closest_intersection)
            closest_intersection = child_intersection;

    // Return closest intersection in tree rooted at this node
    return closest_intersection
}
```

Ray-Scene Intersection

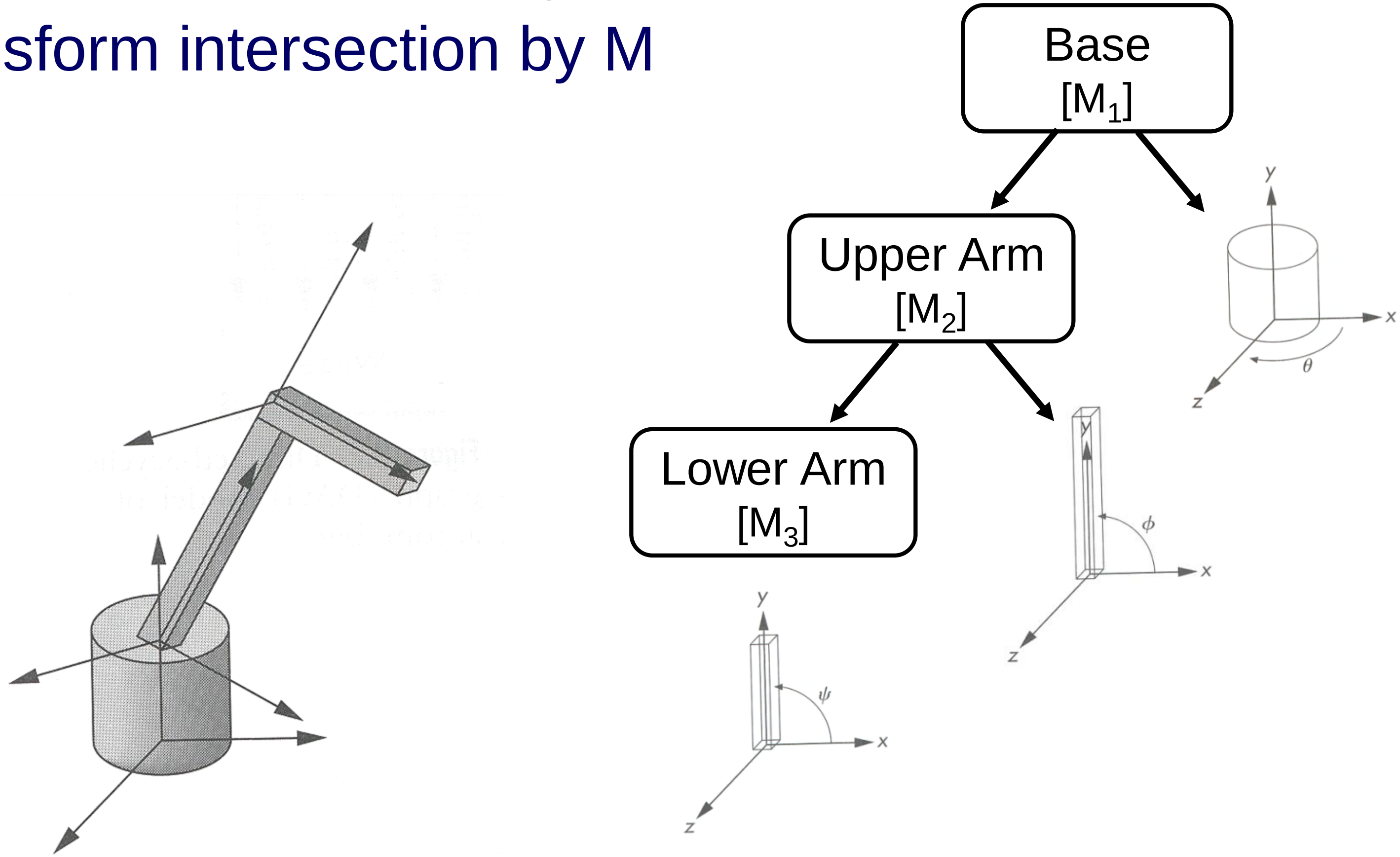
- Scene graph can have transformations





Ray-Scene Intersection

- Scene graph node can have transformations
  - Transform ray (not primitives) by inverse of M
  - Intersect in coordinate system of node
  - Transform intersection by M



Ray-Scene Intersection II

```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
        // Transform ray by inverse of node's transformation

        // Check for intersection with shape

        // Check for intersection with children nodes

        // Transform intersection by node's transformation

        // Return closest intersection in tree rooted at this node
}
```

Ray-Scene Intersection II

```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
      // Transform ray by inverse of node's transformation

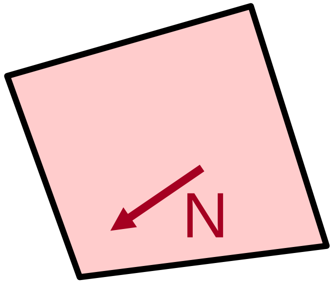
      // Check for intersection with shape

      // Check for intersection with children nodes

      // Transform intersection by node's transformation

      // Return closest intersection in tree rooted at this node
}
```

Note: directions (including ray direction and surface normal N) must be transformed by inverse transpose of M



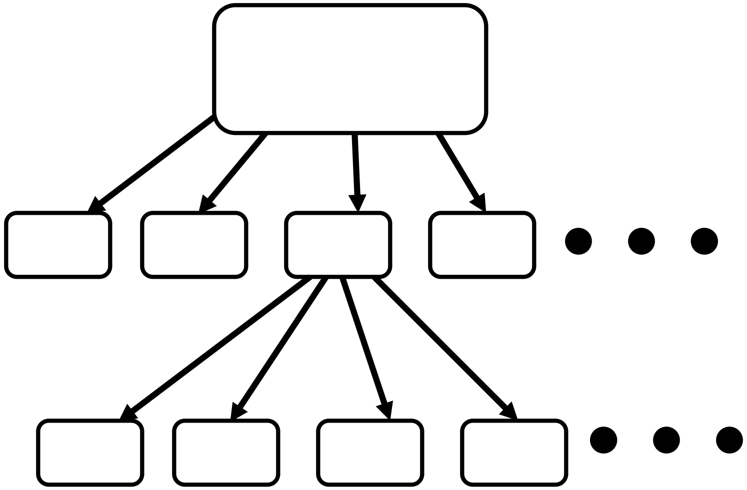
Ray Intersection

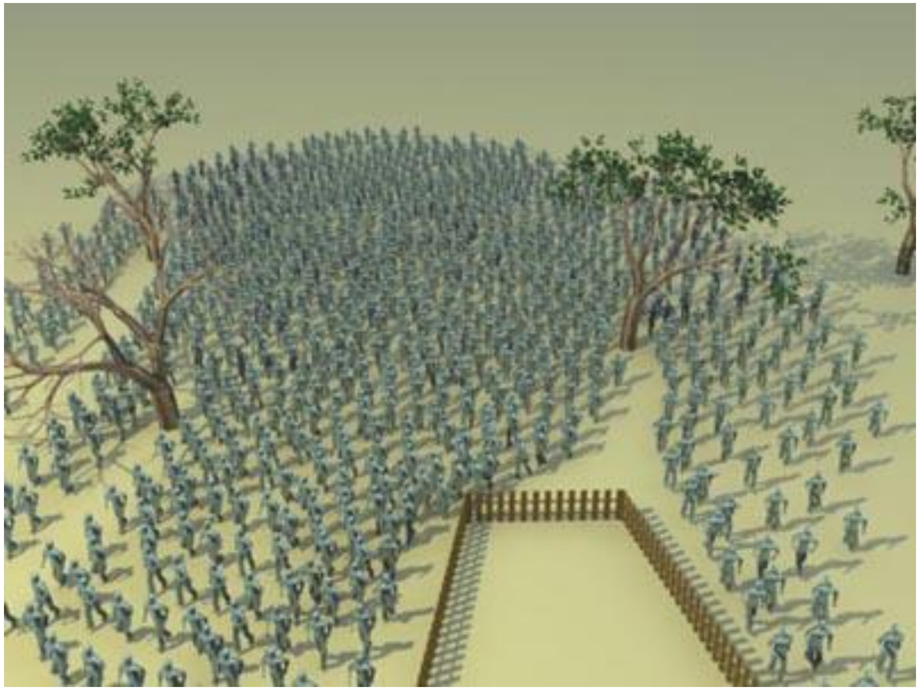
- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

Ray Intersection Acceleration

- What if there are a lot of nodes?

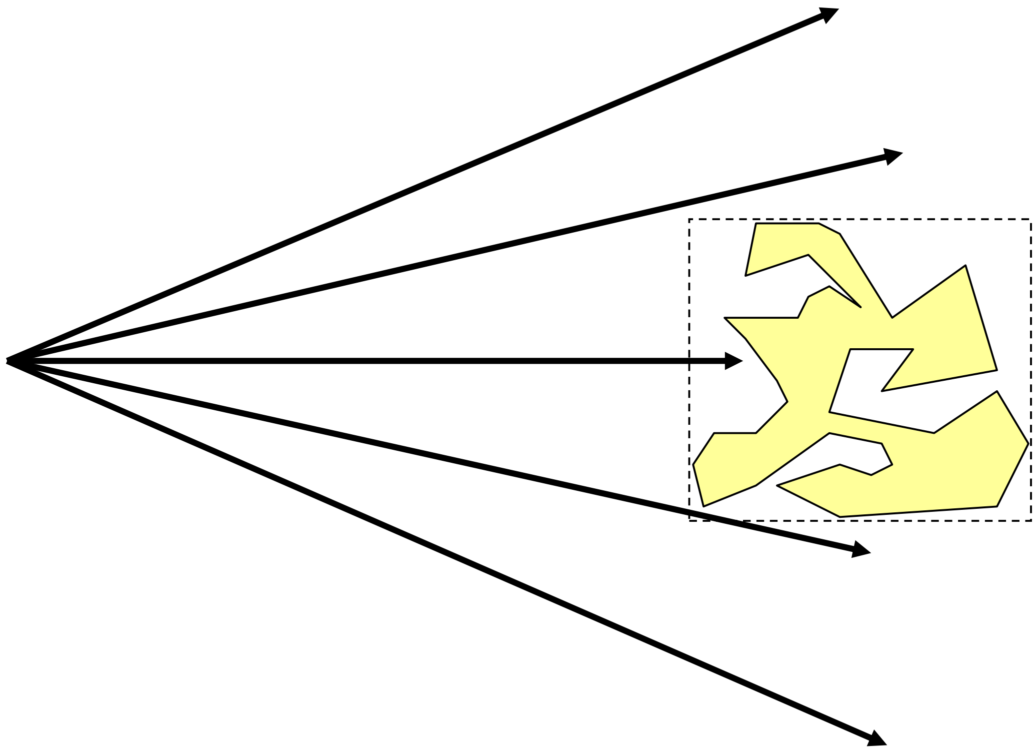




http://www.3dm3.com

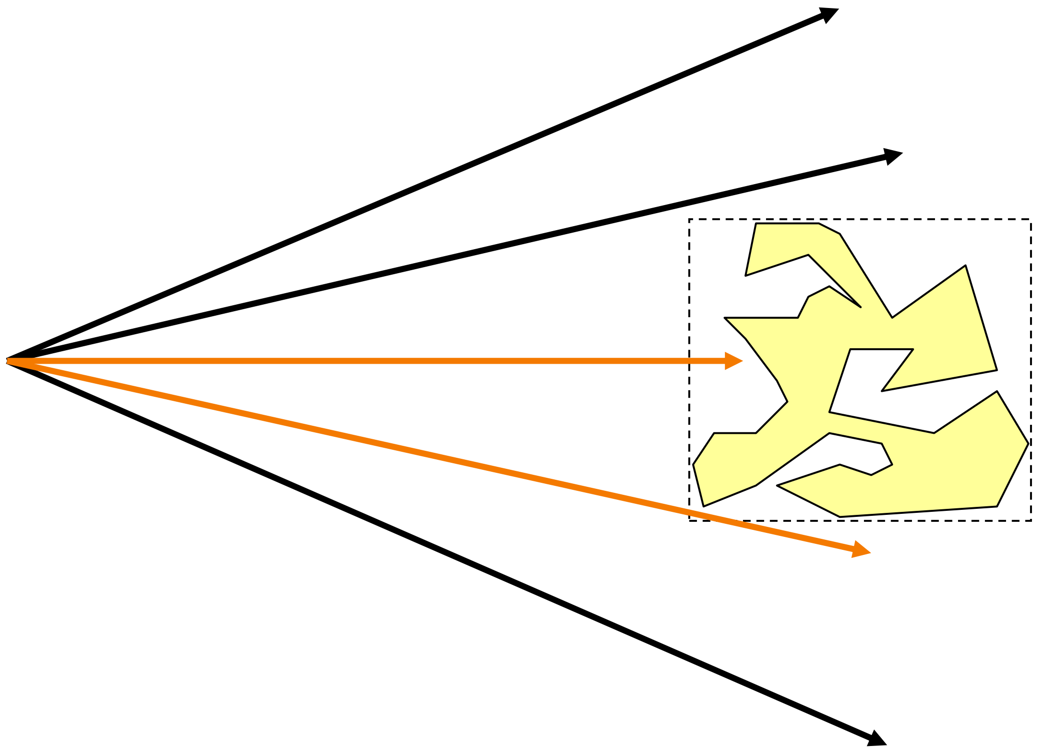
Bounding Volumes

- Check for intersection with simple bounding volume first



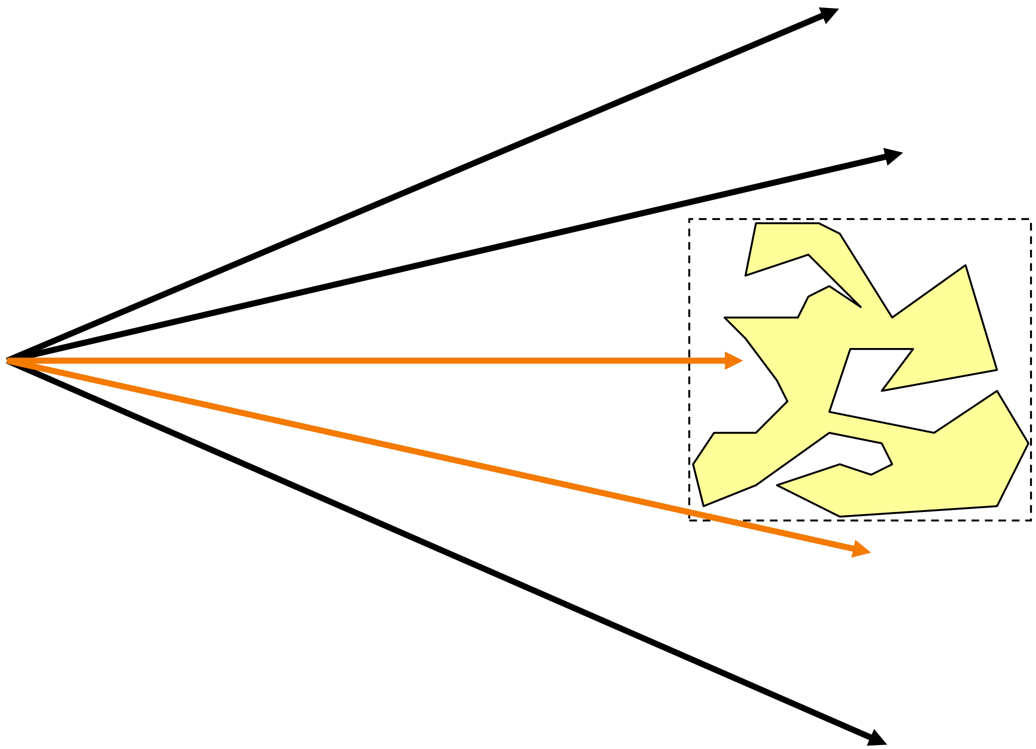
Bounding Volumes

- Check for intersection with bounding volume first



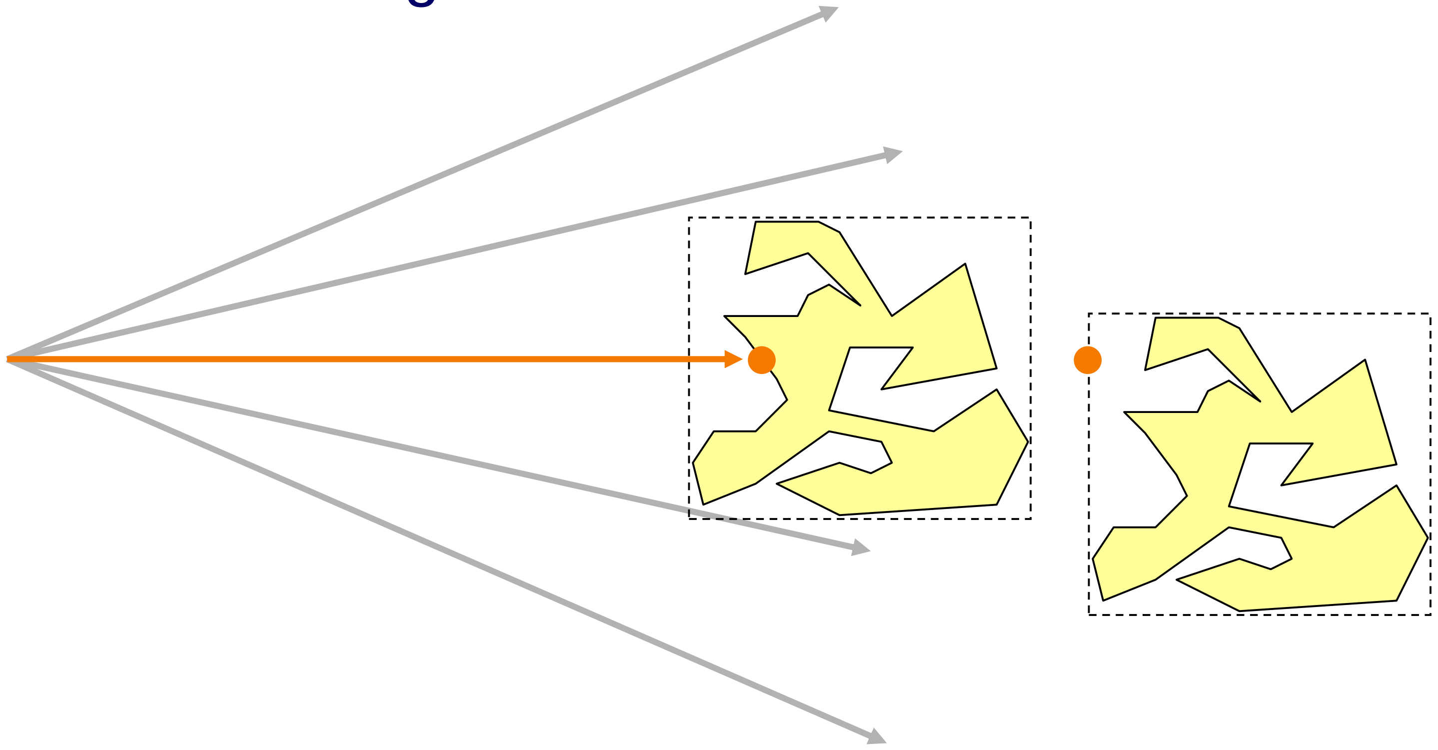
Bounding Volumes

- Check for intersection with bounding volume first
  - If ray doesn't intersect bounding volume, then it can't intersect its contents



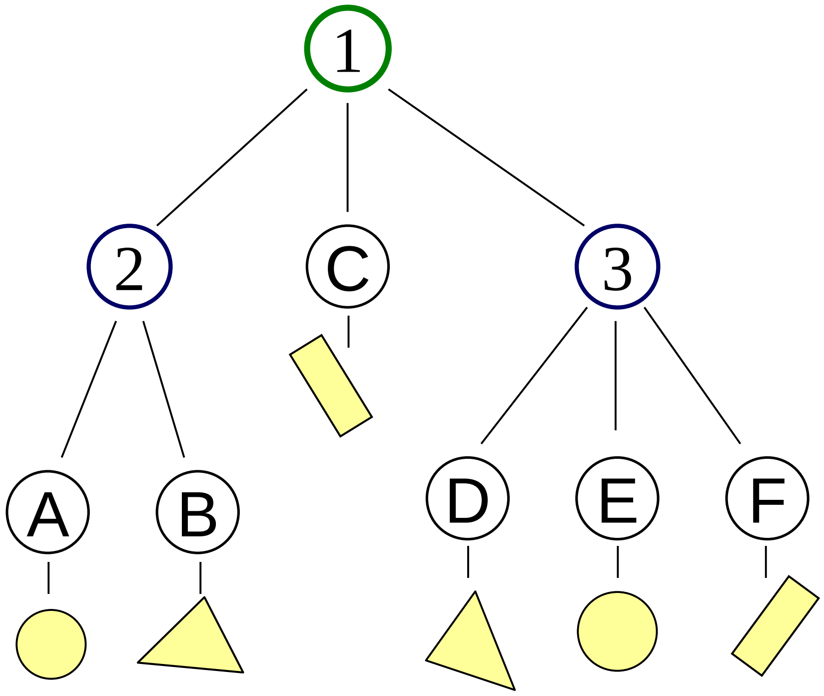
Bounding Volumes

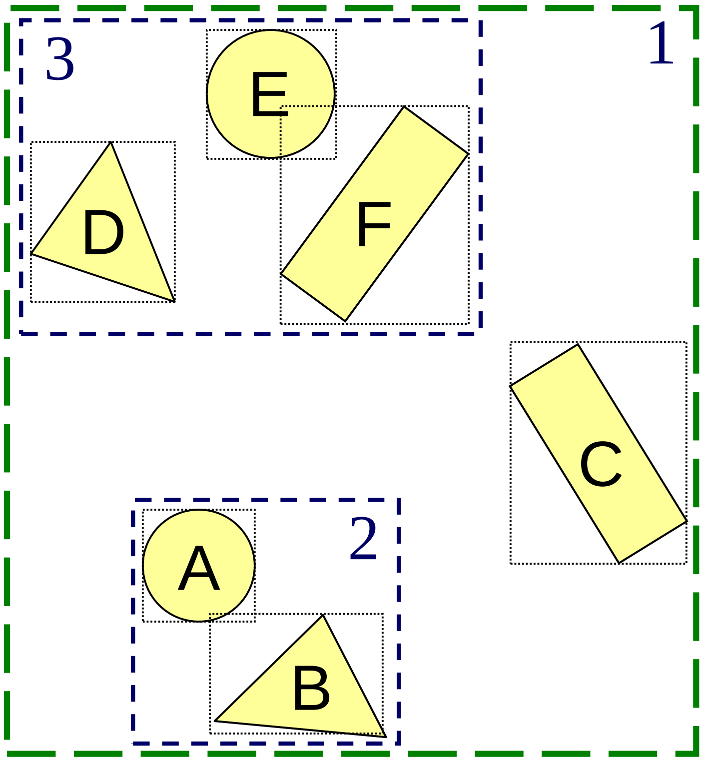
- Check for intersection with bounding volume first
  - If already found a primitive intersection closer than intersection with bounding box, then skip checking contents of bounding box



Bounding Volume Hierarchies

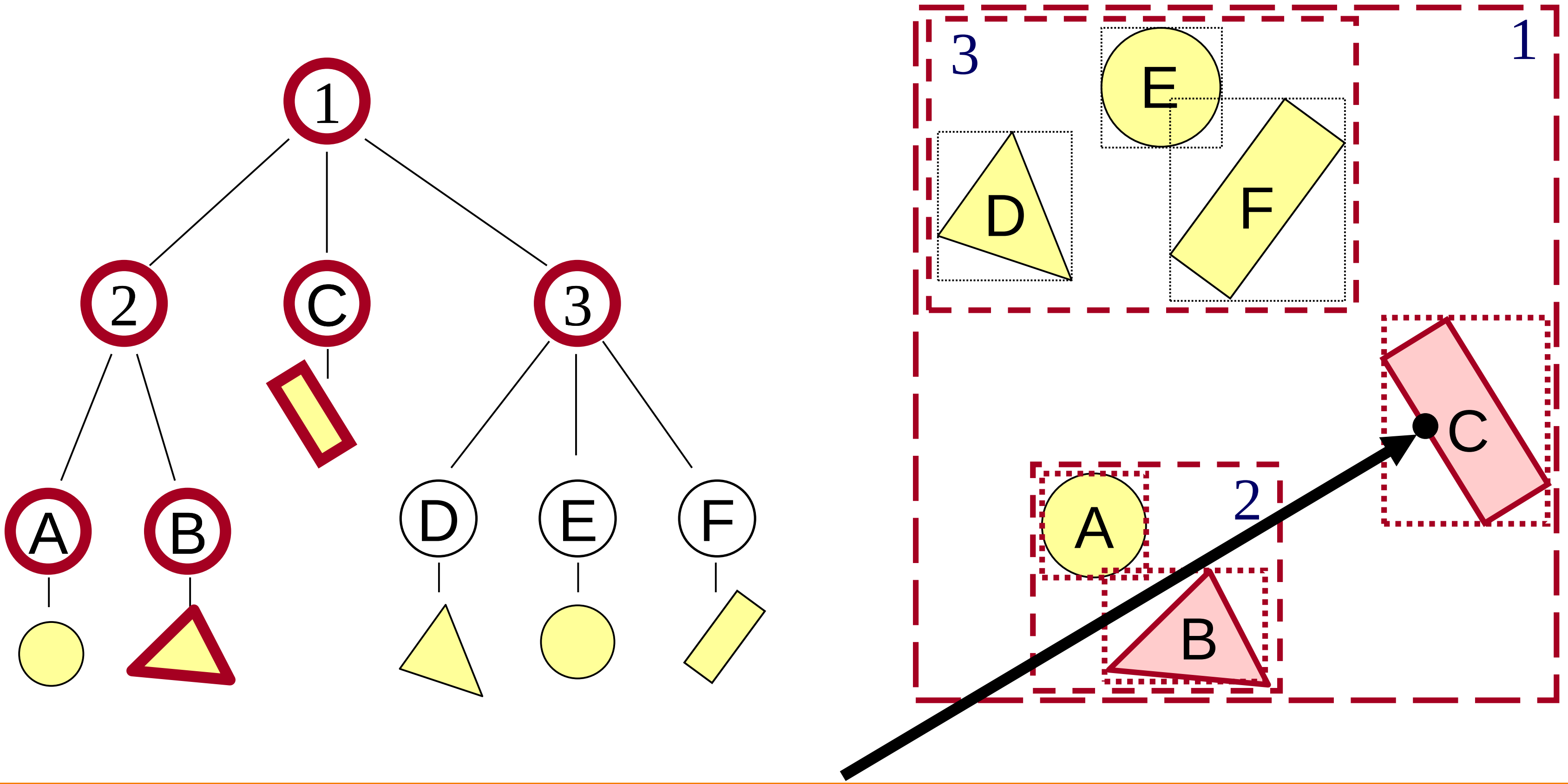
- Scene graph has hierarchy of bounding volumes
  - Bounding volume of interior node contains all children





Bounding Volume Hierarchies

- Checking bounding volumes hierarchically (within each node) can greatly accelerate ray intersection



Bounding Volume Hierarchies

```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Transform ray by inverse of node's transformation
    // Check for intersection with shape

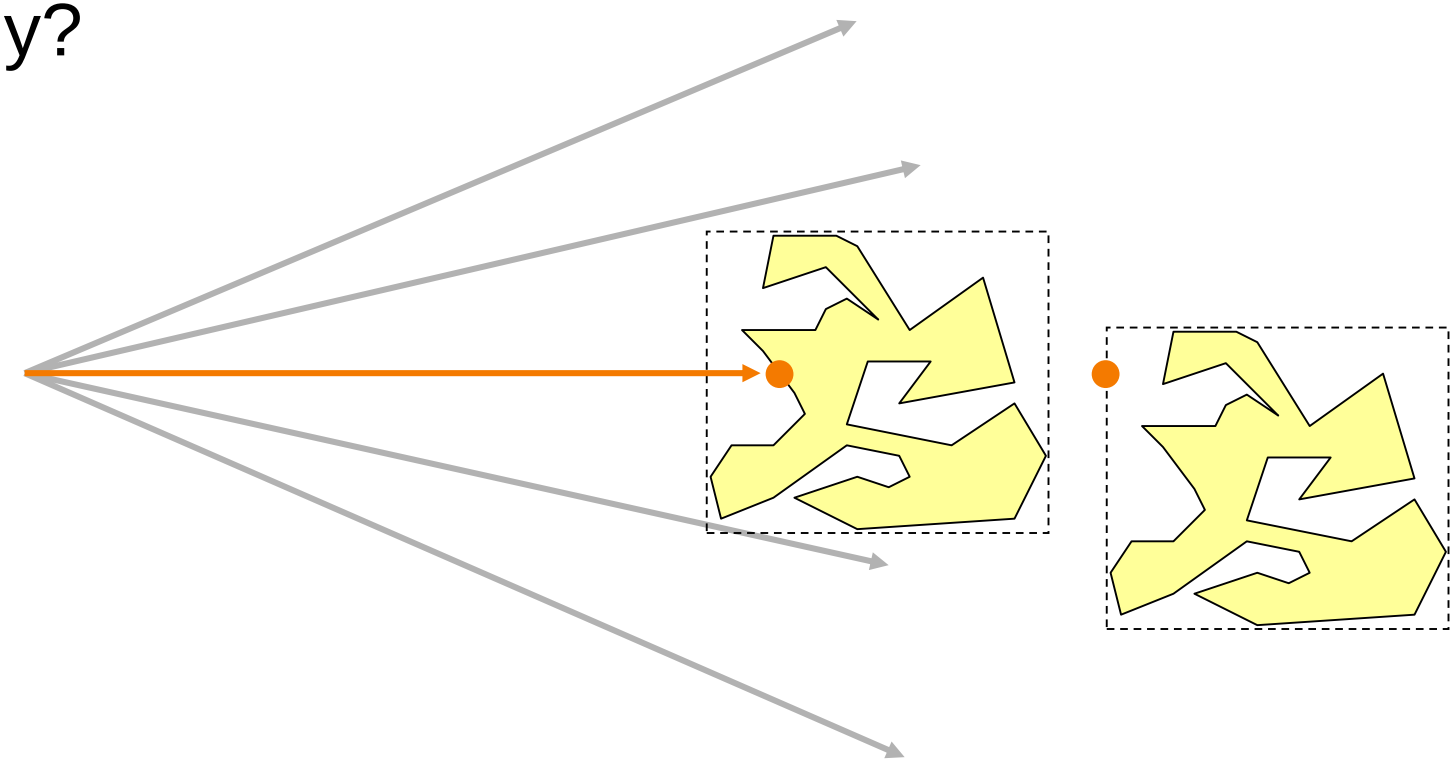
    // Check for intersection with  children nodes
    for each child node
        // Check for intersection with child bounding box first
        bbox_intersection = Intersect child's bounding box with ray
        if (bbox_intersection is a miss or further than closest_intersection) continue

        // Check for intersection with child contents
        child_intersection = ComputeIntersection(scene, child, ray);
        if (child_intersection is a hit and is closer than closest_intersection)
            closest_intersection = child_intersection;

    // Transform intersection by node's transformation
    // Return closest intersection in tree rooted at this node
}
```

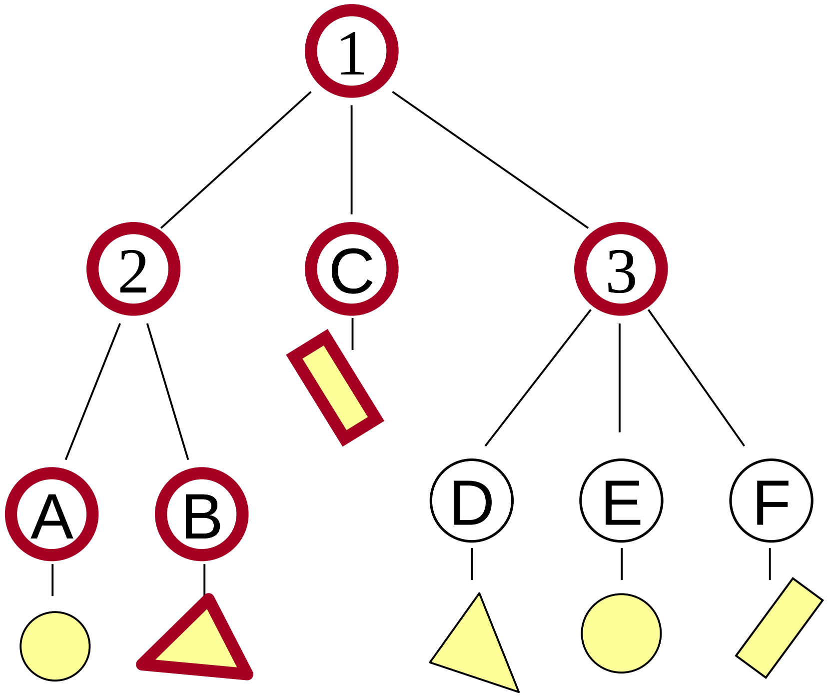
Sort Bounding Volume Intersections

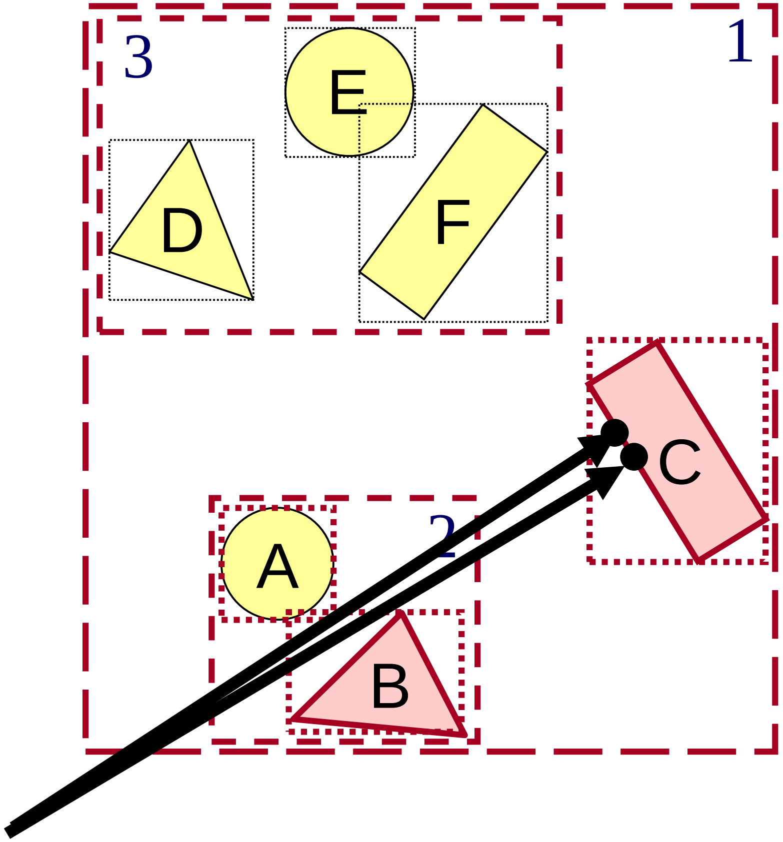
- Sort child bounding volume intersections and then visit child nodes in front-to-back order
- Why?



Cache Node Intersections

- For each node, store closest child intersection from previous ray and check that node first





Bounding Volumes

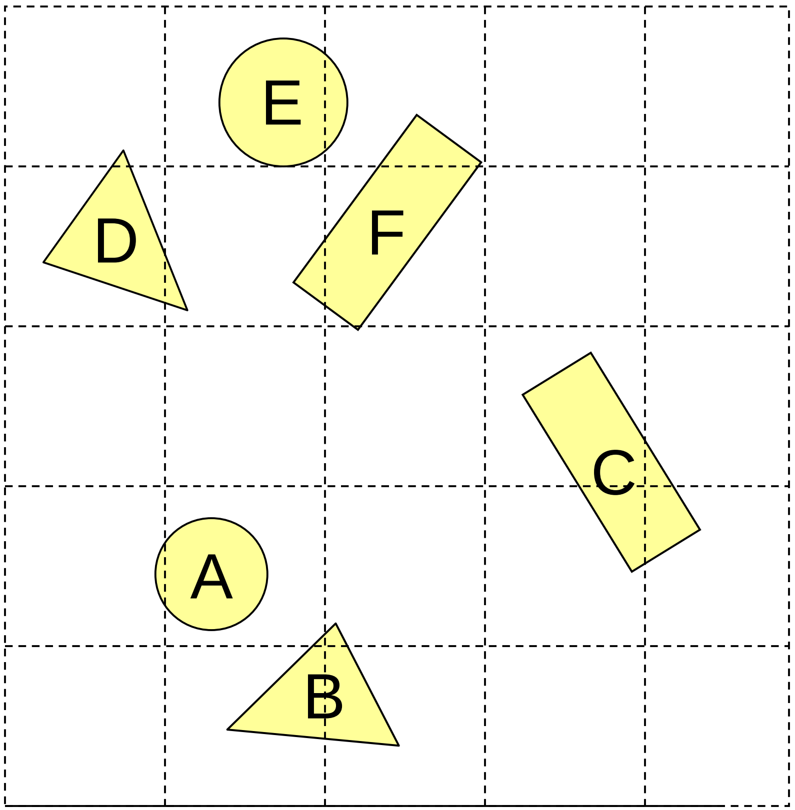
- Common primitives are:
  - Axis-aligned bounding box
  - Sphere
- What are the tradeoffs?
  - Sphere has simple/efficient intersection code
  - Bounding box is generally "tighter"

Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene
- Ray Intersection Acceleration
  - Bounding volumes
  - ➢ Uniform grids
  - Octrees
  - BSP trees

Uniform Grid

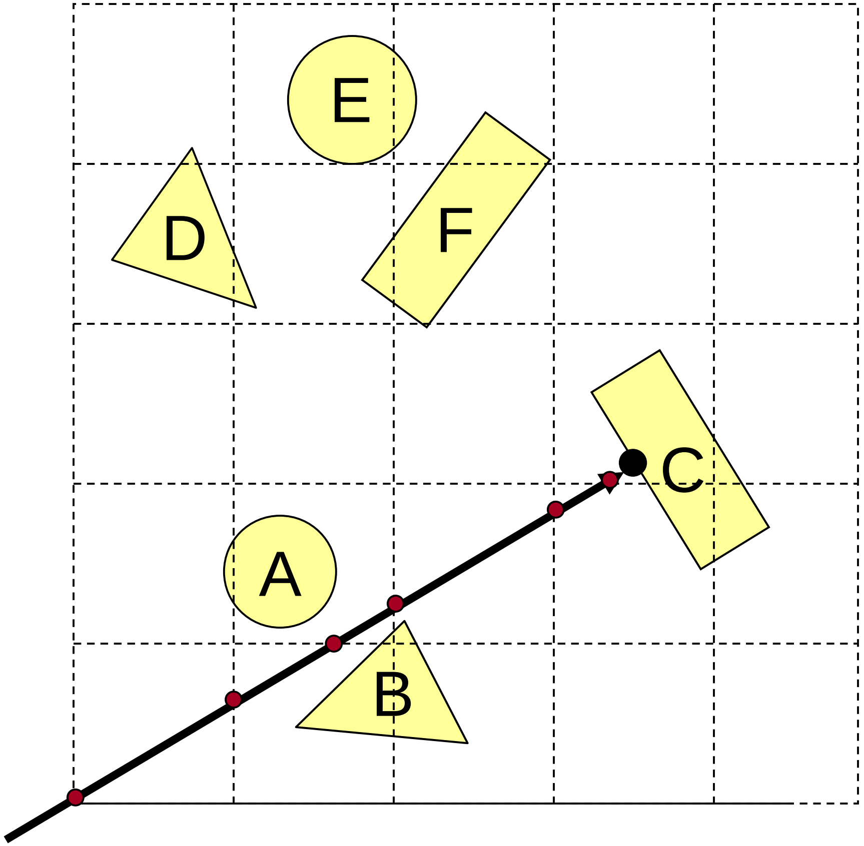
- Construct uniform grid over scene
  - Index primitives according to overlaps with grid cells



Uniform Grid

- Trace rays through grid cells
  - Fast
  - Incremental

Only check primitives in intersected grid cells

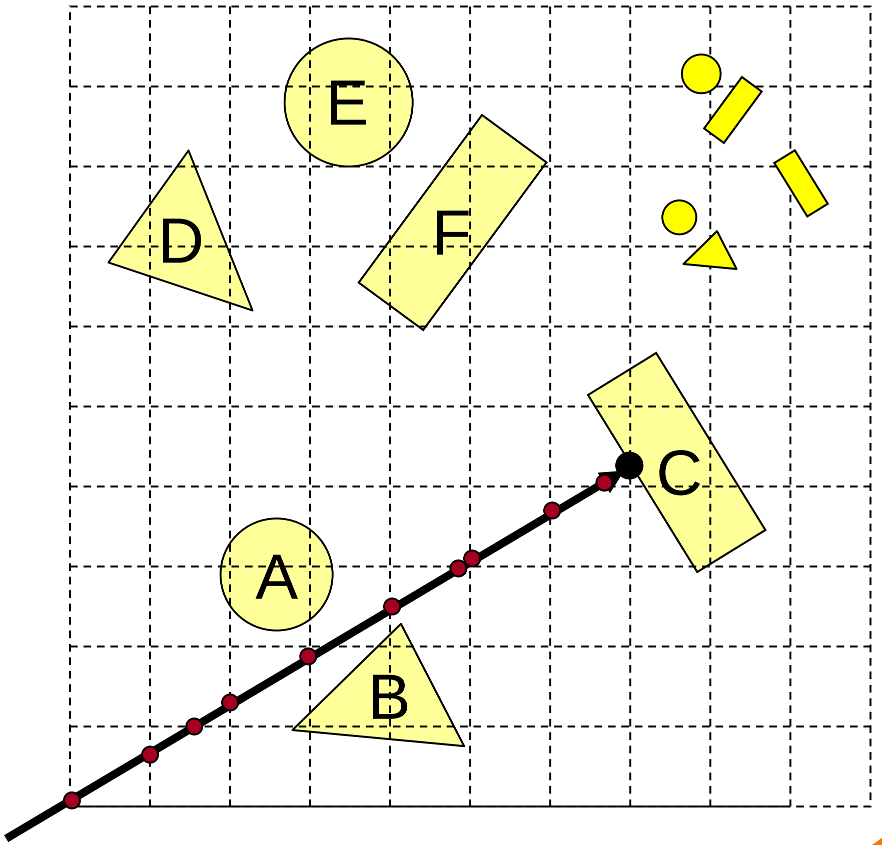


Uniform Grid

- Potential problem:
  - How choose suitable grid resolution?

Too little benefit
if grid is too coarse

Too much cost
if grid is too fine



Ray Intersection

- Ray Intersection
  - Sphere
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  - Box
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- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
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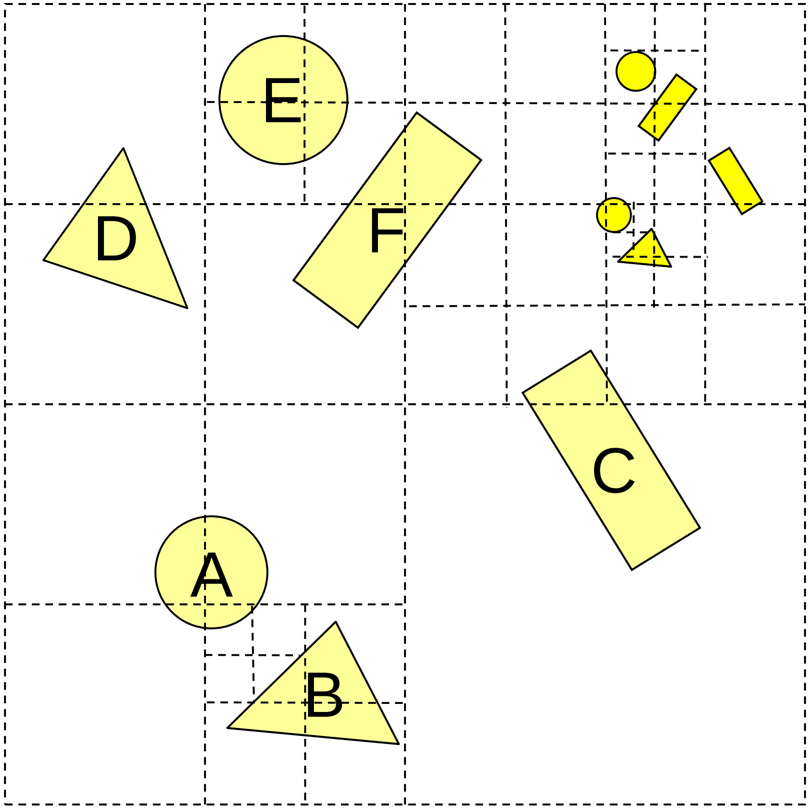
  - BSP trees

Octree

- Construct adaptive grid over scene
  - Recursively subdivide box-shaped cells into 8 octants
  - Index primitives by overlaps with cells

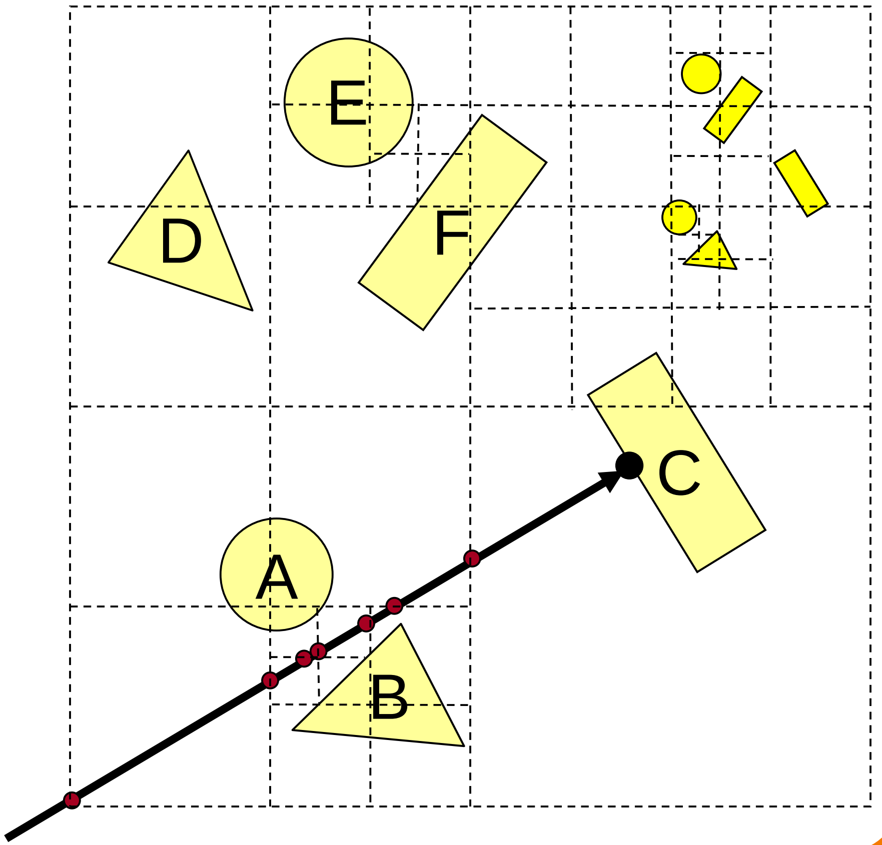
Generally fewer cells



Octree

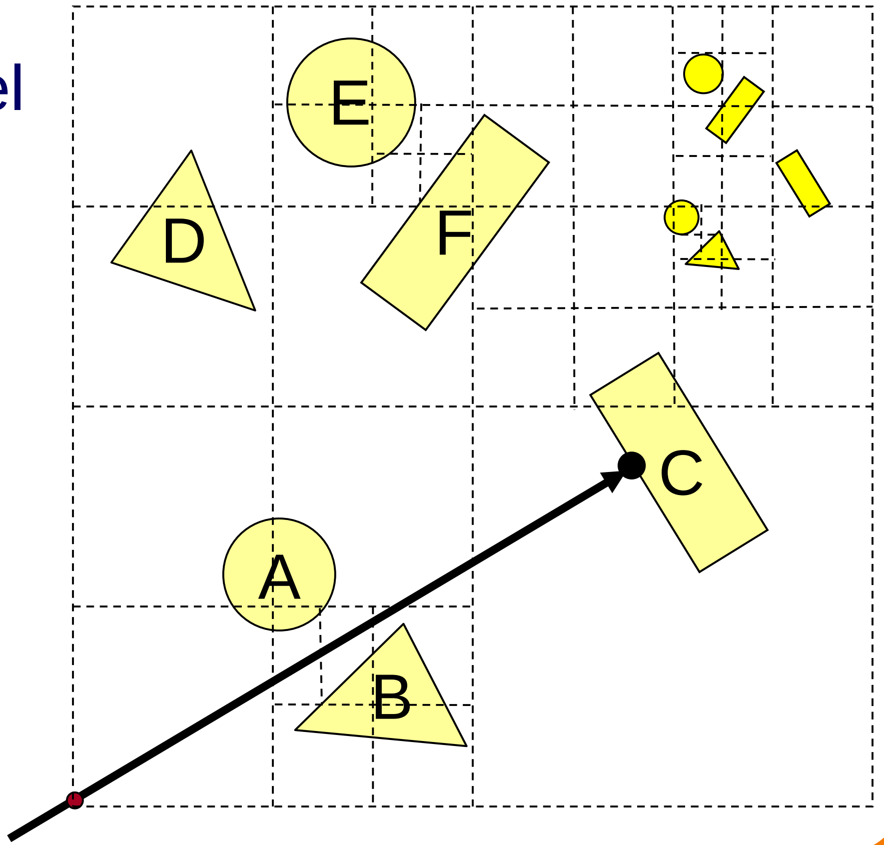
- Trace rays through neighbor cells
  - Fewer cells

Trade-off fewer cells for more expensive traversal



Octree

- Or, check rays versus octree boxes hierarchically
  - Computing octree boxes while descending tree
  - Sort eight boxes front-to-back at each level
  - Check primitives/children inside box



Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

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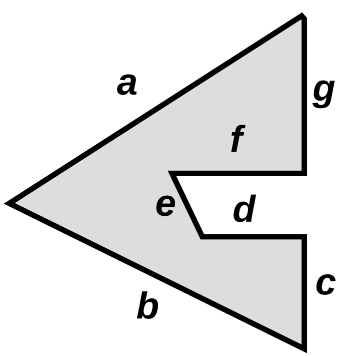
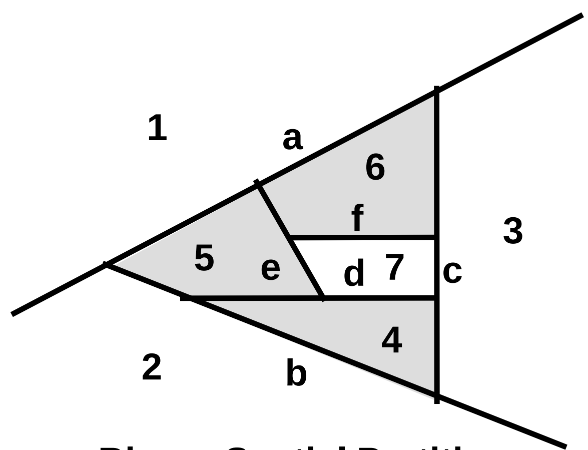
  - BSP trees

Binary Space Partition (BSP) Tree

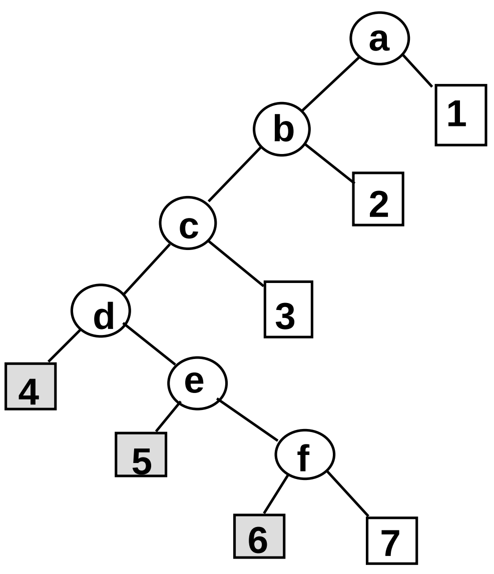
- Recursively partition space by planes
  - BSP tree nodes store partition plane and set of polygons lying on that partition plane
  - Every part of every polygon lies on a partition plane



Object



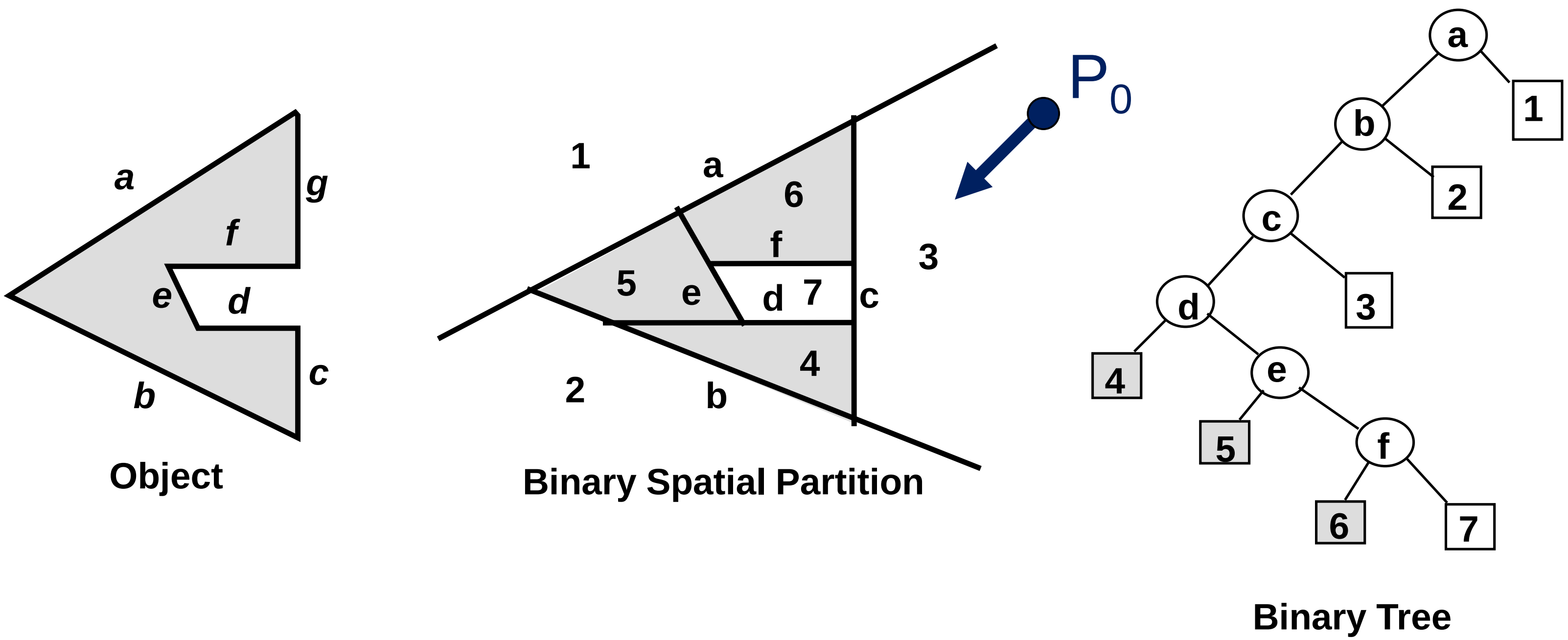
Binary Spatial Partition



Binary Tree

Binary Space Partition (BSP) Tree

- Traverse nodes of BSP tree front-to-back
  - Visit halfspace (child node) containing P_0
  - Intersect polygons lying on partition plane
  - Visit halfspace (other child node) not containing P_0



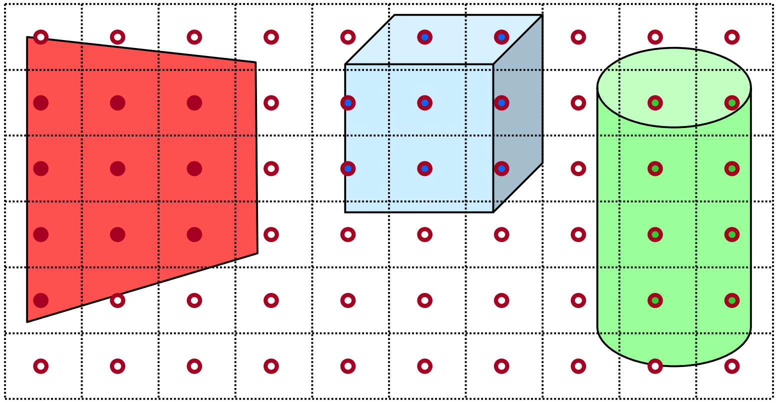
Binary Space Partition (BSP) Tree

```
R3Intersection
ComputeBSPIntersection(R3Ray *ray, BspNode *node, double min_t, double max_t)
{
        // Compute parametric value of ray-plane intersection
        t = ray parameter for intersection with split plane of node
        if  (t < min_t) || (t < max_t)) return no_intersection;

        // Compute side of partition plane that contains ray start point
        int side = (SignedDistance(node->plane, ray.Start()) < 0) ? 0 : 1;
        intersection1 = ComputeBSPIntersection(ray, node->child[side],  min_t, t);
        if (intersection1 is a hit) return intersection1;
        intersection2 = ComputePolygonsIntersection(ray, node->polygons);
        if (intersection2 is a hit) return intersection2;
        intersection3 = ComputeBSPIntersection(ray, node->child[1-side], t, max_t);
        return intersection 3;
}
```

Other Accelerations

- Screen space coherence – check > 1 ray at once
  - Beam tracing
  - Pencil tracing
  - Cone tracing



- Memory coherence
  - Large scenes
- Parallelism
  - Ray casting is “embarrassingly parallelizable”
  - Assignment 3 (raytracer) runs program per-pixel
- etc.

Acceleration

- Intersection acceleration techniques are important
  - Bounding volume hierarchies
  - Spatial partitions
- General concepts
  - Sort objects spatially
  - Make trivial rejections quick
  - Perform checks hierarchically
  - Utilize coherence when possible

Expected time is sub-linear in number of primitives

Summary

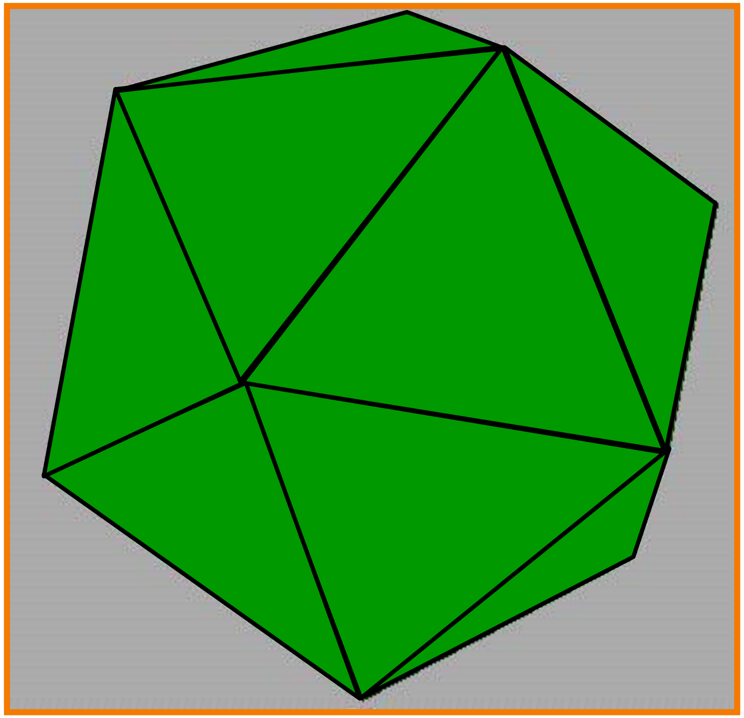
- Writing a simple ray casting renderer is easy
  - Generate rays
  - Intersection tests
  - Lighting calculations

```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
	R2Image *image = new R2Image(width, height);
	for (int i = 0; i < width; i++) {
		for (int j = 0; j < height; j++) {
			R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
			R3Rgb radiance = ComputeRadiance(scene,  &ray);
			image->SetPixel(i, j, radiance);
		}
	}
	return image;
}
```

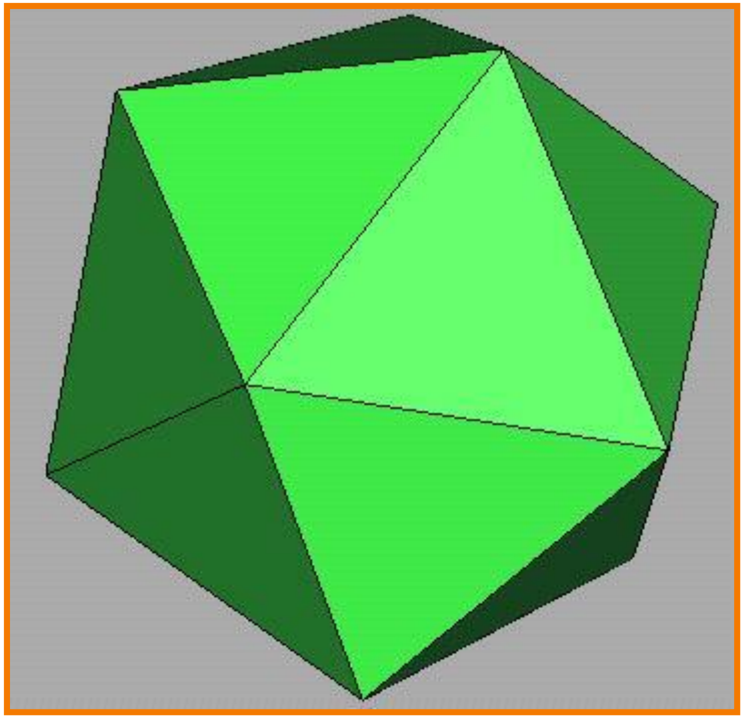
Heckbert's Business Card Ray Tracer

```
typedef struct{double x,y,z}vec;vec U,black,amb={.02,.02,.02};struct sphere{ vec cen,color;
double rad,kd,ks,kt,kl,ir}*s,*best,sph[]={0.,6.,.5,1.,1.,1.,.9, .05,.2,.85,0.,1.7,-1.,8.,-.5,1.,.5,.2,1.,
.7,.3,0.,.05,1.2,1.,8.,-.5,.1,.8,.8, 1.,.3,.7,0.,0.,1.2,3.,-6.,15.,1.,.8,1.,7.,0.,0.,0.,.6,1.5,-3.,-3.,12.,
.8,1., 1.,5.,0.,0.,0.,.5,1.5,};yx;double u,b,tmin,sqrt(),tan();double vdot(A,B)vec A ,B;{return A.x
*B.x+A.y*B.y+A.z*B.z;}vec vcomb(a,A,B)double a;vec A,B;{B.x+=a* A.x;B.y+=a*A.y;B.z+=a*A.z;
return B;}vec vunit(A)vec A;{return vcomb(1./sqrt( vdot(A,A)),A,black);}struct sphere*intersect
(P,D)vec P,D;{best=0;tmin=1e30;s= sph+5;while(s-->sph)b=vdot(D,U=vcomb(-1.,P,s->cen)),
u=b*b-vdot(U,U)+s->rad*s ->rad,u=u>0?sqrt(u):1e31,u=b-u>1e-7?b-u:b+u,tmin=u>=1e-7&&
u<tmin?best=s,u: tmin;return best;}vec trace(level,P,D)vec P,D;{double d,eta,e;vec N,color;
struct sphere*s,*l;if(!level--)return black;if(s=intersect(P,D));else return amb;color=amb;eta=
s->ir;d= -vdot(D,N=vunit(vcomb(-1.,P=vcomb(tmin,D,P),s->cen )));if(d<0)N=vcomb(-1.,N,black),
eta=1/eta,d= -d;l=sph+5;while(l-->sph)if((e=l ->kl*vdot(N,U=vunit(vcomb(-1.,P,l->cen))))>0&&
intersect(P,U)==l)color=vcomb(e ,l->color,color);U=s->color;color.x*=U.x;color.y*=U.y;color.z
*=U.z;e=1-eta* eta*(1-d*d);return vcomb(s->kt,e>0?trace(level,P,vcomb(eta,D,vcomb(eta*d-
sqrt (e),N,black))):black,vcomb(s->ks,trace(level,P,vcomb(2*d,N,D)),vcomb(s->kd, color,vcomb
(s->kl,U,black))));}main(){printf("%d %d\n",32,32);while(yx<32*32) U.x=yx%32-32/2,U.z=32/2-
yx++/32,U.y=32/2/tan(25/114.5915590261),U=vcomb(255., trace(3,black,vunit(U)),black),printf
("%.0f %.0f %.0f\n",U);}/*minray!*/
```

Next Time is Illumination!



Without Illumination



With Illumination