COS320: Compiling Techniques

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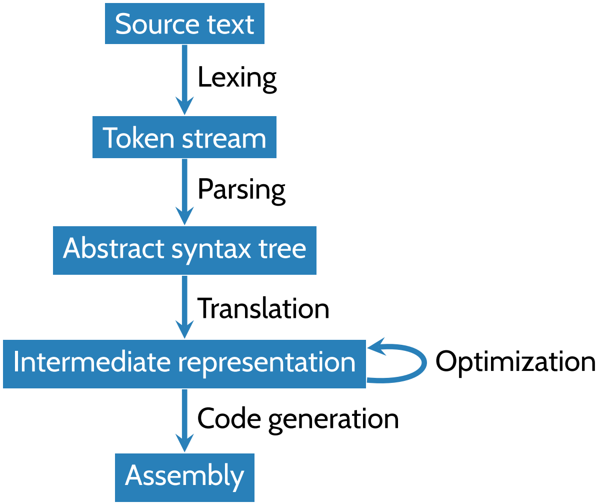
Lexing

Compiler phases (simplified)



- The *lexing* (or *lexical analysis*) phase of a compiler breaks a stream of characters (source text) into a stream of *tokens*.
    - Whitespace and comments often discarded
- A *token* is a sequence of characters treated as a unit.
  Each token is associated with a *token type*:
    - *identifier tokens*: `x`, `y`, `foo`, ...
    - *integer tokens*: `0`, `1`, `-14`, `512`, ...
    - *if tokens*: `if`
    - ...
- Algebraic datatypes are a convenient representation for tokens

```
type token = IDENT of string
           | INT of int
           | IF
           | ...
```

```
// compute absolute value
if (x < 0) {
  return -x;
} else {
  return x;
}
```

↓Lexer

```
IF, LPAREN, IDENT "x", LT, INT 0, RPAREN, LBRACE,
RETURN, MINUS, IDENT "x", SEMI,
RBRACE, ELSE, LBRACE,
RETURN, IDENT "x", SEMI,
RBRACE
```

Implementing a lexer

- Option 1: write by hand
- Option 2: use a *lexer generator*
  - Write a *lexical specification* in a domain-specific language
  - Lexer generator compiles specification to a lexer (in language of choice)
- Many lexer generators available
  - lex, flex, ocamllex, jflex, …

Formal Languages

- An *alphabet* Σ is a finite set of symbols (e.g., $\{0, 1\}$, ASCII, unicode, tokens).
- A *word* (or *string*) over Σ is a finite sequence $w = w_1 w_2 w_3 ... w_n$, with each $w_i \in \Sigma$.
  - The *empty word* ϵ is a word over any alphabet
  - The set of all words over Σ is typically denoted Σ^*
  - E.g., $01001 \in \{0, 1\}^*$, *covfefe* $\in \{a, ..., z\}^*$
- A *language* over Σ is a set of words over Σ
  - Integer literals form a language over $\{0, ..., 9, -\}$
  - The keywords of OCaml form a (finite) language over ASCII
  - Syntactically-valid Java programs forms an (infinite) language over Unicode

Regular expressions (regex)

- Regular expressions are one mechanism for describing languages
- Abstract syntax of regular expressions:

<RegExp> ::=	ϵ	Empty word
	$\mid \Sigma$	Letter
	$\mid$ <RegExp><RegExp>	Concatenation
	$\mid$ <RegExp>$\mid$<RegExp>	Alternative
	$\mid$ <RegExp>*	Repetition

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- Meaning of regular expressions:

$$\mathcal{L}(\epsilon) = \{\epsilon\}$$
$$\mathcal{L}(a) = \{a\}$$
$$\mathcal{L}(R_1 R_2) = \{uv : u \in \mathcal{L}(R_1) \wedge v \in \mathcal{L}(R_2)\}$$
$$\mathcal{L}(R_1 | R_2) = \mathcal{L}(R_1) \cup \mathcal{L}(R_2)$$
$$\mathcal{L}(R^*) = \{\epsilon\} \cup \mathcal{L}(R) \cup \mathcal{L}(RR) \cup \mathcal{L}(RRR) \cup \ldots$$

`ocamllex` regex concrete syntax

- 'a': letter
- "abc": string (equiv. `'a''b''c'`)
- R+: one or more repetitions of R (equiv. `RR*`)
- R?: zero or one R (equiv. `R|`ϵ)
- `['a'-'z']`: character range (equiv. `'a'|'b'|...|'z'`)
- `R as x`: bind string matched by `R` to variable `x`

Lexer generators

Lexer generators take as input a lexical specification, and output code that tokenizes a character stream w.r.t. that specification

Example lexical specification:

$$\overbrace{\textbf{identifier}}^{\text{token type}} = \overbrace{[a - zA - Z][a - zA - Z0 - 9]^*}^{\text{pattern}}$$

$$\textbf{integer} = [1 - 9][0 - 9]^*$$

$$\textbf{plus} = +$$

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- "foo+42+bar" $\rightarrow$ $\underbrace{\textbf{identifier}}_{\text{token type}}$ $\underbrace{\text{"foo"}}_{\text{lexeme}}$, **plus** "+", **integer** "42", **plus** "+", **identifier** "bar"

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\textbf{integer} &= [1-9][0-9]^* \\
\textbf{plus} &= +
\end{aligned}
$$

- “foo+42+bar” $\rightarrow$ $\underbrace{\textbf{identifier}}_{\text{token type}}$ $\underbrace{\text{“foo”}}_{\text{lexeme}}$, **plus** “+”, **integer** “42”, **plus** “+”, **identifier** “bar”
- Typically, lexical spec associates an *action* to each token type, which is code that is evaluted on the lexeme (often: produce a token value)

Disambiguation

- May be more than one way to lex a string:

$$
\begin{aligned}
IF &= \texttt{if} \\
IDENT &= \texttt{[a-zA-Z][a-zA-Z0-9]}^* \\
INT &= \texttt{[1-9][0-9]}^* \\
LT &= \texttt{<} \\
&\ldots
\end{aligned}
$$

  - Input string `ifx<10`: $\boxed{\texttt{IDENT "ifx", LT, INT 10}}$ *or* $\boxed{\texttt{IF, IDENT "x", LT, INT 10}}$?
  - Input string `if x<9`: $\boxed{\texttt{IF, IDENT "x", LT, INT 9}}$ *or* $\boxed{\texttt{IDENT "if", IDENT "x", LT, INT 9}}$?

Disambiguation

- May be more than one way to lex a string:

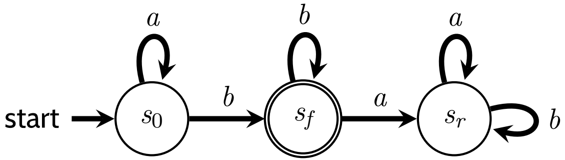
$$\begin{aligned} IF &= \texttt{if} \\ IDENT &= \texttt{[a-zA-Z][a-zA-Z0-9]}^* \\ INT &= \texttt{[1-9][0-9]}^* \\ LT &= \texttt{<} \\ &\dots \end{aligned}$$

  - Input string `ifx<10`: `IDENT "ifx", LT, INT 10` *or* `IF, IDENT "x", LT, INT 10`?
  - Input string `if x<9`: `IF, IDENT "x", LT, INT 9` *or* `IDENT "if", IDENT "x", LT, INT 9`?
- The lexer is greedy: always prefer longest match
- Order matters: prefer earlier patterns

Lexer generator pipeline

- Lexical specification is compiled to a *deterministic finite automaton* (DFA), which can be executed efficiently
- Typical pipeline: lexical specification $\rightarrow$ *non*deterministic FA $\rightarrow$ DFA
- Kleene's theorem: regular expressions, NFAs, and DFAs describe the same class of languages
    - A language is *regular* if it is accepted by a regular expression (equiv., NFA, DFA).

Deterministic finite automata (DFA)

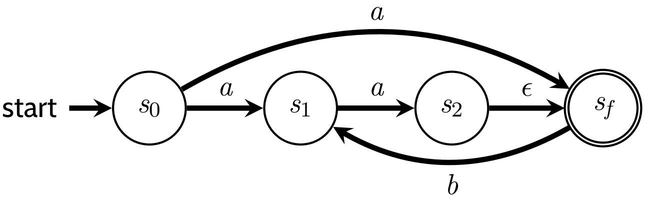


A *deterministic finite automaton* (DFA) $A = (Q, \Sigma, \delta, s, F)$ consists of

- Q: finite set of states
- Σ: finite alphabet
- $\delta : Q \times \Sigma \to Q$: transition function
  - Every state has *exactly* one outgoing edge per letter
- $s \in Q$: initial state
- $F \subseteq Q$: final states

DFA accepts a string $w = w_1 \ldots w_n \in \Sigma^*$ iff $\delta(\ldots\delta(\delta(s, w_1), w_2), \ldots, w_n) \in F$.

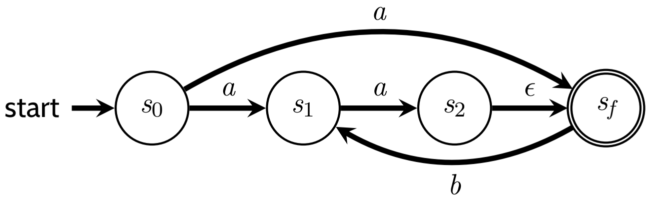
Non-deterministic finite automata



A *non-deterministic finite automaton* (NFA) $A = (Q, \Sigma, \Delta, s, F)$ generalization of a DFA, where

- $\Delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times Q$: transition *relation*
  - A state can have *more than one* outgoing edge for a given letter
  - A state can have *no* outgoing edges for a given letter
  - A state can have ϵ-transitions (read no input, but change state)

Non-deterministic finite automata



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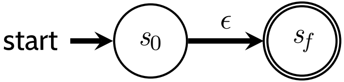
NFA accepts a string $w = w_1 \ldots w_n \in \Sigma^*$ iff there exists a w-labeled path from q_0 to an accepting state (i.e., there is some sequence $(q_0, u_1, q_1), (q_1, u_2, q_2), \ldots, (q_{m-1}, u_m, q_m)$ with $q_0 = s$, $q_m \in F$, and $u_1 u_2 \ldots u_m = w$.

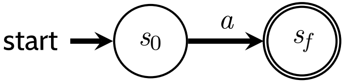
Regex $\rightarrow$ NFA

Case: ϵ (empty word)



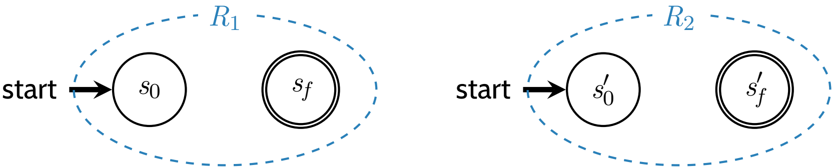
Regex $\to$ NFA

Case: a (letter)



Regex $\rightarrow$ NFA

Case: $R_1 R_2$ (concatenation)



Regex $\rightarrow$ NFA

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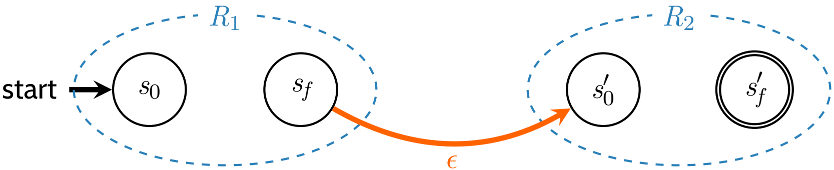
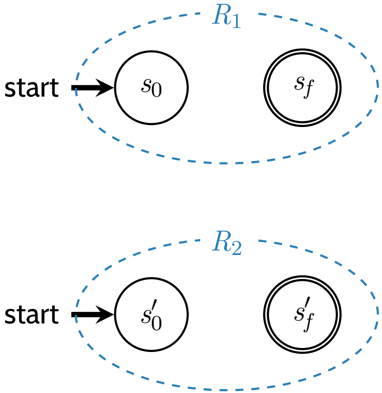


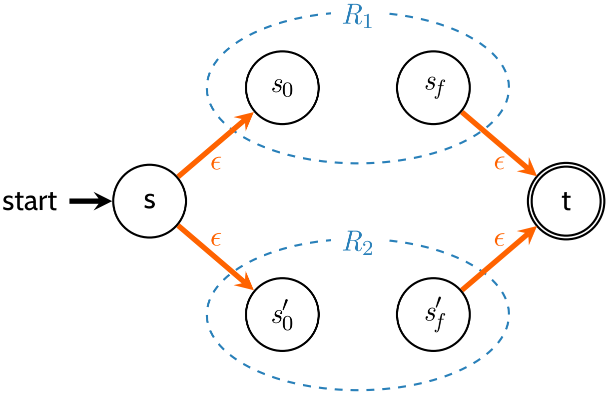
Regex $\to$ NFA

Case: $R_1 | R_2$ (alternative)



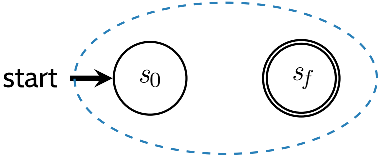
Regex $\rightarrow$ NFA

Case: $R_1 | R_2$ (alternative)



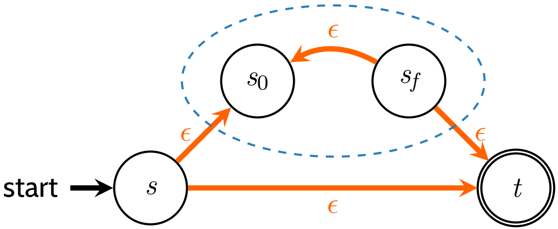
Regex $\rightarrow$ NFA

Case: R^* (iteration)



Regex $\to$ NFA

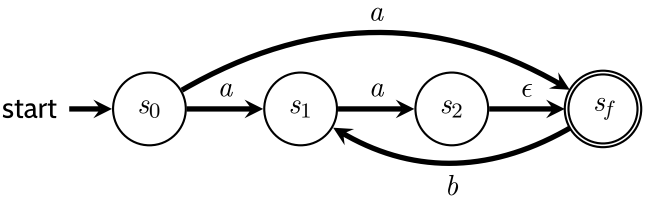
Case: R^* (iteration)

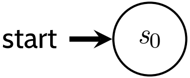


NFA $\rightarrow$ DFA

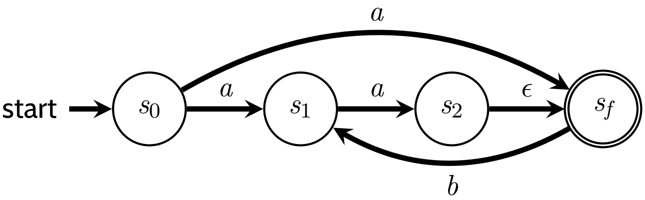
- For any NFA, there is a DFA that recognizes the same language
- **Intuition**: the DFA simulates all possible paths of the NFA simultaneously
  - There is an unbounded number of paths *but* we only care about the "end state" of each path, not its history
  - States of the DFA track the set of possible states the NFA could be in
  - DFA accepts when *some* path accepts

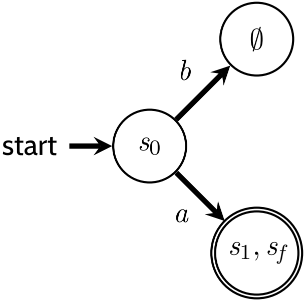
NFA $\rightarrow$ DFA





NFA → DFA





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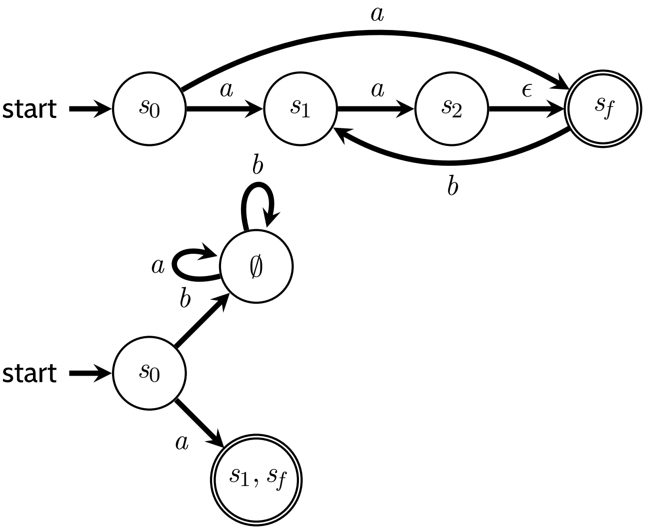

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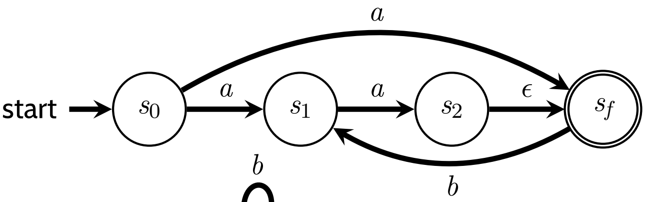

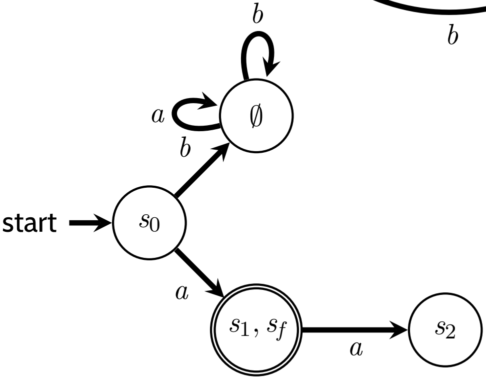

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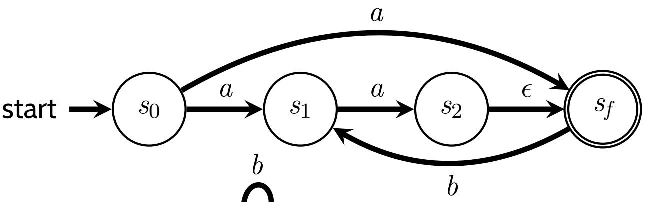
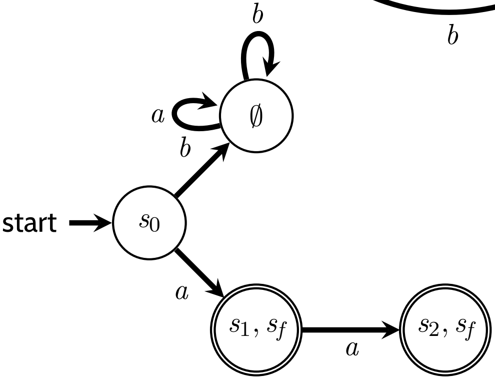
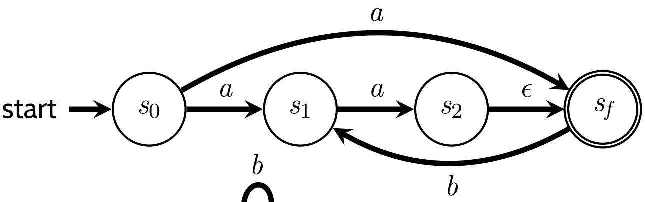
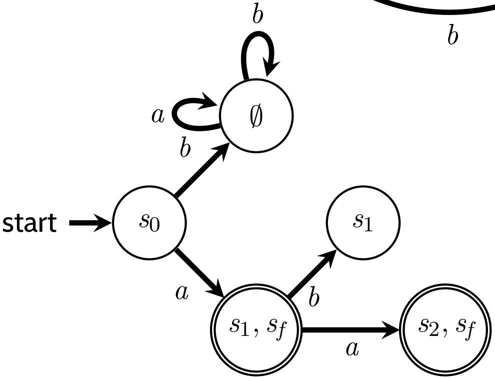

start
s_0
a
s_1
a
s_2
ϵ
s_f
a
b

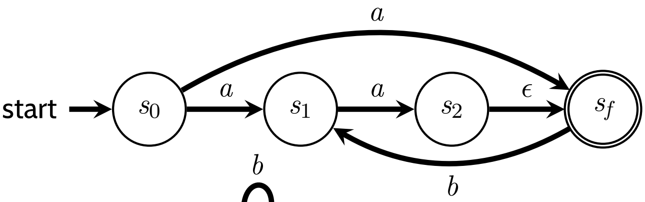

start
s_0
b
$\emptyset$
a
b
b
a
s_1, s_f
a
s_2, s_f

NFA $\rightarrow$ DFA





NFA → DFA



start
s_0
a
s_1
a
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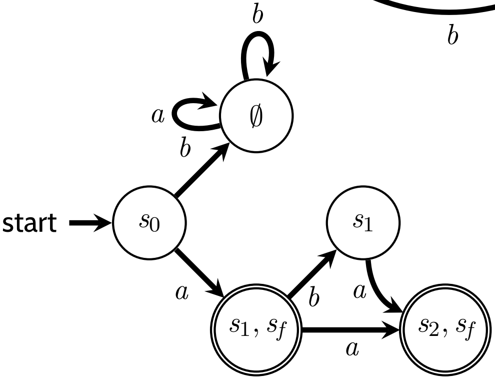
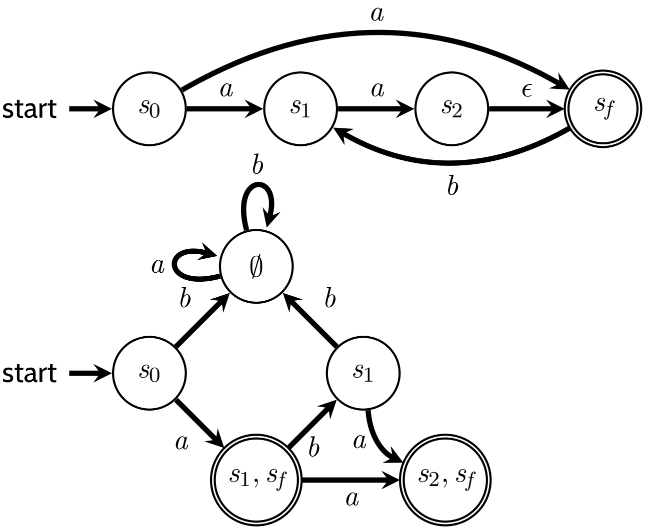
s_f
a
b

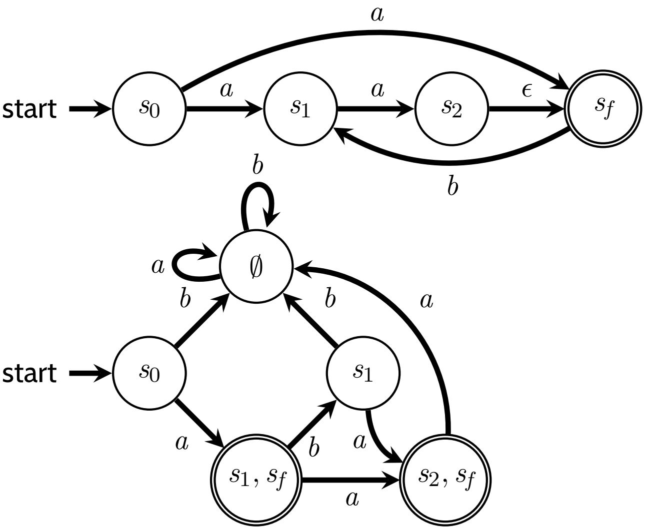


b
a
$\emptyset$
b
start
s_0
s_1
a
b
a
s_1, s_f
a
s_2, s_f

NFA $\rightarrow$ DFA

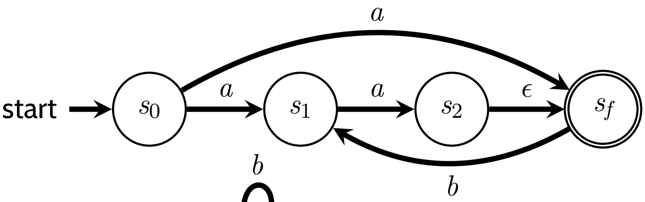


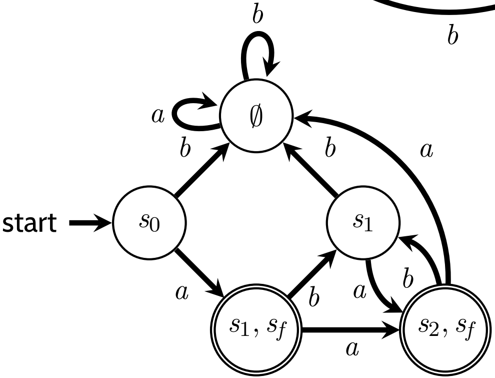
NFA → DFA



start
s_0
s_1
s_2
s_f
a
a
a
ϵ
b
start
s_0
$\emptyset$
s_1
s_1, s_f
s_2, s_f
a
b
b
b
b
a
a
b
a
a

NFA $\rightarrow$ DFA





NFA $\to$ DFA, formally

- Have: NFA $A = (Q, \Sigma, \delta, s, F)$. Want: DFA $A' = (Q', \Sigma, \delta', s', F')$ that accepts same language.
- For any $S \subseteq Q$, define the ϵ-closure of S to be the set of states reachable from S by ϵ transitions (incl. S)
  ϵ-$cl(S)$ = smallest set that contains S and such that $\forall (q, \epsilon, q') \in \Delta,\ q \in S \Rightarrow q' \in S$

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- Construct DFA as follows:
  - $Q' =$ set of all ϵ-closed subsets of Q
  - $\delta'(S, a) = \epsilon$-closure of $\{q_2 : \exists q_1 \in S.(q_1, a, q_2) \in \Delta\}$
  - $s' = \epsilon$-closure of $\{s\}$
  - $F' = \{S \in Q' : S \cap F \neq \emptyset\}$

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- Crucial optimization: only construct states that are reachable from s'

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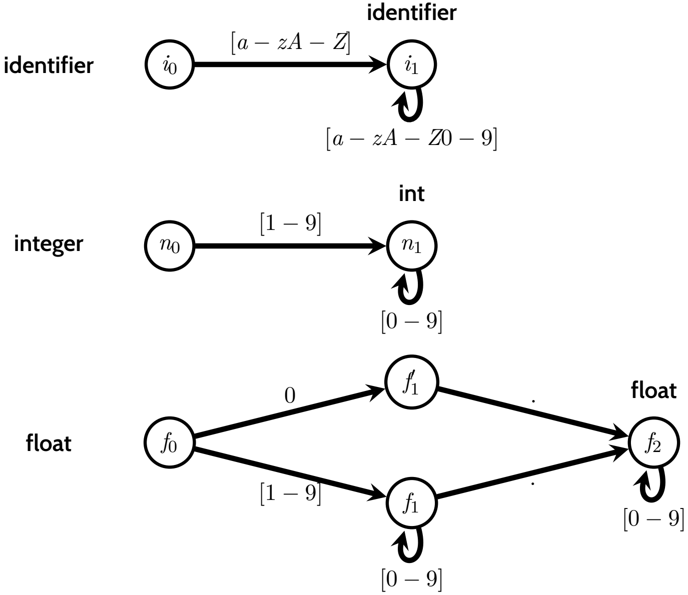
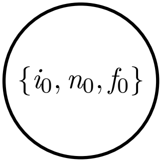
- Crucial optimization: only construct states that are reachable from s'
- Less crucial, still important: minimize DFA (Hopcroft's algorithm, $O(n \log n)$)

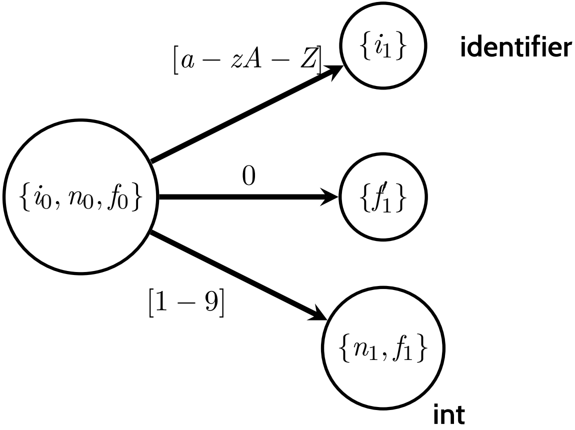
Lexical specification $\rightarrow$ String classifier

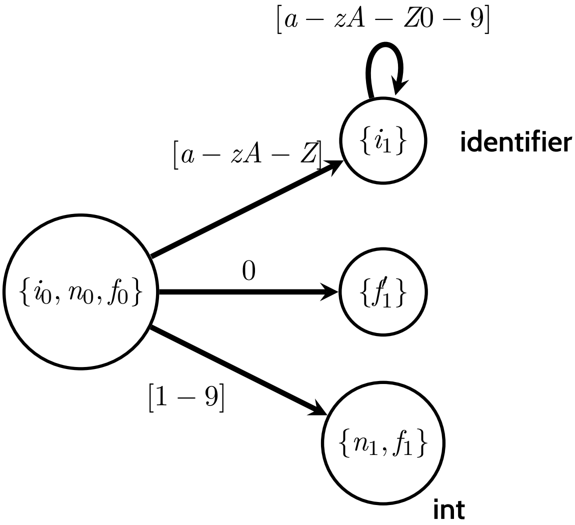
- Want: partial function *match* mapping strings to token types
  - *match*(s) = highest-priority token type whose pattern matches s (undef otherwise)
- Process:
  1. Convert each pattern to an NFA. Label accepting states w/ token types.
  2. Take the union of all NFAs
  3. Convert to DFA
     - States of the DFA labeled with *sets* of token types.
     - Take highest priority.

$$\begin{aligned}\textbf{identifier} &= [a-zA-Z][a-zA-Z0-9]^* \\ \textbf{integer} &= [1-9][0-9]^* \\ \textbf{float} &= ([1-9][0-9]^*|0).[0-9]^+\end{aligned}$$


identifier
identifier
$[a-zA-Z]$
i_0
i_1
$[a-zA-Z0-9]$
int
integer
$[1-9]$
n_0
n_1
$[0-9]$
float
float
0
f_0
f_1'
$.$
f_2
$[1-9]$
f_1
$.$
$[0-9]$
$[0-9]$


$\{i_0, n_0, f_0\}$


$\{i_0, n_0, f_0\}$
$[a-zA-Z]$
$\{i_1\}$
identifier
0
$\{f_1'\}$
$[1-9]$
$\{n_1, f_1\}$
int



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identifier
$[a-zA-Z]$
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0
$\{f_1'\}$
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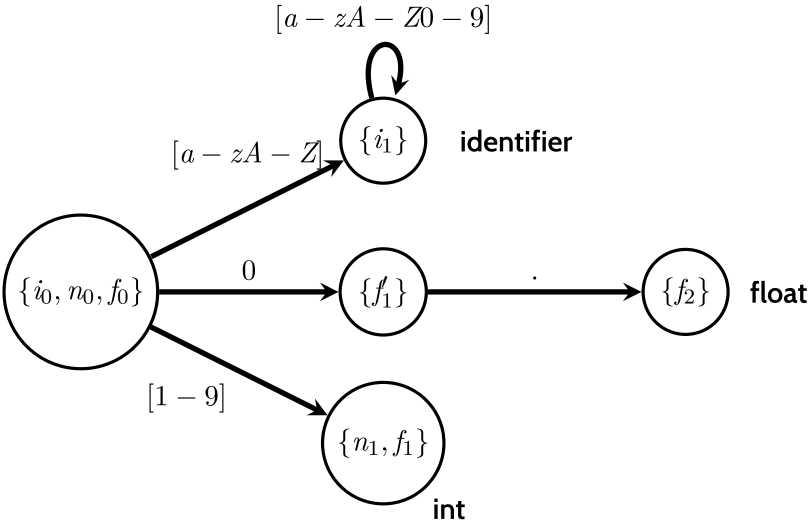
int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$\{f_2\}$
float
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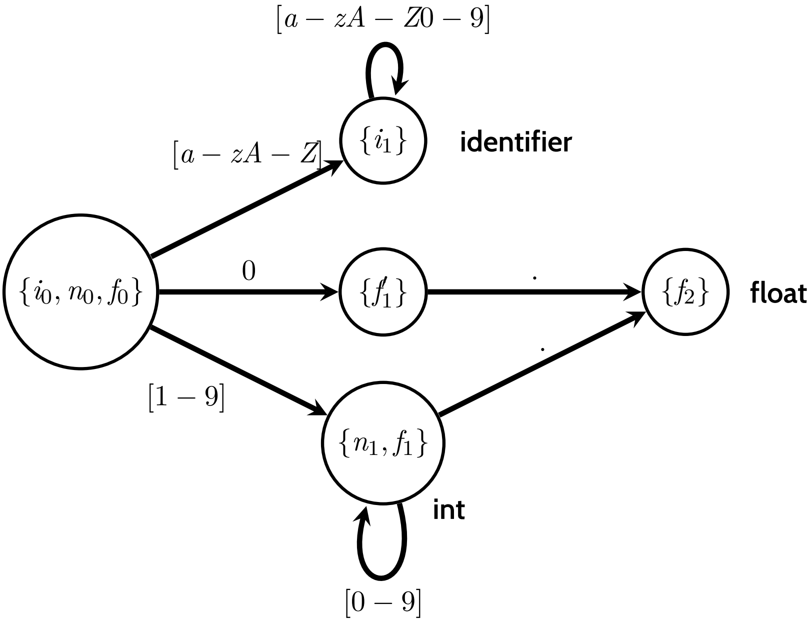
int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$\{f_2\}$
float
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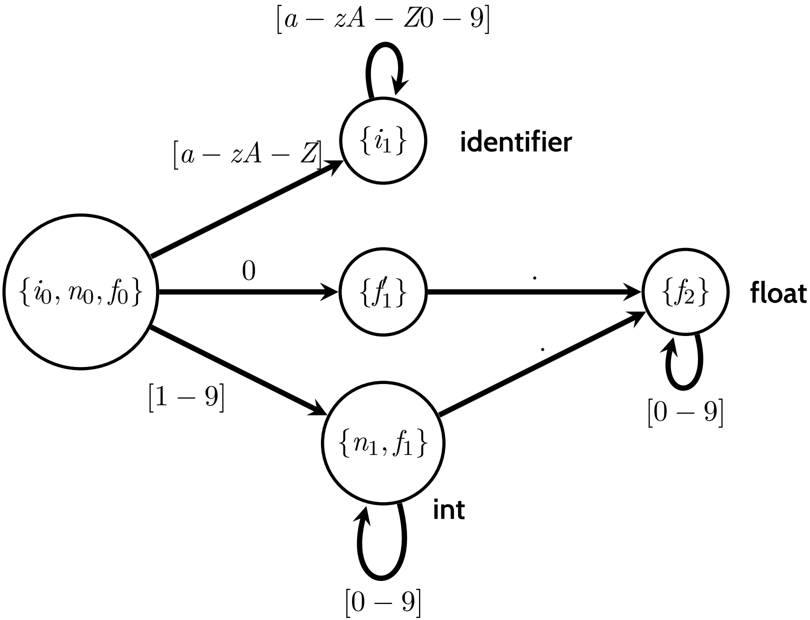
$[1-9]$
$\{n_1, f_1\}$
int
$[0-9]$



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$\{f_1'\}$
.
$\{f_2\}$
float
.
$[1-9]$
$\{n_1, f_1\}$
int
$[0-9]$
$[0-9]$