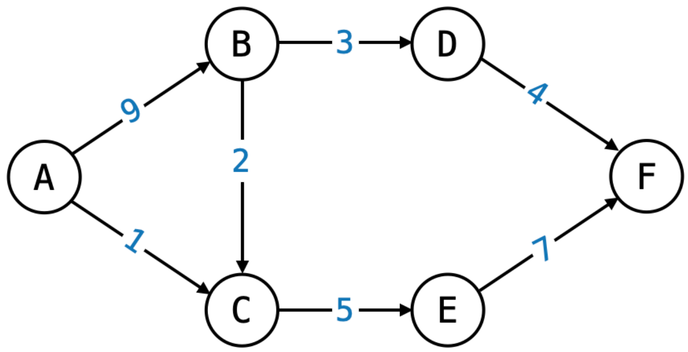
EXERCISE 1: Shortest Paths

Consider the following digraph:



A. Assuming ***A*** is the source vertex, relax all the vertices in ***topological order*** to find the shortest path from ***A*** to all the vertices in the graph.

Topological Order :

1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____

Length of shortest path from ***A*** to each vertex :

A: _____ B: _____ C: _____ D: _____ E: _____ F: _____

B. In what order will the vertices be removed from the priority queue (and relaxed) if **Dijkstra**'s algorithm is used?

1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____

C. Bellman-Ford's algorithm performs $V - 1$ passes; relaxing all vertices in the graph at each pass.

- Find a vertex order that leads to the ***minimum*** number of passes before finding the shortest paths on this graph.

- Find a vertex order that leads to the ***maximum*** number of passes before finding the shortest paths on this graph.

EXERCISE 2: Seam Carving

Consider the given 3x4 image and the corresponding energies matrix.

- A *Vertical Seam* is a path of pixels connected from the top row to the bottom row , where a pixel at column x and row y can only be connected to the pixels $(x-1, y+1)$, $(x, y+1)$ and $(x+1, y+1)$.
- The *Seam Energy* is the sum of the energies of the pixels in the seam.
- A *Minimum Energy Vertical Seam* is the vertical seam with the minimum energy.

(15,10,16)	(31,15,19)	(15,10,3)
(5,18,0)	(80,18,0)	(120,100,80)
(35,20,12)	(36,17,13)	(15,10,3)
(5,1,13)	(13,1,16)	(120,110,40)

RGB Values of the 3x4 Image

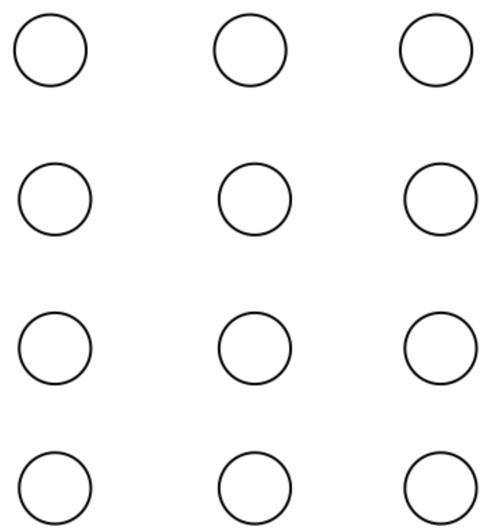
32	72	45
123	163	75
32	75	41
156	161	9

Energy Values (Rounded)

A. Mark the *minimum energy vertical seam* in the given energies matrix. What is the energy of this seam?

B. In order to find the minimum energy vertical seam, you will have to find the shortest path from any pixel in the top row to any pixel in the bottom row.

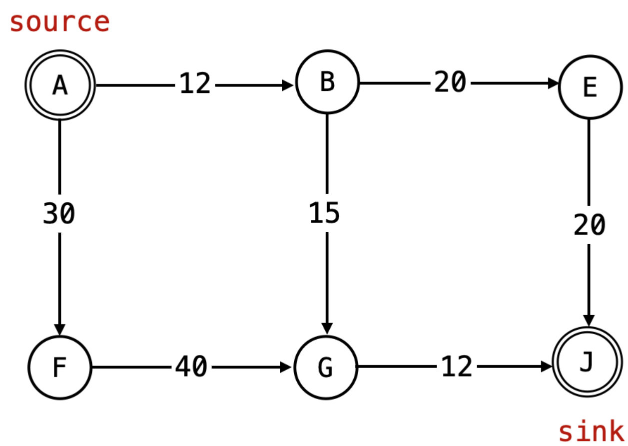
Draw the implicit graph which the energies matrix represents. Show all the edges and edge weights.



C. Assume that the image is of size $W \times H$, what is the order of growth of the running time to find the minimum energy vertical Seam? (use *W* and *H*)

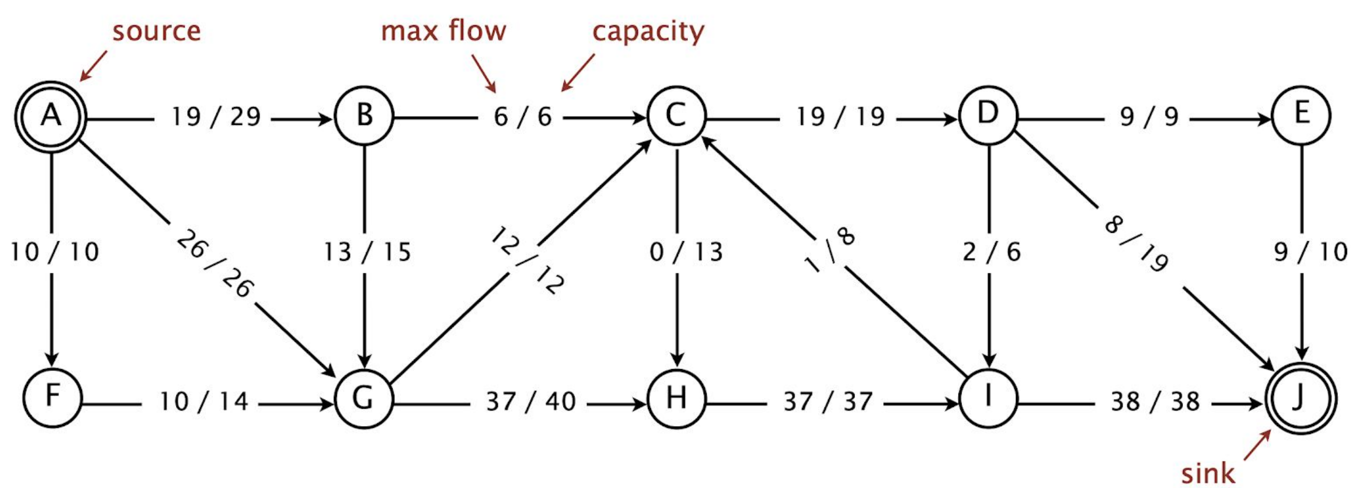
EXERCISE 3: Maximum Flow

A. Use **Ford-Fulkerson** to find the maximum flow in the following network.



B. List all the *possible* ways Ford-Fulkerson could run on the above network. For each possible run, list the vertices on each augmenting path.

C. Consider the following flow network and the maximum flow f^*.



- What is the **capacity** of the cut $\{A, B, C\}$? What is the net **flow** across this cut?
- Which vertices are on the source side of the **minimum cut**?
- What is the **capacity** of the minimum cut? What is the net **flow** across the minimum cut?
- What is the maximum flow f^* ?