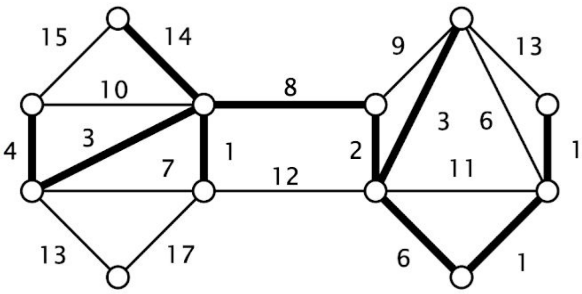
Online (auto-graded) version: https://stepik.org/lesson/219467

EXERCISE 1: Minimum Spanning Trees

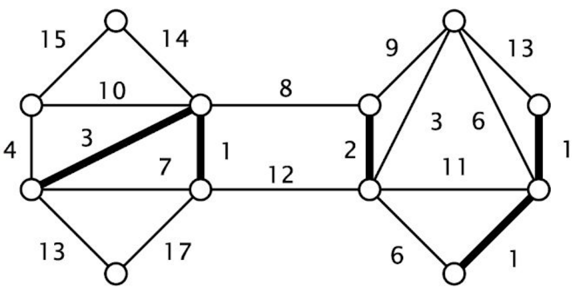
Each of the figures below represents a *partial* spanning tree. Determine whether it could possibly be obtained from (a prematurely stopped) *Kruskal's* algorithm, (a prematurely stopped) *Prim's* algorithm, *both* or *neither*.

	Kruskal	Prim	Both	Neither
(A)	○	○	○	○
(B)	○	○	○	○
(C)	○	○	○	○
(D)	○	○	○	○

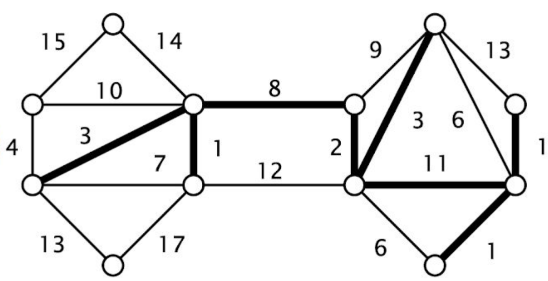
(A)



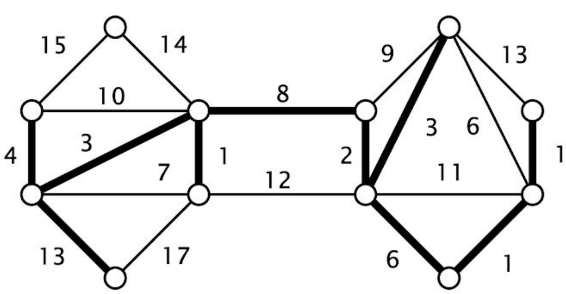
(B)



(C)



(D)

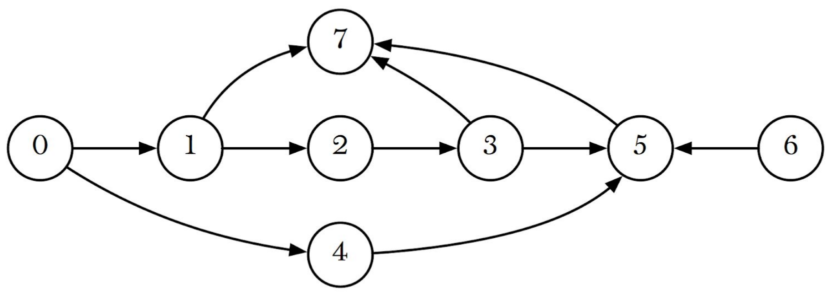


EXERCISE 2: Shortest Common Ancestor

In a directed graph, a vertex x is an ancestor of v if there exists a (directed) path from v to x. Given two vertices v and w in a rooted directed acyclic graph (DAG), a shortest common ancestor `sca(v, w)` is a vertex x which:

- is an ancestor to both v and w;
- minimizes the sum of the distances from v to x and w to x (this path, which goes from v to x to w, is the shortest ancestral path between v and w).

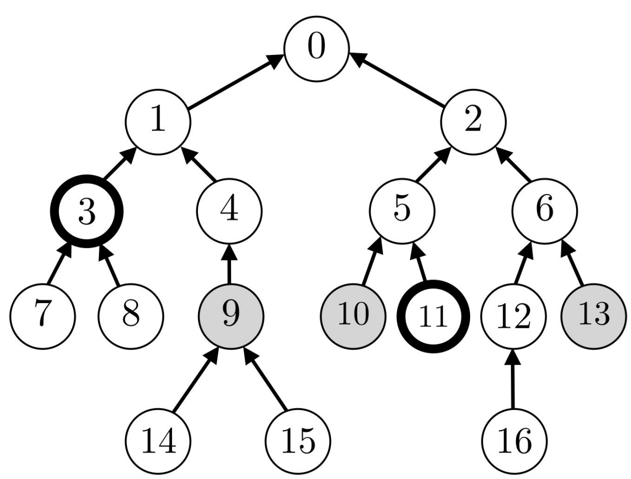
A. In the following digraph, find the shortest common ancestor of vertices `1` and `4`, and give the sum of the path lengths from these vertices to all common ancestors, and then circle the shortest.



B. Describe an algorithm for calculating the shortest common ancestor of two vertices v and w. Your algorithm should run in linear time (proportional to $V + E$).

C. How would your algorithm differ if we are interested in the shortest ancestral path between two ***sets*** of vertices A and B instead of two vertices? I.e. between any vertex v in A and any vertex w in B.

In the example, $A = 3, 11$ and $B = 9, 10, 13$. The shortest common ancestor is 5 (between 10 and 11).



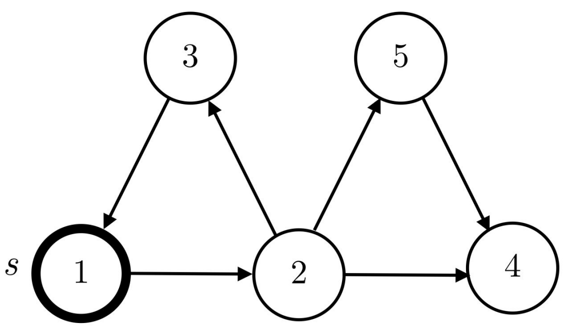
EXERCISE 3: Detecting Directed Cycles

A. Consider the graph G given below and the marked vertex s. Show in the given box what the output would be if `depthFirstSearch` is called on G and s.

```java
private boolean[] marked;

public void depthFirstSearch(Digraph G, int s) {
      marked = new boolean[G.V()];
      dfs(G, s);
}

private void dfs(Digraph G, int v) {
      marked[v] = true;
      StdOut.println("Started " v);
      for (int w : G.adj(v)) {
            if (!marked[w])
                  dfs(G, w);
      }
      StdOut.println("Finished " + v);
}
```



B. Consider the following modified version of the `dfs` method. Explain with the simplest counterexample why this code is not a correct cycle detection code.

```java
private void dfs(Digraph G, int v) {
      marked[v] = true;

      for (int w : G.adj(v)) {
            if (!marked[w])
                  dfs(G, w);
            else StdOut.print("Cycle found!");
      }
}
```

C. Briefly describe how depth-first search could be modified to detect cycles in a digraph.

D. Fill the blank lines in the following DFS code so that it prints "`Cycle found!`" if and only if there is a cycle in the graph. Assume that the graph is connected.

```
private boolean[] marked;
private boolean[] onStack;

public void checkCycles(Digraph G, int s) {
      marked = new boolean[G.V()];
      ___________________________________________
      dfs(G, s);
}

private void dfs(Graph G, int v) {
      marked[v] = true;
      ___________________________________________
      for (int w : G.adj(v)) {
            if (!marked[w])
                  dfs(G, w);
            else if (________________________)
                  StdOut.print("Cycle found!");
      }
      ___________________________________________
}
```