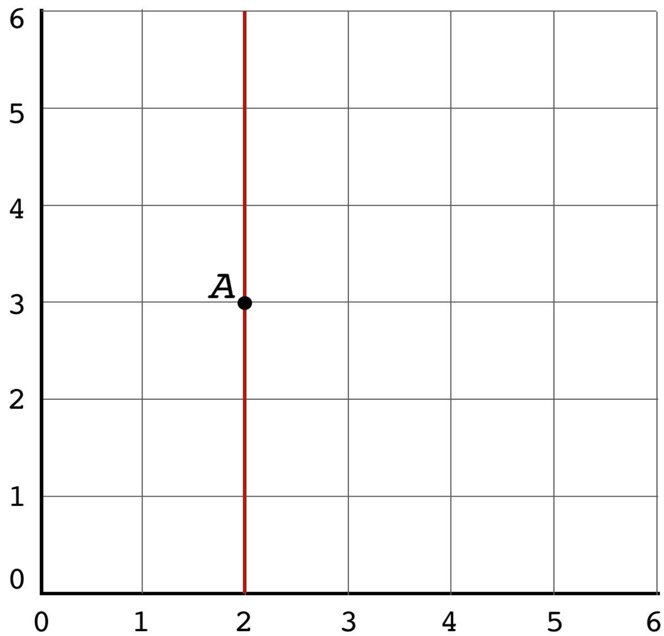
EXERCISE 1: Kd-Trees

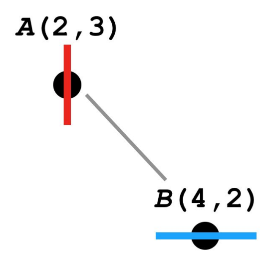
(a) Draw the Kd-tree that results from inserting the following points:

`[A(2, 3), B(4, 2), C(4, 5), D(3, 3), E(1, 5), F(4, 4), G(1, 1)]`

Draw each point on the grid, as well as the vertical or horizontal line that runs through the point and partitions the plane, or a subregion of it.

Note: While inserting, go left if the coordinate of the inserted point is less than the coordinate of the current node. Go right if it is greater than ***or equal***.



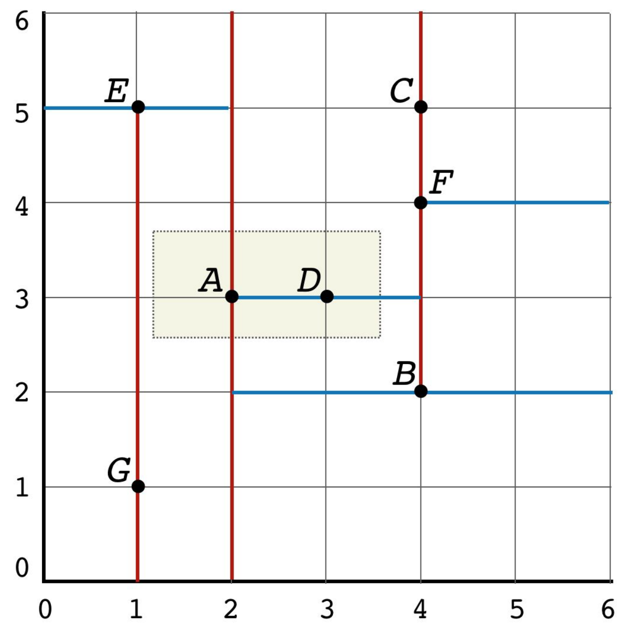


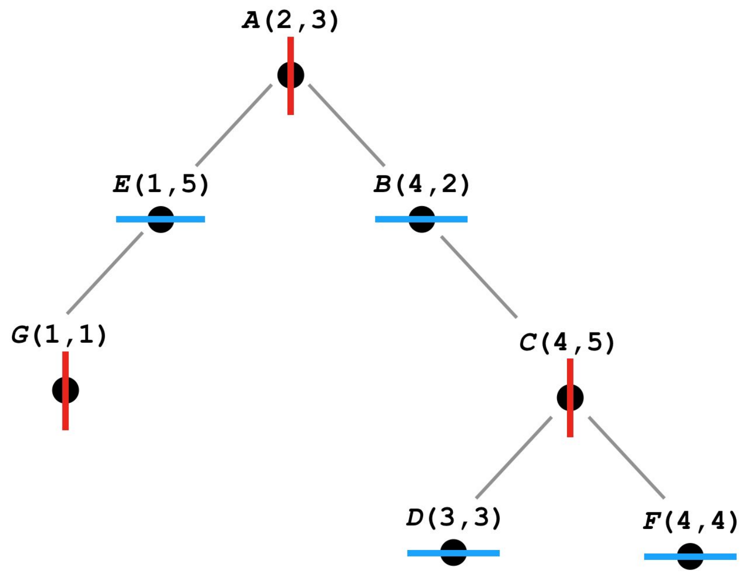
(b) Give each point's bounding rectangle `[(xmin, ymin), (xmax, ymax)]`.

A(2, 3)	$[(-\infty, -\infty), (+\infty, +\infty)]$
B(4, 2)	
C(4, 5)	
D(3, 3)	
F(4, 4)	$[(4, 2), (+\infty, +\infty)]$
E(1, 5)	$[(-\infty, -\infty), (2, +\infty)]$
G(1, 1)	

(c) Number the tree nodes according to the visiting order when performing a *range query* using the rectangle shown below. Label pruned subtrees with ✗.

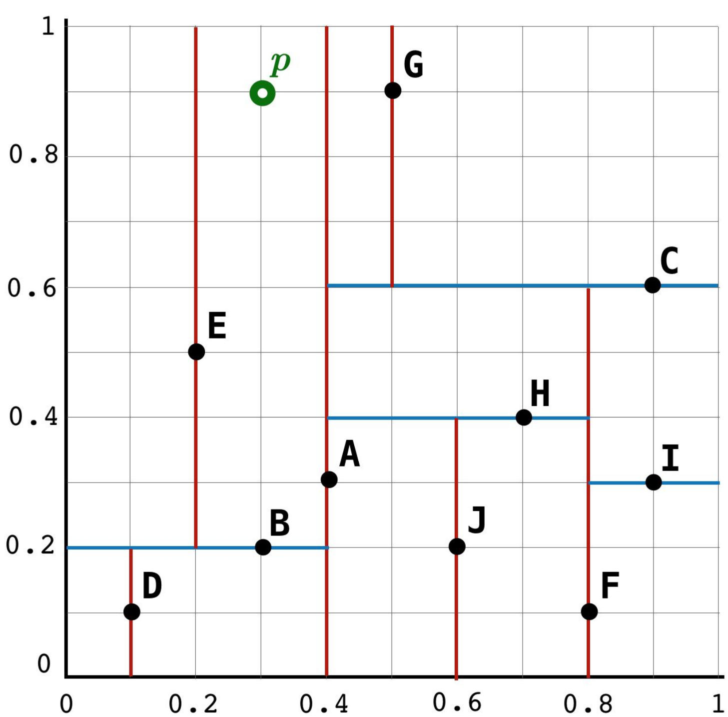
Remember. The range search algorithm recursively searches in both the left and right subtrees unless the bounding rectangle of the *current* node does not intersect the query rectangle.

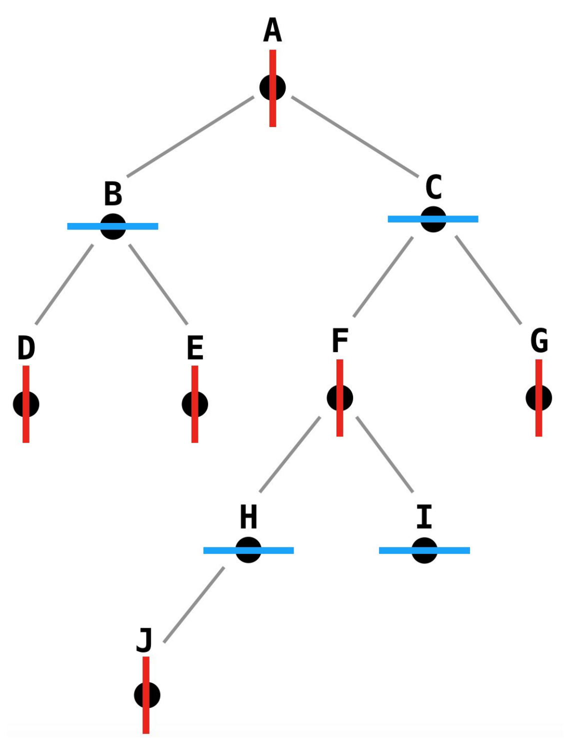




(d) Number the tree nodes according to the visiting order when performing a *nearest neighbor (NN) query* using the point ***p*** and the Kd-Tree shown below. Label pruned subtrees with ✗.

Remember. The NN algorithm recursively searches in *both* the left and right subtrees unless the distance between *p* and the bounding rectangle of the *current* node is not less than the distance between *p* and the nearest point found so far.





EXERCISE 2: Operations on Binary Trees

Consider the following Binary Search Tree class for storing integers.

```java
public class BinarySearchTree {
      private Node root;
      private class Node {
            private int key;
            private Node left, right;

            private int size;       // # of nodes in subtree rooted here

            public Node(int key, int size) {
                  this.key = key;
                  this.size = size;
            }
      }

      private int size(Node x) { /* returns x.size or 0 if x is null. */ }
      // ... other public and private methods
}
```

(a) What modifications should be made to the `put()` method to update the subtree counts?

```java
public void put(Key key) {
    root = put(root, key);
}
private Node put(Node x, Key key) {
    if (x == null) return new Node(key, 1);

    if (key < x.key) x.left = put(x.left, key);
    else if (key > x.key) x.right = put(x.right, key);



    return x;
}
```

(b) Assume the tree is a *left-leaning red-black BST.* Is the modification you have introduced to `put()` in part (a) enough to maintain the subtree counts? Why?

(c) Implement method `rank()`, which returns the number of keys in the BST that are strictly less than the given key.

```
public int rank(int key) {
    return rank(key, root);
}

private int rank(int key, Node x) {
}
```

Extra (Optional) Exercise:

(d) Implement `int rangeCount(int lo, int hi)`. This method should return the number of keys in the BST that are between `lo` and `hi` (inclusive).

```
// Returns the number of keys in the symbol table in the given range.
public int rangeCount(int lo, int hi)
}
```

More Ordered Operations on BSTs:

Implement method `Node select(int rank)`, which returns the node in the tree with the key of a given `rank`.

(See pages 406-409 in the textbook)

```
private Node select(Node x, int rank) {
      if (x == null)
            return null;

      int leftSize = size(x.left);

      if (leftSize > rank) return select(x.left, rank);
      else if (leftSize < rank) return select(x.right, rank - leftSize - 1);
      else return x;
}
```

Java References Reminder. While updating the champion in the NN method, many students face bugs that are avoidable if the following is understood:

- If the recursive method returns a value, make sure to catch the return value and use it whenever you make a recursive call.
- If the recursive method receives an argument that needs to be updated, note that the following works:

  ```
  void myMethod(Type1 arg1, Type2 result) {
      result.setX(someValue);
      ...
  ```

  But the following does not work:

  ```
  void myMethod(Type1 arg1, Type2 result) {
      result = new Type2(someValue);
      ...
  ```

  In the first example, `result` is a reference, and the method is changing X in the object referenced by `result`, so the effect will be applied to the object passed to the method as an argument.

  In the second example, `result` is a reference that is pointing to the object passed to the method as an argument. By doing `result = ...`, we make the reference `result` point to a new object other than the one passed as an argument. In other words, the object passed as an argument will not be affected by the operation.