From AM Radio to Digital I/Q Modulation

COS 463: Wireless Networks
Lecture 12

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[Parts adapted from H. Balakrishnan, M. Perrott, C. Terman]

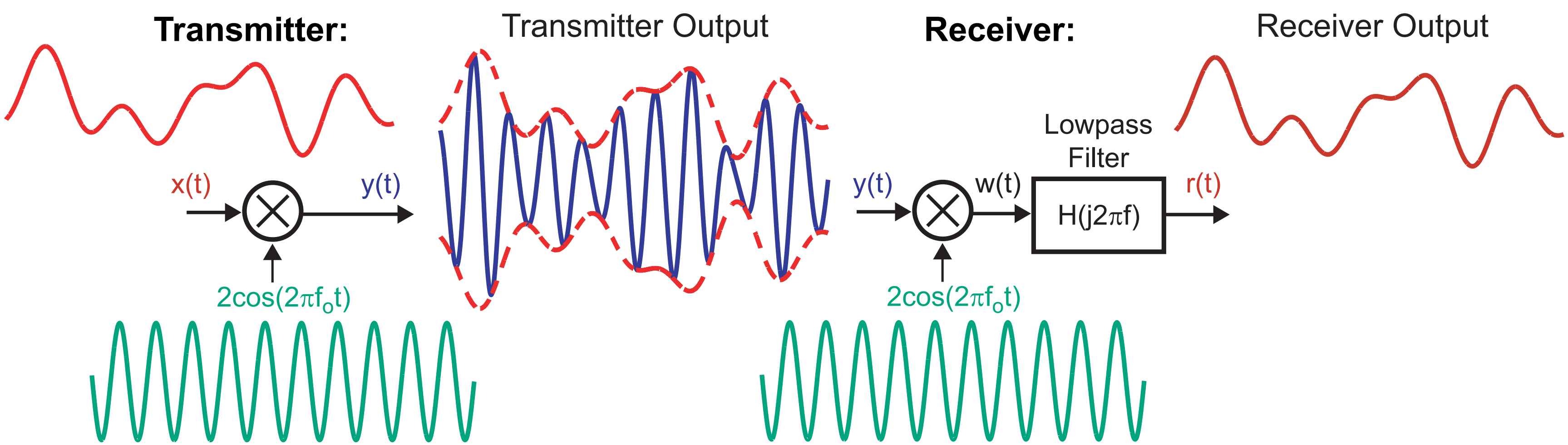
Roadmap

1. **Analog I/Q modulation**

2. Discrete-time processing of continuous signals
   – The Digital Abstraction
   – Quantization: Discretizing *values*
   – Sequences: Discretizing *time*

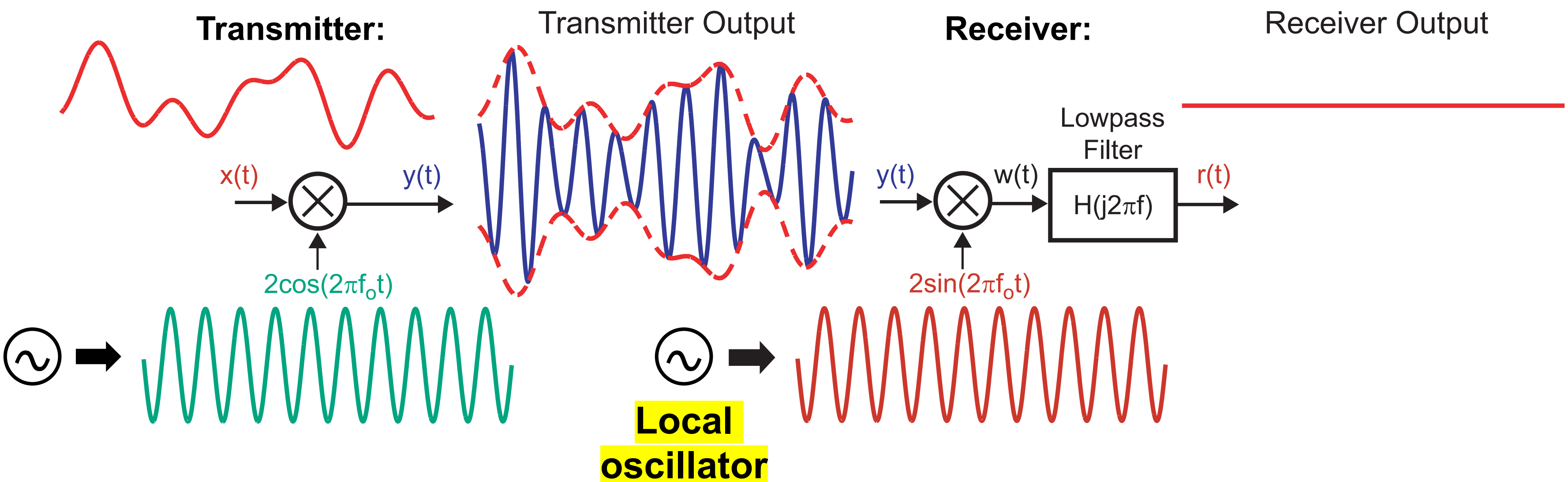
3. Digital I/Q Modulation

Review: AM Modulation & Demodulation



- **Multiplication** (*i.e.* ***mixing***) operation **shifts in frequency**
- **Low-pass filtering** passes **only the desired baseband signal** at the receiver
- **Works with cos or sin carrier signal**

Impact of a 90º phase shift

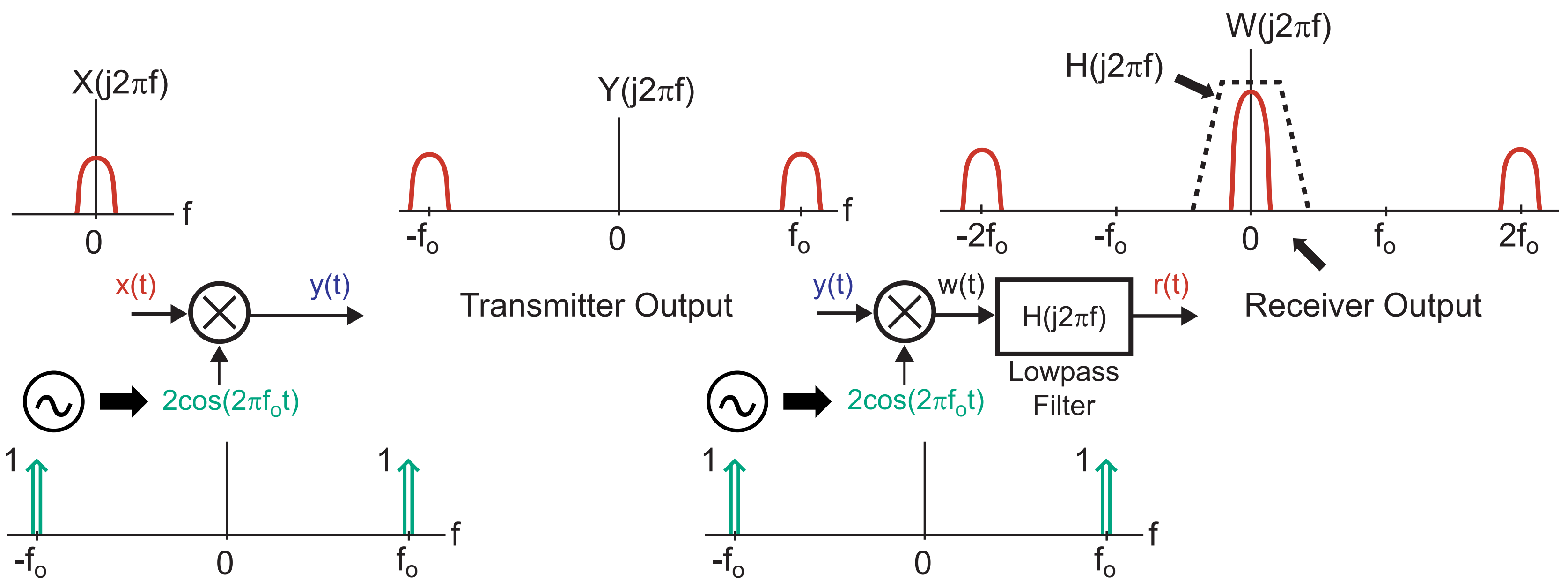


- **Suppose** receiver uses **sine,** instead of **cosine:** **no output**

$$\sin(2\pi f_0 t) \times \cos(2\pi f_0 t) = \tfrac{1}{2}\sin(4\pi f_0 t)$$

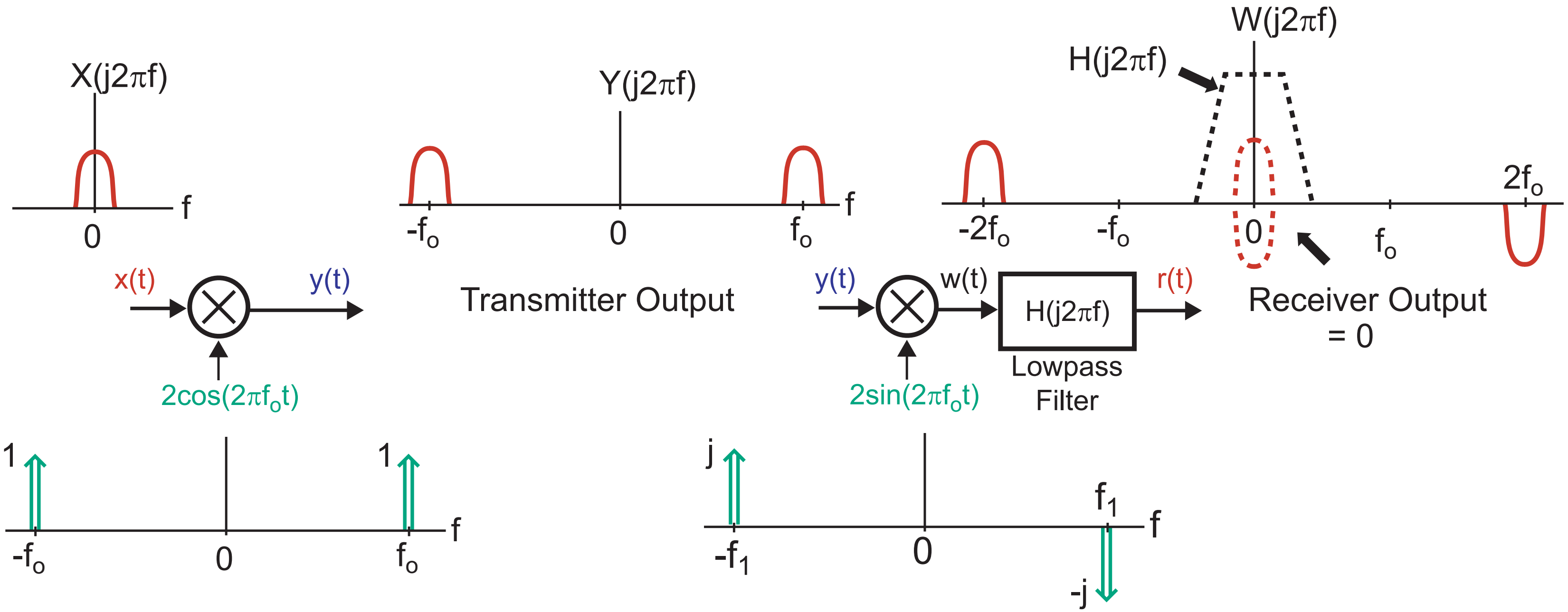
- Need to **synchronize phase** of transmitter and receiver ***local oscillators*** (***coherent*** demodulation)

Coherent Demodulation: Frequency-domain analysis



- Transmitter and receiver oscillators **phase-synchronized**
- Demodulated copies **add constructively at baseband** (close to $f = 0$)

90º Phase Shift: Frequency-domain analysis

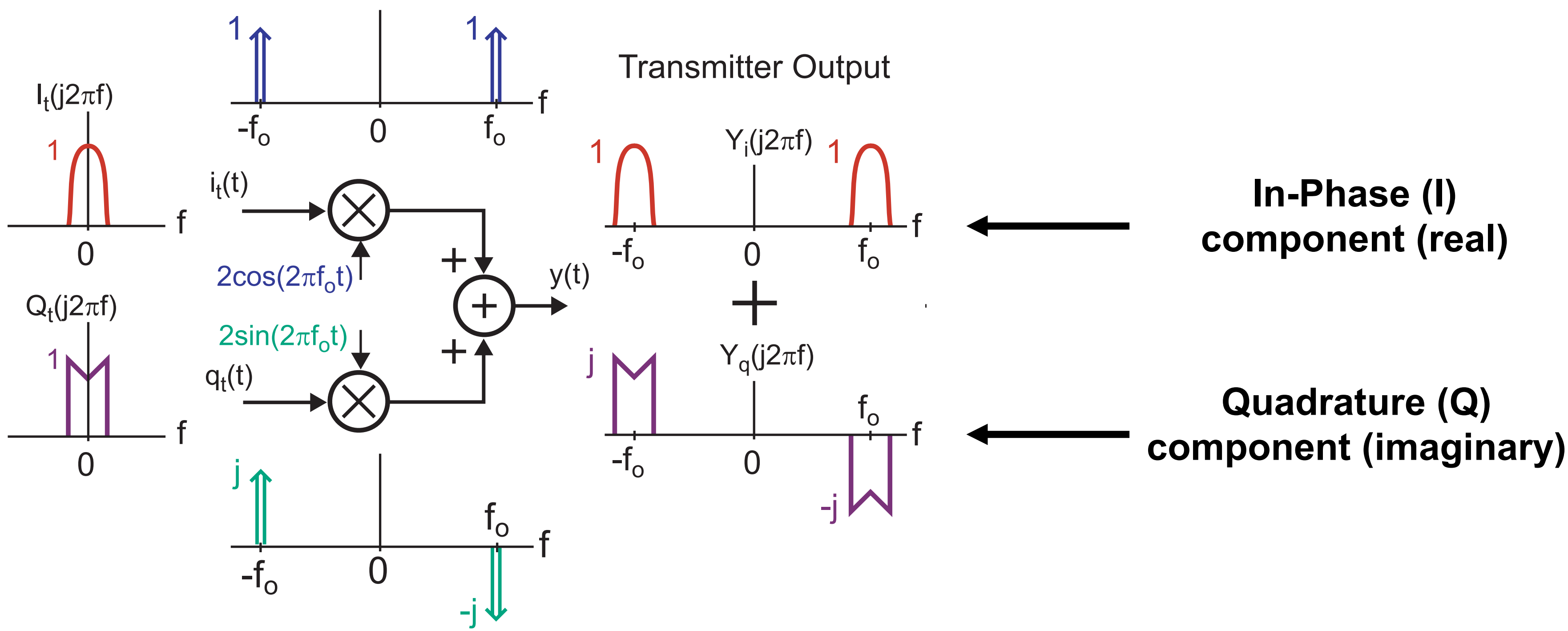


- Transmitter, receiver oscillators **are offset in phase by 90º**
- Demodulated copies **add destructively** at **baseband** (close to $f = 0$)
  - **Zero output from receiver**

- **But: Opportunity** to create **two separate channels!**

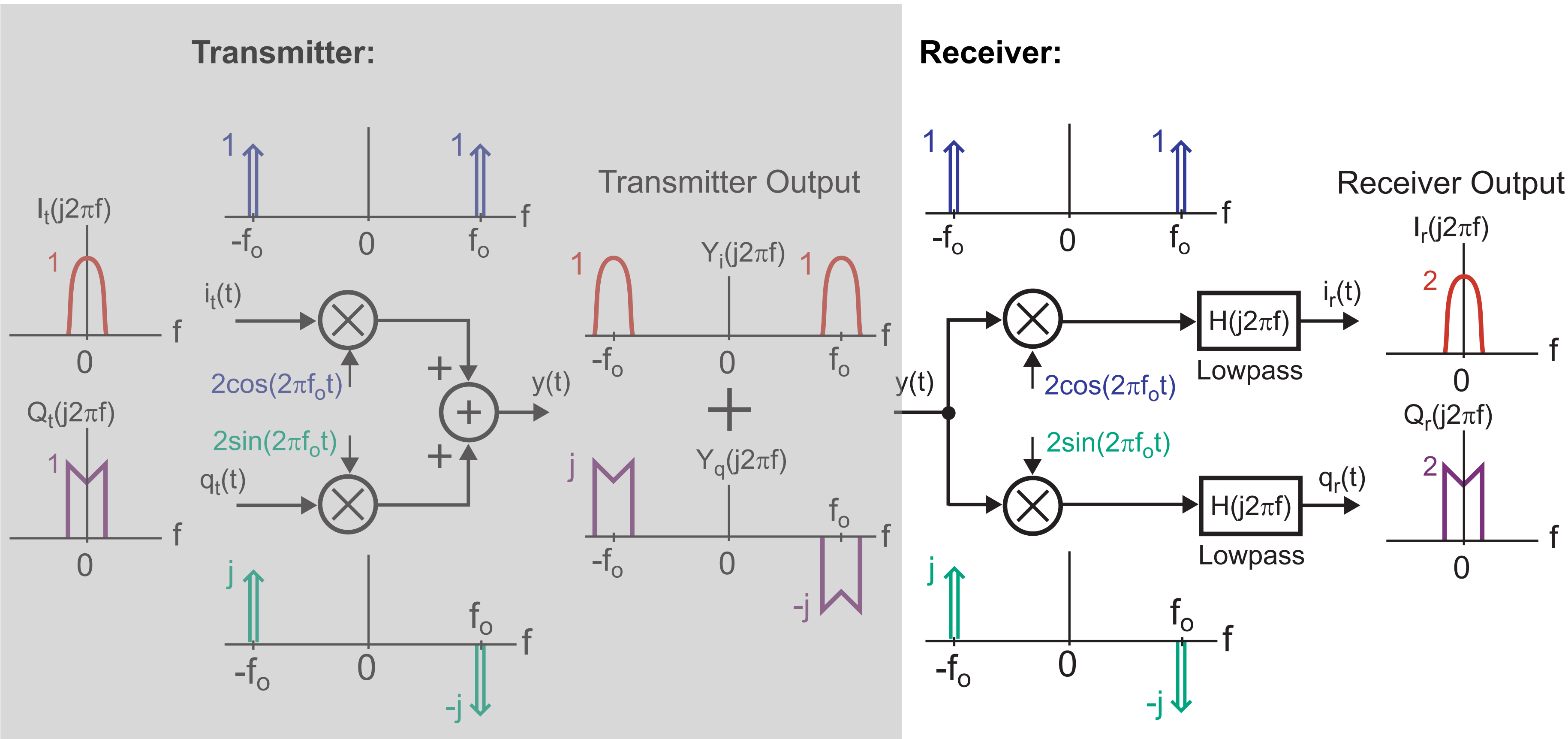
In-Phase/Quadrature Modulation

- Modulate each with a cosine & sine, then **sum** the result



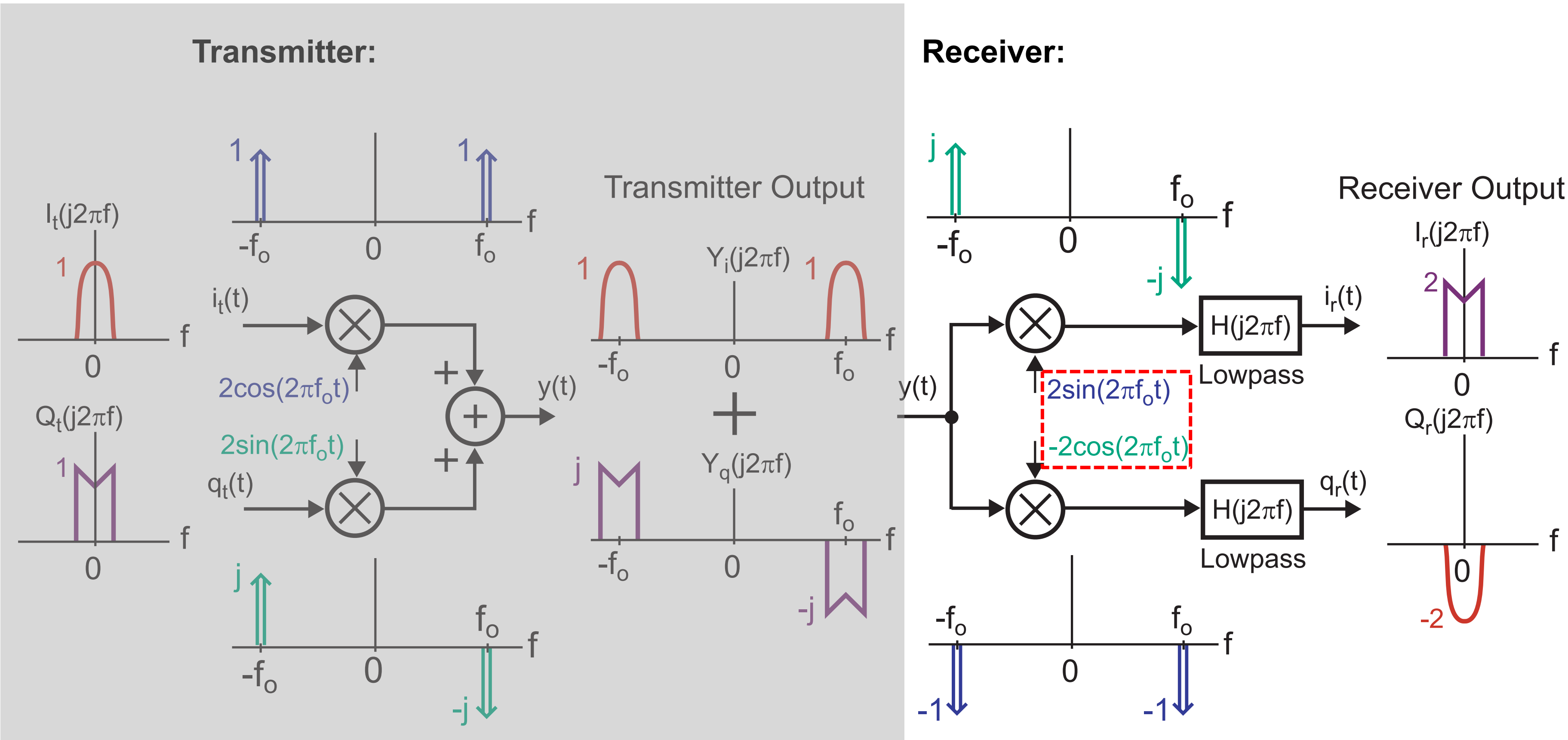
- I, Q signals occupy the **same frequency band**
  - One is **real** (**for cos**), one is **imaginary** (**for sin**)

In-Phase/Quadrature Demodulation



- Demodulate with **both** a sine and a cosine
  - **Both I and Q channels are recovered!**

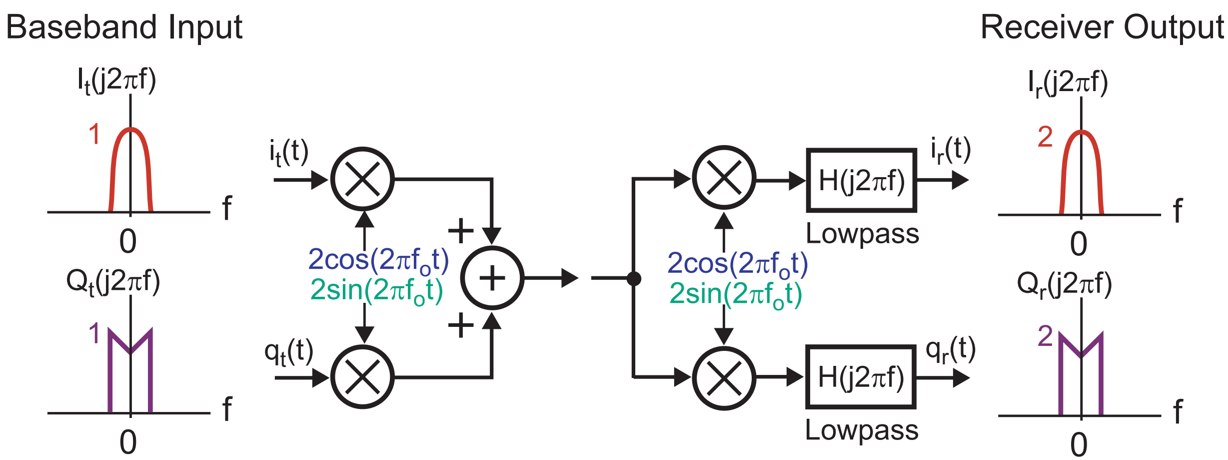
I/Q Demodulation: 90º Phase Shift



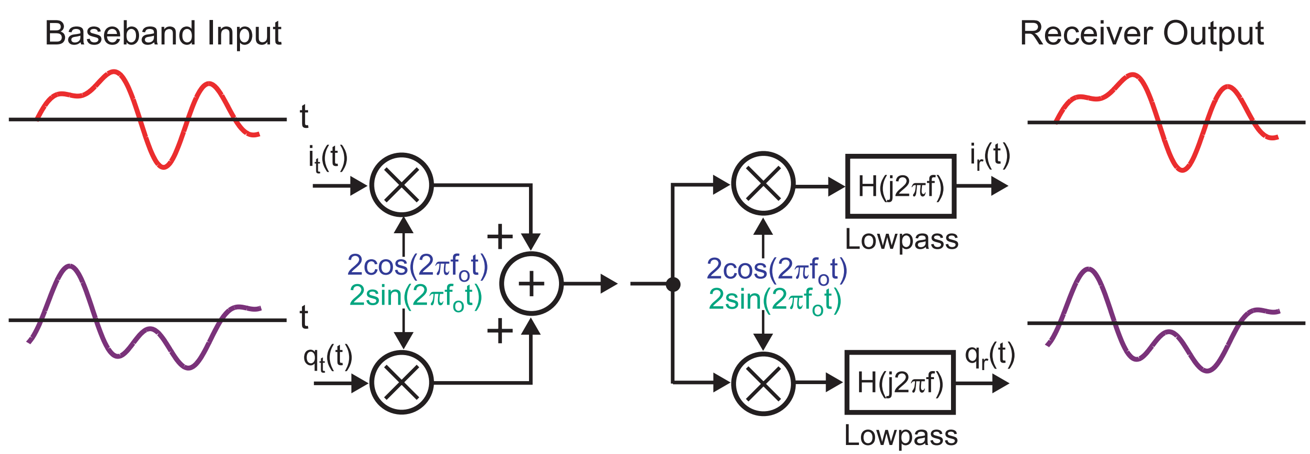
- I and Q channels get **swapped** at receiver
  - **Key observation:** No information is lost!

Summary of Analog I/Q modulation

- **Frequency domain view**



- **Time domain view**



I/Q modulation: Wrap-up

- I/Q modulation allows **twice the amount of information** to be sent **compared to basic AM**

- Impact of **phase offset** is to **swap I/Q**

- Impact of **frequency offset** is I/Q swapping (**small offset**)
  - Or **catastrophic corruption** (large offset) of received signal

Roadmap

1. Analog I/Q modulation

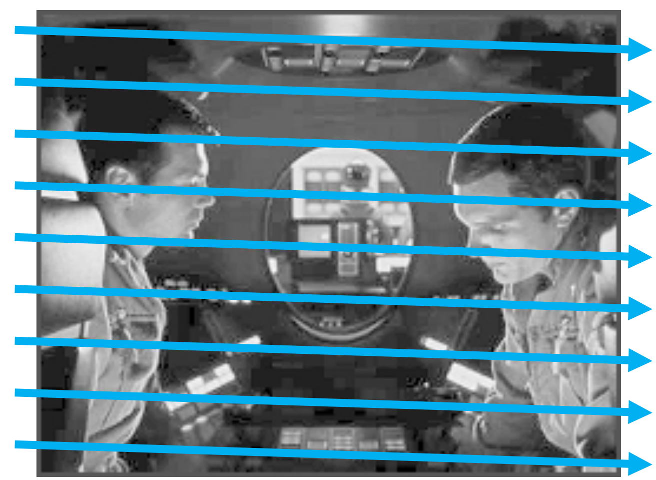
2. **Discrete-time processing of continuous signals**
   – The Digital Abstraction
   – Quantization: Discretizing *values*
   – Sequences: Discretizing *time*

3. Digital I/Q Modulation

Representing information with voltage

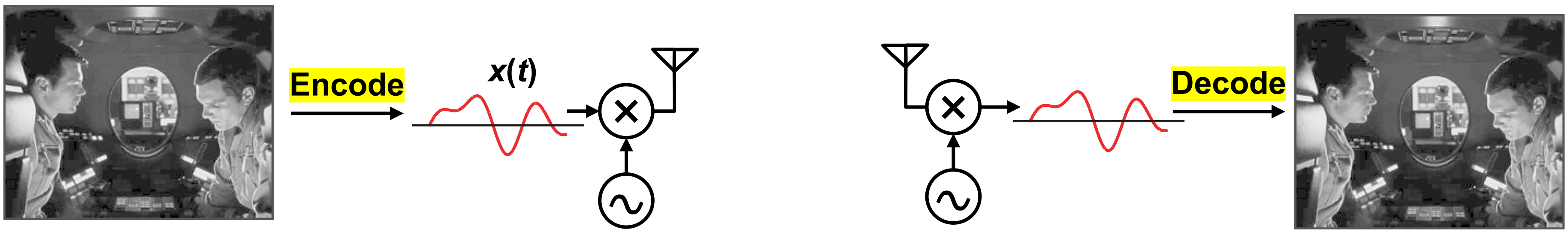
- **Encoding of pixel** at location (x, y) in a Black & White picture:
  - 0 Volts = Black
  - 1 Volt = White
  - 0.37 V = 37% Gray
  - etc...

- **Encoding of a picture:**
  - Scan points in prescribed order
  - Generate a continuous voltage waveform (**baseband signal**)

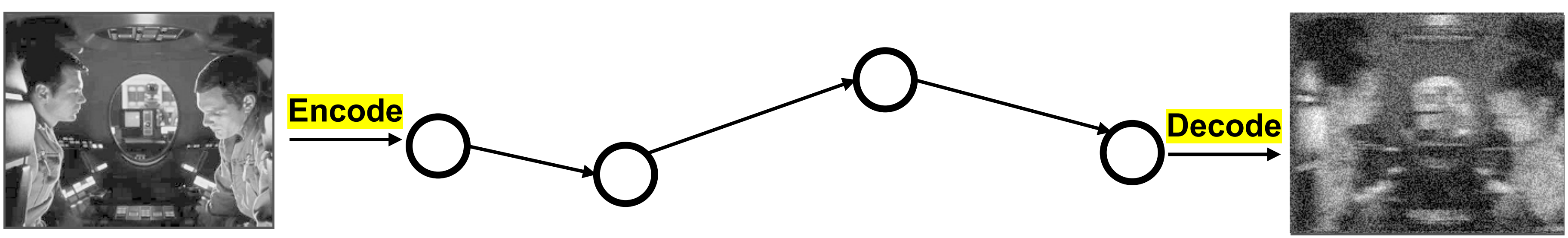


Let's build a P2P television network

One hop:



Many hops:



Why did our network fail?

1. Transmitter doesn't work right
2. Receiver doesn't work right
3. Theory is imperfect
4. System architecture isn't right

- **Answer:** All of the above!

- Noise, transmitter/receiver imperfections **inevitable**, so must **design system to tolerate** some amount of error

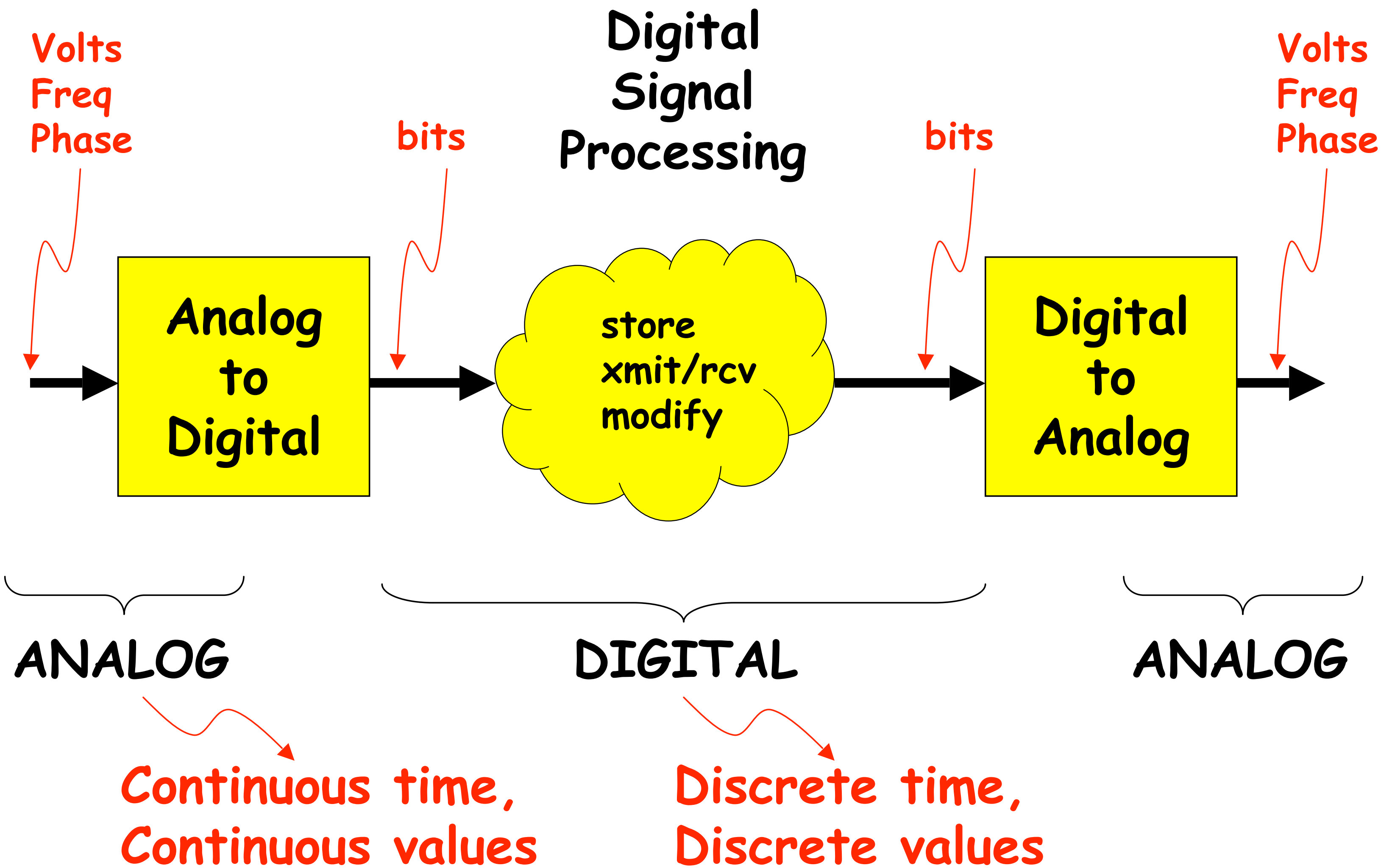
Analog communications issues

- **Problem:** It's hard to **distinguish legitimate analog waveforms from corrupted** ones
  - Every waveform is potentially legitimate!

- **Small errors accumulate** at each hop

- Endpoints can't help, so need to **eliminate errors at each hop**

Plan: *Mixed Signal* Architecture



Discrete time, Discrete values

- Continuous-time, continuous-valued **waveform** is being converted into a discrete-valued **sequence**
  - **Only certain values** are ”allowed” to be used
- Questions yet to be answered:

1. **How many discrete values** should we use?
2. **How rapidly in time** should we sample the waveform?

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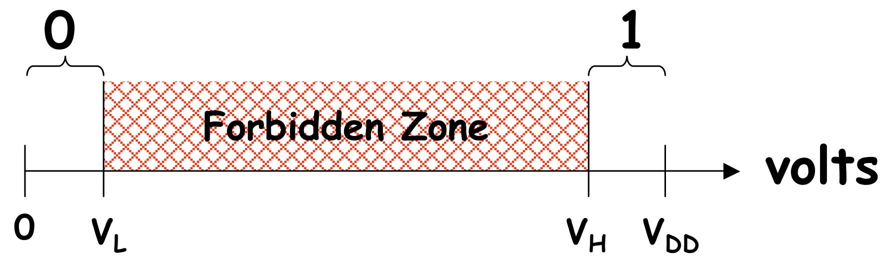
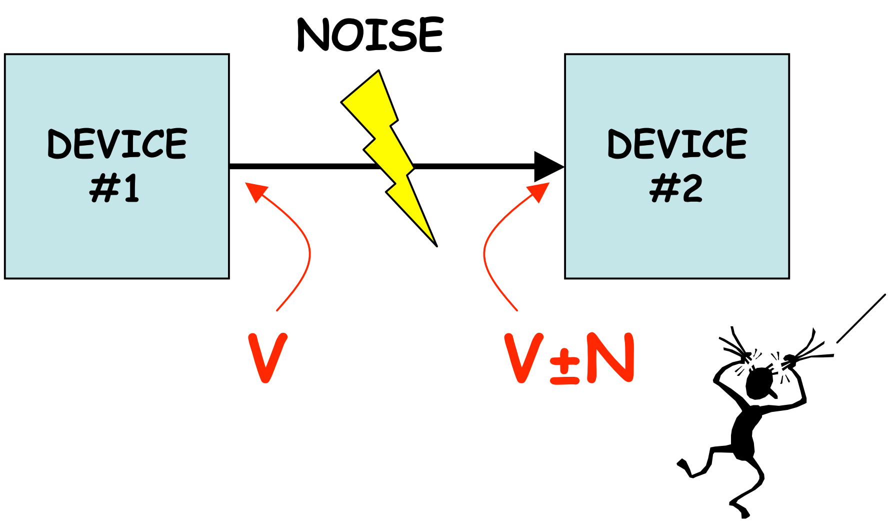
3. Digital I/Q Modulation

Encoding Information Digitally

- **First encoding attempt:**

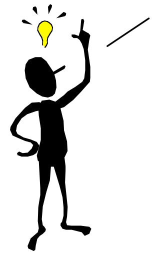


- But, what happens in the **presence of noise?**

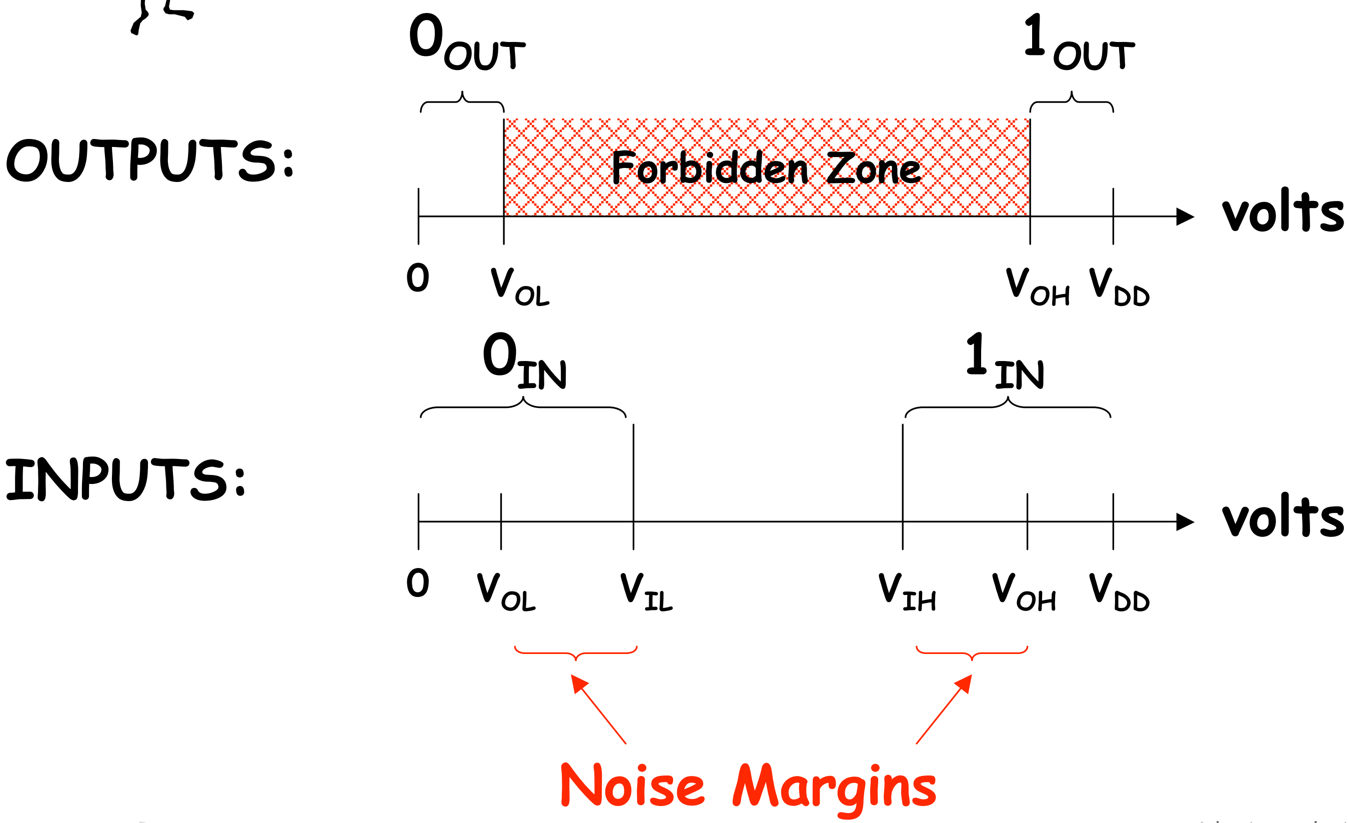


So an output voltage just below V_L might become an <u>illegal</u> input voltage in the forbidden zone!

Big Idea: Noise Margins



Let's leave room for bad things to happen! So we'll design devices restore marginally valid input signals. They must accept marginal inputs and provide unquestionable outputs (i.e., to leave room for noise).



Roadmap

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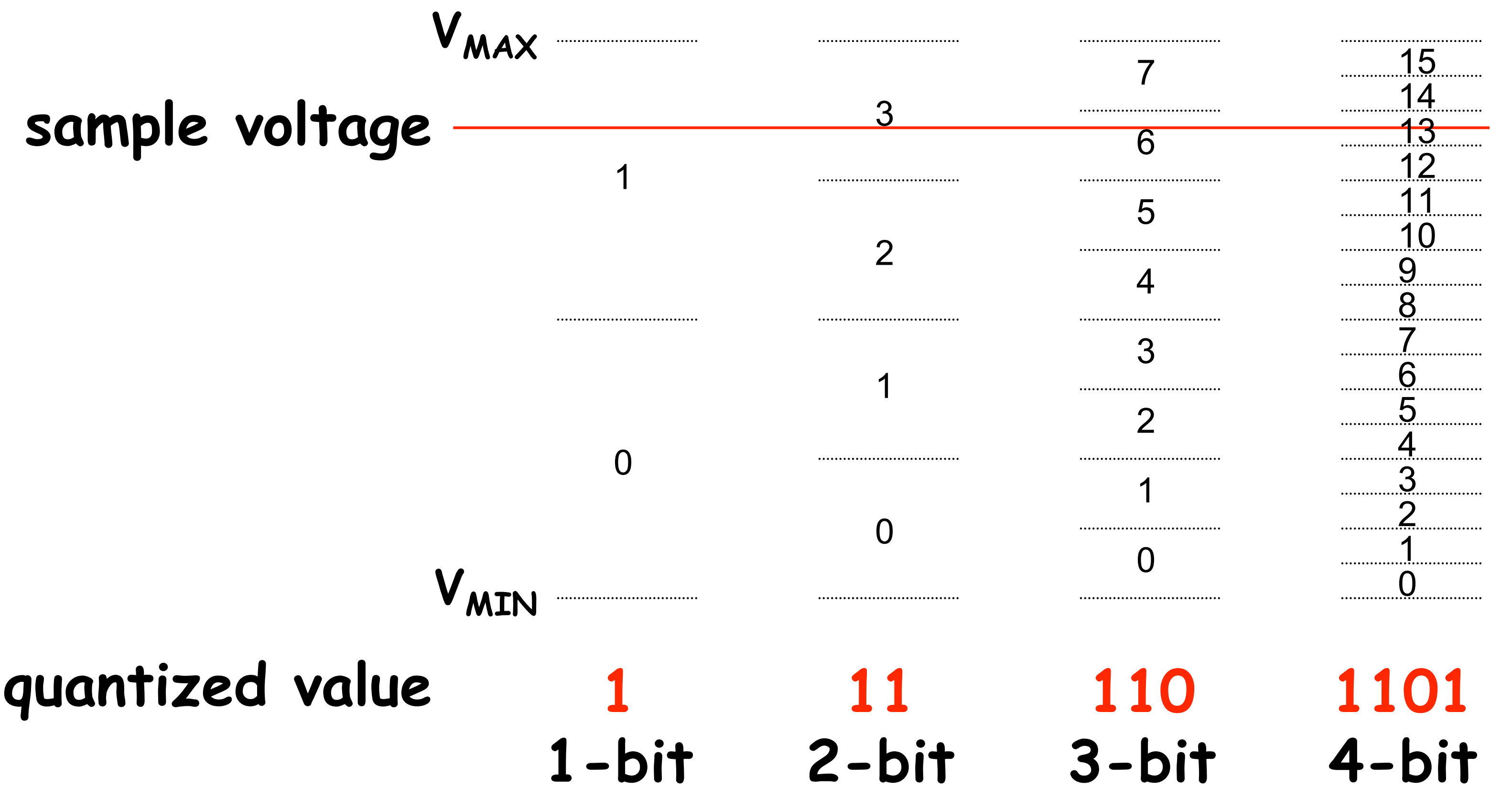
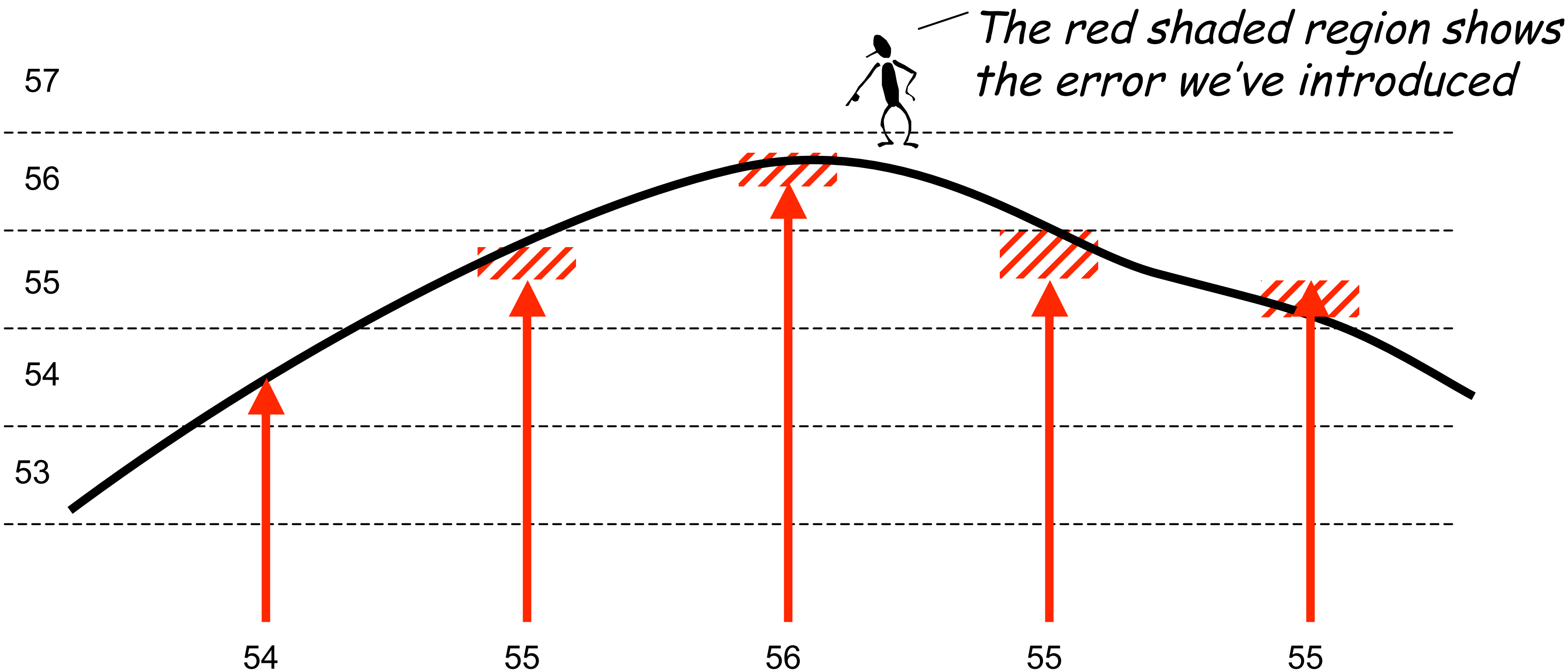
Quantization: How many discrete values?



Quantization Error



- Introduce **at most ½ interval length** of quantization error

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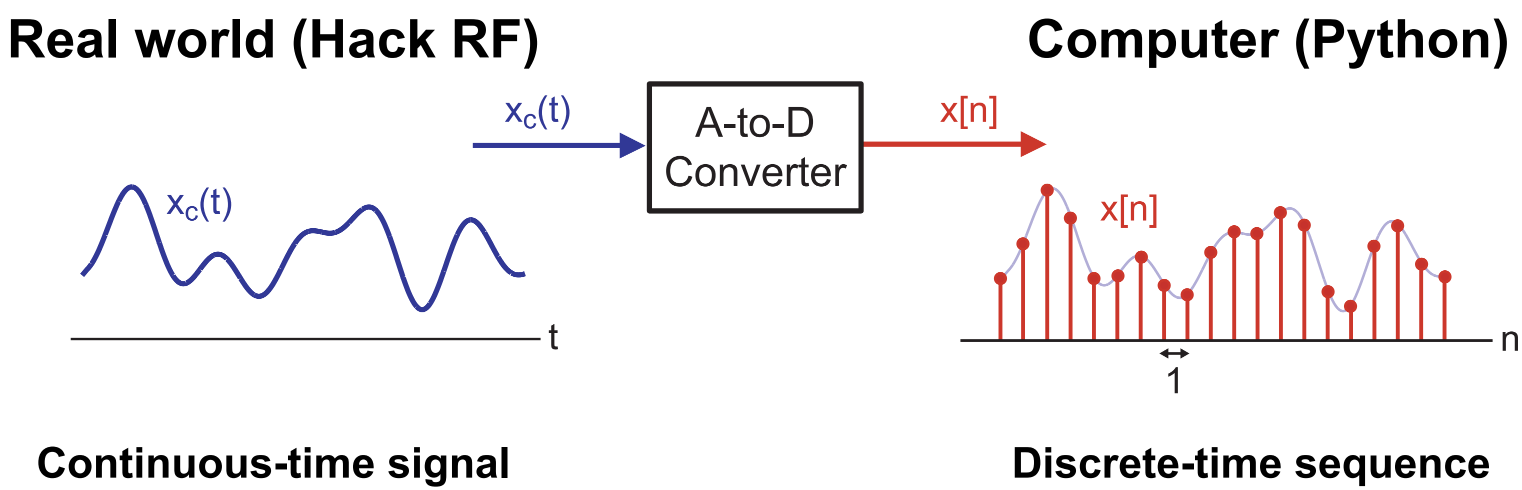
3. Digital I/Q Modulation

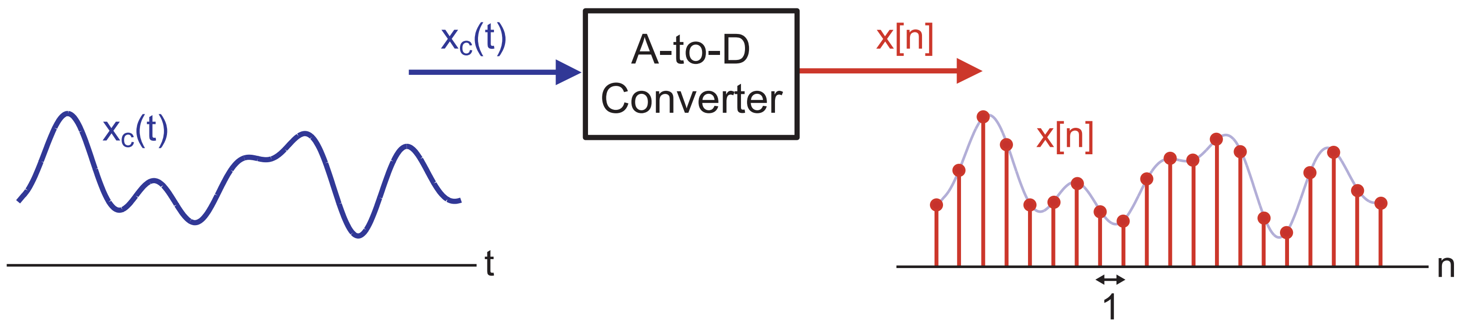
Need for Continuous-to-Discrete Conversion



- The boundary between analog and digital
  - Real world filled with continuous-time signals
  - Computers operate on **discrete-time sequences**
- Crossing analog to digital boundary requires **conversion between the two**

How Fast to Sample?

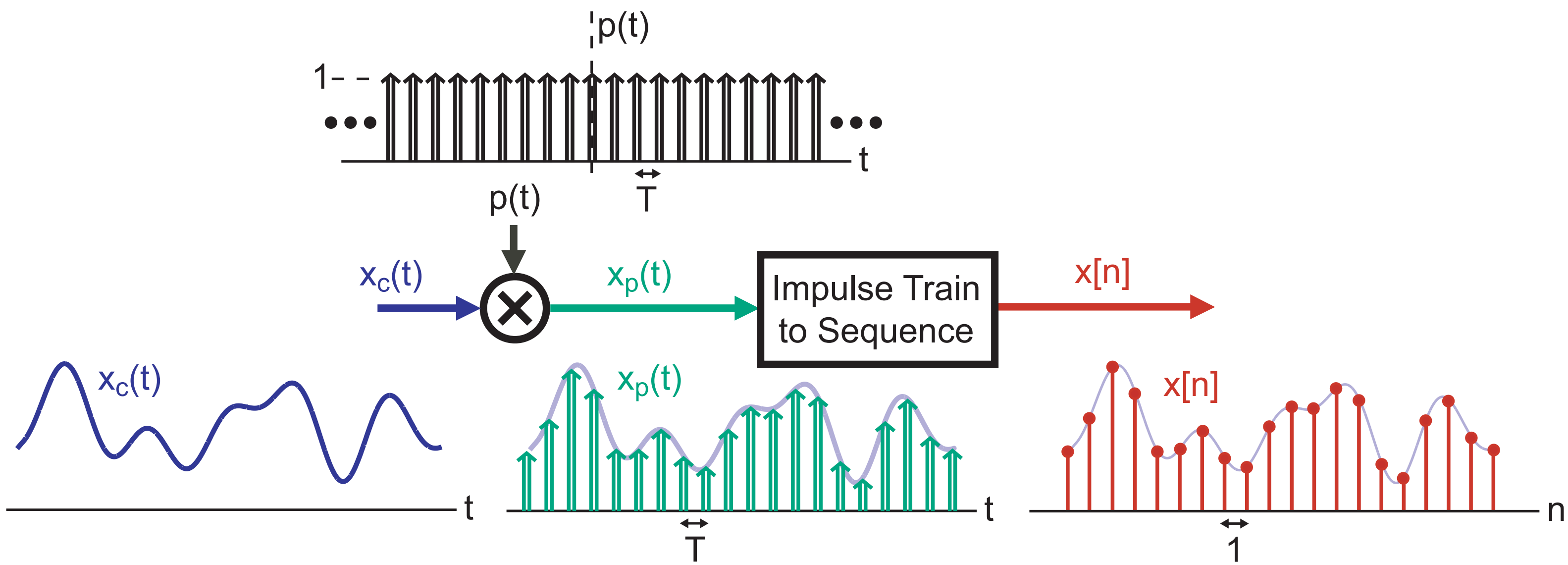
Real world (Hack RF) **Computer (Python)**



Continuous-time (CT) signal **Discrete-time (DT) sequence**

- **Too slow (*undersampling*):**
  - DT sequence **loses information** about CT signal
- **Too fast (*oversampling*):**
  - DT sequence contains **redundant, useless information**

Modelling Continuous-to-Discrete Conversion

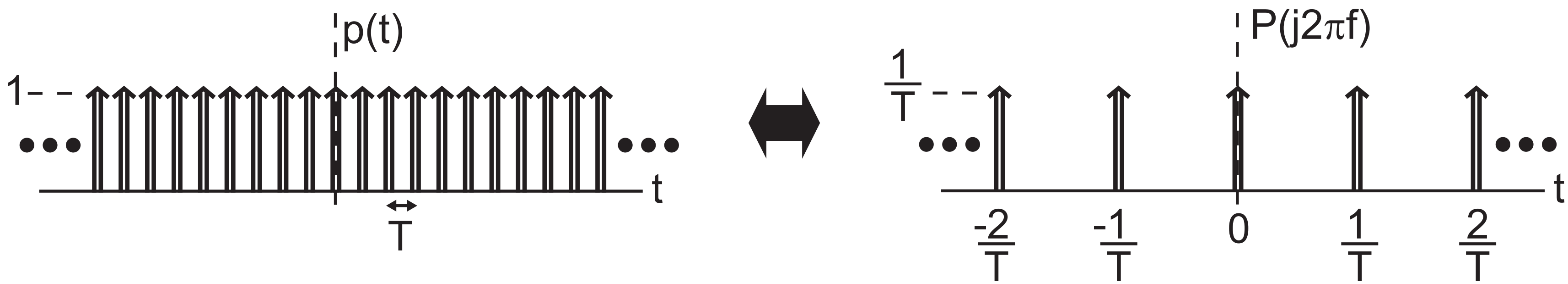


- **Two-step process:**

1. **Sample** continuous-time signal every ***T*** seconds
   – Model as multiplication of signal with impulse train

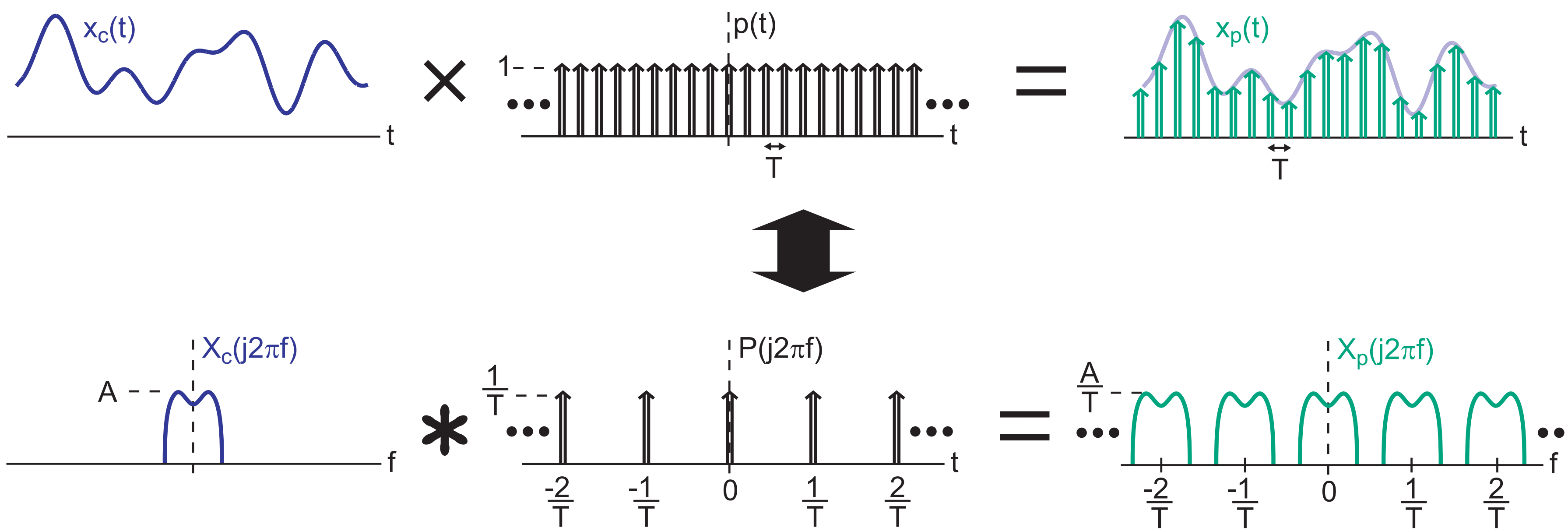
2. **Create sequence** from amplitude of scaled impulses
   – Model as rescaling of time axis (T → 1)

Fourier Transform of Impulse Train



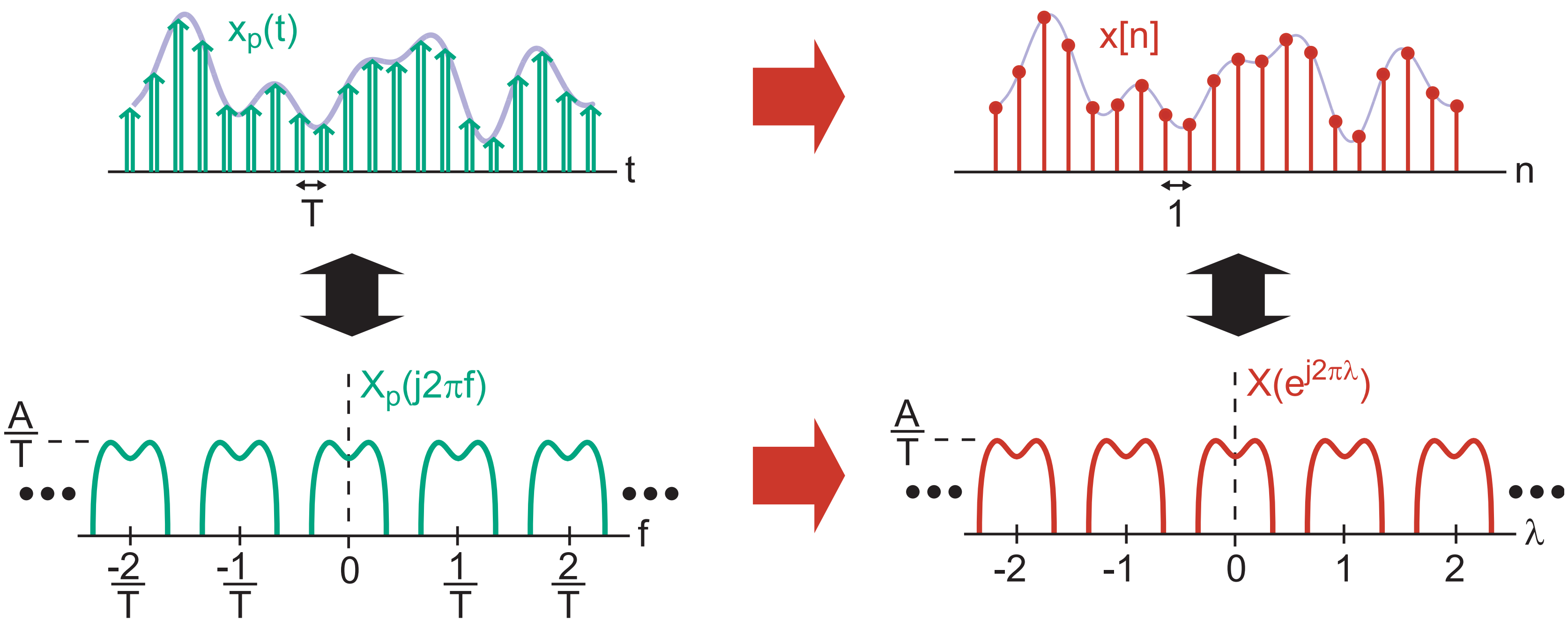
- **Impulse train in time** corresponds to **impulse train in frequency**
  - Spacing in time of **T seconds** corresponds to **1/T Hz** spacing in frequency

Frequency Domain view of Sampling



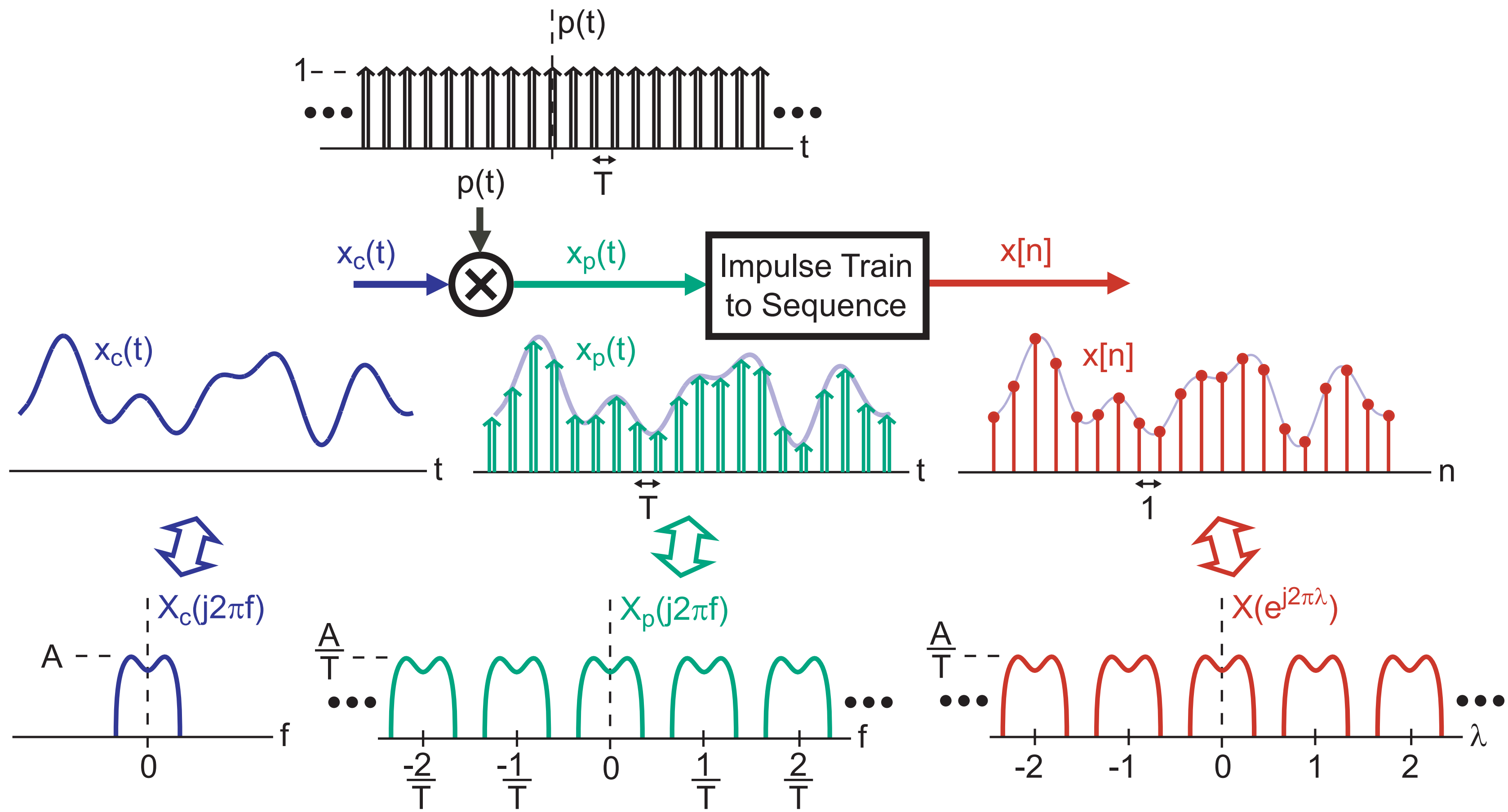
- Sampling in time leads to a **periodic Fourier Transform**, with period 1/T

Frequency Domain View of a Sequence



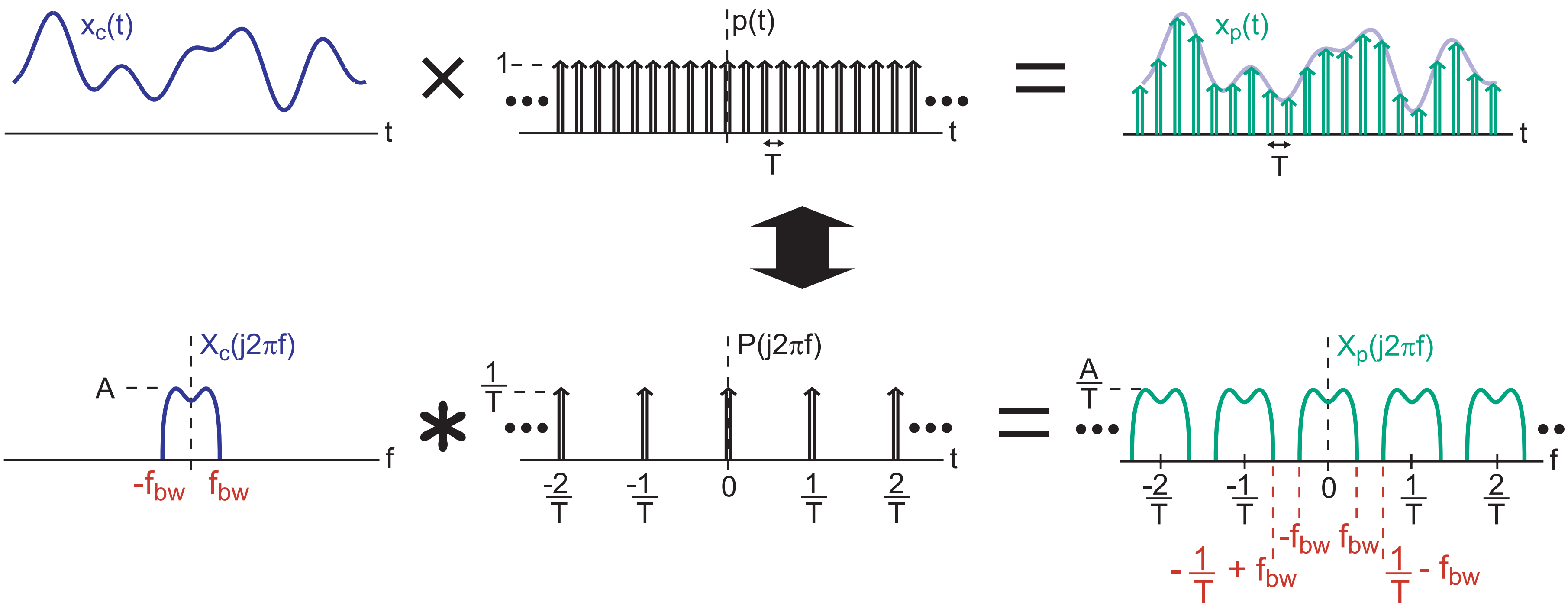
- Conversion to a sequence **amounts to setting T = 1**
- Resulting Fourier Transform is now **periodic** with a **period of one**

Summary: Continuous-to-Discrete Conversion



- Sampling leads to **periodicity** in frequency domain

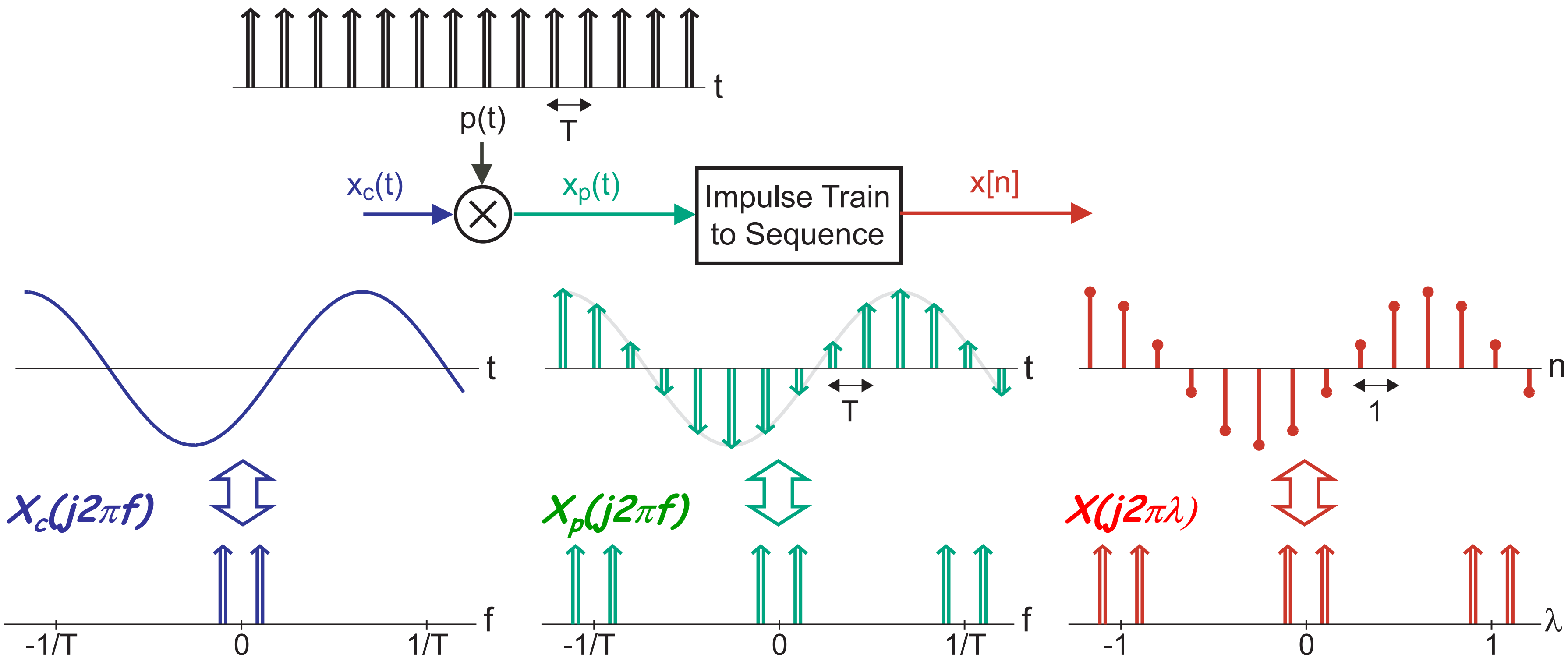
The Sampling Theorem



- **No overlap in frequency domain** (aliasing) when $-{}^{1}/_{T} - f_{bw} \geq f_{bw}$ or ${}^{1}/_{T} \geq 2f_{bw}$

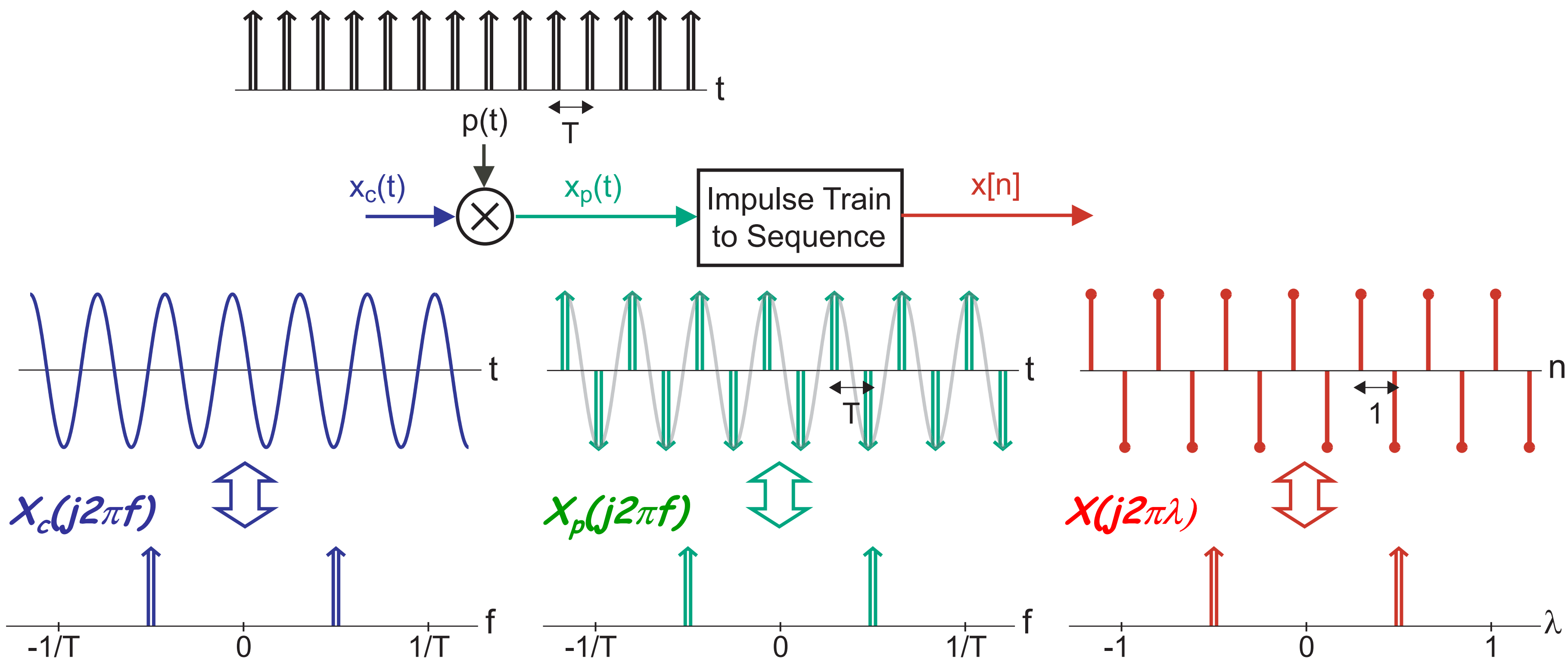
- We refer to the minimum 1/T that avoids aliasing as the ***Nyquist sampling frequency*** of a signal

Sine Wave Example: Sampling Above Nyquist Rate



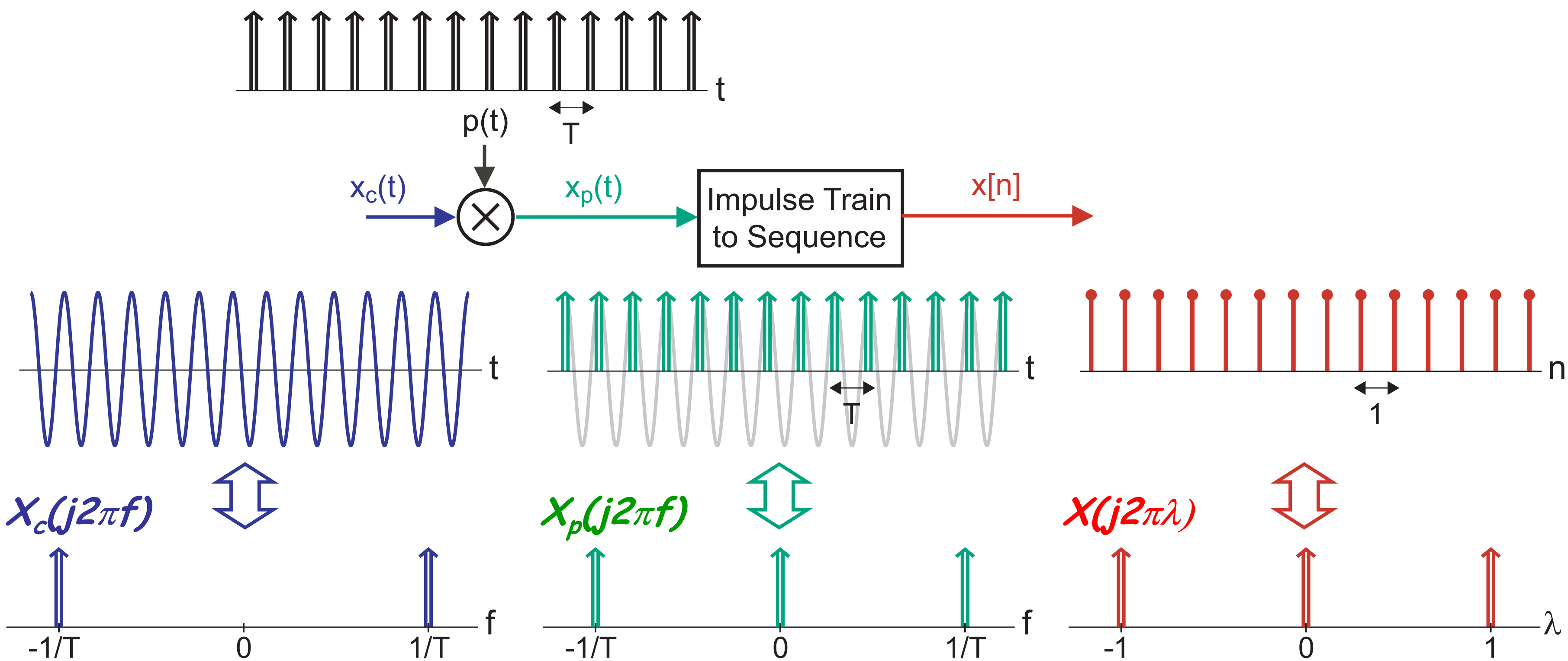
- **Time domain:** Resulting sequence **maintains same period** as input signal

Sine Wave Example: Sampling at Nyquist Rate



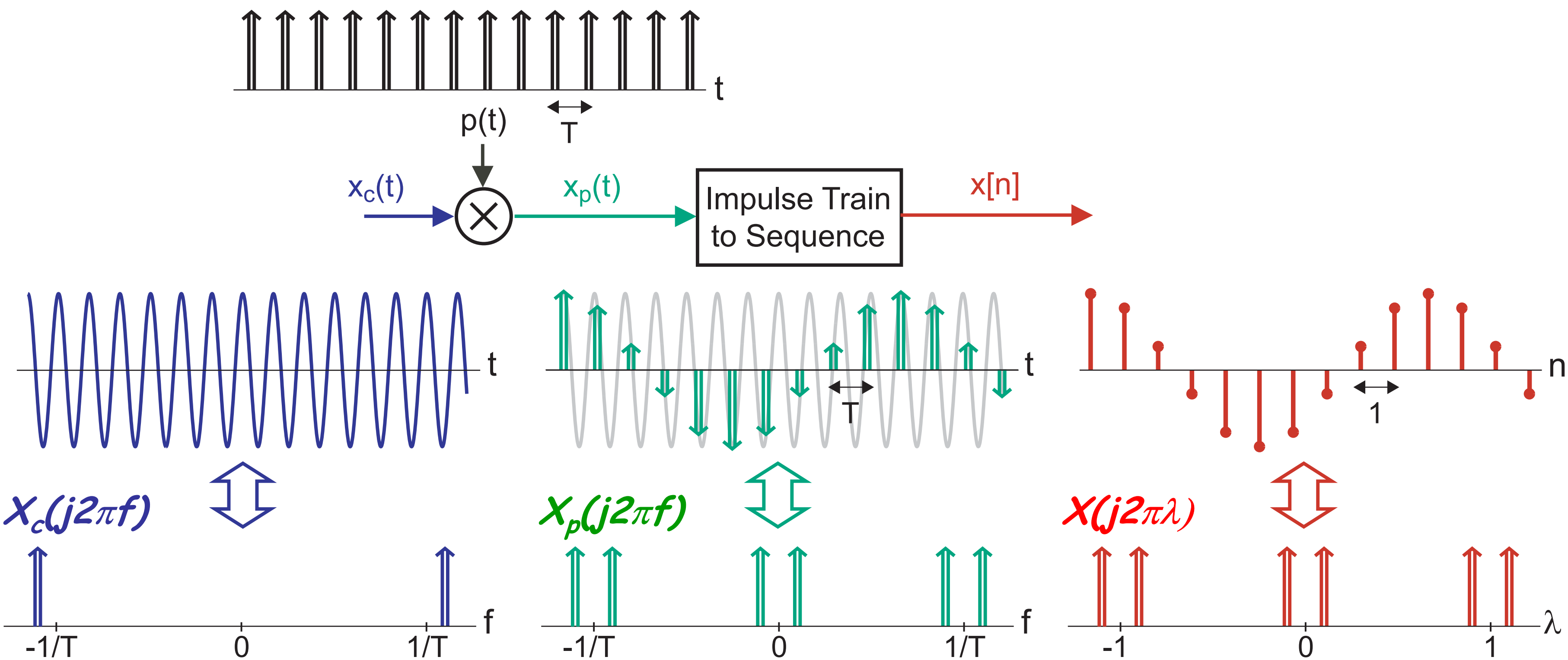
- **Time Domain:** Resulting sequence **maintains same period** as input signal

Sine Wave Example: Sampling at Half the Nyquist Rate



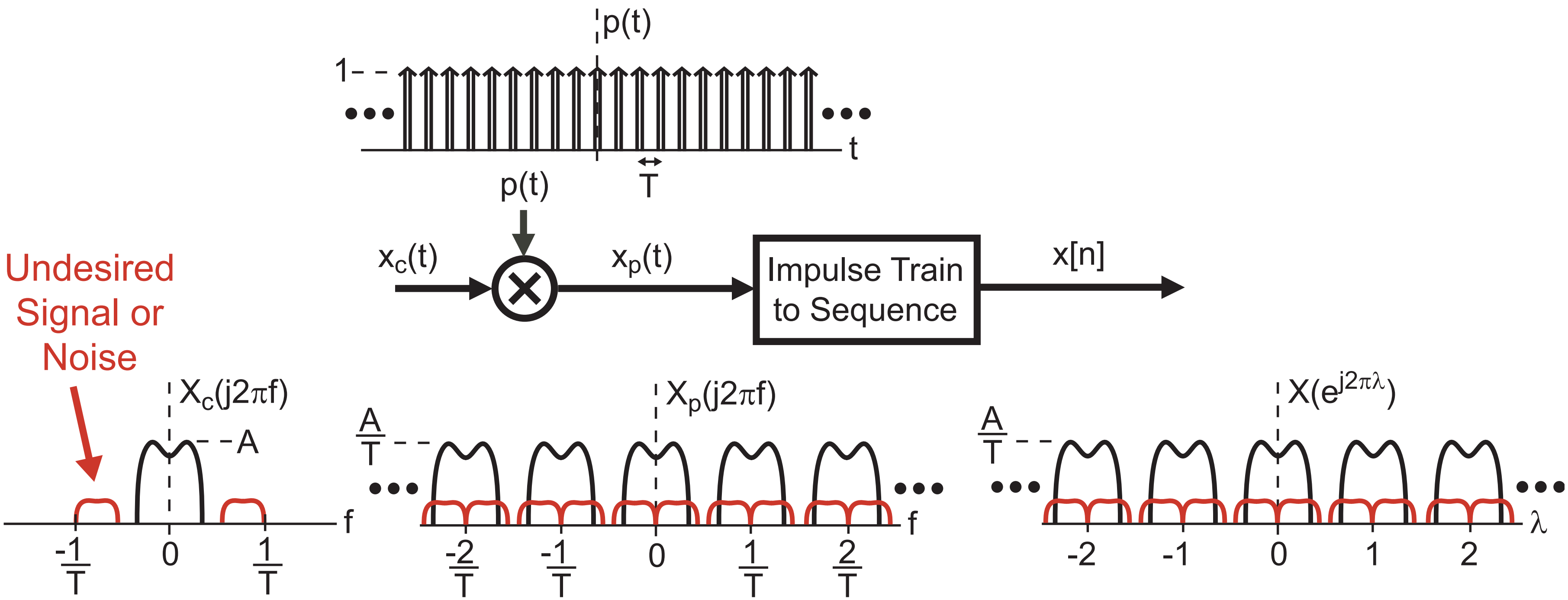
- Sequence now **appears as constant (zero frequency) signal**
- **Frequency domain: Aliasing to 0**

Sine Wave Example: Sampling Below the Nyquist Rate



- Resulting sequence now a **sine wave with a different period than the input**
- **Frequency domain: Aliasing** to a lower frequency

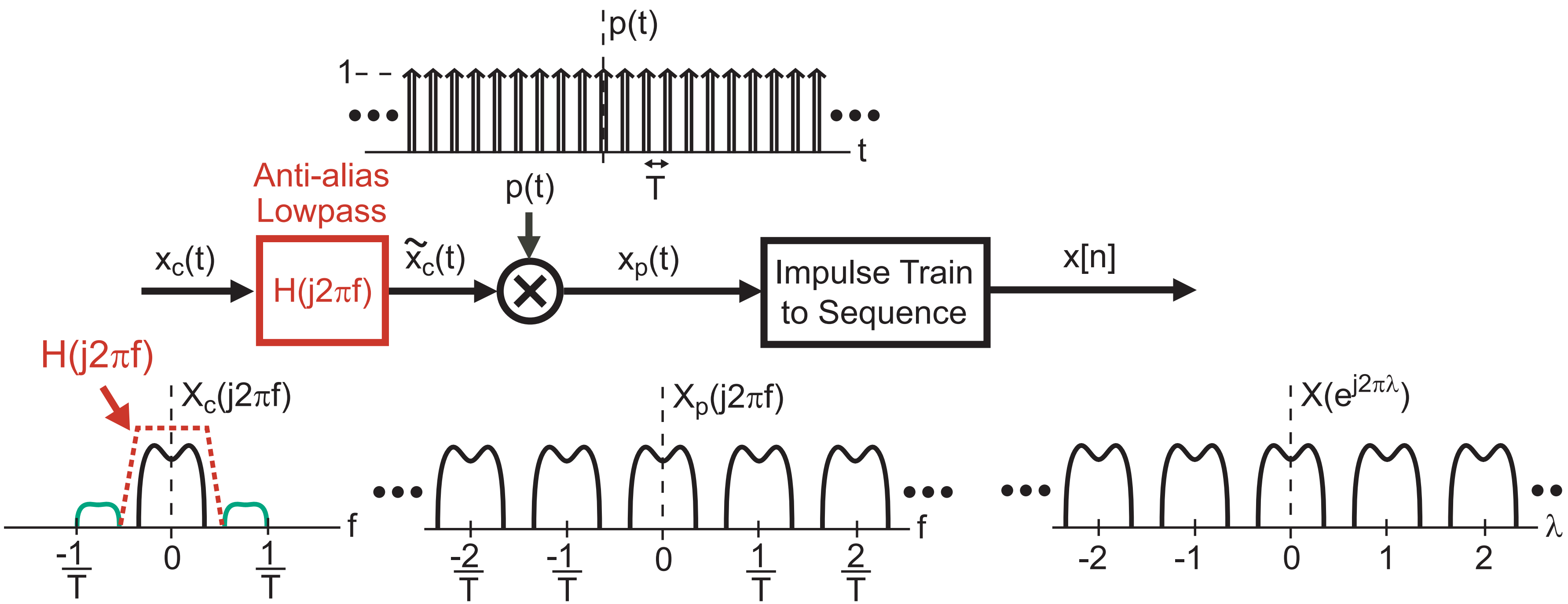
The Issue of High Frequency Noise



- Typically **set sample rate** to exceed **desired signal's** bandwidth

- Real systems can introduce **noise/other interference at high frequencies**
  - Sampling causes **noise to alias** into desired frequency band

Anti-Alias Filtering



- Practical systems **use a continuous-time filter** before the sampling operation
  - **Removes noise/interference >1/2T** in frequency, **prevents aliasing**

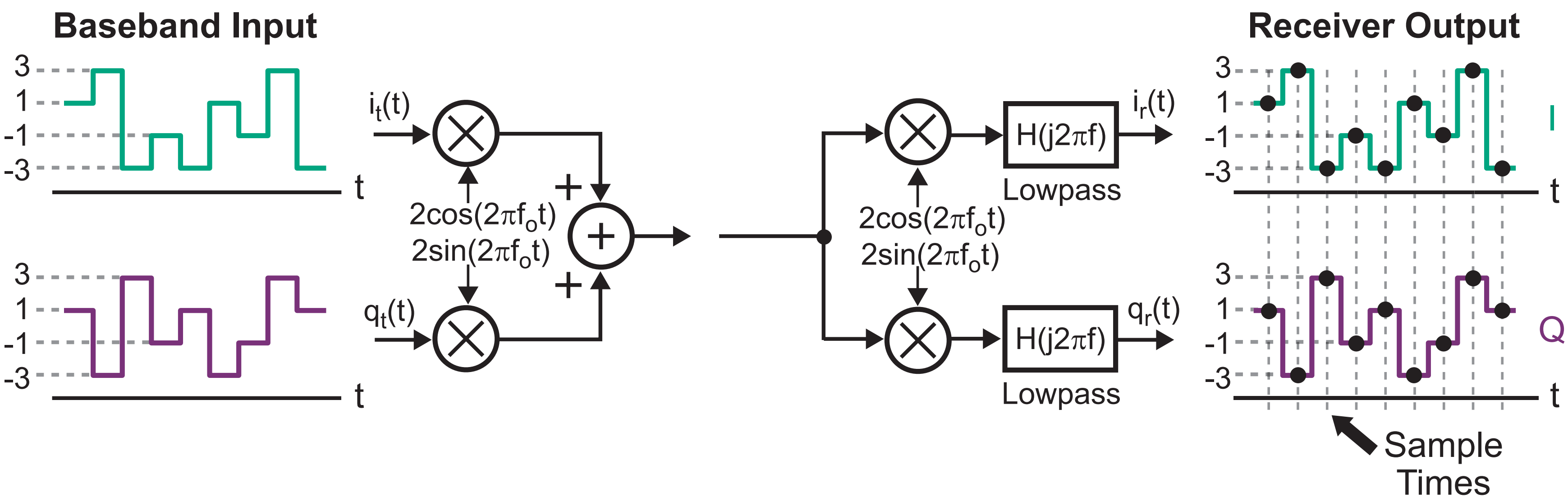
Summary: Advantages of Going Digital

- Allows **error correction** to be achieved
  - Less sensitivity to radio channel imperfections
- **Enables compression** of information
  - More efficient use of the channel
- Supports a **wide variety of information** content
  - Voice, text, video can all be represented as digital bit streams

Roadmap

1. Analog I/Q modulation
2. Discrete-time processing of continuous signals
3. **Digital I/Q Modulation**

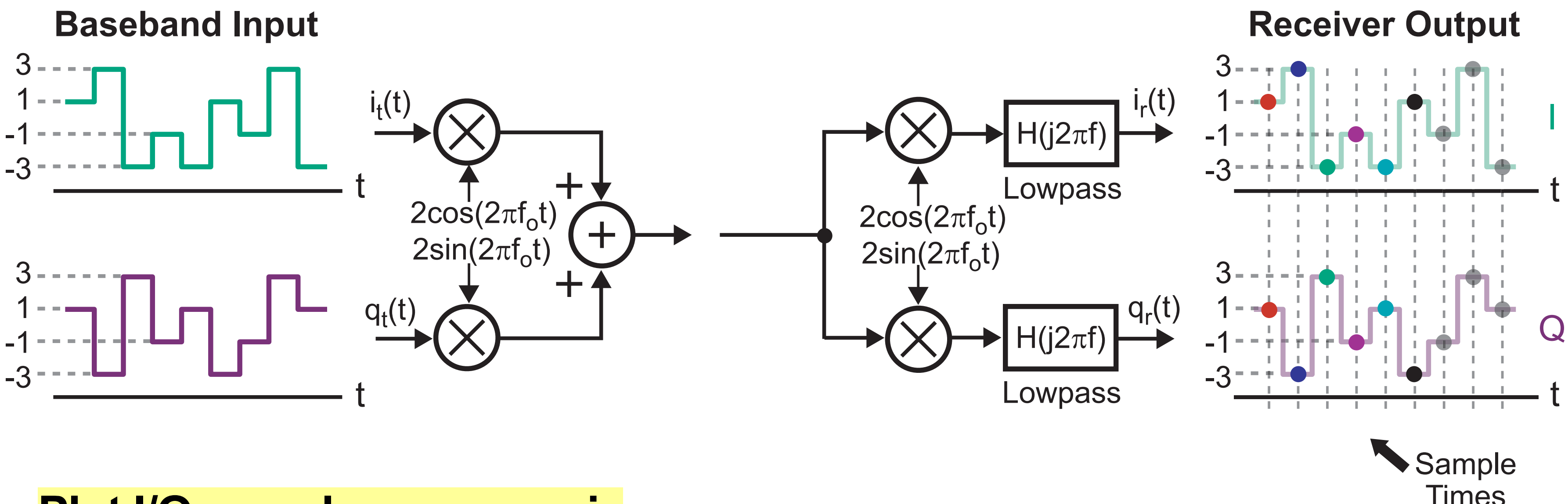
Digital I/Q modulation



- Leverage analog communication channel to send **discrete-valued *symbols***
  - **Example:** Send symbol from set { -3, -1, 1, 3 } on both I and Q channels

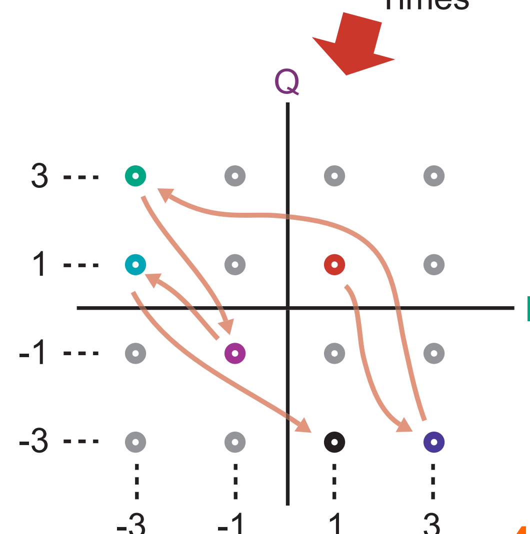
- At receiver, **sample I/Q waveforms** every ***symbol period***
  - Associate each sampled I/Q value with symbols from same set on both I and Q channels

Constellation Diagrams

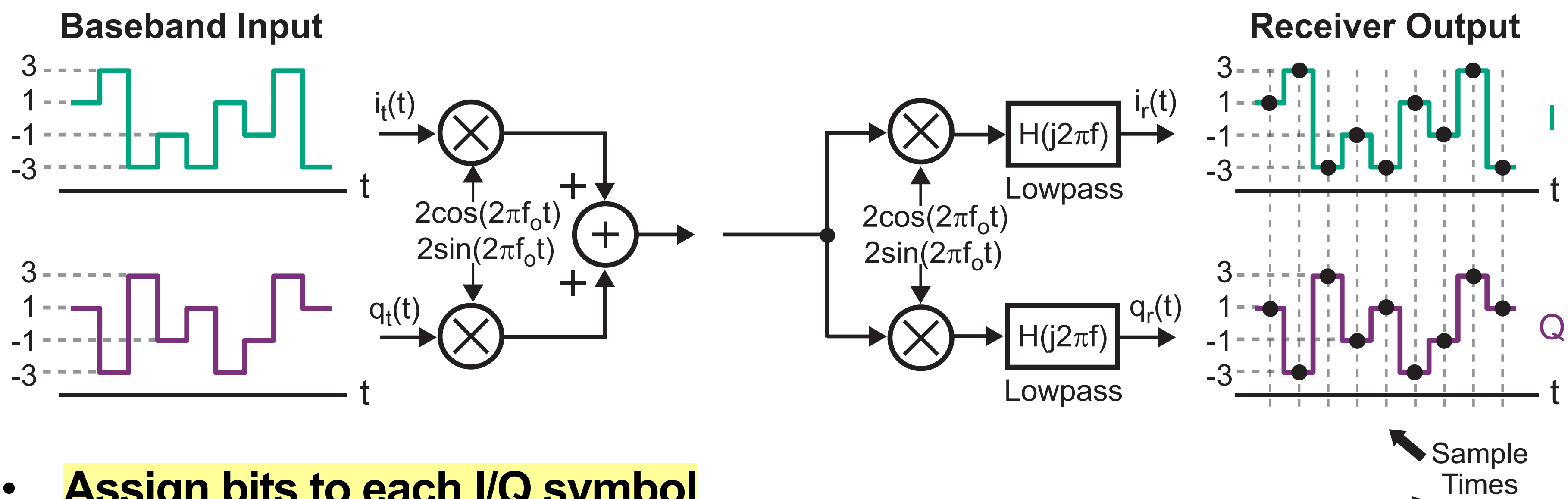


- **Plot I/Q samples on x-y axis**
  - As samples are plotted, constellation diagram eventually displays all possible symbol values

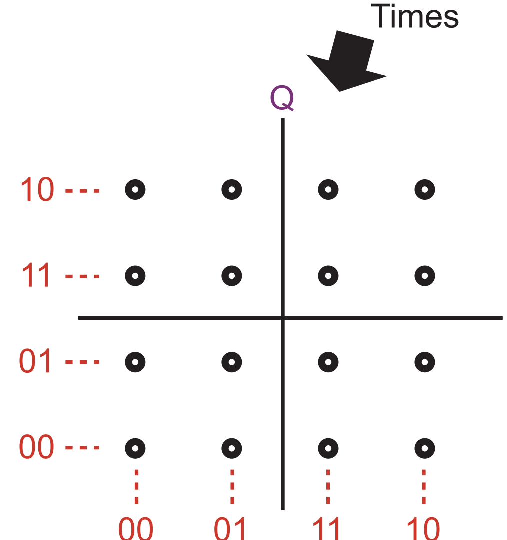
- Provides a sense of **how easy it is to distinguish** between different symbols



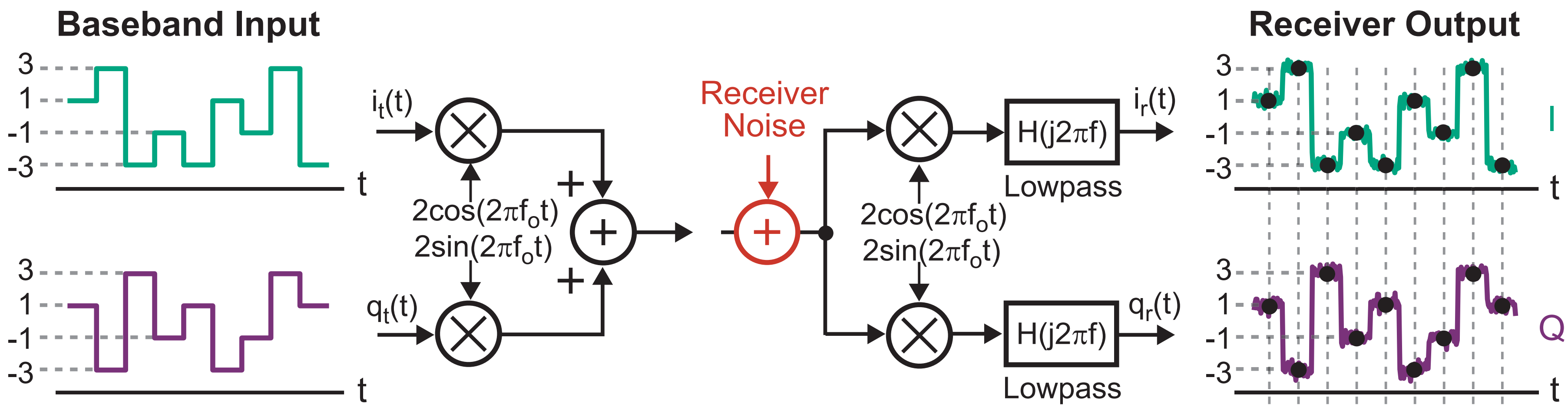
Sending Digital Bits



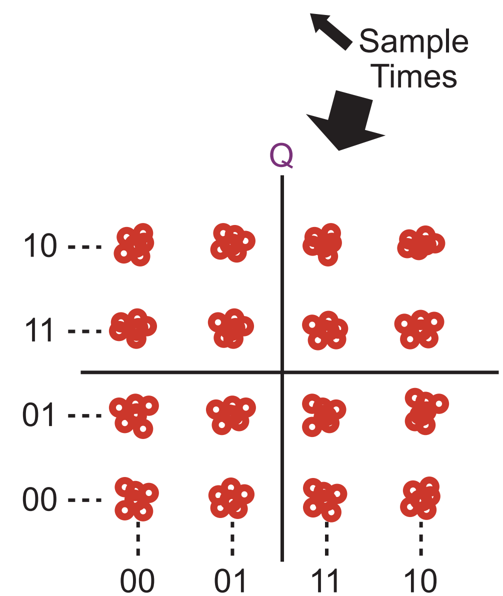
- Assign bits to each I/Q symbol
  - **Example:** I/Q = {1, 3} translates to bits 1110
  - ***Gray coding*** minimizes bit errors when symbol errors are made
    - **Example:** I/Q = {3, 3} translates to bits 1010 (one bit flip from {1, 3})



The Impact of Noise

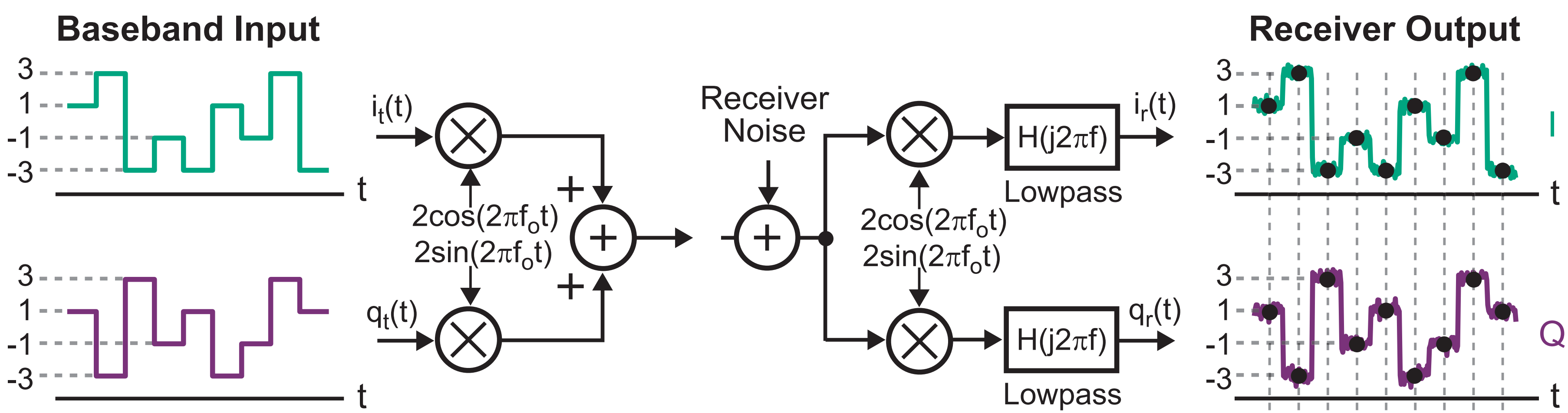


- **At receiver, noise perturbs sampled I/Q values**
  - Constellation points no longer consist of single points for each symbol



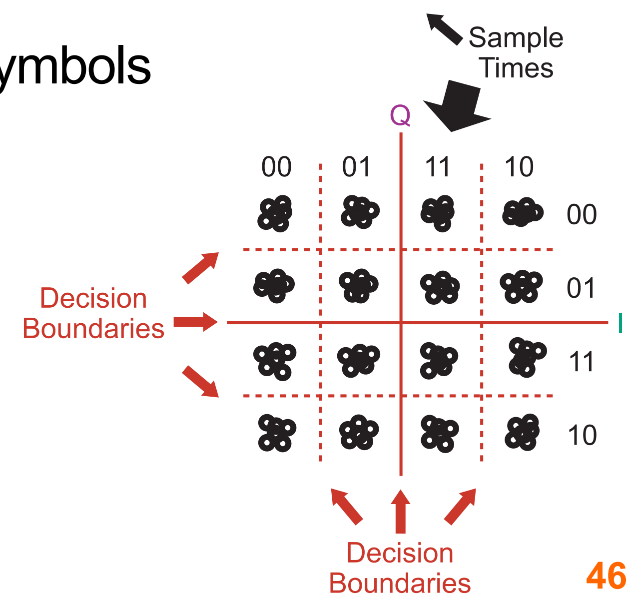
- **Issue:** What's the **best way of matching received I/Q samples** with their corresponding transmitted symbols?

Symbol Selection based on Slicing

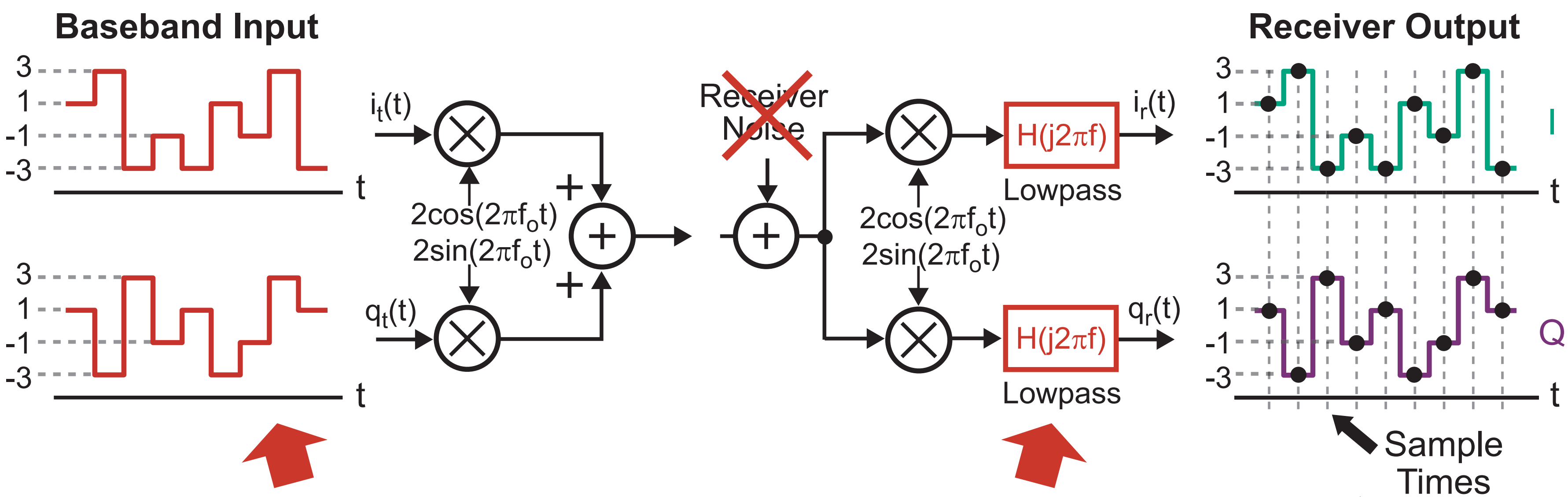


- Receiver matches I/Q samples to corresponding symbols based on ***decision boundaries***

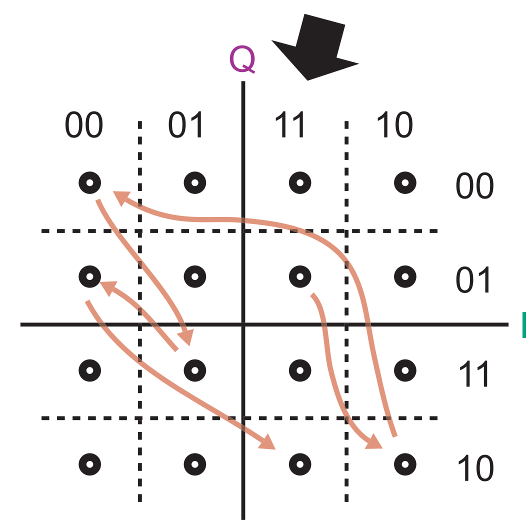
- Decision boundaries are also called ***slicing levels***



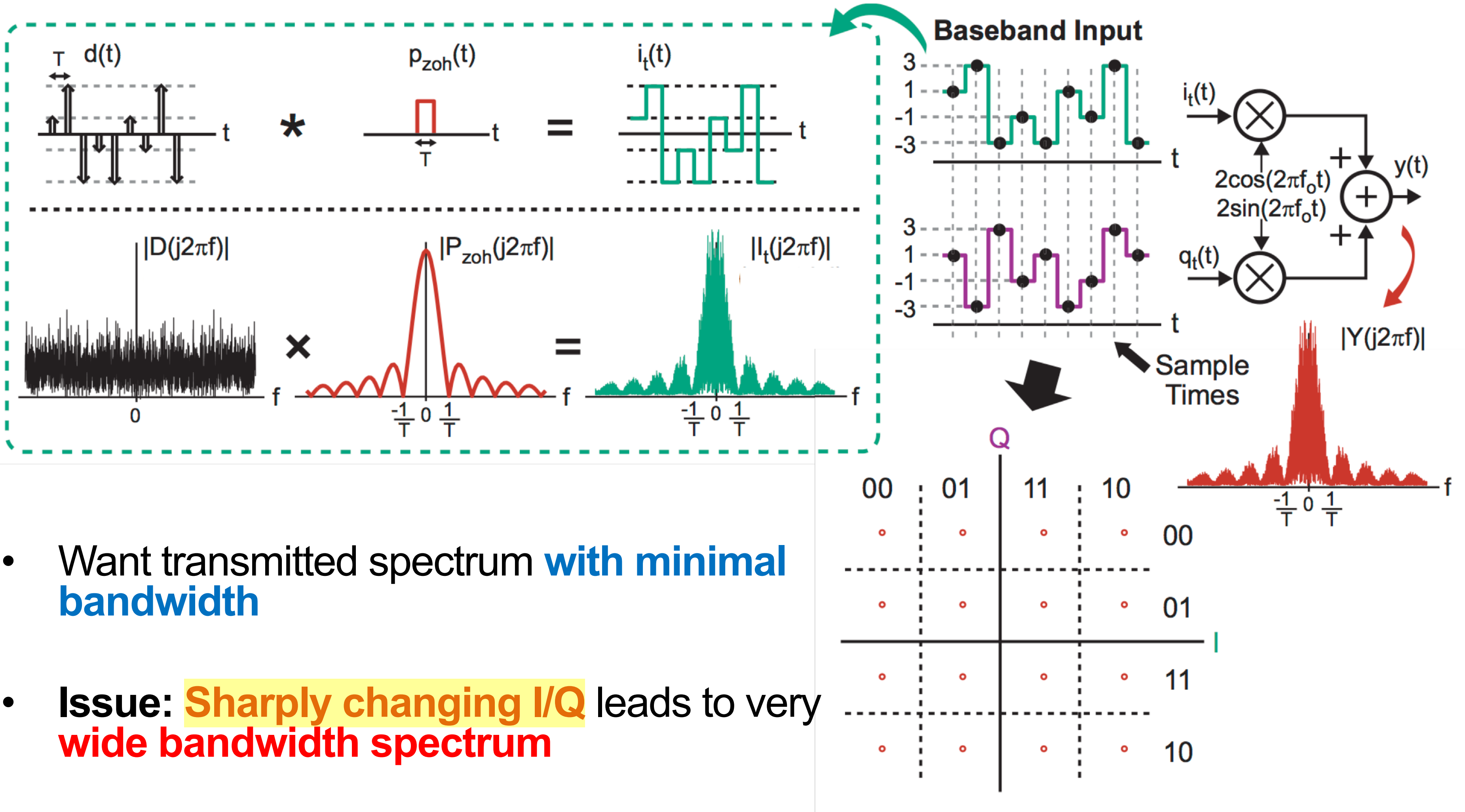
Transitioning Between Symbols



- Transition behavior influenced by transmit I/Q input waveforms **and** receive filter
  - **Today:** Focus on **what the transmitter does**
  - **Ignore impact of noise** for this analysis



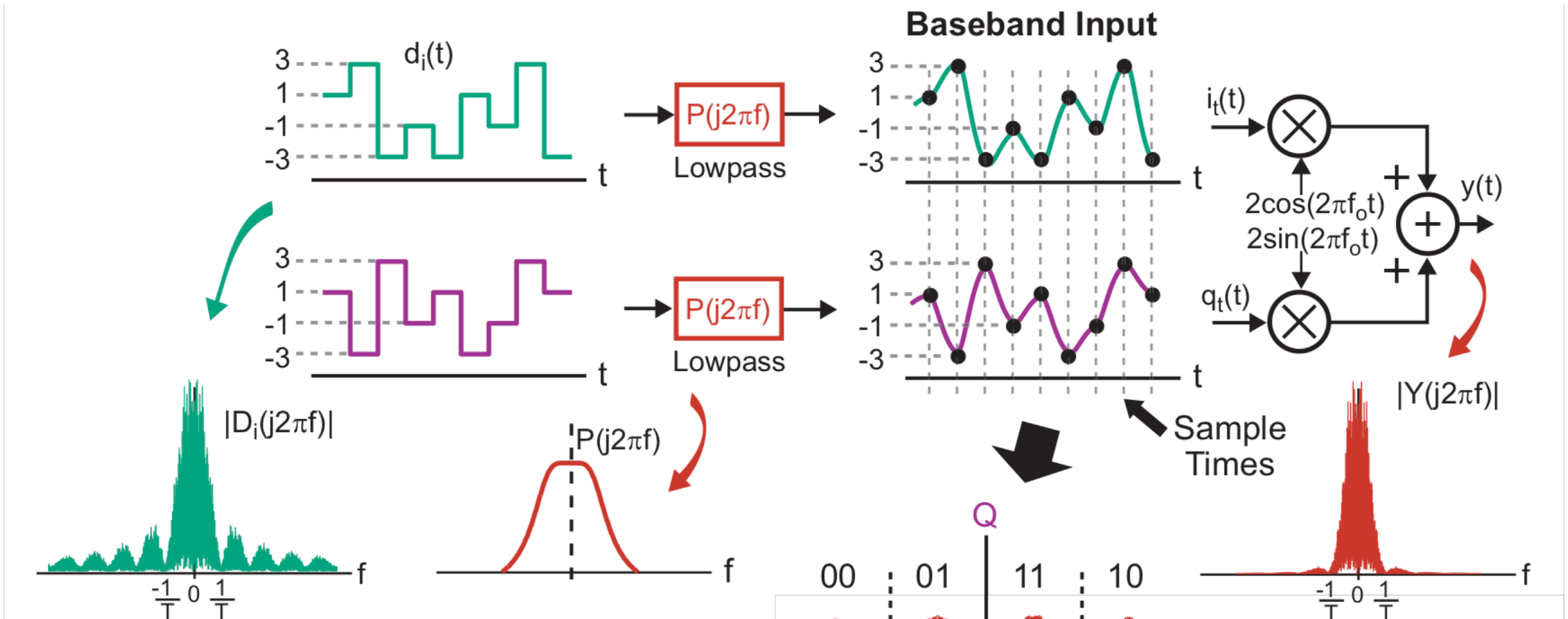
Transitions and the Transmitted Spectrum

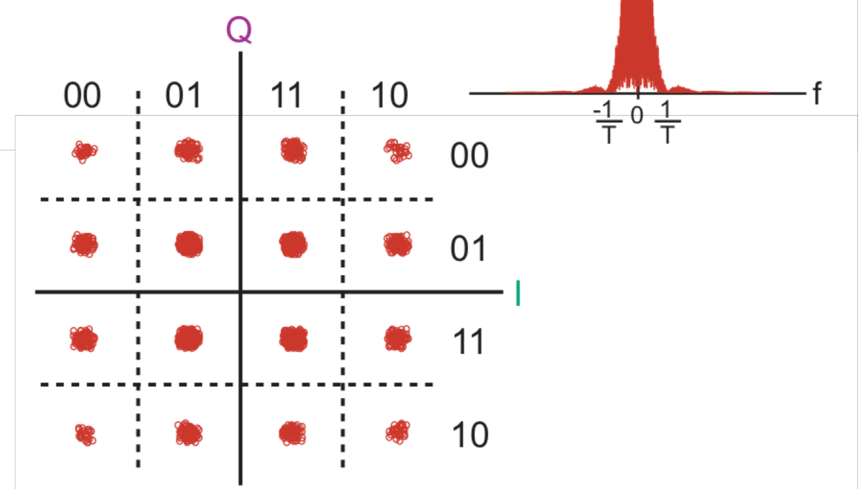


- Want transmitted spectrum **with minimal bandwidth**

- **Issue: Sharply changing I/Q** leads to very **wide bandwidth spectrum**

Impact of Transmit Filter

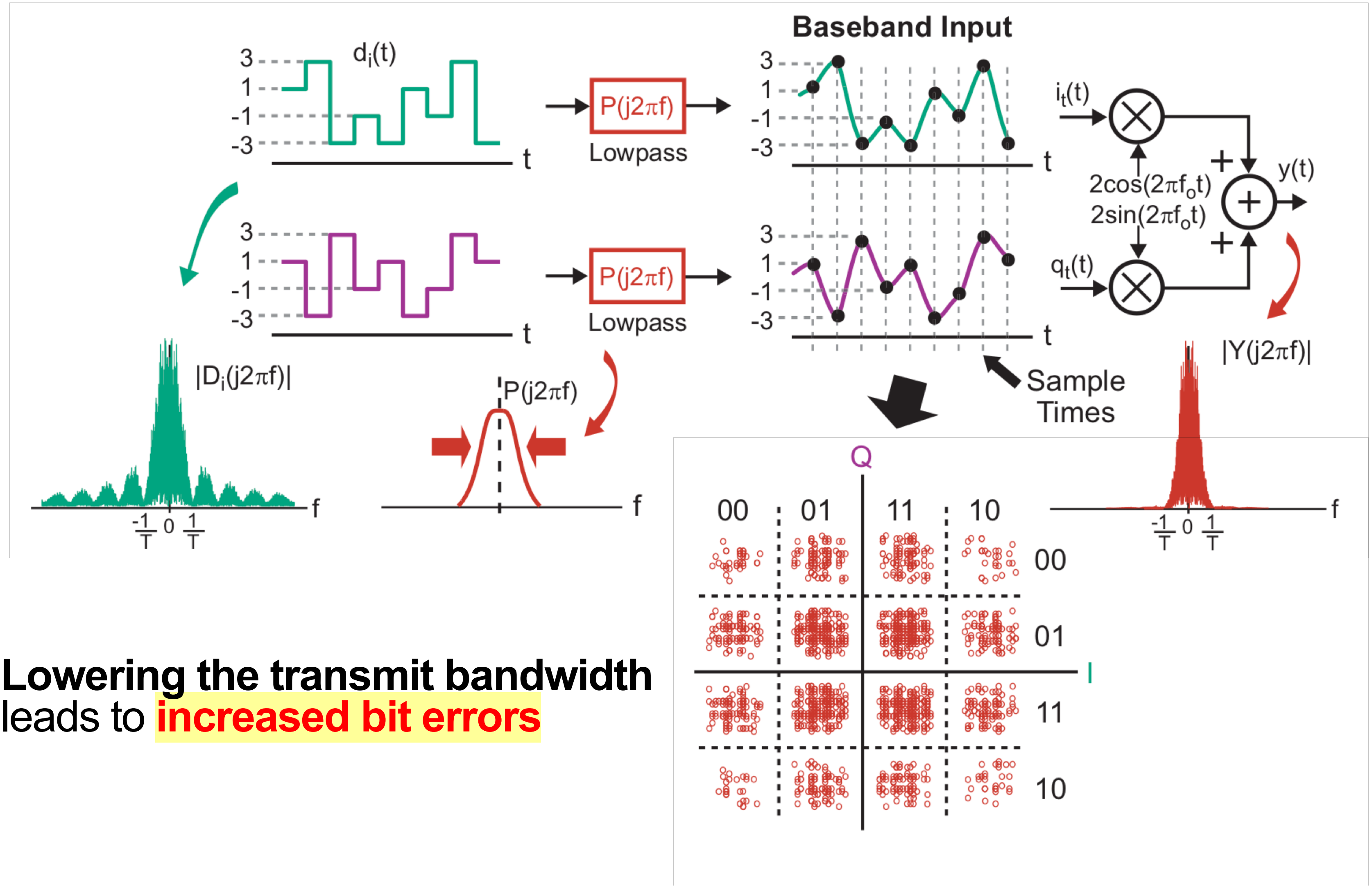




- Transmit filter **shapes the pulses**, **enables reduced bandwidth** for transmitted spectrum

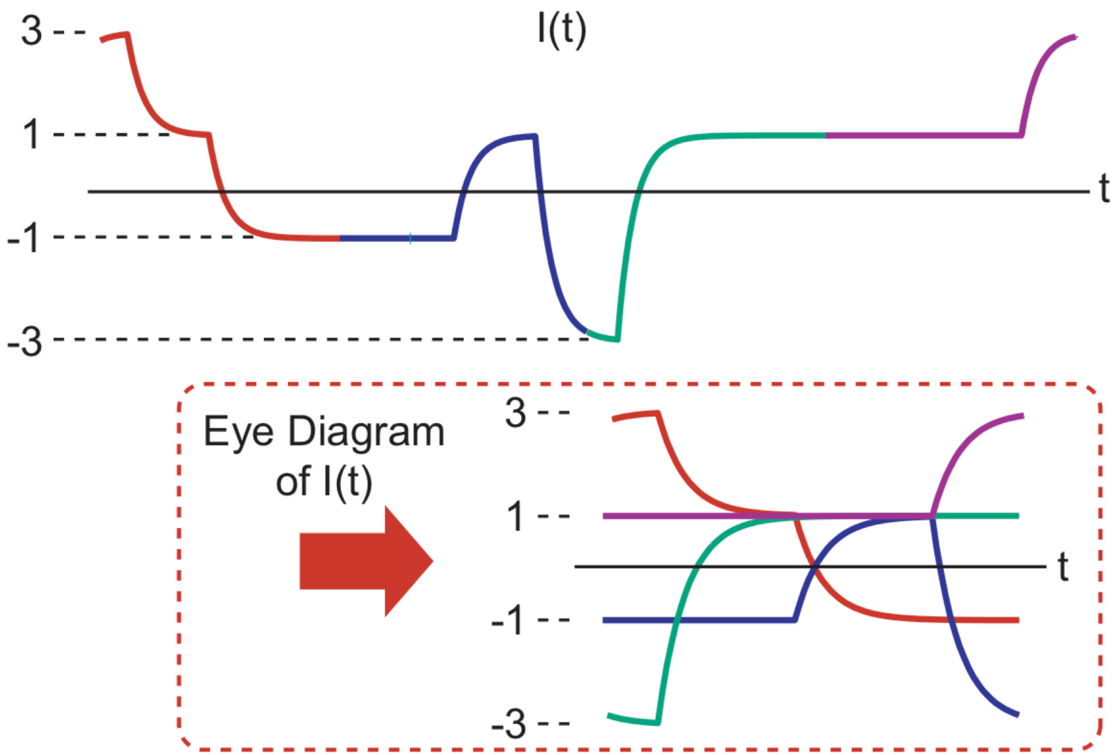
- **Next Issue:** Can lead to **bit errors**

Impact of Low Bandwidth Transmit Filter



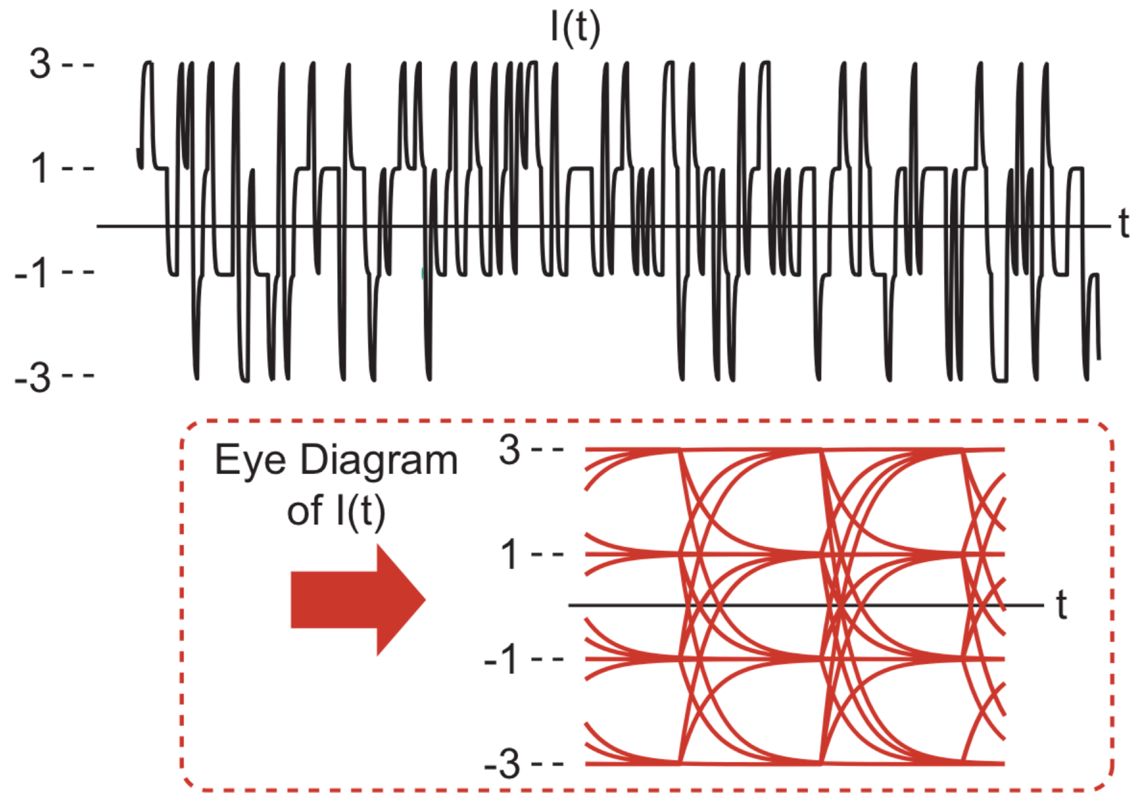
- **Lowering the transmit bandwidth** leads to **increased bit errors**

Eye Diagrams



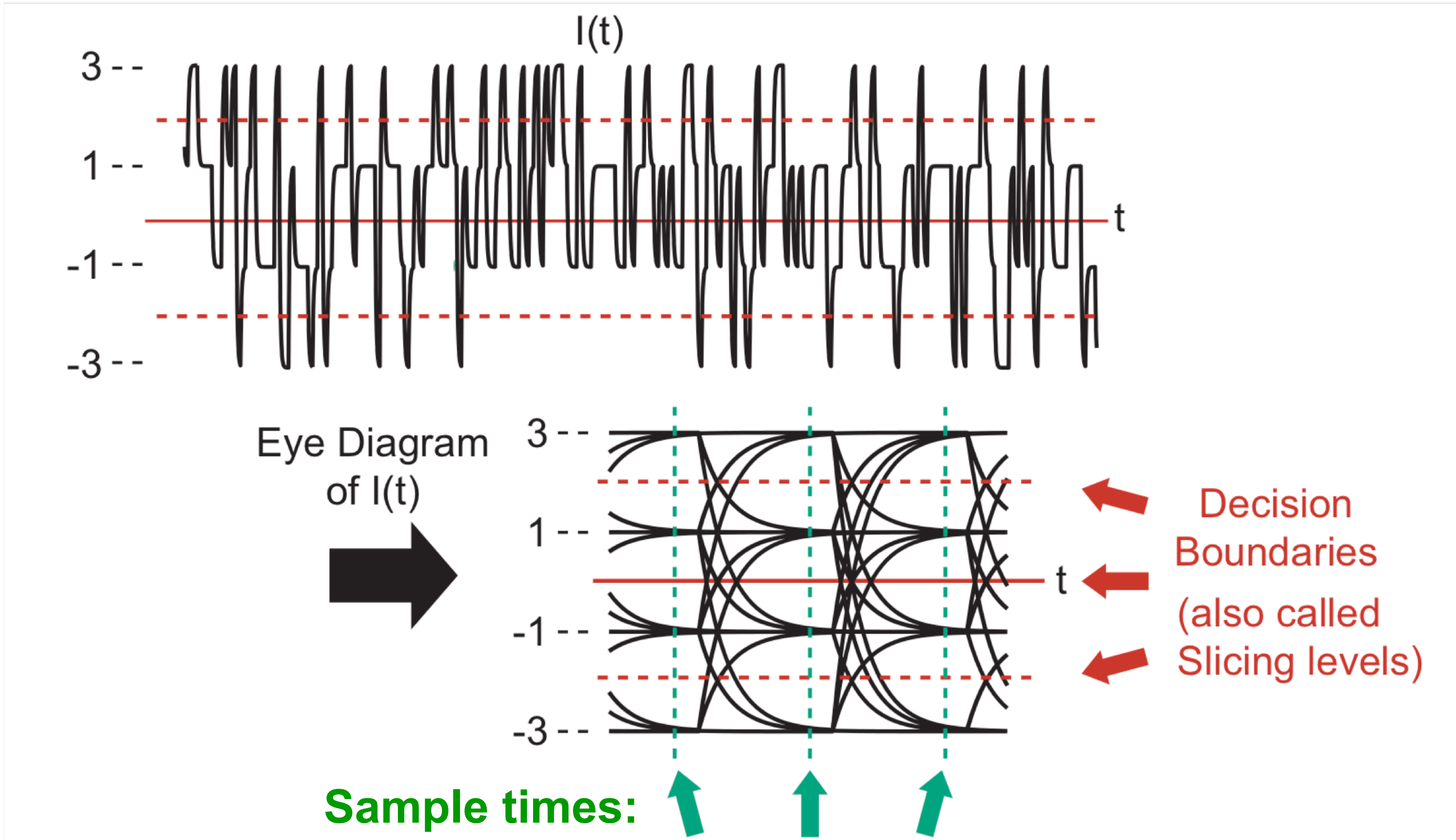
- ***Eye Diagram:*** **Wrap signal back onto itself** in **periodic time intervals,** retaining all "traces"

Looking at Many Symbols



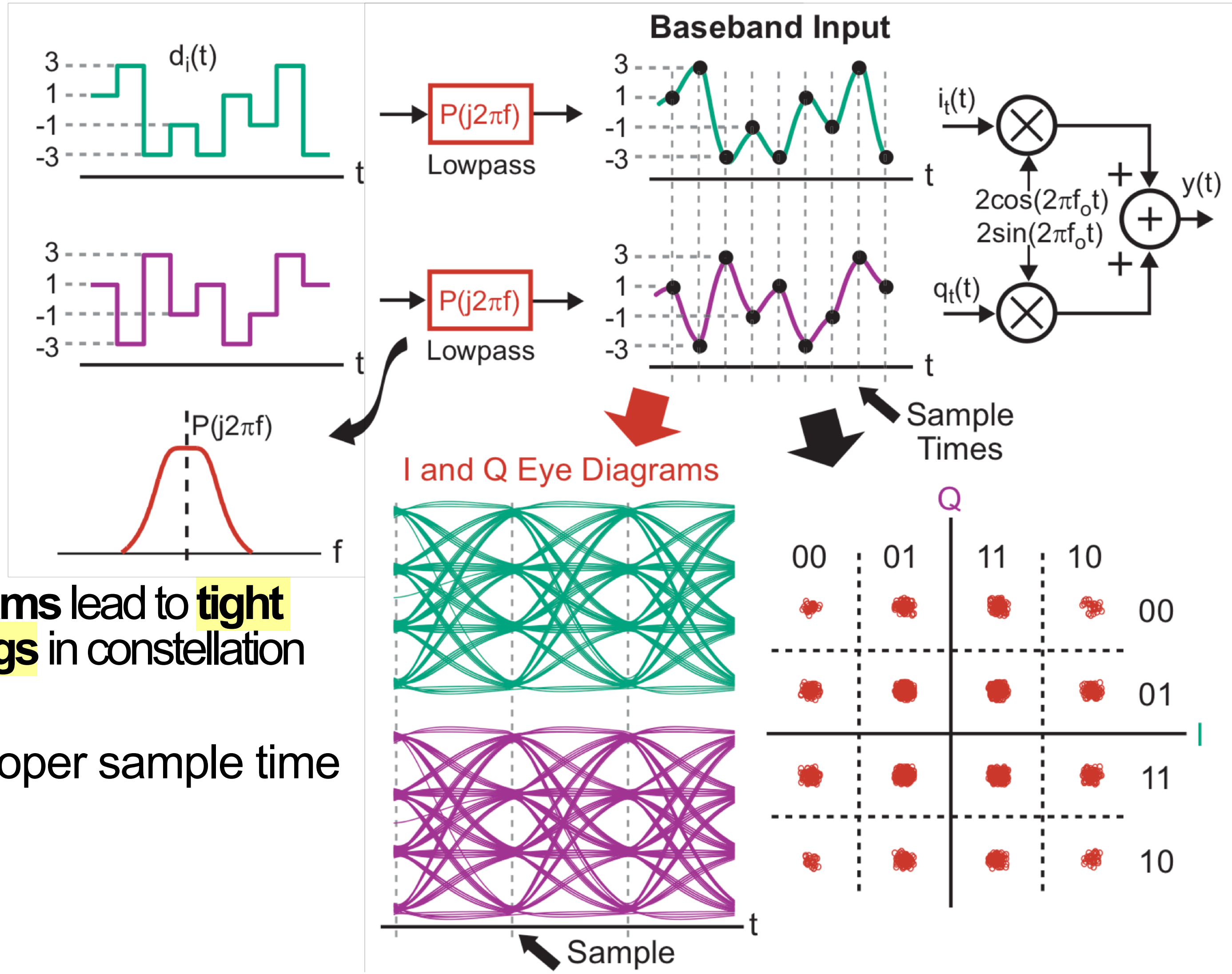
- Increasing the number of symbols eventually **reveals all possible symbol transition trajectories**
  - Intuitively **displays the impact of filtering** on transmitted signal

Assessing the Quality of an Eye Diagram



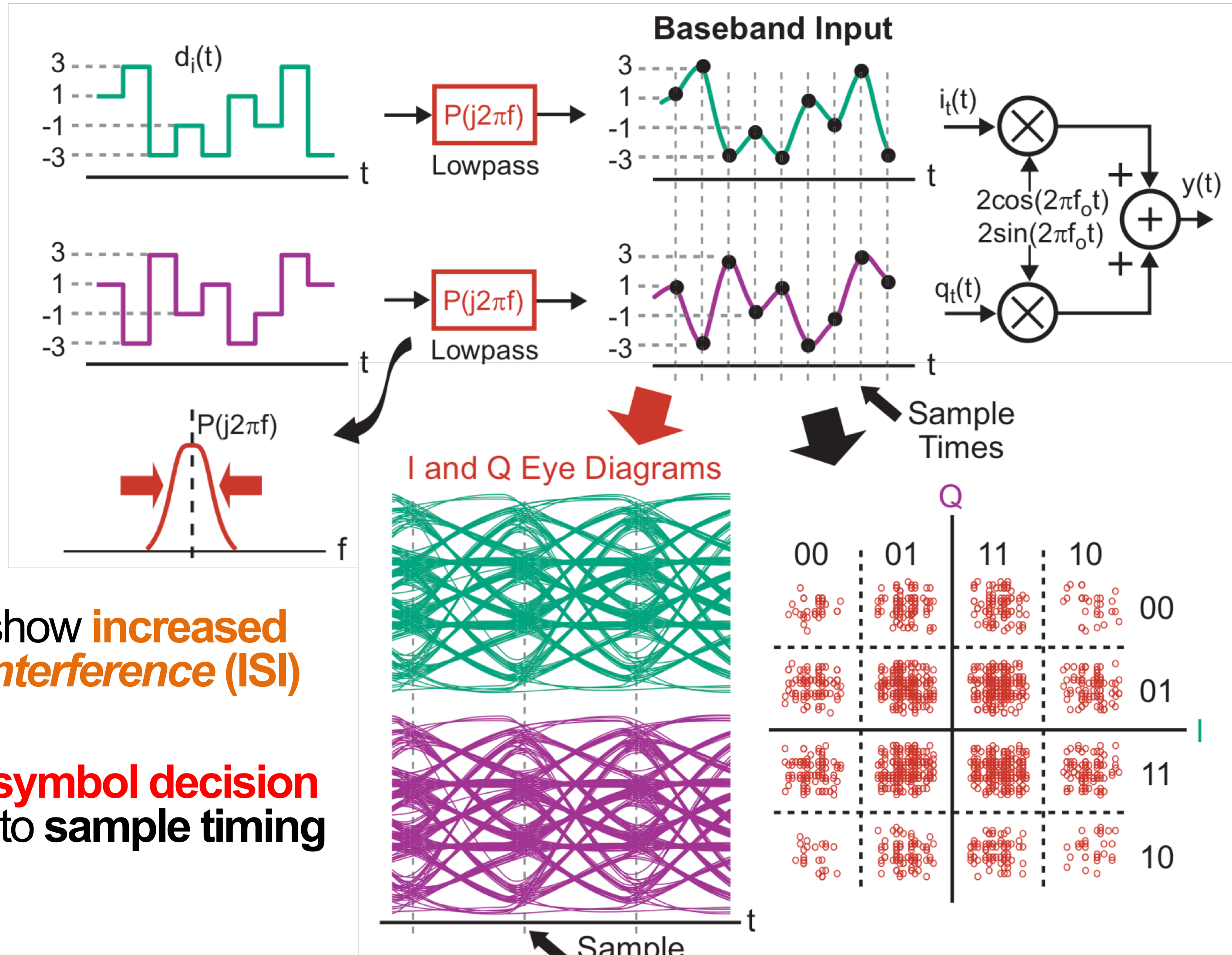
- Eye diagram **allows visual inspection of the impact of sample time and decision boundary** choices
  - **Large "eye opening"** implies less vulnerability to symbol errors

Relating Eye Diagrams to Constellation



- **Open eye diagrams** lead to **tight symbol groupings** in constellation
  - Assumes proper sample time placement

Impact of Low Transmit Bandwidth



- Eye diagrams show **increased *inter-symbol interference* (ISI)**
  - Also show **symbol decision sensitivity** to **sample timing placement**

Digital Modulation: Summary

- **Digital modulation:** Sends **discrete-valued symbols** through an **analog** communication channel
  - Receiver must **sample** I/Q signals at the appropriate time
  - Receiver matches I/Q sample values to corresponding symbols based on **decision regions**
  - **Constellation diagrams** are a convenient tool to see likelihood of bit errors being made
- **Choice of transmit filter:** **Tradeoff** between achieving a **minimal bandwidth** transmitted spectrum and **minimal ISI**
  - **Eye diagrams:** A convenient tool to see effects of ISI and sensitivity of bit errors to sample time choice

Thursday Topic:
Digital Communications II:
Receiver Processing, Performance