



# Global Illumination

COS 426, Spring 2019  
Princeton University

# Overview



- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - Shadows
  - Inter-object reflections
  - Rendering equation
  - Recursive ray tracing
  - More advanced ray tracing
  - Radiosity

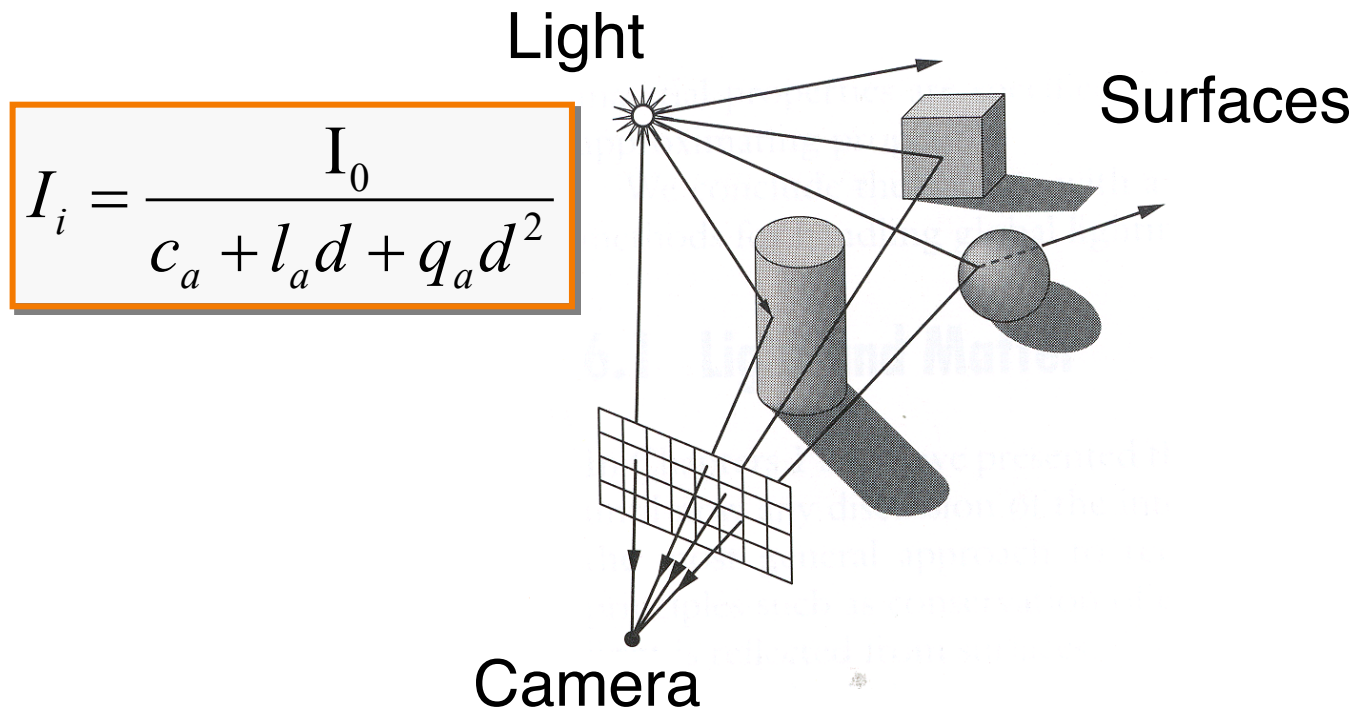
*Kajiya 1986*



*Greg Ward*

# Direct Illumination (last lecture)

- For each ray traced from camera
  - Sum radiance reflected from each light



$$I = I_E + K_A I_{AL} + \sum_i \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) I_i$$

# Direct illumination example



Red's Dream [Pixar 1987]

# Overview

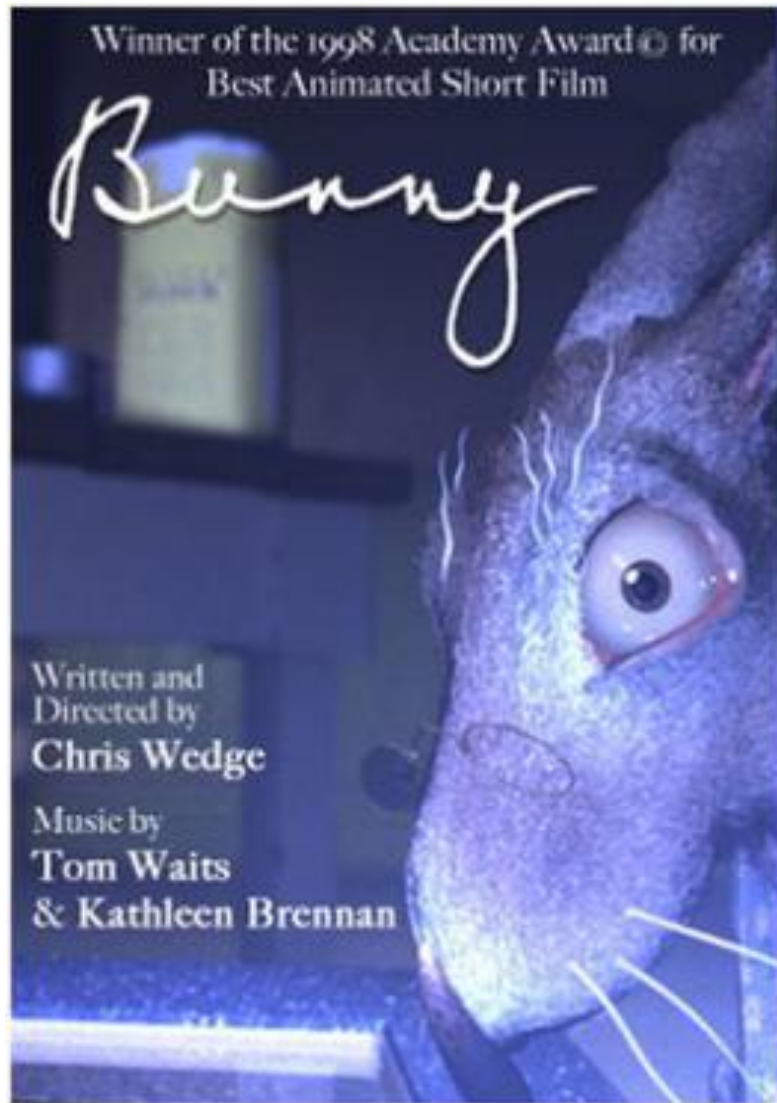


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Greg Ward

# Global illumination example



# Overview



- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - **Shadows**
    - Inter-object reflections
    - Rendering equation
    - Recursive ray tracing
    - More advanced ray tracing
    - Radiosity

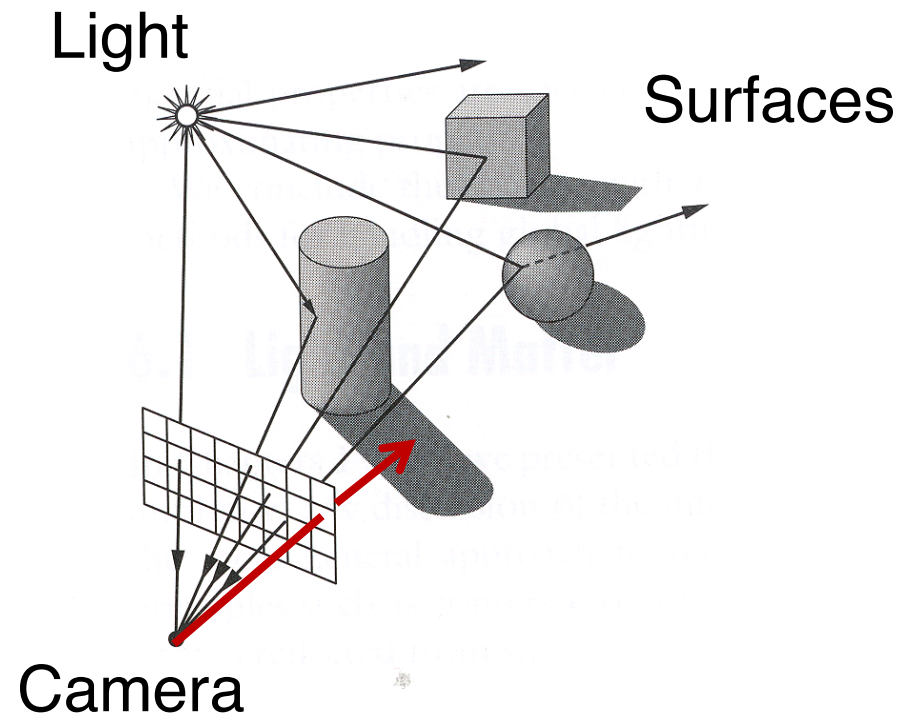


Greg Ward

# Shadows



- Hard shadows from point light sources

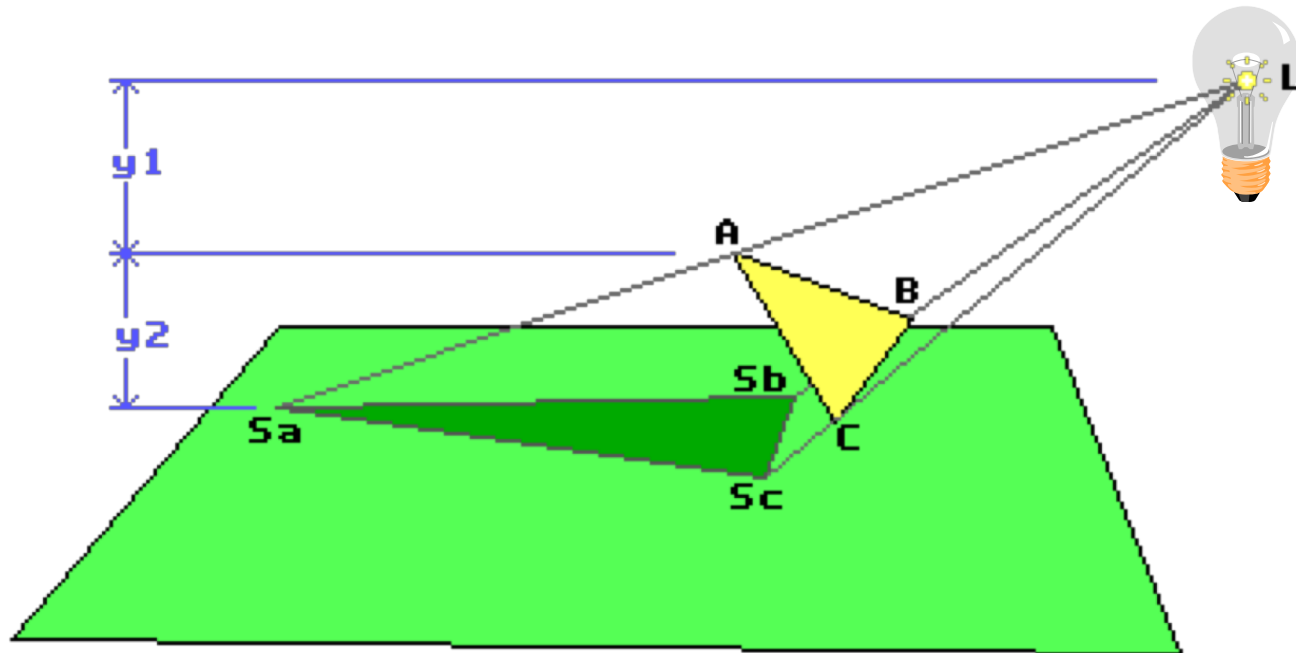




# Shadows

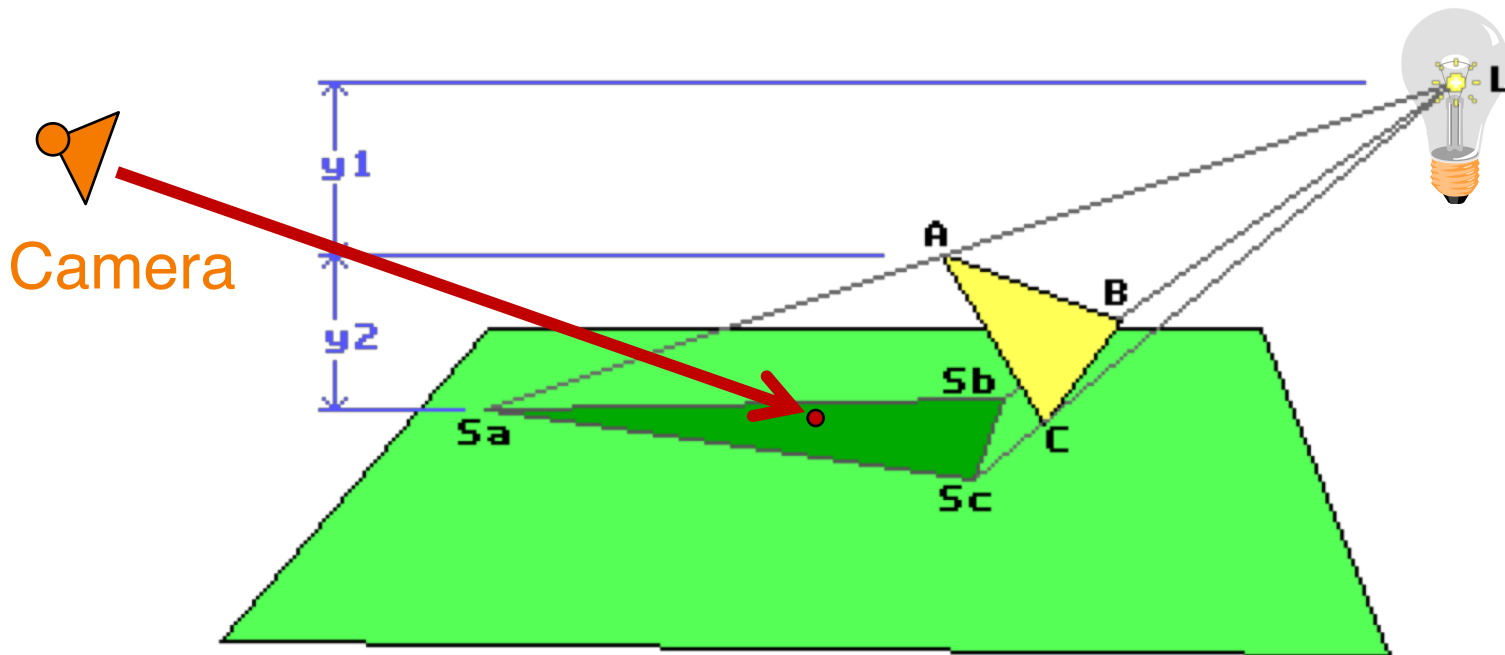


- Hard shadows from point light sources



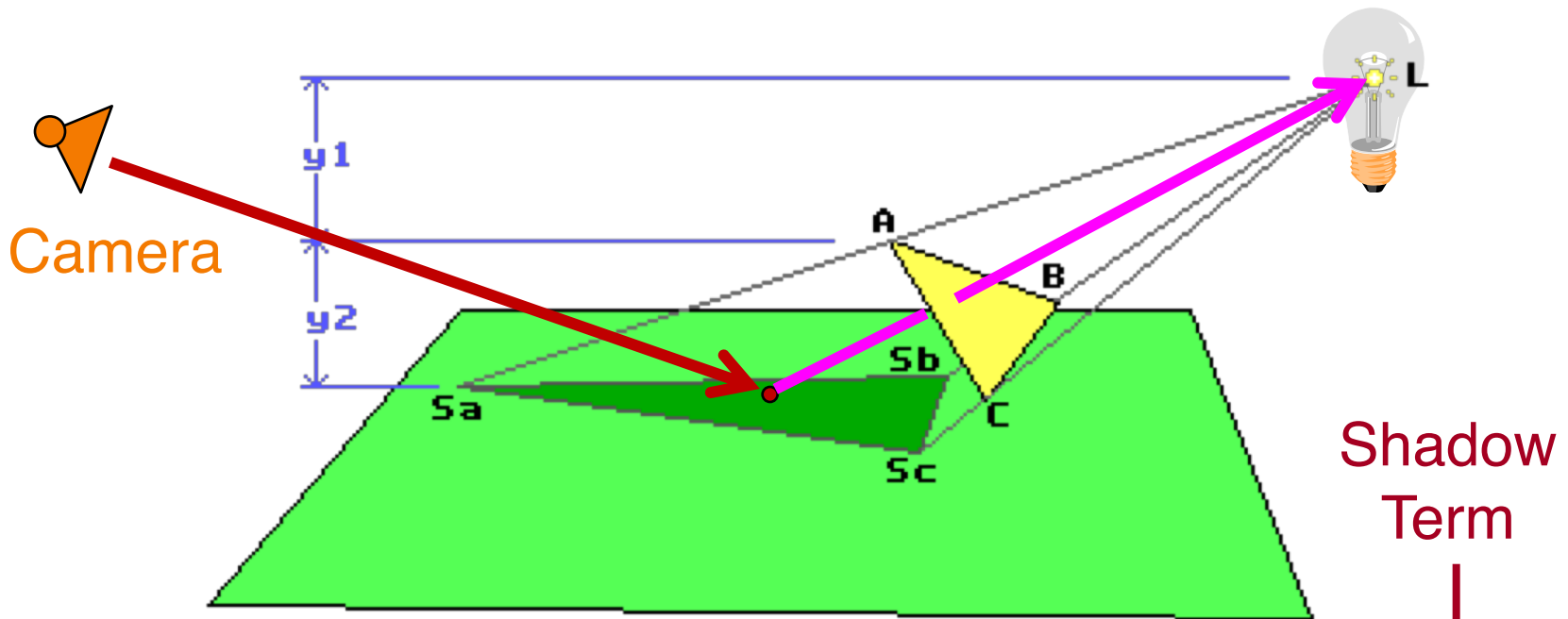
# Shadows

- Hard shadows from point light sources



# Shadows

- Hard shadows from point light sources
  - Cast ray towards light;  $S_L=0$  if blocked,  $S_L=1$  otherwise

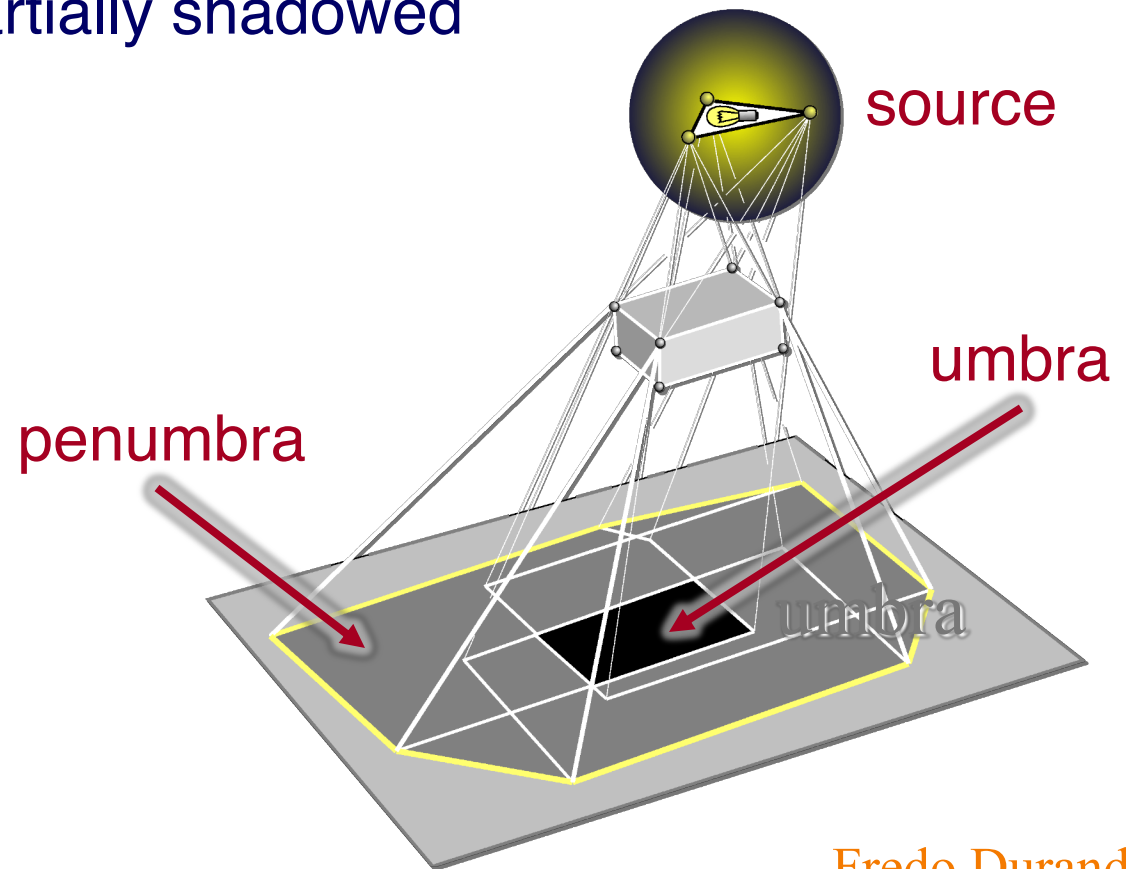


$$I = I_E + K_A I_{AL} + \sum_{i \in \text{lights}} \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_i I_i$$

# Shadows

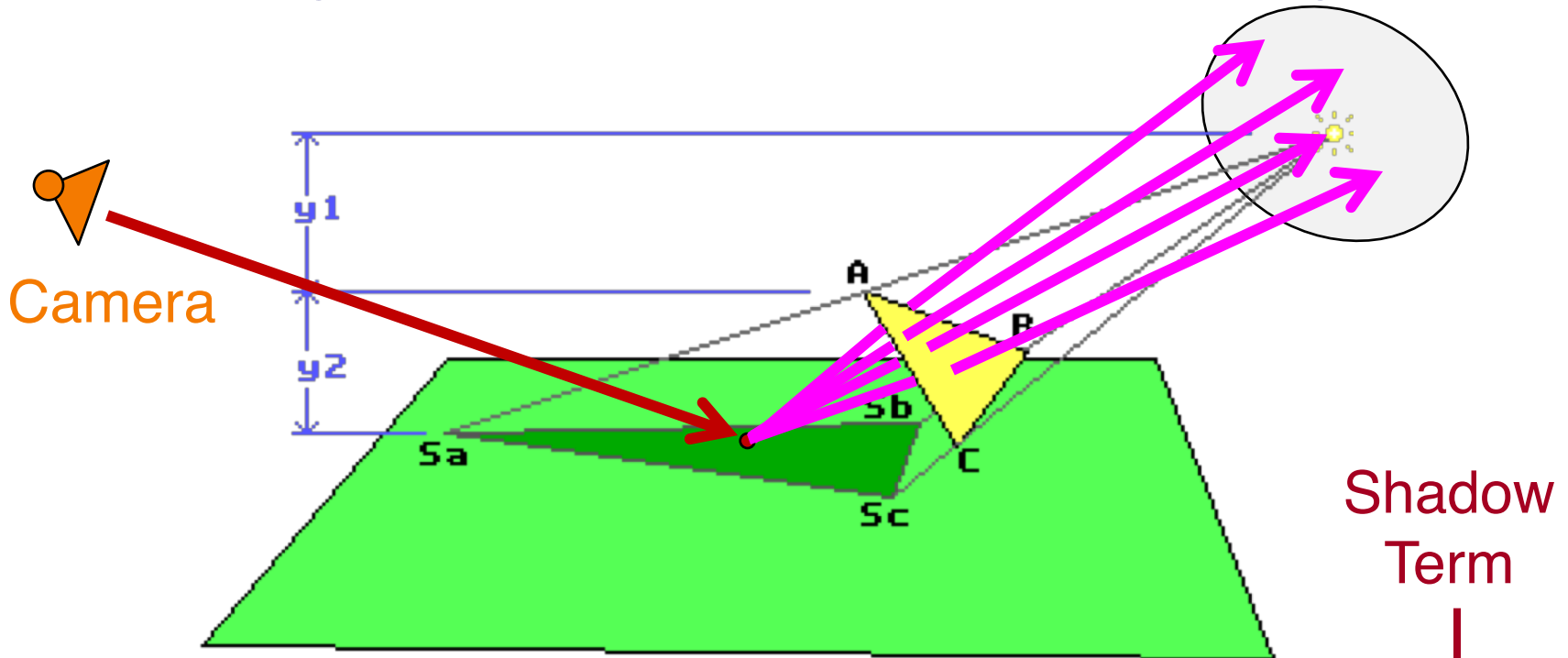


- Soft shadows from area light sources
  - Umbra = fully shadowed
  - Penumbra = partially shadowed



# Shadows

- Soft shadows from area light sources
  - Average illumination for  $M$  sample rays per light

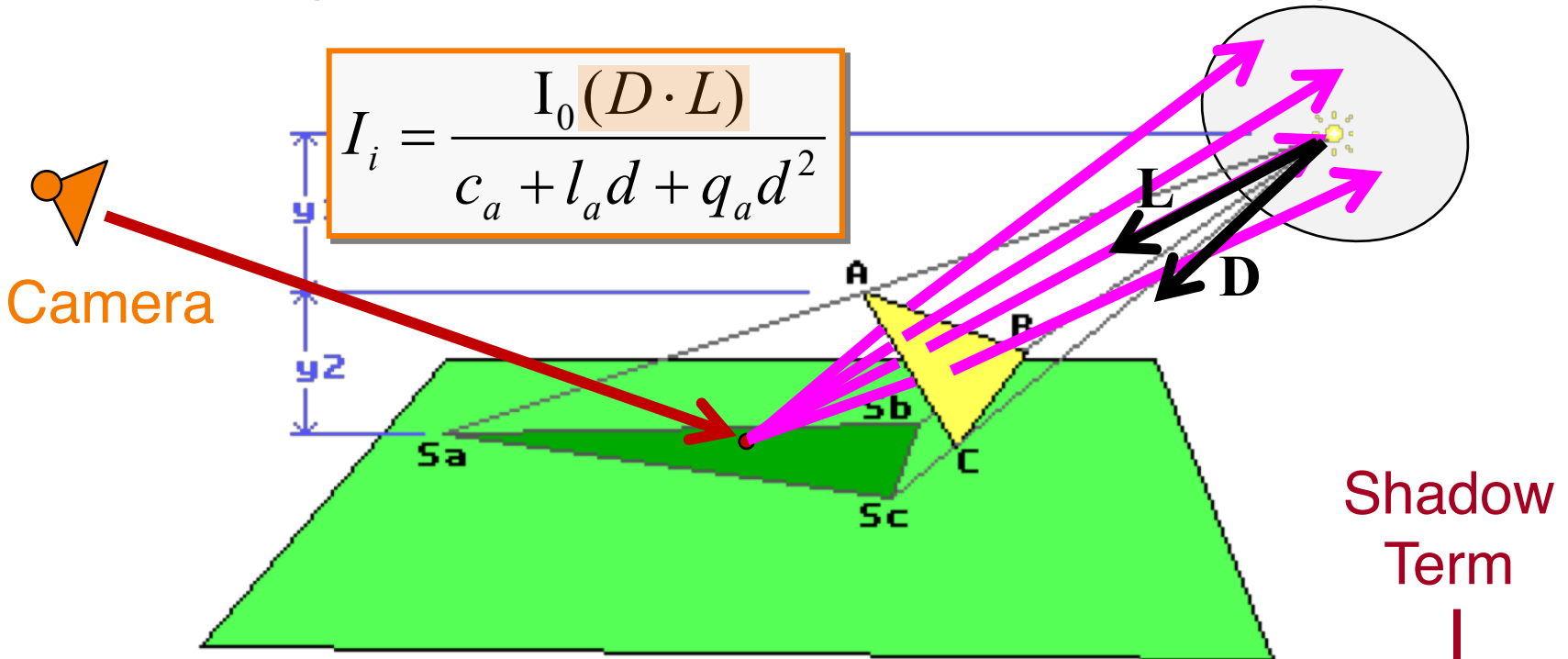


Shadow Term

$$I = \dots + \sum_{i \in \text{AreaLights}} \frac{1}{M} \sum_{j \in \text{samples}} \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$

# Shadows

- Soft shadows from circular area light sources
  - Average illumination for  $M$  sample rays per light

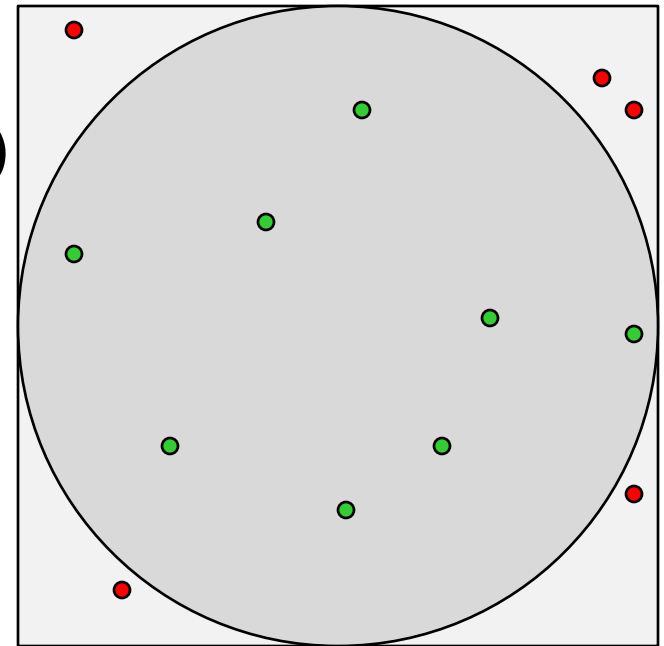


$$I = \dots + \sum_{i \in \text{AreaLights}} \frac{1}{M} \sum_{j \in \text{samples}} \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$

# Shadows



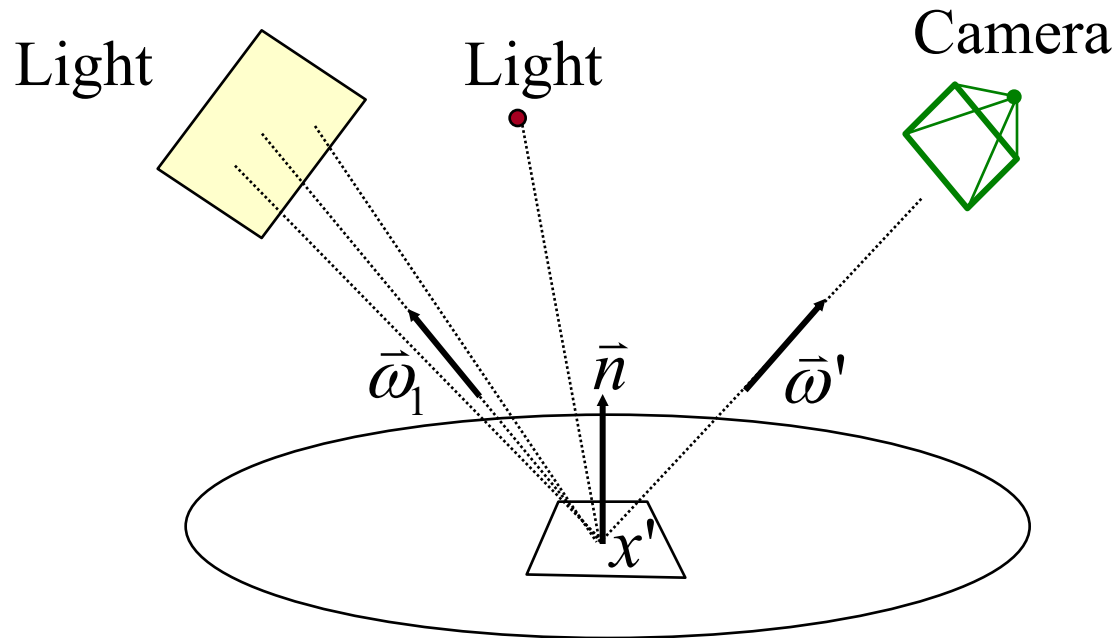
- Soft shadows from circular area light sources
  - Average illumination for M sample rays per light
    - Generate M random sample points on area light (e.g., with rejection sampling)
    - Compute illumination for every sample
    - Average



$$I = \dots + \sum_{i \in \text{AreaLights}} \frac{1}{M} \sum_{j \in \text{samples}} \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$

# Direct Illumination

- Illumination from polygonal area light sources
  - Average illumination for  $M$  sample rays per light



$$I = \dots + \sum_{i \in \text{AreaLights}} \frac{1}{M} \sum_{j \in \text{samples}} \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_{ij} I_{ij}$$



# Overview

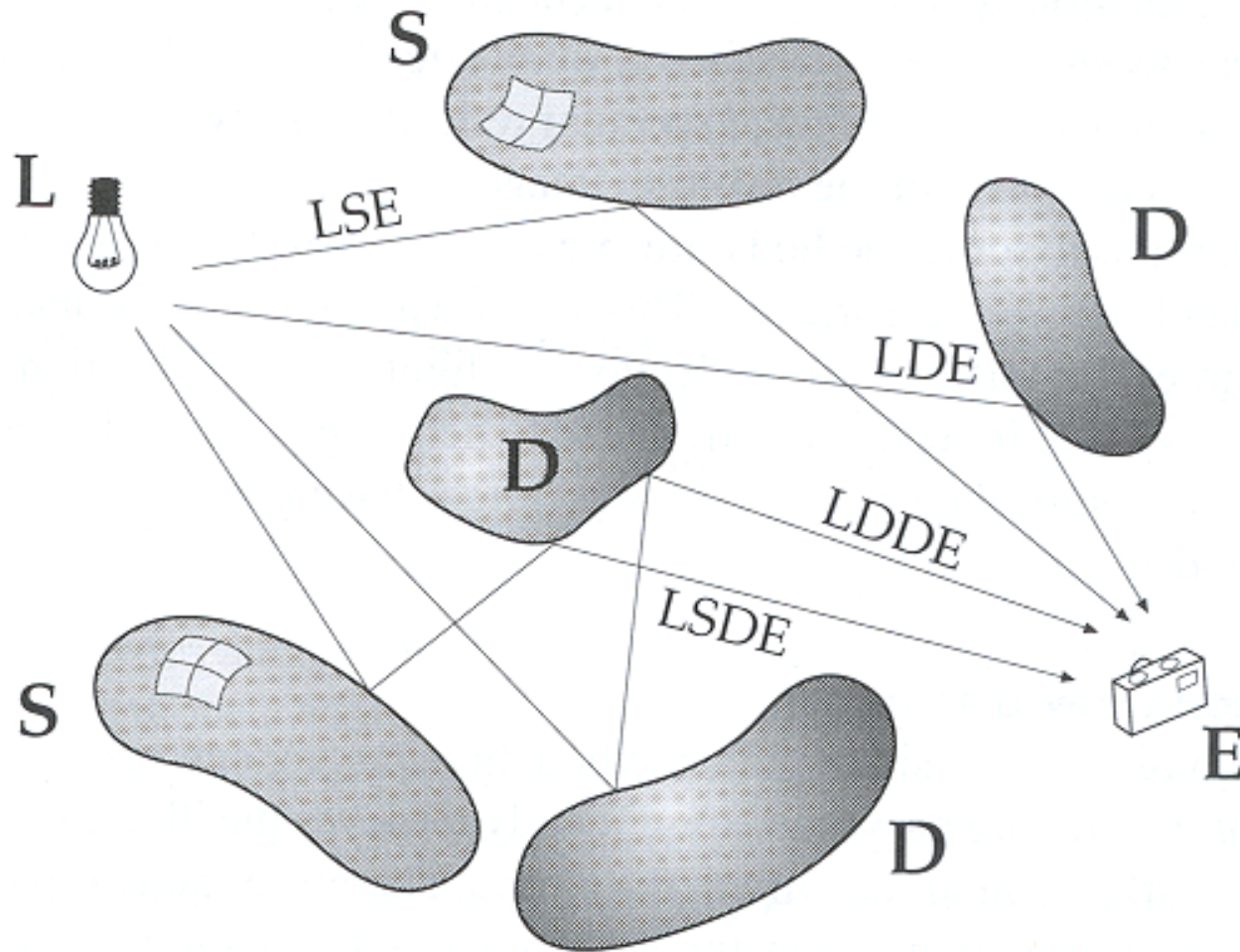


- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - Shadows
  - **Inter-object reflections**
  - Rendering equation
  - Recursive ray tracing
  - More advanced ray tracing
  - Radiosity



Greg Ward

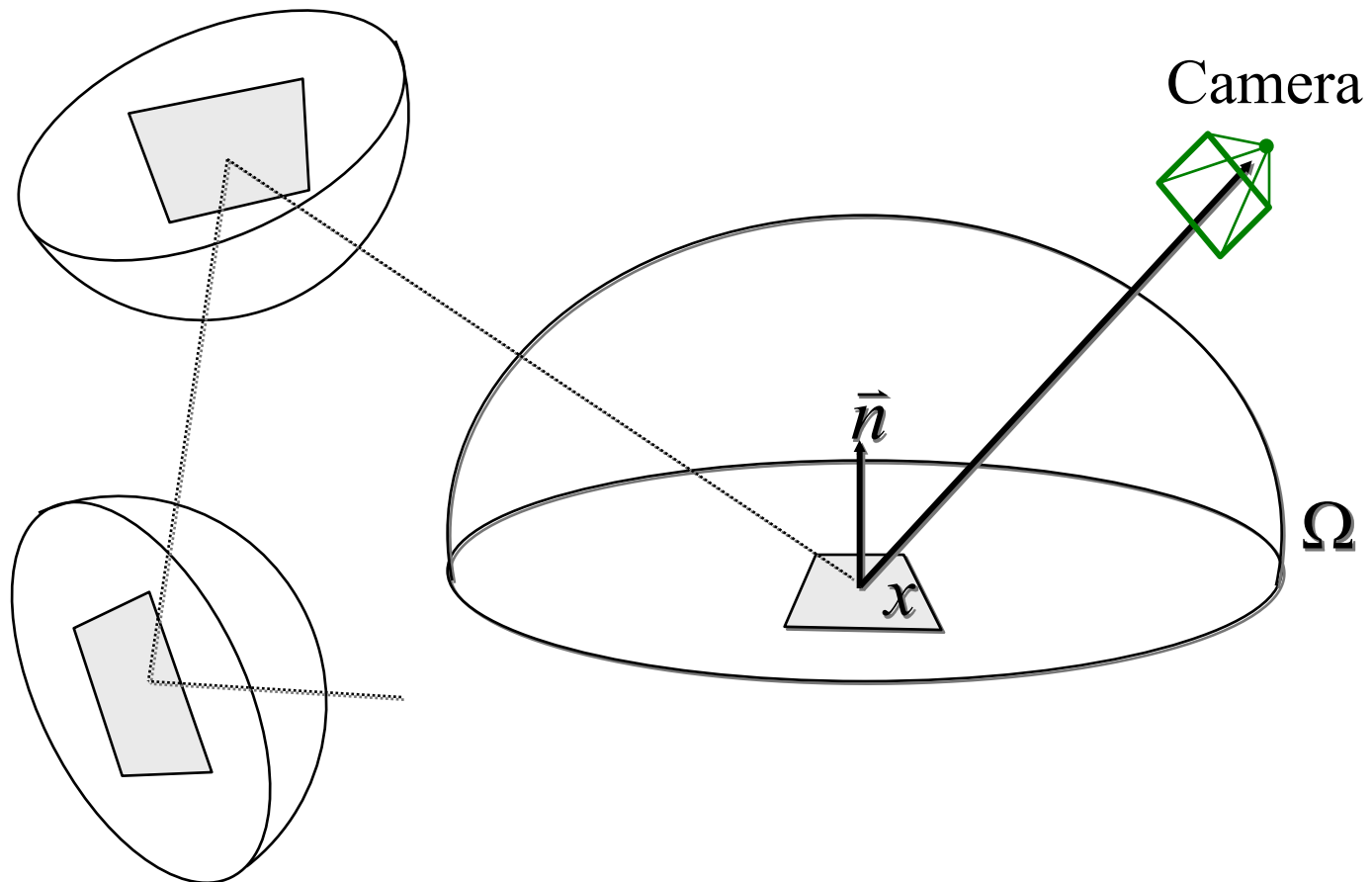
# Inter-Object Reflection



# Inter-Object Reflection



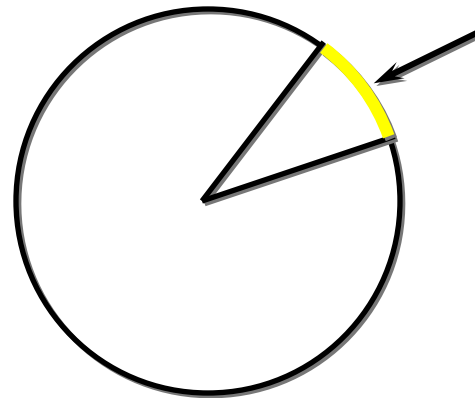
- **Radiance** leaving point  $x$  on surface is sum of reflected **irradiance** arriving from other surfaces



# Solid Angle



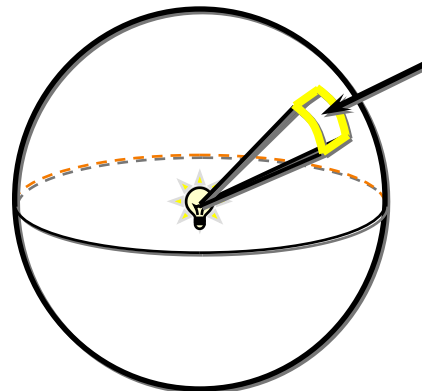
- Angle in radians



Length  $l$   
Angle  $\theta = l/r$

(Full circle is  $2\pi$  radians.)

- Solid angle in **steradians**

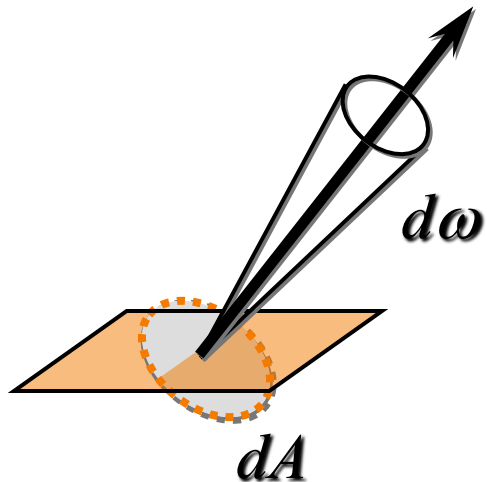


Area  $A$   
Solid angle  $\omega = A/r^2$

(Full sphere is  $4\pi$  steradians.)

# Light Emitted from a Surface

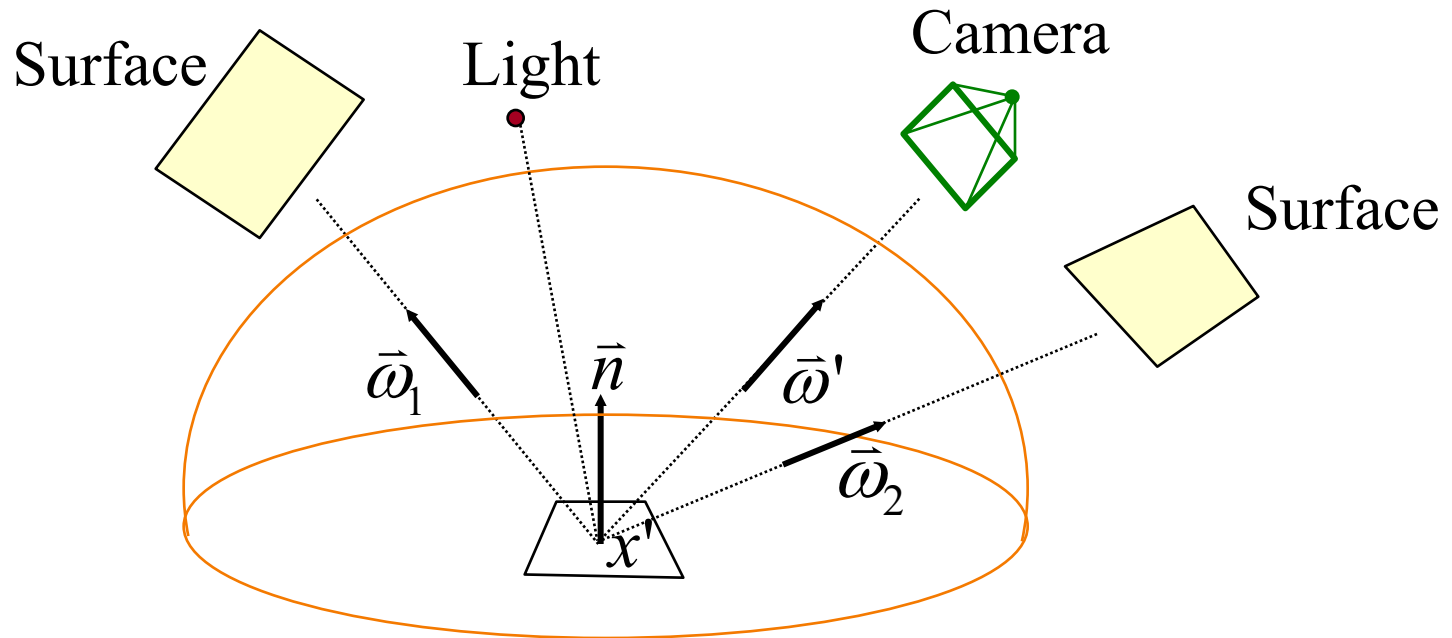
- Power per unit area per unit solid angle – *Radiance* ( $L$ )
  - Measured in  $\text{W}/\text{m}^2/\text{sr}$



$$L = \frac{d\Phi}{dA d\omega}$$

# Rendering Equation [Kajiya 86]

- Compute radiance in outgoing direction by integrating reflections over all incoming directions

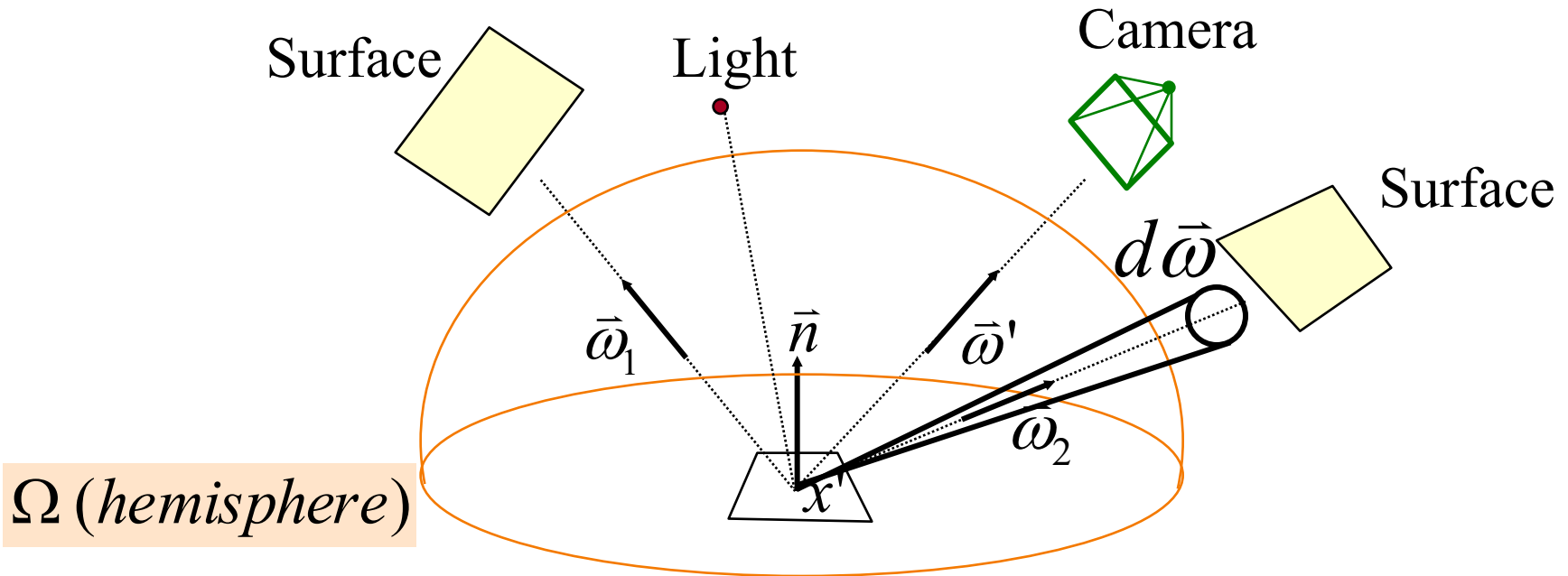


$$L_o(x', \vec{\omega}') = L_e(x', \vec{\omega}') + \int_{\Omega} f_r(x', \vec{\omega}, \vec{\omega}') (\vec{\omega} \cdot \vec{n}) L_i(x', \vec{\omega}) d\vec{\omega}$$

# Rendering Equation [Kajiya 86]



- Compute radiance in outgoing direction by integrating reflections over all incoming directions



$$L_o(x', \bar{\omega}') = L_e(x', \bar{\omega}') + \int_{\Omega} f_r(x', \bar{\omega}, \bar{\omega}') (\bar{\omega} \cdot \bar{n}) L_i(x', \bar{\omega}) d\bar{\omega}$$

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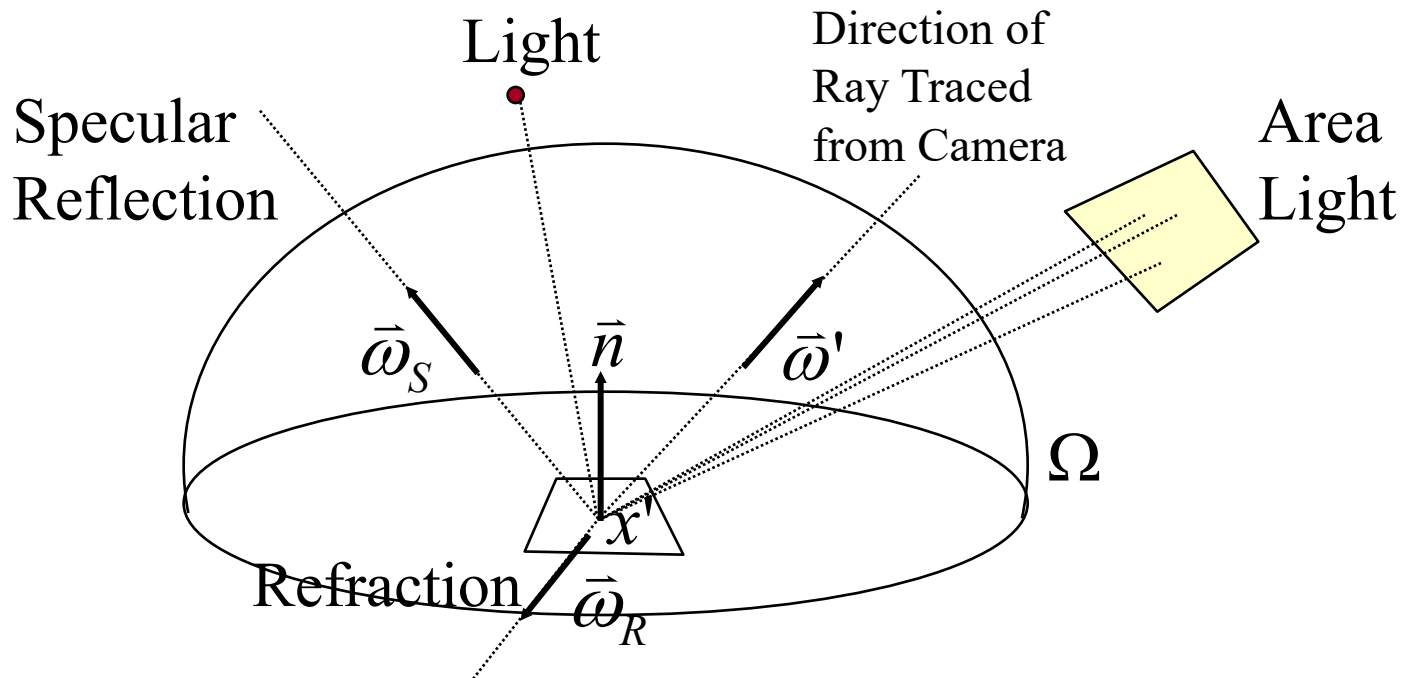
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# Recursive Ray Tracing



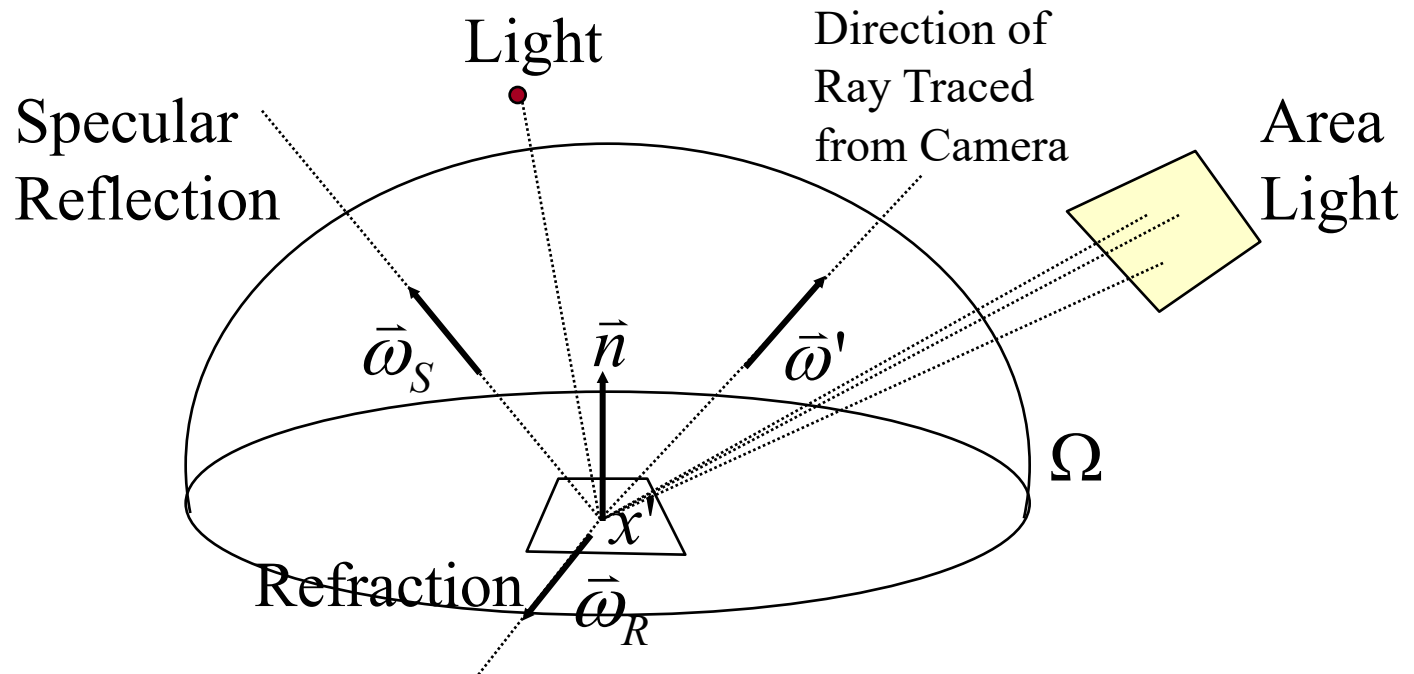
- Assume only significant irradiance is in directions of light sources, specular reflection, and refraction



$$L_o(x', \vec{\omega}') = L_e(x', \vec{\omega}') + \int_{\Omega} f_r(x', \vec{\omega}, \vec{\omega}') (\vec{\omega} \cdot \vec{n}) L_i(x', \vec{\omega}) d\vec{\omega}$$

# Recursive Ray Tracing

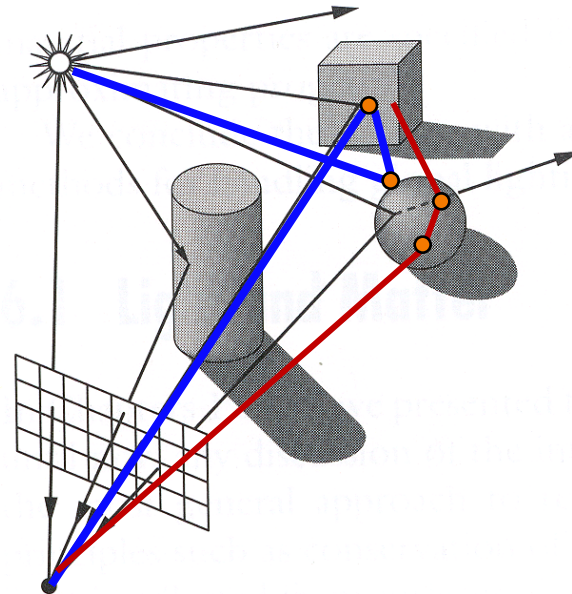
- Compute radiance in outgoing direction by summing reflections from directions of lights specular reflections, and refractions



$$I = I_E + K_A I_{AL} + \sum_L \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

# Recursive Ray Tracing

- Same as ray casting, but trace secondary rays for specular (mirror) reflection and refraction



$$I = I_E + K_A I_{AL} + \sum_L \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

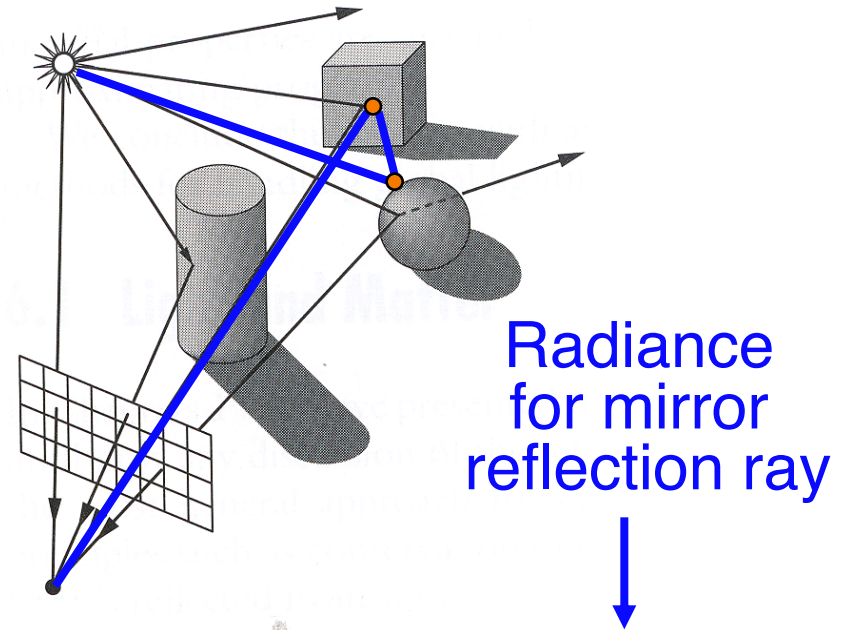
# Specular Reflection



- Trace secondary ray in direction of mirror reflection
  - Evaluate radiance along secondary ray and include it into illumination model



[wikimedia.org/wiki/en/c1/Cloud\\_Gate\\_\(The\\_Bean\)\\_from\\_east'.jpg](https://commons.wikimedia.org/wiki/File:Cloud_Gate_(The_Bean)_from_east'.jpg)

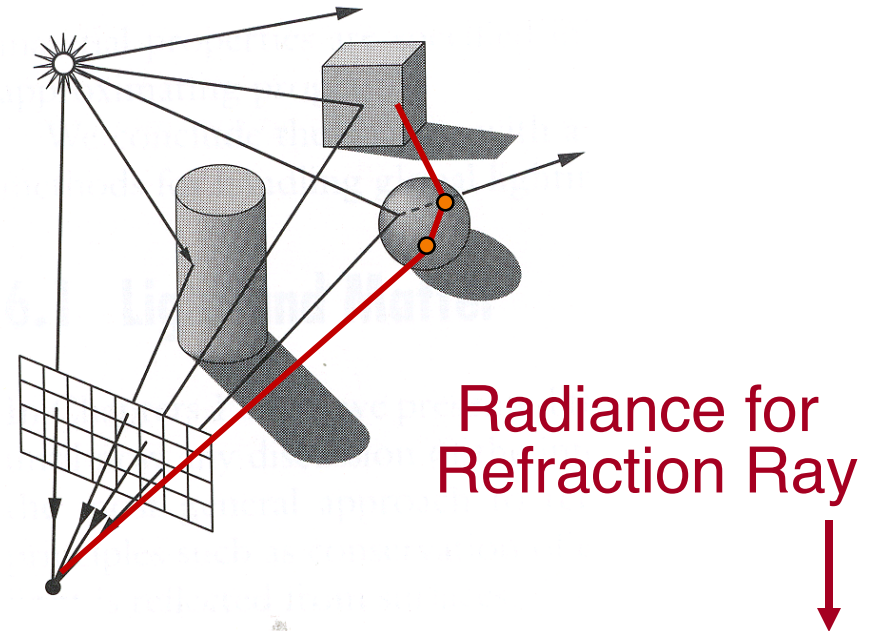


$$I = I_E + K_A I_{AL} + \sum_L \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

# Refraction



- Trace secondary ray in direction of refraction
  - Evaluate radiance along secondary ray and include it into illumination model



$$I = I_E + K_A I_{AL} + \sum_L \left( K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

# Recursive Ray Tracing

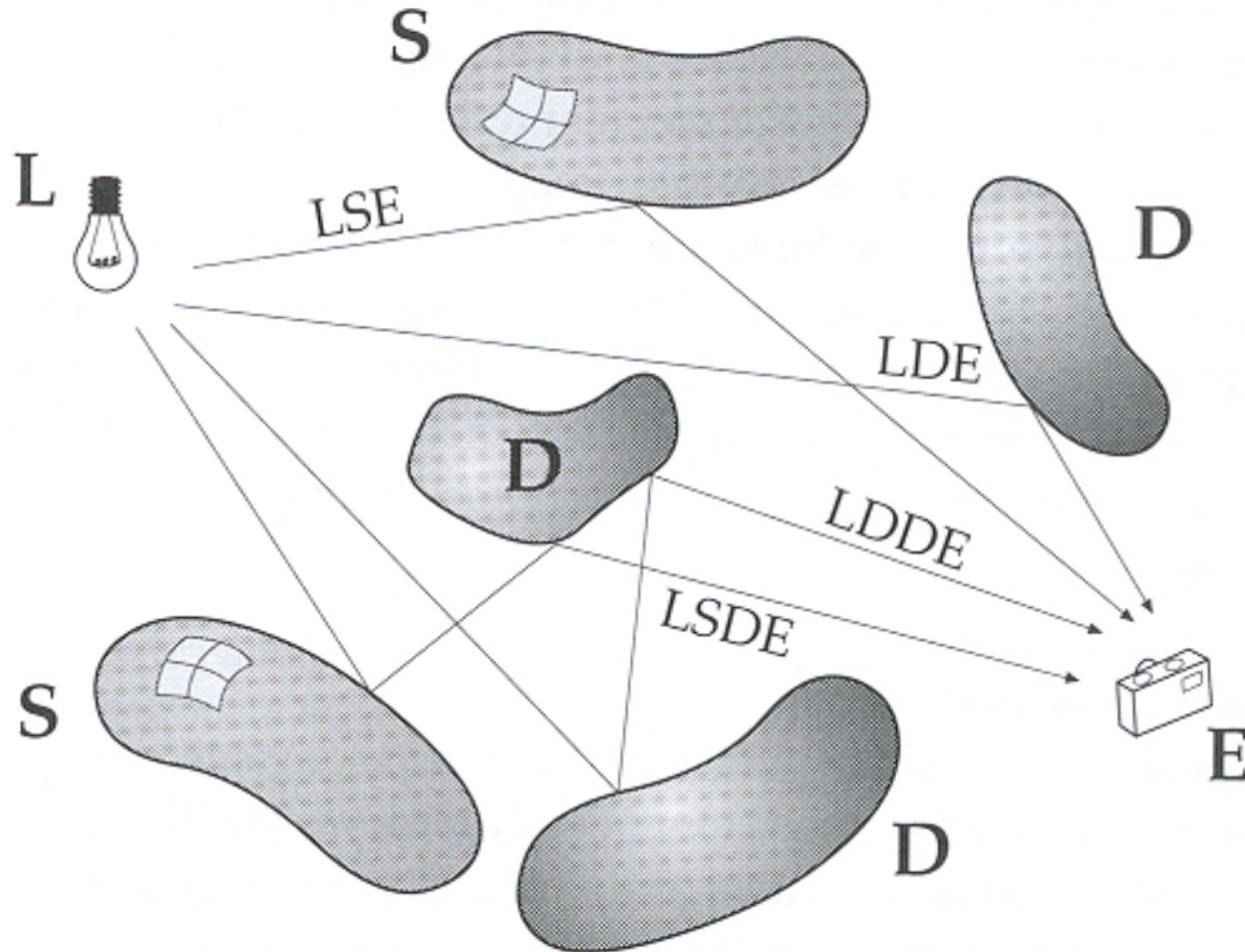


- ComputeRadiance is called recursively

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray, R3Intersection& hit)
{
    R3Ray specular_ray = SpecularRay(ray, hit);
    R3Ray refractive_ray = RefractiveRay(ray, hit);
    R3Rgb radiance = Phong(scene, ray, hit) +
                    Ks * ComputeRadiance(scene, specular_ray) +
                    Kt * ComputeRadiance(scene, refractive_ray);
    return radiance;
}
```

# Recursive Ray Tracing

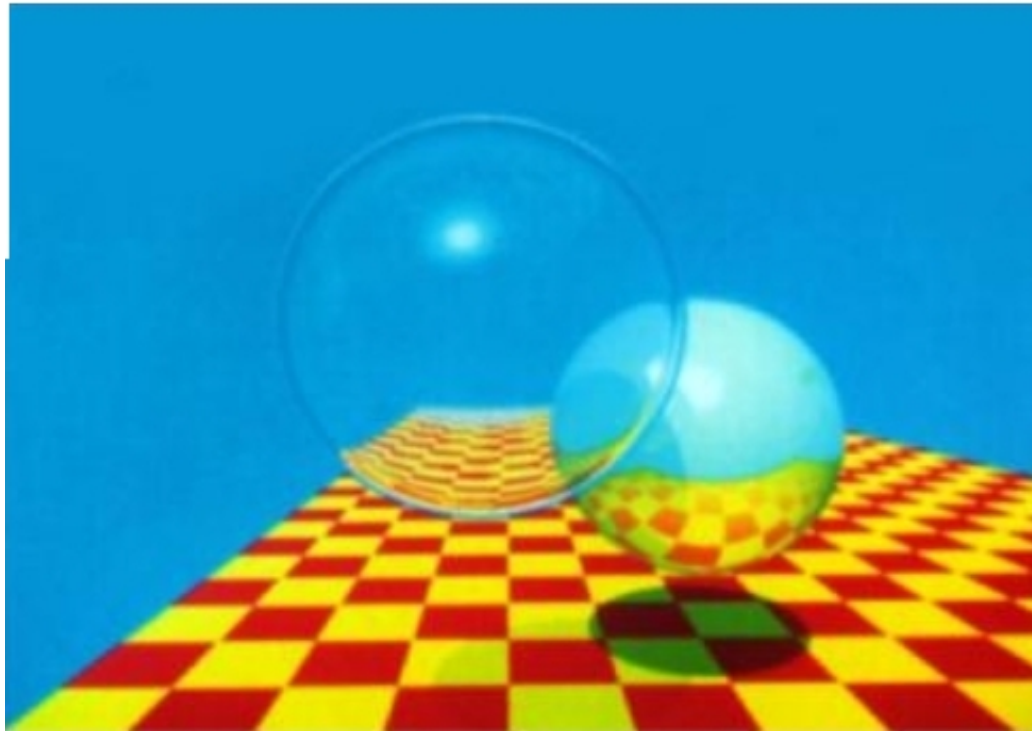
- Which paths?



# Recursive Ray Tracing



- Specular reflection and refraction --  $LD(S|R)^*E$





# Overview

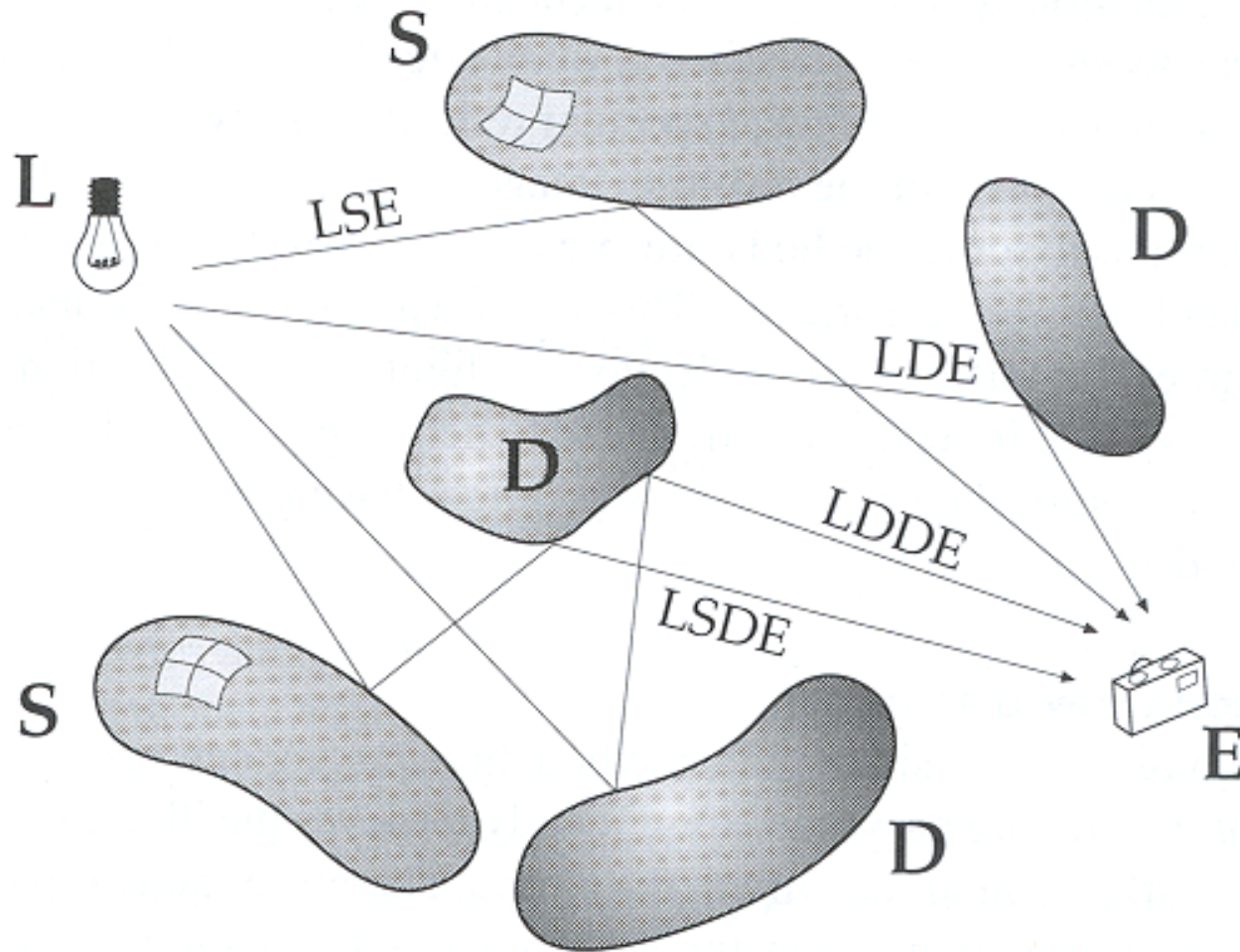


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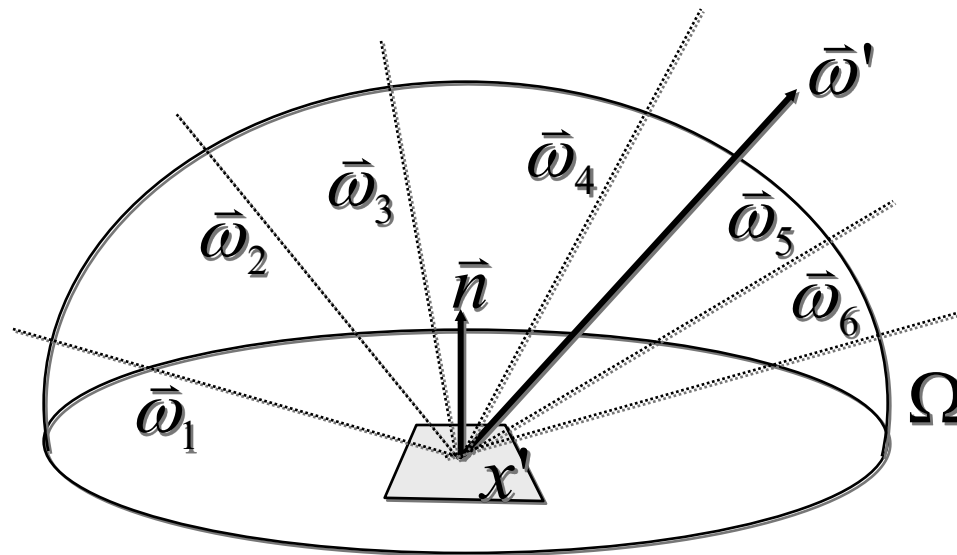
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# Beyond Recursive Ray Tracing



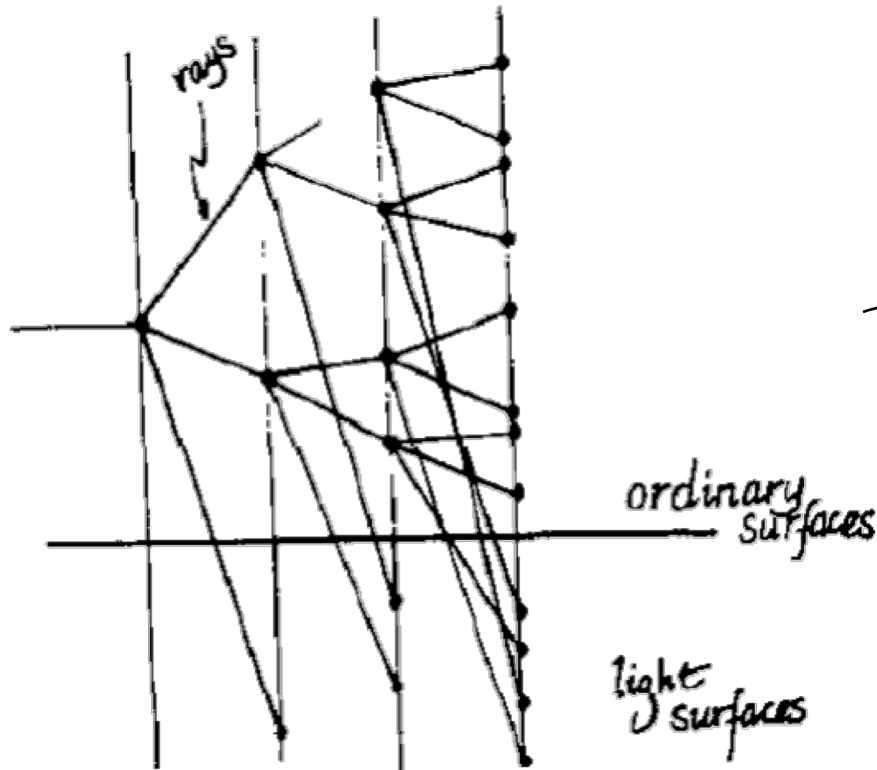
# Distributed Ray Tracing

- Estimate integral for each reflection by sampling incoming directions

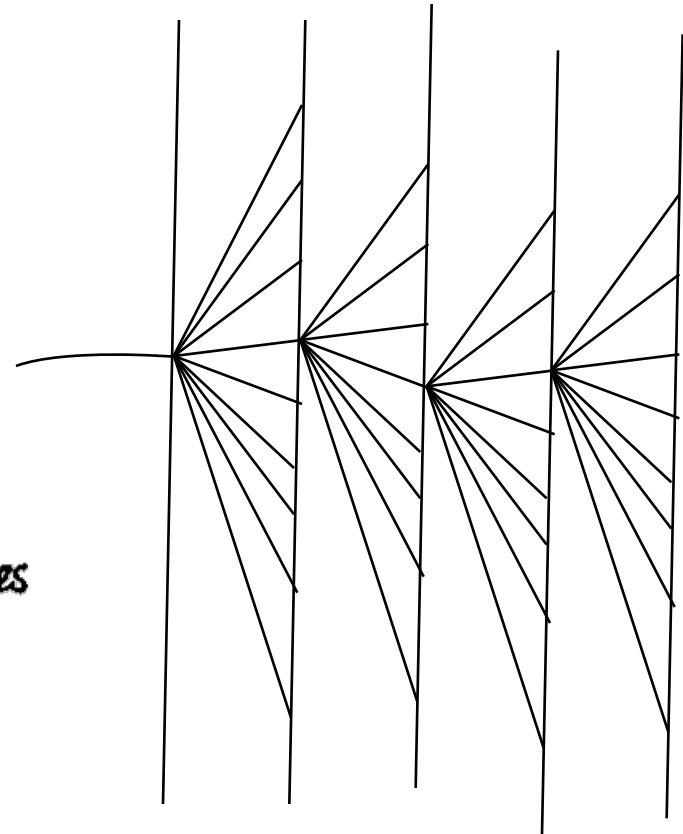


$$L_o(x', \bar{\omega}') = L_e(x', \bar{\omega}') + \sum_{\text{samples}} f_r(x', \bar{\omega}, \bar{\omega}') (\bar{\omega} \cdot \bar{n}) L_i(x', \bar{\omega}) d\bar{\omega}$$

# Ordinary Ray Tracing vs. Distribution Ray Tracing



Ray tracing

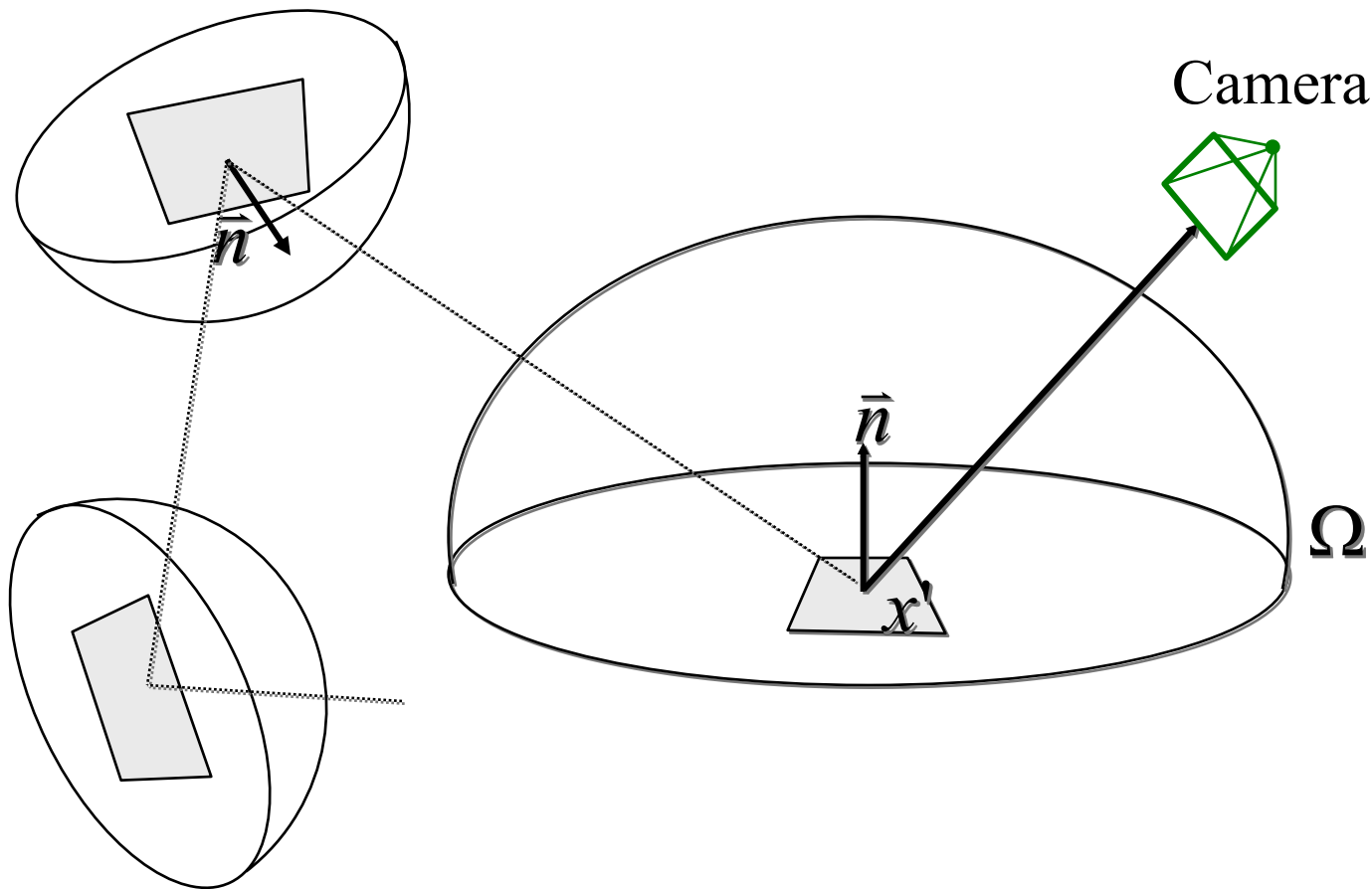


Distributed ray tracing

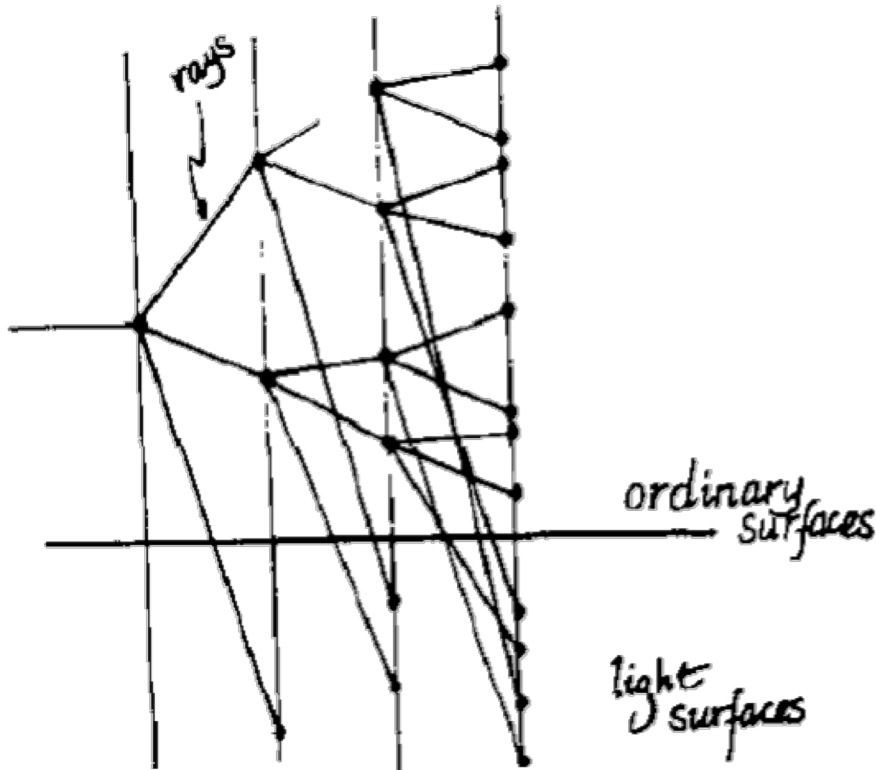
# Monte Carlo Path Tracing



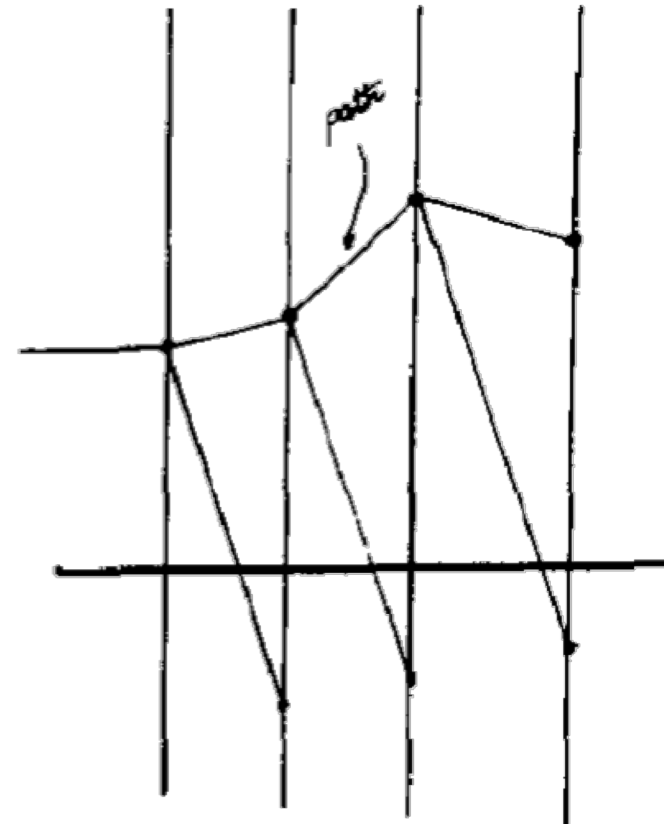
- Estimate integral for each pixel by sampling paths from camera



# Ray Tracing vs. Path Tracing



Ray tracing



Path tracing

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# Radiosity

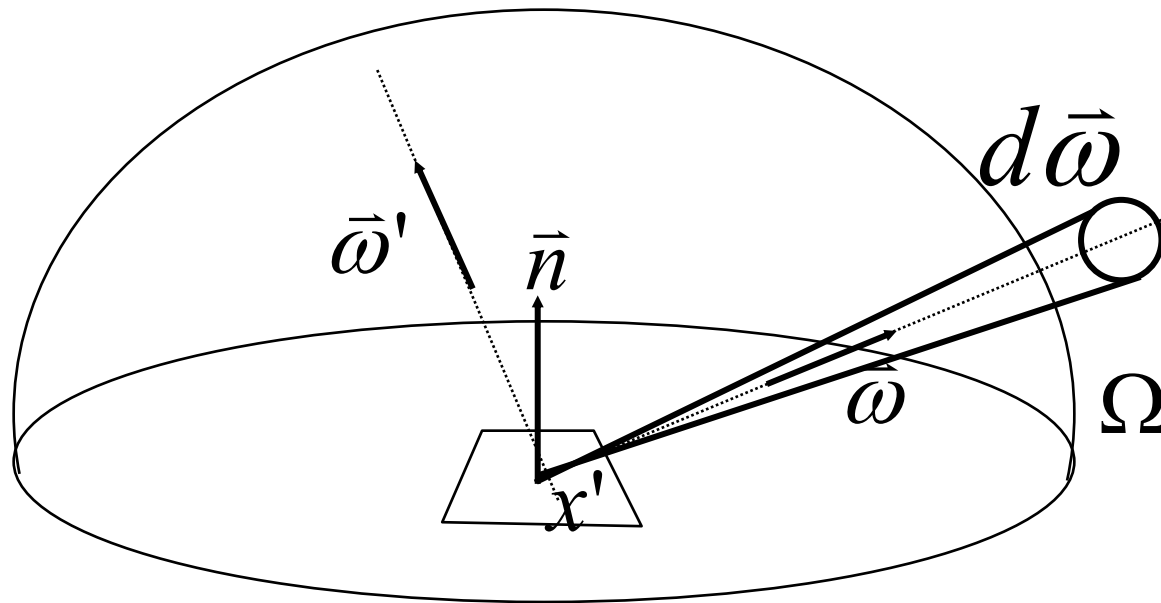


- Indirect diffuse illumination –  $LD^*E$



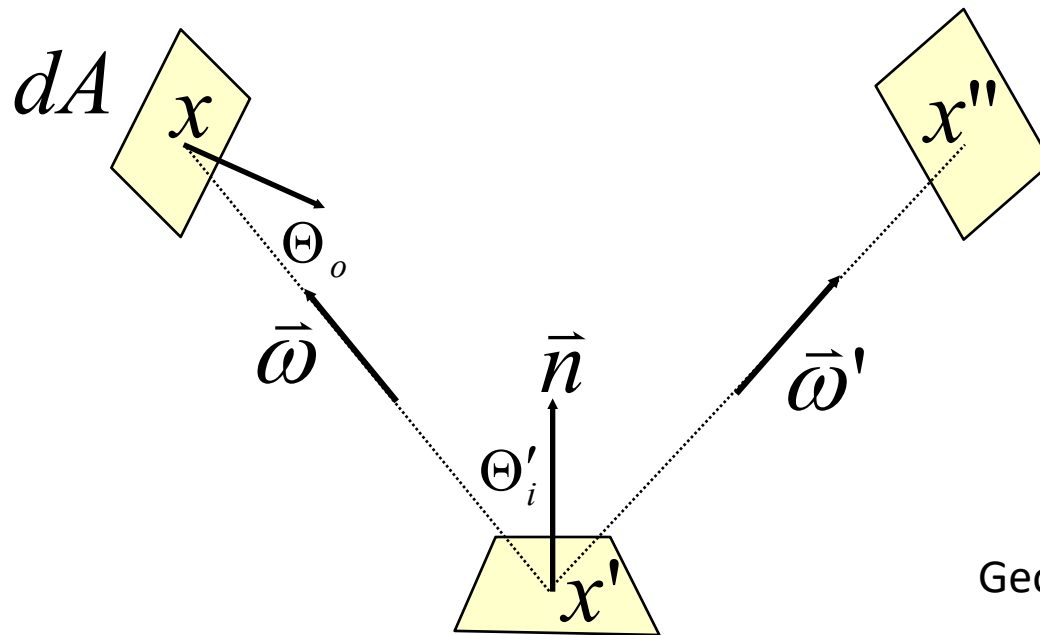


# Rendering Equation (1)



$$L_o(x', \bar{\omega}') = L_e(x', \bar{\omega}') + \int_{\Omega} f_r(x', \bar{\omega}, \bar{\omega}') (\bar{\omega} \cdot \bar{n}) L_i(x', \bar{\omega}) d\bar{\omega}$$

# Rendering Equation (2)



Geometry (G)

$$G(x, x') = \frac{\cos \Theta'_i \cos \Theta_o}{\|x - x'\|^2}$$

Visibility (V)

$$L(x' \rightarrow x'') = L_e(x' \rightarrow x'') + \int_S f_r(x \rightarrow x' \rightarrow x'') L(x \rightarrow x') V(x, x') G(x, x') dA$$

*Kajiya 1986*

# Radiosity Equation



$$L(x' \rightarrow x'') = L_e(x' \rightarrow x'') + \int_S f_r(x \rightarrow x' \rightarrow x'') L(x \rightarrow x') V(x, x') G(x, x') dA$$

Assume everything  
is Lambertian

$$\rho(x') = f_r(x \rightarrow x' \rightarrow x'') \pi$$

$$L(x') = L_e(x') + \frac{\rho(x')}{\pi} \int_S L(x) V(x, x') G(x, x') dA$$

Convert to  
Radiosities

$$B = \int_{\Omega} L_o \cos \theta d\omega$$

$$L = \frac{B}{\pi}$$

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi} \int_S B(x) V(x, x') G(x, x') dA$$

# Radiosity Approximation

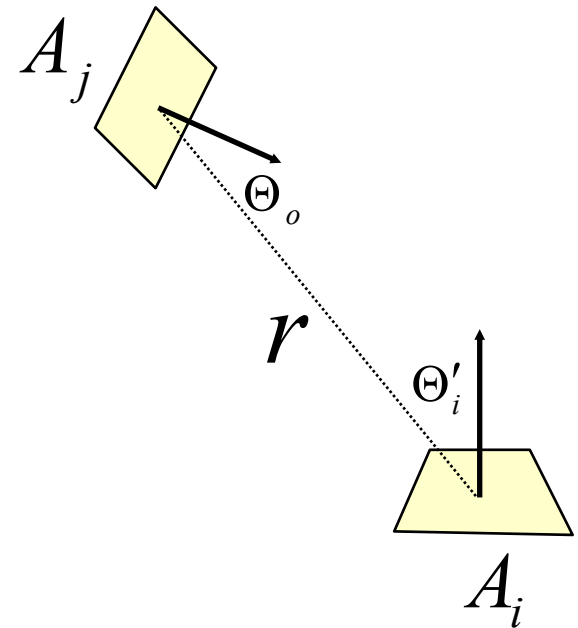
$$B(x') = B_e(x') + \frac{\rho(x')}{\pi} \int_S B(x) V(x, x') G(x, x') dA$$

Discretize the surfaces  
into “elements”

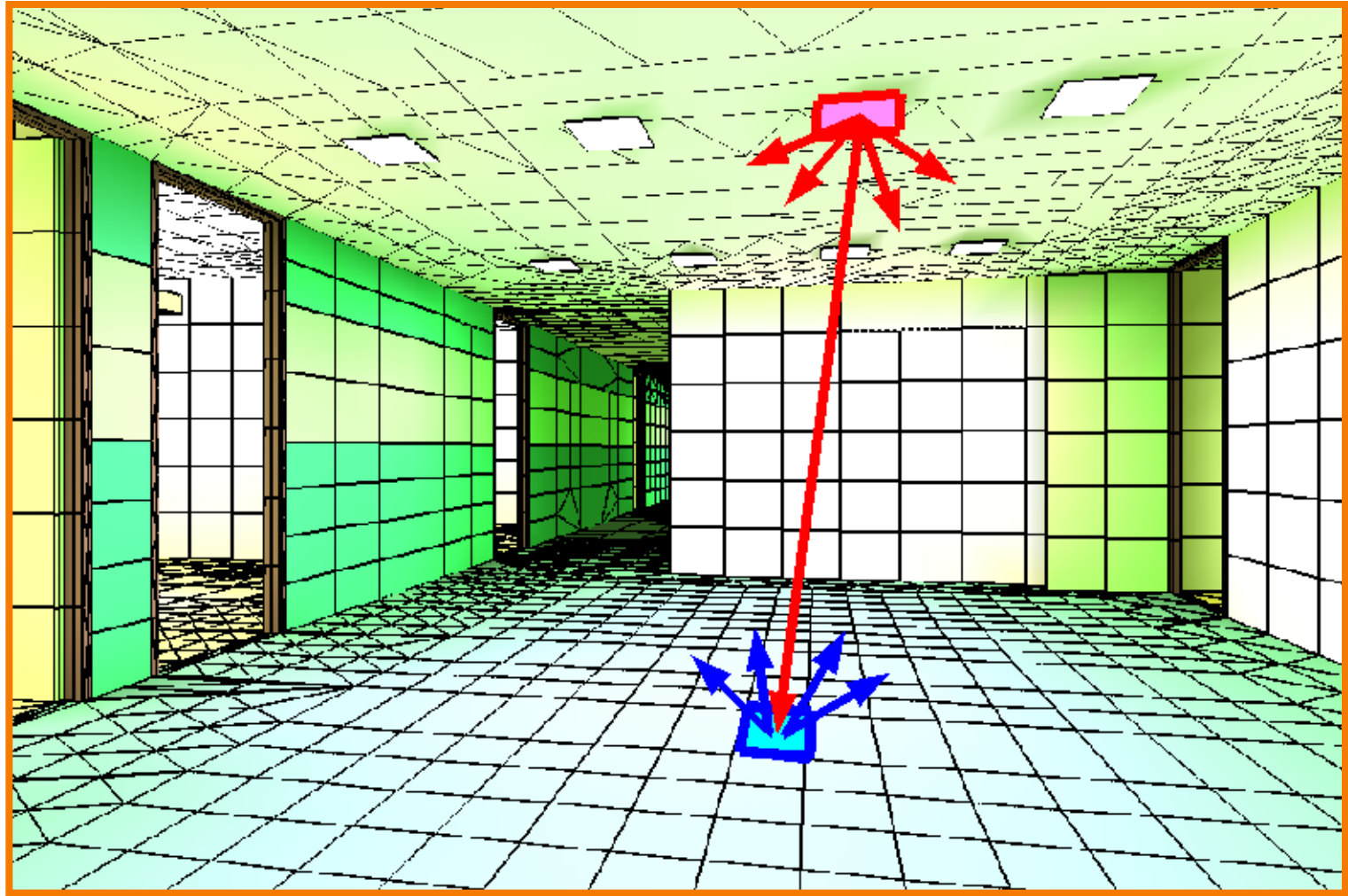
$$B_i = E_i + \rho_i \sum_{j=1}^N B_j F_{ij}$$

where “form factor” F is:

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{V_{ij} \cos \Theta'_i \cos \Theta_o}{\pi r^2} dA_j dA_i$$



# Radiosity Approximation



# Form Factor



## On the Form Factor between Two Polygons

Peter Schröder

Pat Hanrahan

Department of Computer Science  
Princeton University



1993

### Abstract

Form factors are used in radiosity to describe the fraction of diffusely reflected light leaving one surface and arriving at another. They are a fundamental geometric property used for computation. Many special configurations admit closed form solutions. However, the important case of the form factor between two polygons in three space has had no known closed form solution. We give such a solution for the case of general (planar, convex or concave, possibly containing holes) polygons.

**CR Categories and Subject Descriptors:** I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism – Radiosity; J.2 [Physical Sciences and Engineering]: Engineering.

**Additional Key Words and Phrases:** Closed form solution; form factor; polygons.

### 1 Introduction

When using the radiosity technique, the form factor plays a central role. It is the fraction of light that is emitted from

In this paper we present a formula for the form factor integral between two general polygons. The derivation of this formula is quite involved, and the interested reader is referred to [9] for a detailed derivation. The purpose of this paper is to bring this result to the attention of the graphics community.

### 2 Closed form solution

The form factor integral can be reduced to a double contour integral by two applications of Stokes' theorem [6]

$$\begin{aligned}\pi A_1 F_{12} &= \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{\|\vec{r}\|^2} dA_2 dA_1 \\ &= \frac{1}{4} \int_{\partial A_1} \int_{\partial A_2} \ln(\vec{r} \cdot \vec{r}) d\vec{x}_2 \cdot d\vec{x}_1\end{aligned}$$

where  $\theta_1$ ,  $\theta_2$  are the angles between the normal vector of the respective surface and a radius vector  $\vec{r}$ , which connects two points on the surfaces. The above equation holds for all surfaces such as the same contour of the

- contour integral reduces
- [5] HANRAHAN, P., SALZMAN, D., AND AUPPERLE, L. A Rapid Hierarchical Radiosity Algorithm. *Computer Graphics* 25, 4 (July 1991), 197–206.
  - [6] HERMAN, R. A. *A Treatise on Geometrical Optics*. Cambridge University Press, 1900.
  - [7] LAMBERT. *Photometria sive de mensura et gradibus luminis, colorum et umbrae*. 1760. German translation by E. Anding in *Ostwald's Klassiker der Exakten Wissenschaften*, Vol. 31-33, Leipzig, 1892.

# System of Equations



$$B_i = E_i + \rho_i \sum_{j=1}^N B_j F_{ij}$$

$$E_i = B_i - \rho_i \sum_{j=1}^N B_j F_{ij}$$

$$B_i - \rho_i \sum_{j=1}^N B_j F_{ij} = E_i$$

$$\begin{bmatrix} 1 - \rho_1 F_{1,1} & \cdot & \cdot & \cdot & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1 - \rho_2 F_{2,2} & \cdot & \cdot & -\rho_2 F_{2,n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -\rho_{n-1} F_{n-1,1} & \cdot & \cdot & \cdot & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & \cdot & \cdot & \cdot & 1 - \rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_n \end{bmatrix}$$

$$\left( 1 - \rho_i \sum_{j=1}^N F_{ij} \right) B_i - \rho_i \sum_{j=1}^N F_{ij} B_j = E_i$$

$$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^N F_{ji} B_j A_j$$

← This is an energy balance equation

# Compare with Direct Illumination



Direct Illumination



Radiosity



# Radiosity



- Application
  - Interior lighting design
  - $LD^*E$
- Issues
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving large linear system of equations
  - Meshing surfaces into elements
  - Rendering images



Henrik Wann Jensen

# Summary



- Global illumination
  - Rendering equation
- Solution methods
  - Sampling
    - Ray tracing
    - Distributed ray tracing
    - Monte Carlo path tracing
  - Discretization
    - Radiosity

Photorealistic rendering with global illumination is an integration problem