COS 426, Spring 2012
Midterm 1

Name:

NetID:

Honor Code pledge:

Signature:

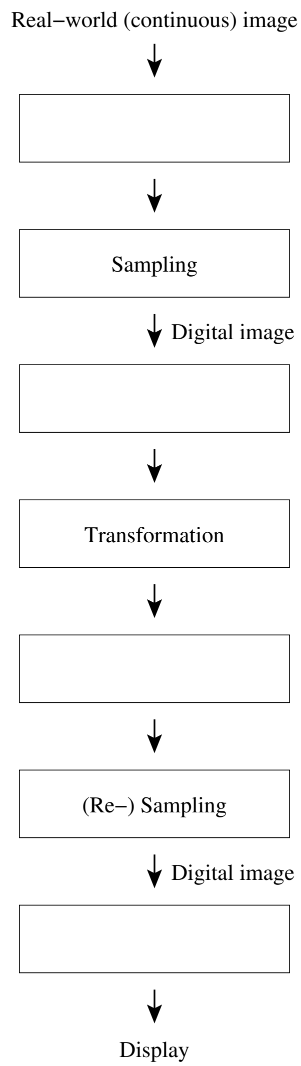
This exam consists of 6 questions. Do all of your work on these pages (use the back for scratch space), giving the answer in the space provided. This is a closed-book exam, but you may use one page of notes during the exam. **Put your NetID on every page (1 point), and write out and sign the Honor Code pledge before turning in the test:**

"I pledge my honor that I have not violated the Honor Code during this examination."

Question	**Score**
1	
2	
3	
4	
5	
6	
NetID on every page	
Total	

1. Resampling (20 pts)

Consider the following conceptual image warping pipeline:



(a) Fill in each of the blank boxes as either a ***bandlimiting*** filter or a ***reconstruction*** filter. (Some of these may operate in the digital realm, while others may be physical.)

(b) The original input to this pipeline is an image of a brick wall. Describe a possible visual artifact of omitting the last bandlimiting filter.

(c) Could the effect in (b) occur when scaling the image to be larger, smaller, both, or neither?

2. Compositing (16 pts)

Consider (R,G,B,α) colors

A = (1, 1, 0, 0.9)

B = (1, 0, 1, 0.4)

C = (0, 1, 1, 1.0)

Compute the following:

(a) A **over** B

(b) B **over** C

(c) A **over** B **over** C

(d) When composing a series of many "**over**" operations, what is the net amount by which the color of the ***lowest*** layer is multiplied, as a function of the alphas of all the layers?

3. Surface representations (10 pts)

Rank from smallest (1) to largest (5) the size required by each of the following 3D surface representations to store a model of a sphere. For all sampled representations, assume that samples are placed roughly the same distance apart (e.g., 1 mm sample spacing for a 100 mm sphere).

_____________ Triangle mesh with indexed face set

_____________ Triangle mesh with half-edges

_____________ Point cloud

_____________ Implicit function stored an a voxel grid

_____________ Algebraic (quadric polynomial) implicit function

4. Surface continuity (10 pts)

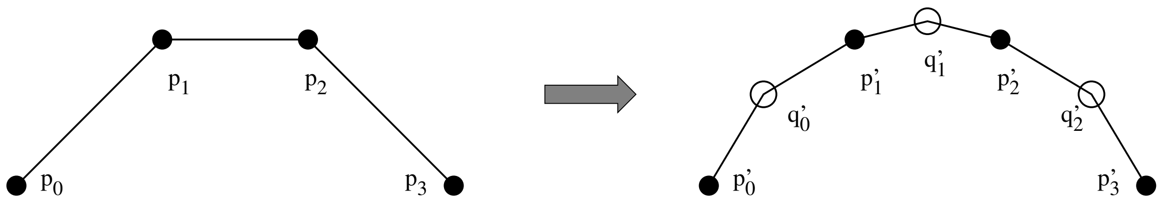
You are given two infinitely smooth (C^∞) surfaces that intersect along a single curve. Consider the shape U that is their CSG union:

a) What is the ***minimum*** degree of surface continuity anywhere on U?

b) What is the ***maximum*** degree of surface continuity anywhere on U?

5. Subdivision (18 pts)

In addition to the subdivision surfaces considered in class, it is possible to define subdivision curves in 2D. Consider the following three schemes, each consisting of a ***topology refinement*** step that inserts a point on each existing edge, and a ***geometry refinement*** step that has particular rules for the positions of new points q'_i and the updated positions p'_i of old points p_i:



Scheme #1:

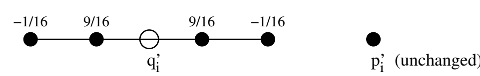
$$\text{new points } q'_i \leftarrow \tfrac{1}{2}p_i + \tfrac{1}{2}p_{i+1}$$
$$\text{old points } p'_i \leftarrow p_i$$

Scheme #2:

$$\text{new points } q'_i \leftarrow \tfrac{1}{2}p_i + \tfrac{1}{2}p_{i+1}$$
$$\text{old points } p'_i \leftarrow \tfrac{1}{8}\,p_{i-1} + \tfrac{3}{4}\,p_i + \tfrac{1}{8}\,p_{i+1}$$

Scheme #3:

$$\text{new points } q'_i \leftarrow -\tfrac{1}{16}p_{i-1} + \tfrac{9}{16}p_i + \tfrac{9}{16}p_{i+1} - \tfrac{1}{16}p_{i+2}$$
$$\text{old points } p'_i \leftarrow p_i$$



(a) Is each scheme ***interpolating*** or ***approximating***?

Scheme #1: _______________ Scheme #2: _______________ Scheme #3: _______________

(b) Does each scheme have the ***convex hull*** property? (yes/no)

Scheme #1: _______________ Scheme #2: _______________ Scheme #3: _______________

(c) Would you expect each scheme to be smooth (at least C^1) in the limit? (yes/no)
(***Hint:*** only one scheme is ***not*** C^1 — simulate a round on a simple initial curve to determine which.)

Scheme #1: _______________ Scheme #2: _______________ Scheme #3: _______________

6. Transformations (25 pts)

(a) Given 2D points represented as column vectors in 3D homogeneous coordinates:

$$p_1 = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \quad p_2 = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \quad p_3 = \begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix}$$

Find a 3×3 affine transformation matrix $\mathbf{M}(p_1, p_2, p_3)$ that maps:

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mapsto p_1 \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \mapsto p_2 \quad \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \mapsto p_3$$

Feel free to leave your result as the product of simpler matrices, if that's easier.

(b) Now find an affine transformation matrix that maps arbitrary points p_4, p_5, and p_6, to p_1, p_2, and p_3, respectively. Feel free to write your result in terms of simpler pieces (such as $\mathbf{M}(\cdot, \cdot, \cdot)$ defined above), if that's easier.